Statistics -2

9709

Hypothesis Testing

Exercise (march and June series 2019 – 2002)

With marking scheme

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1) march 2019 Q6

The time taken by volunteers to complete a certain task is normally distributed. In the past the time, in minutes, has had mean 91.4 and standard deviation 6.4. A new, similar task is introduced and the times, t minutes, taken by a random sample of 6 volunteers to complete the new task are summarised by $\Sigma t = 568.5$. Andrea plans to carry out a test, at the 5% significance level, of whether the mean time for the new task is different from the mean time for the old task.

(i) Give a reason why Andrea should use a two-tail test. [1]

(ii) State the probability that a Type I error is made, and explain the meaning of a Type I error in this context. [2]

You may assume that the times taken for the new task are normally distributed.

(iii) Stating another necessary assumption, carry out the test. [7]

2) march 2018 Q5

Packets of Frugums contain 30 sweets. The manufacturer claims that, on average, 17% of the sweets are orange flavoured. Angela suspects that the average is actually less than 17%. In order to test the manufacturer's claim, she buys a packet of Frugums. If there are fewer than 3 orange flavoured sweets in the packet, she will conclude that the claim is false.

(i) State appropriate null and alternative hypotheses. [1]

(ii) Explain what is meant by a Type I error in this situation. [1]

(iii) Calculate the probability of a Type I error. [3]

(iv) Given that the true percentage of orange flavoured sweets is 5%, calculate the probability of a Type II error. [3]

3) march 2017 Q2

Karim has noted the lifespans, in weeks, of a large random sample of certain insects. He carries out a test, at the 1% significance level, for the population mean, μ. Karim's null hypothesis is $\mu = 6.4$.

(i) Given that Karim's test is two-tail, state the alternative hypothesis. [1]

Karim finds that the value of the test statistic is $z = 2.43$.

(ii) Explain what conclusion he should draw. [2]

(iii) Explain briefly when a one-tail test is appropriate, rather than a two-tail test. [1]

4) march 2016 Q3

Jill shoots arrows at a target. Last week, 65% of her shots hit the target. This week Jill claims that she has improved. Out of her first 20 shots this week, she hits the target with 18 shots. Assuming shots are independent, test Jill's claim at the 1% significance level. [5]

5) June 2019 /71/Q3

Sumitra has a six-sided die. She suspects that it is biased so that it shows a six less often than it would if it were fair. She decides to test the die by throwing it 30 times and noting the number of throws on which it shows a six.

(i) It shows a six on exactly 2 throws. Use a binomial distribution to carry out the test at the 5% significance level. [5]

(ii) Later, Sumitra repeats the test at the 5% significance level by throwing the die 30 times again. Find the probability of a Type I error in this second test. [2]

6) june 2019/73/Q5

The amount of money, in dollars, spent by a customer on one visit to a certain shop is modelled by the distribution $N(\mu, 1.94)$. In the past, the value of μ has been found to be 20.00, but following a rearrangement in the shop, the manager suspects that the value of μ has changed. He takes a random sample of 6 customers and notes how much they each spend, in dollars. The results are as follows.

17.60 23.50 17.30 22.00 31.00 15.50

The manager carries out a hypothesis test using a significance level of $\alpha\%$. The test does not support his suspicion. Find the largest possible value of α. [6]

7) june 2018/71/Q5(iii)

A geologist suspects that rocks in another area have a mean mass which is less than 14.2 kg. A random sample of 100 rocks in this area has sample mean 13.5 kg. Assuming that the standard deviation for rocks in this area is also 3.1 kg, test at the 2% significance level whether the geologist is correct. [5]

8) june 2018/72/Q7

A ten-sided spinner has edges numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Sanjeev claims that the spinner is biased so that it lands on the 10 more often than it would if it were unbiased. In an experiment, the spinner landed on the 10 in 3 out of 9 spins.

(i) Test at the 1% significance level whether Sanjeev's claim is justified. [5]

(ii) Explain why a Type I error cannot have been made. [1]

In fact the spinner is biased so that the probability that it will land on the 10 on any spin is 0.5.

