Statistics -2

9709

Continuous Random Variables

Exercise (march and June series 2019 – 2002)

With marking scheme

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Random variables

1) S/2002/7/Q7

A factory is supplied with grain at the beginning of each week. The weekly demand, X thousand tonnes, for grain from this factory is a continuous random variable having the probability density function given by

$$f(x) = \begin{cases} 2(1-x) & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

Find

(i) the mean value of X, [3]

(ii) the variance of X, [3]

(iii) the quantity of grain in tonnes that the factory should have in stock at the beginning of a week, in order to be 98% certain that the demand in that week will be met. [5]

2) S/2003/7/Q4

A random variable X has probability density function given by

$$f(x) = \begin{cases} 1 - \frac{1}{2}x & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find $P(X > 1.5)$. [2]

(ii) Find the mean of X. [2]

(iii) Find the median of X. [3]

3) S/2004/7/Q7

The queuing time, T minutes, for a person queuing at a supermarket checkout has probability density function given by

$$f(t) = \begin{cases} ct(25 - t^2) & 0 \leqslant t \leqslant 5, \\ 0 & \text{otherwise,} \end{cases}$$

where c is a constant.

(i) Show that the value of c is $\frac{4}{625}$. [3]

(ii) Find the probability that a person will have to queue for between 2 and 4 minutes. [3]

(iii) Find the mean queuing time. [4]

4) S/2005/7/Q7

The random variable X denotes the number of hours of cloud cover per day at a weather forecasting centre. The probability density function of X is given by

$$f(x) = \begin{cases} \frac{(x-18)^2}{k} & 0 \leqslant x \leqslant 24, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = 2016$. [3]

(ii) On how many days in a year of 365 days can the centre expect to have less than 2 hours of cloud cover? [3]

(iii) Find the mean number of hours of cloud cover per day. [4]

5) S/2006/7/Q5

The random variable X has probability density function given by

$$f(x) = \begin{cases} 4x^k & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a positive constant.

(i) Show that $k = 3$. [2]

(ii) Show that the mean of X is 0.8 and find the variance of X. [4]

(iii) Find the upper quartile of X. [2]

(iv) Find the interquartile range of X. [2]

6) S/2007/7/Q7

The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{3}{4}(x^2 - 1) & 1 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Sketch the probability density function of X. [2]

(ii) Show that the mean, μ, of X is 1.6875. [3]

(iii) Show that the standard deviation, σ, of X is 0.2288, correct to 4 decimal places. [3]

(iv) Find $P(1 \leqslant X \leqslant \mu + \sigma)$. [3]

7) S/2008/7/Q7

If Usha is stung by a bee she always develops an allergic reaction. The time taken in minutes for Usha to develop the reaction can be modelled using the probability density function given by

$$f(t) = \begin{cases} \dfrac{k}{t+1} & 0 \leqslant t \leqslant 4, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \dfrac{1}{\ln 5}$. [4]

(ii) Find the probability that it takes more than 3 minutes for Usha to develop a reaction. [3]

(iii) Find the median time for Usha to develop a reaction. [3]

8) S/2009/7/Q5

The time in minutes taken by candidates to answer a question in an examination has probability density function given by

$$f(t) = \begin{cases} k(6t - t^2) & 3 \leqslant t \leqslant 6, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

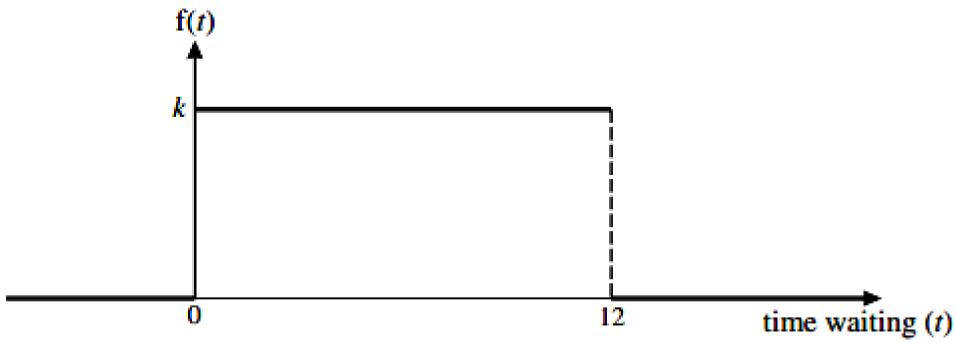
(i) Show that $k = \frac{1}{18}$. [3]

(ii) Find the mean time. [3]

(iii) Find the probability that a candidate, chosen at random, takes longer than 5 minutes to answer the question. [2]

(iv) Is the upper quartile of the times greater than 5 minutes, equal to 5 minutes or less than 5 minutes? Give a reason for your answer. [2]

9) S/2010/71/Q1



Fred arrives at random times on a station platform. The times in minutes he has to wait for the next train are modelled by the continuous random variable for which the probability density function f is shown above.

(i) State the value of k. [1]

(ii) Explain briefly what this graph tells you about the arrival times of trains. [1]

10) S/2010/71/Q5

The random variable T denotes the time in seconds for which a firework burns before exploding. The probability density function of T is given by

$$f(t) = \begin{cases} ke^{0.2t} & 0 \leqslant t \leqslant 5, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \dfrac{1}{5(e - 1)}$. [3]

(ii) Sketch the probability density function. [2]

(iii) 80% of fireworks burn for longer than a certain time before they explode. Find this time. [3]

11) S/2010/73/Q5

The time, in minutes, taken by volunteers to complete a task is modelled by the random variable X with probability density function given by

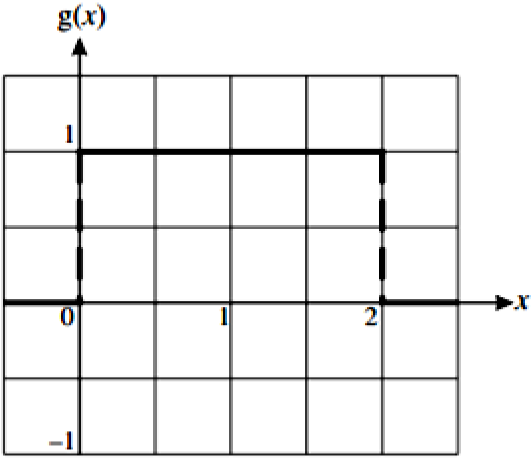
$$f(x) = \begin{cases} \dfrac{k}{x^4} & x \geqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

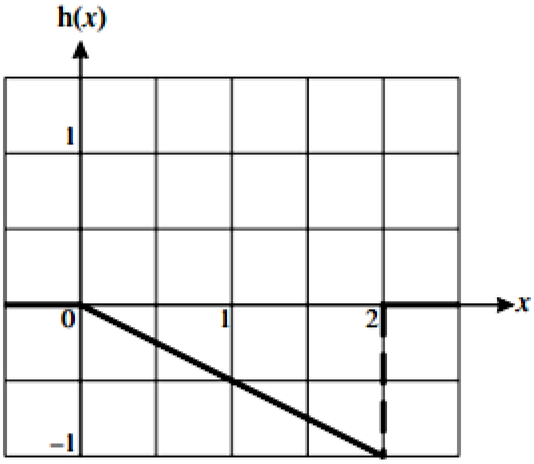
(i) Show that $k = 3$. [2]

(ii) Find $E(X)$ and $Var(X)$. [6]

12) S/2011/71/Q4

(a)





The diagrams show the graphs of two functions, g and h. For each of the functions g and h, give a reason why it cannot be a probability density function. [2]

(b) The distance, in kilometres, travelled in a given time by a cyclist is represented by the continuous random variable X with probability density function given by

$$f(x) = \begin{cases} \dfrac{30}{x^2} & 10 \leqslant x \leqslant 15, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that $E(X) = 30 \ln 1.5$. [3]

(ii) Find the median of X. Find also the probability that X lies between the median and the mean. [5]

13) S/2011/72/Q7

A random variable X has probability density function given by

$$f(x) = \begin{cases} k(1-x) & -1 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{1}{2}$. [2]

(ii) Find $P(X > \frac{1}{2})$. [1]

(iii) Find the mean of X. [3]

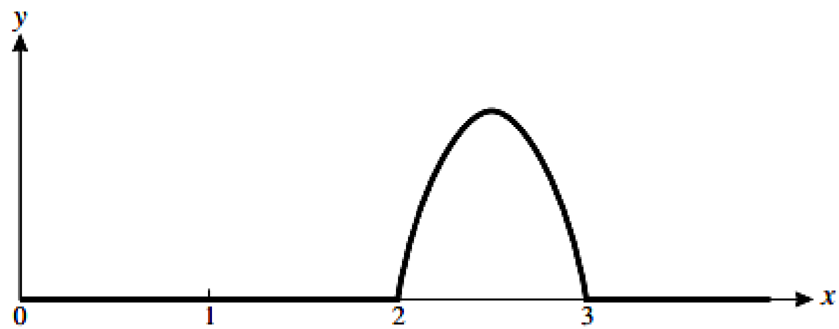
(iv) Find a such that $P(X < a) = \frac{1}{4}$. [3]

14) S/2011/73/Q6

The distance travelled, in kilometres, by a Grippo brake pad before it needs to be replaced is modelled by $10\,000X$, where X is a random variable having the probability density function

$$f(x) = \begin{cases} -k(x^2 - 5x + 6) & 2 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

The graph of $y = f(x)$ is shown in the diagram.



(i) Show that $k = 6$. [2]

(ii) State the value of $E(X)$ and find $\mathrm{Var}(X)$. [4]

(iii) Sami fits four new Grippo brake pads on his car. Find the probability that at least one of these brake pads will need to be replaced after travelling less than 22 000 km. [3]

15) S/2012/71/Q4

The random variable X has probability density function given by

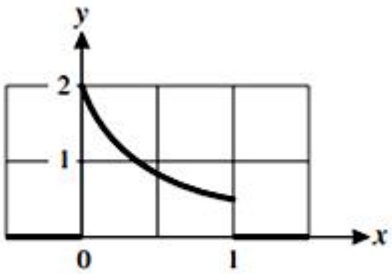
$$f(x) = \begin{cases} \dfrac{k}{(x+1)^2} & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = 2$. [2]

(ii) Find a such that $P(X < a) = \frac{1}{5}$. [3]

(iii)



The diagram shows the graph of $y = f(x)$. The median of X is denoted by m. Use the diagram to explain whether $m < 0.5$, $m = 0.5$ or $m > 0.5$. [2]

16) S/2012/72/Q6

At a certain shop the weekly demand, in kilograms, for flour is modelled by the random variable X with probability density function given by

$$f(x) = \begin{cases} kx^{-\frac{1}{2}} & 4 \leqslant x \leqslant 25, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

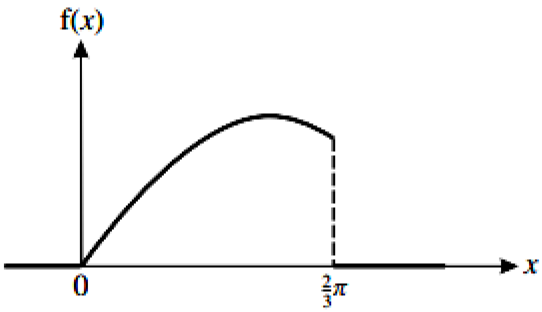
(i) Show that $k = \frac{1}{6}$. [2]

(ii) Calculate the mean weekly demand for flour at the shop. [3]

(iii) At the beginning of one week, the shop has 20 kg of flour in stock. Find the probability that this will not be enough to meet the demand for that week. [2]

(iv) Give a reason why the model may not be realistic. [1]

17) S/2012/73/Q7



A random variable X has probability density function given by

$$f(x) = \begin{cases} k \sin x & 0 \leqslant x \leqslant \frac{2}{3}\pi, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant, as shown in the diagram.

(i) Show that $k = \frac{2}{3}$. [2]

(ii) Show that the median of X is 1.32, correct to 3 significant figures. [4]

(iii) Find $E(X)$. [4]

18) S/2013/71/Q6

The time in minutes taken by people to read a certain booklet is modelled by the random variable T with probability density function given by

$$f(t) = \begin{cases} \dfrac{1}{2\sqrt{t}} & 4 \leqslant t \leqslant 9, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the time within which 90% of people finish reading the booklet. [3]

(ii) Find $E(T)$ and $Var(T)$. [6]

19) S/2013/72/Q2

A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{2}{3}x & 1 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find $E(X)$. [3]

(ii) Find $P(X < E(X))$. [2]

(iii) Hence explain whether the mean of X is less than, equal to or greater than the median of X. [2]

20) S/2013/73/Q5

A random variable X has probability density function given by

$$f(x) = \begin{cases} \dfrac{k}{x^3} & x \geqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = 2$. [2]

(ii) Find $P(1 \leqslant X \leqslant 2)$. [2]

(iii) Find $E(X)$. [3]

21) S/2014/71/Q7

A random variable X has probability density function given by

$$f(x) = \begin{cases} \dfrac{k}{x} & 1 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are positive constants.

(i) Show that $k = \dfrac{1}{\ln a}$. [3]

(ii) Find $E(X)$ in terms of a. [3]

(iii) Find the median of X in terms of a. [4]

22) S/2014/72/Q6

The time, T hours, spent by people on a visit to a museum has probability density function

$$f(t) = \begin{cases} kt(16 - t^2) & 0 \leqslant t \leqslant 4, \\ 0 & \text{otherwise,} \end{cases}$$

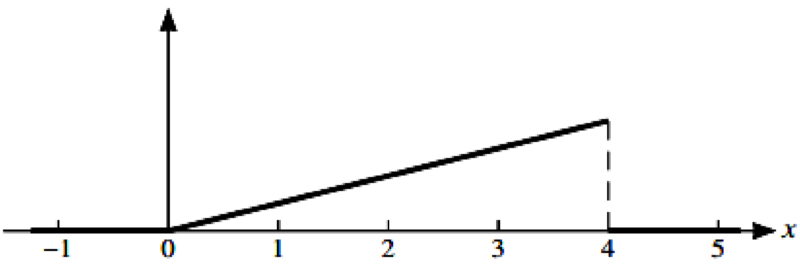
where k is a constant.

(i) Show that $k = \frac{1}{64}$. [3]

(ii) Calculate the probability that two randomly chosen people each spend less than 1 hour on a visit to the museum. [4]

(iii) Find the mean time spent on a visit to the museum. [3]

23) S/2014/73/Q2



A random variable X takes values between 0 and 4 only and has probability density function as shown in the diagram. Calculate the median of X. [3]

24) S/2014/72/Q5

The lifetime, X years, of a certain type of battery has probability density function given by

$$f(x) = \begin{cases} \dfrac{k}{x^2} & 1 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

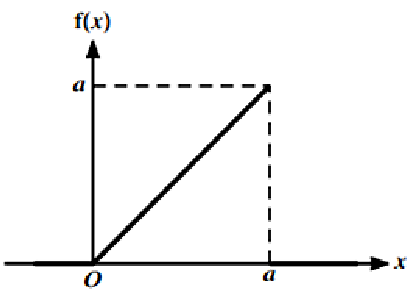
where k and a are positive constants.