(iii) Another test at the 1% significance level, also based on 9 spins, is carried out. Calculate the probability of a Type II error. [6]

9) june 2018/73/Q5

The time taken for a particular train journey is normally distributed. In the past, the time had mean 2.4 hours and standard deviation 0.3 hours. A new timetable is introduced and on 30 randomly chosen occasions the time for this journey is measured. The mean time for these 30 occasions is found to be 2.3 hours.

(i) Stating any assumption(s), test, at the 5% significance level, whether the mean time for this journey has changed. [6]

(ii) A similar test at the 5% significance level was carried out using the times from another randomly chosen 30 occasions.

(a) State the probability of a Type I error. [1]

(b) State what is meant by a Type II error in this context. [1]

10) june 2017/71/Q2

Past experience has shown that the heights of a certain variety of plant have mean 64.0 cm and standard deviation 3.8 cm. During a particularly hot summer, it was expected that the heights of plants of this variety would be less than usual. In order to test whether this was the case, a botanist recorded the heights of a random sample of 100 plants and found that the value of the sample mean was 63.3 cm. Stating a necessary assumption, carry out the test at the 2.5% significance level. [6]

11) june 2017/72/Q4

It is claimed that 1 in every 4 packets of certain biscuits contains a free gift. Marisa and André both suspect that the true proportion is less than 1 in 4.

(i) Marisa chooses 20 packets at random. She decides that if fewer than 3 contain free gifts, she will conclude that the claim is not justified. Use a binomial distribution to find the probability of a Type I error. [2]

(ii) André chooses 25 packets at random. He decides to carry out a significance test at the 1% level, using a binomial distribution. Given that only 1 of the 25 packets contains a free gift, carry out the test. [5]

12) june 2017/73/Q7

In the past the number of accidents per month on a certain road was modelled by a random variable with distribution Po(0.47). After the introduction of speed restrictions, the government wished to test, at the 5% significance level, whether the mean number of accidents had decreased. They noted the number of accidents during the next 12 months. It is assumed that accidents occur randomly and that a Poisson model is still appropriate.

(i) Given that the total number of accidents during the 12 months was 2, carry out the test. [6]

(ii) Explain what is meant by a Type II error in this context. [1]

It is given that the mean number of accidents per month is now in fact 0.05.

(iii) Using another random sample of 12 months the same test is carried out again, with the same significance level. Find the probability of a Type II error. [4]

13) june 2016/71/Q4

In the past, the time spent by customers in a certain shop had mean 12.5 minutes and standard deviation 4.2 minutes. Following a change of layout in the shop, the mean time spent in the shop by a random sample of 50 customers is found to be 13.5 minutes.

(i) Assuming that the standard deviation remains at 4.2 minutes, test at the 5% significance level whether the mean time spent by customers in the shop has changed. [5]

(ii) Another random sample of 50 customers is chosen and a similar test at the 5% significance level is carried out. State the probability of a Type I error. [1]

14) june 2016/72/Q4

At a certain company, computer faults occur randomly and at a constant mean rate. In the past this mean rate has been 2.1 per week. Following an update, the management wish to determine whether the mean rate has changed. During 20 randomly chosen weeks it is found that 54 computer faults occur. Use a suitable approximation to test at the 5% significance level whether the mean rate has changed. [6]

15) june 2016/73/Q2

In the past, the mean annual crop yield from a particular field has been 8.2 tonnes. During the last 16 years, a new fertiliser has been used on the field. The mean yield for these 16 years is 8.7 tonnes. Assume that yields are normally distributed with standard deviation 1.2 tonnes. Carry out a test at the 5% significance level of whether the mean yield has increased. [5]

16) june 2016/73/Q4

The number of sightings of a golden eagle at a certain location has a Poisson distribution with mean 2.5 per week. Drilling for oil is started nearby. A naturalist wishes to test at the 5% significance level whether there are fewer sightings since the drilling began. He notes that during the following 3 weeks there are 2 sightings.