(i) State what the value of a represents in this context. [1]

(ii) Show that $k = \dfrac{a}{a - 1}$. [3]

(iii) Experience has shown that the longest that any battery of this type lasts is 2.5 years. Find the mean lifetime of batteries of this type. [3]

25) S/2015/71/Q1



The random variable X has probability density function, f, as shown in the diagram, where a is a constant. Find the value of a and hence show that $E(X) = 0.943$ correct to 3 significant figures. [5]

26) S/2015/72/Q6

The waiting time, T minutes, for patients at a doctor's surgery has probability density function given by

$$f(t) = \begin{cases} k(225 - t^2) & 0 \leqslant t \leqslant 15, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{1}{2250}$. [3]

(ii) Find the probability that a patient has to wait for more than 10 minutes. [3]

(iii) Find the mean waiting time. [4]

27) S/2015/73/Q7

The probability density function of the random variable X is given by

$$f(x) = \begin{cases} \frac{3}{4}x(c - x) & 0 \leqslant x \leqslant c, \\ 0 & \text{otherwise,} \end{cases}$$

where c is a constant.

(i) Show that $c = 2$. [3]

(ii) Sketch the graph of $y = f(x)$ and state the median of X. [3]

(iii) Find $P(X < 1.5)$. [4]

(iv) Hence write down the value of $P(0.5 < X < 1)$. [1]

28) S/2016/71/Q6

In each turn of a game, a coin is pushed and slides across a table. The distance, X metres, travelled by the coin has probability density function given by

$$f(x) = \begin{cases} kx^2(2 - x) & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) State the greatest possible distance travelled by the coin in one turn. [1]

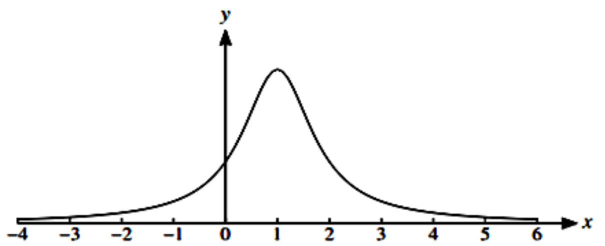
(ii) Show that $k = \frac{3}{4}$. [3]

(iii) Find the mean distance travelled by the coin in one turn. [3]

(iv) Out of 400 turns, find the expected number of turns in which the distance travelled by the coin is less than 1 metre. [3]

29) S/2016/72/Q7

(a)



The diagram shows the graph of the probability density function of a variable X. Given that the graph is symmetrical about the line $x = 1$ and that $P(0 < X < 2) = 0.6$, find $P(X > 0)$. [2]

(b) A flower seller wishes to model the length of time that tulips last when placed in a jug of water. She proposes a model using the random variable X (in hundreds of hours) with probability density function given by

$$f(x) = \begin{cases} k(2.25 - x^2) & 0 \leqslant x \leqslant 1.5, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{4}{9}$. [3]

(ii) Use this model to find the mean number of hours that a tulip lasts in a jug of water. [4]

The flower seller wishes to create a similar model for daffodils. She places a large number of daffodils in jugs of water and the longest time that any daffodil lasts is found to be 290 hours.

(iii) Give a reason why $f(x)$ would not be a suitable model for daffodils. [1]

(iv) The flower seller considers a model for daffodils of the form

$$g(x) = \begin{cases} c(a^2 - x^2) & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where a and c are constants. State a suitable value for a. (There is no need to evaluate c.) [1]

30) S/2016/73/Q5

The time, T minutes, taken by people to complete a test has probability density function given by

$$f(t) = \begin{cases} k(10t - t^2) & 5 \leqslant t \leqslant 10, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

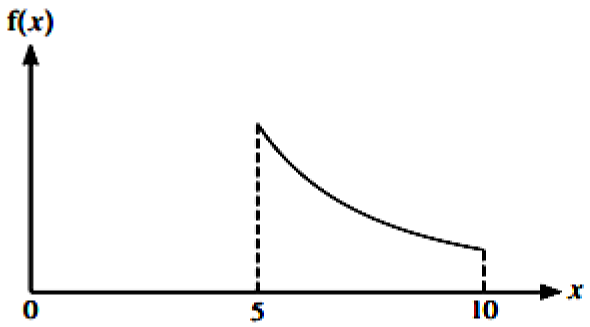
(i) Show that $k = \frac{3}{250}$. [3]

(ii) Find $E(T)$. [3]

(iii) Find the probability that a randomly chosen value of T lies between $E(T)$ and the median of T. [3]

(iv) State the greatest possible length of time taken to complete the test. [1]

31) S/2017/71/Q4



The time, X minutes, taken by a large number of runners to complete a certain race has probability density function f given by

$$f(x) = \begin{cases} \dfrac{k}{x^2} & 5 \leqslant x \leqslant 10, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant, as shown in the diagram.

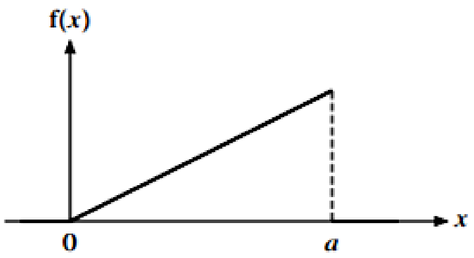
(i) Without calculation, explain how you can tell that there were more runners whose times were below 7.5 minutes than above 7.5 minutes. [1]

(ii) Show that $k = 10$. [3]

(iii) Find $E(X)$. [3]

(iv) Find $\text{Var}(X)$. [2]

32) S/2017/72/Q5



The diagram shows the graph of the probability density function, f, of a random variable X which takes values between 0 and a only. It is given that $P(X < 1) = 0.25$.

(i) Find, in any order,

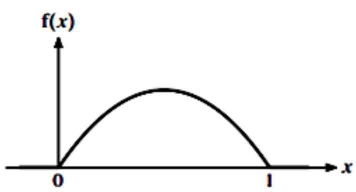
(a) $P(X < 2)$,

(b) the value of a,

(c) $f(x)$.

[5]

33) S/2017/73/Q6



The diagram shows the graph of the probability density function, f, of a continuous random variable X, where f is defined by

$$f(x) = \begin{cases} k(x - x^2) & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that the value of the constant k is 6. [3]

(ii) State the value of $E(X)$ and find $\text{Var}(X)$. [4]

(iii) Find $P(0.4 < X < 2)$. [3]

34) S/2018/71/Q6

The time, in minutes, taken by people to complete a test is modelled by the continuous random variable X with probability density function given by

$$f(x) = \begin{cases} \dfrac{k}{x^2} & 5 \leqslant x \leqslant 10, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

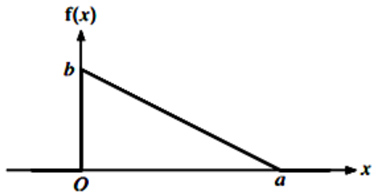
(i) Show that $k = 10$. [3]

(ii) Show that $E(X) = 10 \ln 2$. [2]

(iii) Find $P(X > 9)$. [3]

(iv) Given that $P(X < a) = 0.6$, find a. [3]

35) S/2018/72/Q5



The diagram shows the probability density function, f, of a random variable X, in terms of the constants a and b.

(i) Find b in terms of a. **[2]**

(ii) Show that $f(x) = \dfrac{2}{a} - \dfrac{2}{a^2}x$. **[3]**

(iii) Given that $E(X) = 0.5$, find a. **[4]**

36) S/2018/73/Q7

(a)

A random variable X has probability density function defined by

$$f(x) = \begin{cases} k\left(\dfrac{1}{x^2} + \dfrac{1}{x^3}\right) & 1 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

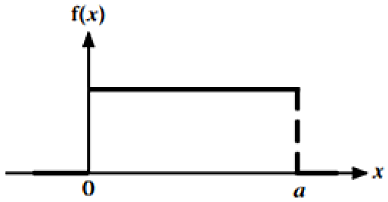
(i) Show that $k = \frac{8}{7}$. [3]

(ii) Find $E(X)$. [3]

(iii) Three values of X are chosen at random. Find the probability that one of these values is less than 1.5 and the other two are greater than 1.5. [5]

37) S/2019/71/Q1

(a)



The diagram shows the graph of the probability density function, f, of a random variable X, where a is a constant greater than 0.5. The graph between $x = 0$ and $x = a$ is a straight line parallel to the x-axis.

(i) Find $P(X < 0.5)$ in terms of a. **[2]**

(ii) Find $E(X)$ in terms of a. **[1]**

(iii) Show that $\text{Var}(X) = \frac{1}{12}a^2$. **[2]**

(b) A random variable T has probability density function given by

$$g(t) = \begin{cases} \dfrac{3}{2(t-1)^2} & 2 \leqslant t \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

Find the value of b such that $P(T \leqslant b) = \frac{3}{4}$. **[4]**

38) S/2019/72/Q6

X is a random variable with probability density function given by

$$f(x) = \begin{cases} \dfrac{a}{x^2} & 1 \leqslant x \leqslant b, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are constants.

(i) Show that $b = \dfrac{a}{a-1}$. [3]

(ii) Given that the median of X is $\frac{3}{2}$, find the values of a and b. [3]

(iii) Use your values of a and b from part **(ii)** to find $E(X)$. [3]

39) S/2019/73/Q6

A function f is defined by

$$f(x) = \begin{cases} \dfrac{3x^2}{a^3} & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(i) Show that f is a probability density function for all positive values of a. [3]

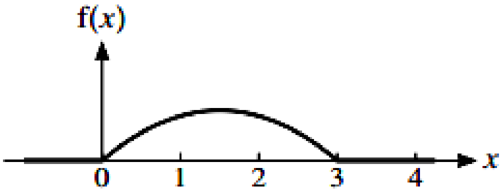
The random variable X has probability density function f and the median of X is 2.

(ii) Show that $a = 2.52$, correct to 3 significant figures. [3]

(iii) Find $E(X)$. [3]

40) M/2016/72/Q7

(a)



The diagram shows the graph of the probability density function, f, of a random variable X, where

$$f(x) = \begin{cases} \frac{2}{9}(3x - x^2) & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(i) State the value of $E(X)$ and find $Var(X)$. [4]

(ii) State the value of $P(1.5 \leqslant X \leqslant 4)$. [1]

(iii) Given that $P(1 \leqslant X \leqslant 2) = \frac{13}{27}$, find $P(X > 2)$. [2]

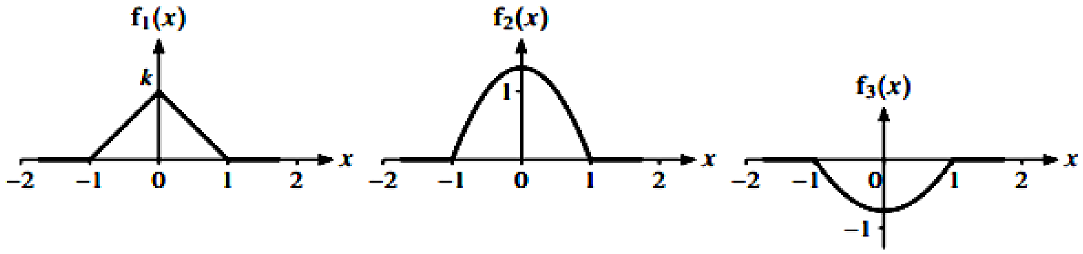
(b) A random variable, W, has probability density function given by

$$g(w) = \begin{cases} aw & 0 \leqslant w \leqslant b, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are constants. Given that the median of W is 2, find a and b. [4]

41) M/2017/72/Q5

(a)



The diagram shows the graphs of three functions, f_1, f_2 and f_3. The function f_1 is a probability density function.

(i) State the value of k. [1]

(ii) For each of the functions f_2 and f_3, state why it cannot be a probability density function. [2]

(b) The probability density function g is defined by

$$g(x) = \begin{cases} 6(a^2 - x^2) & -a \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(i) Show that $a = \frac{1}{2}$. [3]

(ii) State the value of $E(X)$. [1]

(iii) Find $Var(X)$. [2]

42) M/2018/72/Q6

A random variable X has probability density function given by

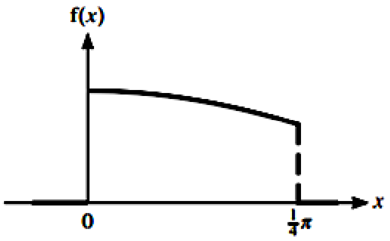
$$f(x) = \begin{cases} 6x(1-x) & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the probability that X does not lie between 0.3 and 0.7. [4]

(ii) Sketch the graph of the probability density function and hence state the value of $E(X)$. [2]

(iii) Find $Var(X)$. [3]

43) M/2019/72/Q7



A random variable X has probability density function given by

$$f(x) = \begin{cases} (\sqrt{2})\cos x & 0 \leqslant x \leqslant \frac{1}{4}\pi, \\ 0 & \text{otherwise,} \end{cases}$$

as shown in the diagram.

(i) Find $P(X > \frac{1}{6}\pi)$. [2]

(ii) Find the median of X. [4]

(iii) Find $E(X)$. [4]

Answers

1)

(i) $E(X) = \int_0^1 2x(1-x)\,dx$

$= \int_0^1 2x - 2x^2\,dx$

$= \left[x^2 - \frac{2x^3}{3}\right] = 0.333$

(ii) $\mathrm{Var}(X) = \int_0^1 2x^2 - 2x^3\,dx - (0.333)^2$

$= \left[\frac{2x^3}{3} - \frac{2x^4}{4}\right] - (0.333)^2$

$= 0.0556$

(iii) $\int_0^x 2(1-x)\,dx = 0.98$

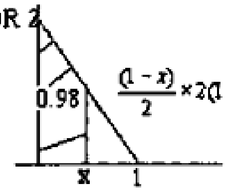
$\left[2x - x^2\right] = 0.98$

$x^2 - 2x + 0.98 = 0$

$x = 0.859$

859 tonnes

OR 2



$\frac{(1-x)}{2} \times 2(1-x) = 0.02$

2)

(i) $P(X > 1.5) = \left[x - \frac{x^2}{4}\right]_{1.5}^{2}$

or $1 - \left[x - \frac{x^2}{4}\right]_{0}^{1.5}$

$= 0.0625$

(ii) $E(X) =$

$\int_0^2 (x - \frac{1}{2}x^2)dx = \left[\frac{x^2}{2} - \frac{x^3}{6}\right]_0^2$

$= 2/3$

(iii) $m - \frac{m^2}{4} = 0.5$

$m = 0.586$ ($2 - \sqrt{2}$)

3)

(i) $c\int_0^5 t(25 - t^2)dt = 1$

$c\left[\frac{25t^2}{2} - \frac{t^4}{4}\right]_0^5 = 1$

$c\left[\frac{625}{2} - \frac{625}{4}\right] = 1 \Rightarrow c = \frac{4}{625}$

(ii) $\int_2^4 ct(25 - t^2)dt = \left[\frac{25ct^2}{2} - \frac{ct^4}{4}\right]_2^4 = c[136] - c[46]$

$= \frac{72}{125}$ (0.576)

(iii) $\int_0^5 ct^2(25 - t^2)dt = \left[\frac{4}{625} \times \frac{25t^3}{3} - \frac{4}{625} \times \frac{t^5}{5}\right]_0^5$

$= \frac{8}{3}$

4)

(i) $\int_0^{24} \frac{(x-18)^2}{k} = 1$

$\left[\frac{(x-18)^3}{3k}\right]_0^{24} = 1$

$\frac{2016}{k} = 1 \Rightarrow k = 2016$ AG

(ii) $p(x < 2) = \int_0^2 \frac{(x-18)^2}{2016}dx$

$= \left[\frac{(x-18)^3}{3 \times 2016}\right]_0^2$

$= \frac{(-16)^3 - (-18)^3}{6048}$

$= 0.2870 (31/108)$

Number of days = 0.287 × 365 = 104 or 105

(iii) mean $= \int_0^{24} \frac{x(x-18)^2}{k}dx$

$= \frac{1}{k}\left[\frac{x^4}{4} - \frac{36x^3}{3} + \frac{324x^2}{2}\right]_0^{24}$

$= 5.14 \left(5\frac{1}{7}\right)$

5)

(i) $\int_0^1 4x^k\,dx = 1$

$\left[\frac{4x^{k+1}}{k+1}\right]_0^1 = 1$

$k = 3$ **AG**

(ii) $E(X) = \int_0^1 4x^4\,dx$

$= \left[\frac{4x^5}{5}\right]_0^1 = 0.8$ **AG**

$\mathrm{Var}(X) = \int_0^1 4x^5\,dx - 0.8^2$

$= \left[\frac{4x^6}{6}\right]_0^1 - 0.8^2 = 0.0267$

(iii) $\int_0^{q_3} 4x^k\,dx = 0.75$

$q_3 = \sqrt[4]{0.75}$ (= 0.931)

$= 0.931$

(iv) $q_1 = \sqrt[4]{0.25}$ (= 0.707)

IQ range = 0.223 (0.22349)

6)

(i) no values outside $1 \leqslant x \leqslant 2$
curve through (1,0) and (2, 2.25)

(ii) $\mu = \int_1^2 3x^3/4 - 3x/4\,dx = \left[3x^4/16 - 3x^2/8\right]_1^2$

$= [3 - 12/8] - [3/16 - 3/8]$

$= 27/16$ (1.6875)

(iii) $\sigma^2 = \int_1^2 3x^4/4 - 3x^2/4\,dx$ − mean²

$= \left[3x^5/20 - 3x^3/12\right]_1^2 - 1.6875^2$

$= [96/20 - 2] - [3/20 - 3/12] - 1.6875^2$

$= 0.052343$

sd = 0.2288 to 4 dp **AG**

(iv) $\int_1^{1.9163} 3x^2/4 - 3/4\,dx$

$= \left[x^3/4 - 3x/4\right]_1^{1.9163}$

$= 0.822$ or 0.821

7)

(i) $\int_0^4 \frac{k}{t+1}dt = 1$

$[k\ln(t+1)]_0^4 = 1$

$k = 1/\ln 5$ AG

(ii) $P(T>3) = \int_3^4 \frac{k}{t+1}dt$

$= [k\ln(t+1)]_3^4$

$= 1 - \ln4/\ln5 = 0.139$

(iii) $\int_0^m \frac{k}{t+1}dt = 0.5$

$[k\ln(t+1)]_0^m = 0.5$

$k\ln(m+1) = 0.5$

$m = 1.24$ min

8)

(i) $\int_3^6 k(6t - t^2)dt = 1$

$k\left[3t^2 - t^3/3\right]_3^6 = 1$

$k([108 - 216/3] - [27 - 9]) = 1$

$k = 1/18$ AG

(ii) mean = $\int_3^6 k(6t^2 - t^3)dt$

$= \left[k(2t^3 - \frac{t^4}{4})\right]_3^6$

$= k(432 - 324) - k(54 - 81/4)$

$= \frac{33}{8}$ (4.13)

(iii) $\int_5^6 k(6t - t^2)dt$

$= k\left[3t^2 - \frac{t^3}{3}\right]_5^6 = k(36 - \frac{100}{3})$

$= \frac{4}{27}$ (0.148)

iv) the area on the left is > 0.75
or **(iii)** is < 0.25
UQ is less than 5

9)

(i) 1/12

(ii) trains arrive every 12 minutes

10)

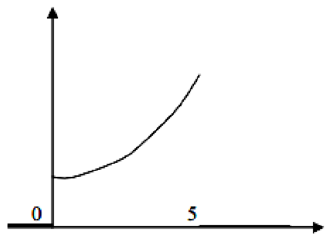
(i) $\int_0^5 ke^{0.2t}dt = 1$

$\left[\frac{k}{0.2}e^{1.0}\right] - \left[\frac{k}{0.2}e^0\right] = 1$

$\frac{k}{0.2}(e-1) = 1$

$k = \frac{1}{5(e-1)}$ AG

(ii)



(iii) $\int_0^T ke^{0.2t}dt = 0.2$

$\left[5ke^{0.2T}\right] - [5k] = 0.2$

$e^{0.2T} = \frac{0.2}{5k} + 1 = 1.344$

$T = 1.48$ (seconds)

11)

(i) $\int_1^\infty \frac{k}{x^4}dx = 1$

$\left[-\frac{k}{3x^3}\right]_1^\infty = 1 \quad 0 + \frac{k}{3} = 1 \Rightarrow k = 3$

(ii) $\int_1^\infty x \times \frac{3}{x^4}dx$

$\left[-\frac{3}{2x^2}\right]_1^\infty$

$= \frac{3}{2}$

$\int_1^\infty x^2 \times \frac{3}{x^4}dx$

$\left[-\frac{3}{x}\right]_1^\infty \ (=3) \ = \frac{3}{4}$

12)

(a) g: Area ≠ 1 or > 1
h: pdf cannot be neg

(b) **(i)** $\int_{10}^{15} \frac{30}{x}\,dx$

$= [30 \ln x]_{10}^{15}$

$= 30(\ln 15 - \ln 10)$

$(= 30\ln 1.5$ **AG**)

(ii) $\int_{10}^{m} \frac{30}{x^2}\,dx = 0.5$

$[-30x^{-1}]_{10}^{m} = 0.5$

$-\frac{30}{m} - (-\frac{30}{10}) = 0.5$

$m = 12$

$\int_{'12'}^{30\ln 1.5} \frac{30}{x^2}\,dx$

$= 0.0337$ (3 sfs)

13)

(i) $\int_{-1}^{1} k(1-x)dx = 1$

$(k[x - \frac{x^2}{2}]_{-1}^{1} = 1)$

$2k = 1$

$(k = \frac{1}{2}$ **AG**)

(ii) $(\int_{0.5}^{1} \frac{1}{2}(1-x)dx = \frac{1}{2}[x - \frac{x^2}{2}]_{0.5}^{1})$

$= \frac{1}{16}$ or 0.0625

(iii) $\int_{-1}^{1} \frac{1}{2}(x - x^2)dx$

$= \frac{1}{2}[\frac{x^2}{2} - \frac{x^3}{3}]_{-1}^{1}$

$= -\frac{1}{3}$ or –0.333

(iv) $\int_{-1}^{a} \frac{1}{2}(1-x)dx = 0.25$

$(\frac{1}{2}[x - \frac{x^2}{2}]_{-1}^{a} = 0.25)$

$(\frac{1}{2}(a - \frac{a^2}{2} - (-1 - \frac{1}{2}) = 0.25)$

$a^2 - 2a - 2 = 0$

$a = 1 - \sqrt{3}$ or –0.732

14)

(i) $-k\int_{2}^{3}(x^2 - 5x + 6)dx = 1$

$(-k(\frac{3^3}{3} - 5\times\frac{3^2}{2} + 6\times 3 - [\frac{2^3}{3} - 5\times\frac{2^2}{2} + 6\times 2]) = 1)$

$-k \times (-\frac{1}{6}) = 1$ or $k \times \frac{1}{6} = 1$

$(k = 6$ **AG**)

(ii) E(X) = 2.5

$-6\int_{2}^{3}(x^4 - 5x^3 + 6x^2)dx$ (= –6 × (–1.05))

$-$ "2.5"^{2}

= 0.05

(iii) $-6\int_{2}^{2.2}(x^2 - 5x + 6)dx$ (= 0.104)

$1 - (1 - \text{"0.104"})^4$

= 0.355/0.356

15)

(i) $\int_{0}^{1} \frac{k}{(x+1)^2}dx = 1$

$-[\frac{k}{(x+1)}]_{0}^{1} = 1$

$-k\left(\frac{1}{2} - 1\right) = 1$

$(k = 2$ **AG**)

(ii) $\int_{0}^{a} \frac{2}{(x+1)^2}dx = \frac{1}{5}$

$-[\frac{2}{(x+1)}]_{0}^{a} = \frac{1}{5}$

$-\left(\frac{2}{a+1} - 2\right) = \frac{1}{5}$

$a = \frac{1}{9}$

(iii) Area below $x = 0.5$ is greater than 0.5
$m < 0.5$

16)

(i) $\int_4^{25} kx^{-\frac{1}{2}}dx = 1$

$\left[\frac{kx^{\frac{1}{2}}}{\frac{1}{2}}\right]_4^{25} = 1$

$2k(5-2) = 1$

($k = \frac{1}{6}$ **AG**)

(ii) $\frac{1}{6}\int_4^{25} x^{\frac{1}{2}}dx$

$= \frac{1}{6}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_4^{25}$ $(= \frac{1}{9}(125-8)$)

$= 13$

(iii) $\frac{1}{6}\int_{20}^{25} x^{-\frac{1}{2}}dx$

$(= \frac{1}{6}\left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_{20}^{25} = \frac{1}{3}(5-\sqrt{20}))$

$= 0.176$ (3 sfs)

(iv) Wkly demand may be > 25 (or < 4)

17)

(i) $k\int_0^{\frac{2\pi}{3}} \sin x\,dx = 1$

$k[-\cos x]_0^{\frac{2\pi}{3}}$

$k[-\cos\frac{2\pi}{3} + \cos 0] = 1$

$k[0.5 + 1] = 1$ $\qquad k = \frac{2}{3}$

(ii) $\frac{2}{3}\int_0^m \sin x\,dx = 0.5$

$\frac{2}{3}[-\cos x]_0^m = 0.5$

$\frac{2}{3}(-\cos m + 1) = 0.5$

$\cos m = 0.25$

$m = 1.32$ (3 sfs) **AG**

(iii) $\frac{2}{3}\int_0^{\frac{2\pi}{3}} x\sin x\,dx$

$= \frac{2}{3}\{[x(-\cos x)]_0^{\frac{2\pi}{3}} - \int_0^{\frac{2\pi}{3}}(-\cos x)dx\}$

$= \frac{2}{3}\{\frac{\pi}{3} - 0 - [-\sin x]_0^{\frac{2\pi}{3}}$

$= \frac{2\pi+3\sqrt{3}}{9}$ or 1.28 (3 sf)

18)

(i) $\frac{1}{2}\int_4^t \frac{1}{\sqrt{t}}dt = 0.9$ or $\frac{1}{2}\int_t^9 \frac{1}{\sqrt{t}}dt = 0.1$

$\left[\sqrt{t}\right]_4^t = 0.9$ or $\left[\sqrt{t}\right]_t^9 = 0.1$

$((\sqrt{t} - 2) = 0.9$ or $(3 - \sqrt{t}) = 0.1)$

$t = 8.41$ (mins) (3 sf)

(ii) $\frac{1}{2}\int_4^9 \frac{t}{\sqrt{t}}dt$ oe

$\frac{1}{2}\left[\frac{t1.5}{1.5}\right]_4^9$ oe

$= \frac{19}{3}$

$\frac{1}{2}\int_4^9 \frac{t2}{\sqrt{t}}dt$ oe

$(= \frac{1}{2}\left[\frac{t2.5}{2.5}\right]_4^9 = \frac{211}{5})$

$= \frac{'211'}{5} - (\frac{'19'}{3})^2$

$= \frac{94}{45}$ or 2.09 (3 sf)

19)

(i) $\frac{2}{3}\int_1^2 x^2dx$

$= \frac{2}{3}\left[\frac{x^3}{3}\right]_1^2$

$= \frac{14}{9}$ or 1.56 o.e.

(ii) $\frac{2}{3}\int_1^{\frac{14}{9}} x\,dx$

$(= \frac{2}{3}\left[\frac{x^3}{3}\right]_1^2)$

$= \frac{115}{243}$ or 0.473 (3 s.f.)

(iii) $\frac{115}{243} < \frac{1}{2}$ o.e.

Hence mean < median

20)

(i) $\int_1^{\infty} \frac{k}{x^3}\,dx = 1$

$\left[-\frac{k}{2x^2}\right]_1^{\infty} = 1$

$0 - (-\frac{k}{2}) = 1$

(ii) $\int_1^2 \frac{2}{x^3}\,dx$

$= \left[-\frac{1}{x^2}\right]_1^2$

$= \frac{3}{4}$

(iii) $\int_1^{\infty} \frac{2}{x^2}\,dx$

$= \left[-\frac{2}{x}\right]_1^{\infty}$

$= 2$

21)

(i) $\int_1^a \frac{k}{x}\,dx = 1$

$k[\ln x]_1^a = 1$

$k\ln a = 1 \quad k = 1/\ln a$

(ii) $\frac{1}{\ln a}\int_1^a 1\,dx$ or $k\int_1^a 1\,dx$

$= \frac{1}{\ln a}[x]_1^a$ or $k[x]_1^a$

$= \frac{1}{\ln a}(a-1)$

(iii) $\frac{1}{\ln a}\int_1^m \frac{1}{x}\,dx = 0.5$

$\frac{1}{\ln a}[\ln x]_1^m = 0.5$

$\frac{1}{\ln a}\ln m = 0.5$

$\ln m = 0.5\ln a$

$m = \sqrt{a}$

22)

(i) $k\int_0^4 (16t - t^3)\,dt = 1$

$k\left[8t^2 - \frac{t^4}{4}\right]_0^4 = 1$

$k(128 - 64) = 1$ o.e.

$k \times 64 = 1$

$\left(k = \frac{1}{64}\right)$ **AG**

(ii) $\frac{1}{64}\int_0^1 (16t - t^3)\,dt$

$= \frac{1}{64}\left[8t^2 - \frac{t^4}{4}\right]_0^1$

$= \frac{1}{64}\left[8 - \frac{1}{4}\right]$

$= \frac{31}{256}$ or 0.121094

$\left(\frac{31}{256}\right)^2 = 0.0147$ (3 s.f.) o.e.

iii $\frac{1}{64}\int_0^4 (16t^2 - t^4)\,dt$

$= \frac{1}{64}\left[\frac{16t^3}{3} - \frac{t^5}{5}\right]_0^4$

$= \frac{1}{64}\left(\frac{1024}{3} - \frac{1024}{5}\right) = \frac{32}{15}$ or 2.13

23)

ht $= \frac{1}{2}$

$\frac{1}{2} \times m \times \left(\frac{m}{4} \times \text{"}\frac{1}{2}\text{"}\right) = \frac{1}{2}$

$m = \sqrt{8}$ or $2\sqrt{2}$ or 2.83 (3 s.f.)

24)

(i) Longest lifetime

(ii) $\int_1^a \frac{k}{x^2}\,dx = 1$

$k\left[-\frac{1}{x}\right]_1^a = 1$

$\left(k\left[-\frac{1}{a} + 1\right] = 1\right)$

$k\left[\frac{-1+a}{a}\right] = 1$ or $k(-1 + a) = a$

$k = \frac{a}{a-1}$ **AG**

(iii) $\frac{5}{3}\int_1^{2.5} \frac{1}{x}\,dx$ or $k\int_1^{2.5} \frac{1}{x}\,dx$

$= \frac{5}{3}[\ln x]_1^{2.5}$ or $k[\ln x]_1^{2.5}$

$= \frac{5}{3}\ln 2.5$ or 1.53 (3 s.f.)

25)

$\frac{1}{2}a^2 = 1$

$a = \sqrt{2}$

$\int_0^{\sqrt{2}} x^2\,dx$

$= \left[\frac{x^3}{3}\right]_0^{\sqrt{2}}$

$= \frac{(\sqrt{2})^3}{3}$ = or $\frac{2^{1.5}}{3}$ or $\frac{2.83}{3}$ or 0.9428

26)

(i) $k\int_0^{15}(225-t^2)\,dt = 1$

$k\left[225t-\frac{t^3}{3}\right]_0^{15} = 1$

$k\times[3375 - 1125\} = 1$ or $k\times 2250 = 1$

(ii) $\frac{1}{2250}\int_{10}^{15}(225-t^2)\,dt$

$\left(= \frac{1}{2250}\left[225t-\frac{t^3}{3}\right]_{10}^{15}\right)$

$= \frac{1}{2250}\,[2250-(2250-\frac{1000}{3})]$

$= \frac{4}{27}$ or 0.148 (3 sf)

(iii) $\frac{1}{2250}\int_0^{15}(225t-t^3)\,dt$

$= \frac{1}{2250}\left[\frac{225t^2}{2}-\frac{t^4}{4}\right]_0^{15}$

$= \frac{1}{2250}\left[\frac{50625}{2}-\frac{50625}{4}\right]$

$= \frac{45}{8}$ or 5.625 or 5.63 (3 sf)

27)

(i) $\frac{3}{4}\int_0^c(cx-x^2)\,dx = 1$

$\frac{3}{4}\left[\frac{cx^2}{2}-\frac{x^3}{3}\right]_0^c = 1$

$\frac{3}{4}\left(\frac{c^3}{2}-\frac{c^3}{3}\right)=1$ or $\frac{3}{4}\times\frac{c^3}{6}=1$ or $\frac{c^3}{8}=1$

(ii) Inverted parabola
Through (0, 0) and (2, 0) and zero elsewhere
Median = 1

(iii) $\frac{3}{4}\int_0^{1.5}(2x-x^2)\,dx$

$=\frac{3}{4}\left[x^2-\frac{x^3}{3}\right]_0^{1.5}$

$\frac{3}{4}\left(1.5^2-\frac{1.5^3}{3}\right)$

$=\frac{27}{32}$ or 0.844 (3 sf)

(iv) $\left(\frac{27}{32}-\frac{1}{2}\text{ or } 0.844-0.5\right)$

$=\frac{11}{32}$ or 0.344 (3 sf)

28)

(i) 2 m

(ii) $k\int_0^2 x^2(2-x)\,dx = 1$

$k\left[\frac{2x^3}{3}-\frac{x^4}{4}\right]_0^2$

$k\times\left[\frac{16}{3}-4\right] = 1$ or $k\times\frac{4}{3} = 1$

(iii) $\frac{3}{4}\int_0^2 x^3(2-x)\,dx$

$=\frac{3}{4}\times\left[\frac{2x^4}{4}-\frac{x^5}{5}\right]_0^2$

1.2 m oe

(iv) $\frac{3}{4}\int_0^1 x^2(2-x)\,dx$

$\left(=\frac{3}{4}\times(\frac{2}{3}-\frac{1}{4})\right)$

$=\frac{5}{16}$ or 0.3125 oe

$400\times\frac{5}{16} = 125$

29)

(a) 0.3 or 1 – 0.6 or 0.4 or 0.2 seen
0.8

(b) (i) $k\int_0^{1.5}(2.25-x^2)\,dx = 1$

$k\left[2.25x-\frac{x^3}{3}\right]_0^{1.5} = 1$

$k\times[3.375-1.125] = 1$ or $k\times\frac{9}{4} = 1$

(ii) $\frac{4}{9}\int_0^{1.5}(2.25x-x^3)\,dx$

$=\frac{4}{9}\left[2.25\frac{x^2}{2}-\frac{x^4}{4}\right]_0^{1.5}$

= 0.5625 or 0.563
Mean no. of hours = 56.25 or 56.3 56 hrs 15 mins

(iii) Max x is 1.5, less than 2.9 or $150 < 290$

(iv) any a such that $2.9 \leqslant a \leqslant 5$

30)

(i) $k\int_5^{10}(10t-t^2)\mathrm{d}t=1$

$k\left[5t^2-\frac{t^3}{3}\right]_5^{10}=1$

$k(500-\frac{1000}{3}-(125-\frac{125}{3}))=1$

$k\times\frac{250}{3}=1$

$(k=\frac{3}{250}$ **AG)**

(ii) $\frac{3}{250}\int_5^{10}(10t^2-t^3)\mathrm{d}t$

$=\frac{3}{250}\left[\frac{10t^3}{3}-\frac{t^4}{4}\right]_5^{10}$

$=\frac{3}{250}(\frac{10000}{3}-\frac{10000}{4}-(\frac{1250}{3}-\frac{625}{4}))$

$=6.875$ or $55/8$

(iii) $P(T<E(T))=\frac{3}{250}\left[5t^2-\frac{t^3}{3}\right]_5^{\text{"6.875"}}$

$=0.5361$

"0.5361" – 0.5

P(T between E(T) & median = 0.0361

(iv) 10 (minutes)

31)

(i) Greater area where $x<7.5$ than $x>7.5$

(ii) $\int_5^{10}\frac{k}{x^2}\mathrm{d}x=1$

$k\left[-\frac{1}{x}\right]_5^{10}=1$

$k\times\frac{1}{10}=1$

$k=10$

32)

(i) $0.5\times1\times h=0.25$

$h=0.5$

grad = 0.5 $\quad f(x)=0.5x$

$0.5\times a\times0.5a=1 \quad a=2 \quad P(X<2)=1$

(ii) $\int_0^m 0.5x\mathrm{d}x=0.5$

$=\left[\frac{x^2}{4}\right]_0^m=0.5$

$m=\sqrt{2}$ or 1.41 (3 sf)

33)(i)

$k\int_0^1(x-x^2)\mathrm{d}x=1 \quad =k\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1=1$

$=k\left[\frac{1}{2}-\frac{1}{3}\right]=1$ or $\frac{k}{6}=1$

(ii) $E(X)=0.5$

$6\int_0^1(x^3-x^4)\,\mathrm{d}x$

$(=6\left[\frac{1}{4}-\frac{1}{5}\right]=0.3)$

"0.3" – "0.5"^{2}

$=0.05(=1/20)$

(iii) $6\int_{0.4}^1(x-x^2)\,\mathrm{d}x$

$=6\{\frac{1}{2}-\frac{1}{3}-(\frac{0.4^2}{2}-\frac{0.4^3}{3})\}$

$=0.648(=81/125)$

34)

(i) $\int_5^{10}\frac{k}{x^2}\mathrm{d}x=1$

$\left[-\frac{k}{x}\right]_5^{10}=1$ oe

$(\frac{k}{5}-\frac{k}{10}=1) \quad k=10$

(ii) $10\int_5^{10}\frac{1}{x}\mathrm{d}x$

$10[\ln x]_5^{10} \quad =10\ln 2$

(iii) $10\int_9^{10}\frac{1}{x^2}\mathrm{d}x$

$(10\left[-\frac{1}{x}\right]_9^{10})$

$10\left[-\frac{1}{10}+\frac{1}{9}\right]$

$=\frac{1}{9}$ or 0.111

(iv) $\int_5^a\frac{k}{x^2}\mathrm{d}x=0.6$

$10\left[-\frac{1}{x}\right]_5^a=0.6$

$10[\frac{1}{5}-\frac{1}{a}]=0.6$

$a=\frac{50}{7}$ or 7.14 (:

35)

(i) $\frac{1}{2} \times a \times b = 1$

$b = \frac{2}{a}$

(ii) grad $= -\frac{2}{a^2}$ or $-\frac{b}{a}$

$y - (\frac{2}{a}) =$ grad $\times x$ or $y =$ grad $\times (x - a)$

$y - (\frac{2}{a}) = -\frac{2}{a^2}x$ or $y = -\frac{2}{a^2}(x - a)$

and $y = \frac{2}{a} - \frac{2}{a^2}x$ **AG**

(iii) $\int_0^a (\frac{2}{a}x - \frac{2}{a^2}x^2)\,dx$

$= [\frac{1}{a}x^2 - \frac{2}{3a^2}x^3]_0^a$

$a - \frac{2}{3}a = 0.5$

$a = 1.5$

36)

(i) $k\int_1^2 (\frac{1}{x^2} + \frac{1}{x^3})\,dx = 1$

$k\left[-\frac{1}{x} - \frac{1}{2x^2}\right]_1^2 = 1$

$k\left[-\frac{1}{2} - \frac{1}{8} + 1 + \frac{1}{2}\right] = 1$

$k = \frac{8}{7}$ **AG**

(ii) $\frac{8}{7}\int_1^2 (\frac{1}{x} + \frac{1}{x^2})\,dx$

$= \frac{8}{7}\left[\ln x - \frac{1}{x}\right]_1^2$

$= \frac{8}{7}(\ln 2 + \frac{1}{2})$ or 1.36

36)

(iii) $\frac{8}{7}\int_1^{1.5} (\frac{1}{x^2} + \frac{1}{x^3})\,dx$

$= \frac{8}{7}\left[-\frac{1}{x} - \frac{1}{2x^2}\right]_1^{1.5}$

$= \frac{44}{63}$ or 0.698.......

$'\frac{44}{63}'(1 - '\frac{44}{63}')^2$

$\times 3$

$= 0.191$

37)

(i) Mean = 115

SD = 40

(ii) Mean = 15 × '115' = 1725

15 × '40'^{2}

SD = √24000

SD = 155 (cents) (3 sf)

38)

(i) $a\int_1^b \frac{1}{x^2}dx = 1$

$a[-\frac{1}{x}]_1^b = 1$

$a[1 - \frac{1}{b}] = 1$ or $a \times \frac{b-1}{b} = 1$

$b = \frac{a}{a-1}$ **AG**

(ii) $a\int_1^{\frac{3}{2}} \frac{1}{x^2}dx = \frac{1}{2}$

$a[-\frac{1}{x}]_1^{\frac{3}{2}} = \frac{1}{2}$

$a[1 - \frac{2}{3}] = \frac{1}{2}$

$a = \frac{3}{2}, b = 3$

(iii) $\frac{3}{2}\int_1^3 \frac{1}{x}dx$

$= \frac{3}{2}[\ln x]_1^3$

$= \frac{3}{2}\ln 3$ or 1.65

39)

(i) $\frac{3}{a^3}\int_0^a x^2 dx$

$(= \frac{3}{a^3}\left[\frac{x^3}{3}\right]_0^a)$

$= \frac{3a^3}{3a^3}$

$= 1$ Hence f is pdf for all a

(ii) $\frac{3}{a^3}\int_0^2 x^2 dx = 0.5$

$\frac{3}{a^3}\left[\frac{x^3}{3}\right]_0^2 = 0.5$

$\frac{3}{a^3}\times\frac{8}{3} = 0.5$ oe

$a^3 = 16$ or $a = \sqrt[3]{16} = 2.52$

(iii) $\frac{3}{16}\int_0^{2.52} x^3 dx$

$= \frac{3}{16}\left[\frac{x^4}{4}\right]_0^{2.52}$

$= \frac{3}{16}\times\frac{40.317}{4}$

$= 1.89$ (3 sf)

42)(i)

$1 - 6\int_{0.3}^{0.7}(x - x^2)\,dx \quad 1 - \left[6(\frac{x^2}{2} - \frac{x^3}{3})\right]_{0.3}^{0.7}$

$1 - 6\left[\frac{0.7^2}{2} - \frac{0.7^3}{3} - \frac{0.3^2}{2} + \frac{0.3^3}{3}\right] = 0.432$ (or 54/125)

(ii) Correct shape between $x = 0$ and 1

$E(X) = 0.5$

(iii) $6\int_0^1(x^3 - x^4)\,dx$

$= \left[6\left(\frac{x^4}{4} - \frac{x^5}{5}\right)\right]_0^1$

$Var(X) = '0.3' - '0.5'^2$
$= 0.05$

40)

(a) (i) $E(X) = 1.5$

$\frac{2}{9}\int_0^3(3x^3 - x^4)dx$

$= \frac{2}{9}\left[\frac{3x^4}{4} - \frac{x^5}{5}\right]_0^3$

$= \frac{2}{9}\left[\frac{243}{4} - \frac{243}{5}\right]$ (= 2.7)

$Var(X)$ (= $2.7 - 1.5^2$) = 0.45 oe

(ii) 0.5

(iii) $(1 - \frac{13}{27}) \div 2$

$= \frac{7}{27}$ or 0.259

(b) $\frac{1}{2}\times 2\times 2a = \frac{1}{2}$ or $\int_0^2 ax\,dx = \frac{1}{2}$

$a = \frac{1}{4}$

$\frac{1}{2}\times b\times\frac{1}{4}b = 1$ or $\int_0^b \frac{1}{4}x\,dx = 1$

or $b = 2\times\sqrt{2}$

$b = 2\sqrt{2}$

41)

(a)(i) $k = 1$

(a)(ii) f_2: area > 1 (area ≠ 1)

f_3: includes negative values of f_3

(b)(i) $6\int_{-a}^{a}(a^2 - x^2)dx = 1$

$6[a^2x - \frac{x^3}{3}]_{-a}^{a} = 1$

$6(2a^3 - \frac{2a^3}{3}) = 1$

$\frac{24a^3}{3} = 1$ or $8a^3 = 1$

$a = 1/2$

(b)(ii) 0

(b)(iii) $6\int_{-0.5}^{0.5}(\frac{x^2}{4} - x^4)dx$

$(= 6\left[\frac{x^3}{12} - \frac{x^5}{5}\right]_{-0.5}^{0.5} = 0.05)$

$Var = 0.05 - 0^2$

$= 0.05$ oe

43)

(i)

$\sqrt{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos x dx$

$= \sqrt{2}\,[\sin x]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$

$= \frac{2-\sqrt{2}}{2}$ oe or 0.293 (3 sf)

(ii)

$\sqrt{2}\int_0^m \cos x dx = 0.5$

$\sqrt{2}\,[\sin x]_0^m = 0.5$

$\sqrt{2}\sin m = 0.5$

$\sin m = \frac{1}{2\sqrt{2}}$ oe

$m = 0.361$ (3 sfs)

(iii)

$\sqrt{2}\int_0^{\frac{\pi}{4}} x\cos x dx$

$= \sqrt{2}\,\{[x(\sin x)]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \sin x dx\}$

$= \sqrt{2}\,\{\frac{\pi}{4\sqrt{2}} - 0 - [-\cos x]_0^{\frac{\pi}{4}}\}$

$= \sqrt{2}\,\{\frac{\pi}{4\sqrt{2}} + \cos\frac{\pi}{4} - 1\}$

$= \frac{\pi}{4} + 1 - \sqrt{2}$ oe or 0.371 (3 sf)