(i) Find the critical region for the test and carry out the test. [5]

(ii) State the probability of a Type I error. [1]

(iii) State why the naturalist could not have made a Type II error. [1]

17) june 2015/71/Q2

Sami claims that he can read minds. He asks each of 50 people to choose one of the 5 letters A, B, C, D or E. He then tells each person which letter he believes they have chosen. He gets 13 correct. Sami says "This shows that I can read minds, because 13 is more than I would have got right if I were just guessing."

(i) State null and alternative hypotheses for a test of Sami's claim. [1]

(ii) Test at the 10% significance level whether Sami's claim is justified. [5]

18) june 2015/71/Q4

In the past, the time taken by vehicles to drive along a particular stretch of road has had mean 12.4 minutes and standard deviation 2.1 minutes. Some new signs are installed and it is expected that the mean time will increase. In order to test whether this is the case, the mean time for a random sample of 50 vehicles is found. You may assume that the standard deviation is unchanged.

(i) The mean time for the sample of 50 vehicles is found to be 12.9 minutes. Test at the 2.5% significance level whether the population mean time has increased. [4]

(ii) State what is meant by a Type II error in this context. [2]

(iii) State what extra piece of information would be needed in order to find the probability of a Type II error. [1]

19) june 2015/72/Q2

Cloth made at a certain factory has been found to have an average of 0.1 faults per square metre. Suki claims that the cloth made by her machine contains, on average, more than 0.1 faults per square metre. In a random sample of 5 m^2 of cloth from Suki's machine, it was found that there were 2 faults. Assuming that the number of faults per square metre has a Poisson distribution,

(i) state null and alternative hypotheses for a test of Suki's claim, [1]

(ii) test at the 10% significance level whether Suki's claim is justified. [4]

20) june 2015/72/Q4

In the past, the flight time, in hours, for a particular flight has had mean 6.20 and standard deviation 0.80. Some new regulations are introduced. In order to test whether these new regulations have had any effect upon flight times, the mean flight time for a random sample of 40 of these flights is found.

(i) State what is meant by a Type I error in this context. [2]

(ii) The mean time for the sample of 40 flights is found to be 5.98 hours. Assuming that the standard deviation of flight times is still 0.80 hours, test at the 5% significance level whether the population mean flight time has changed. [4]

(iii) State, with a reason, which of the errors, Type I or Type II, might have been made in your answer to part **(ii)**. [2]

21) june 2015/73/Q2

Marie claims that she can predict the winning horse at the local races. There are 8 horses in each race. Nadine thinks that Marie is just guessing, so she proposes a test. She asks Marie to predict the winners of the next 10 races and, if she is correct in 3 or more races, Nadine will accept Marie's claim.

(i) State suitable null and alternative hypotheses. [1]

(ii) Calculate the probability of a Type I error. [3]

(iii) State the significance level of the test. [1]

22) june 2014/71/Q3

The lengths, in centimetres, of rods produced in a factory have mean μ and standard deviation 0.2. The value of μ is supposed to be 250, but a manager claims that one machine is producing rods that are too long on average. A random sample of 40 rods from this machine is taken and the sample mean length is found to be 250.06 cm. Test at the 5% significance level whether the manager's claim is justified. [5]

23) june 2014/72/Q3

The number of calls per day to an enquiry desk has a Poisson distribution. In the past the mean has been 5. In order to test whether the mean has changed, the number of calls on a random sample of 10 days was recorded. The total number of calls was found to be 61. Use an approximate distribution to test at the 10% significance level whether the mean has changed. [5]

24) june 2014/73/Q6

A machine is designed to generate random digits between 1 and 5 inclusive. Each digit is supposed to appear with the same probability as the others, but Max claims that the digit 5 is appearing less often than it should. In order to test this claim the manufacturer uses the machine to generate 25 digits and finds that exactly 1 of these digits is a 5.

(i) Carry out a test of Max's claim at the 2.5% significance level. [5]

(ii) Max carried out a similar hypothesis test by generating 1000 digits between 1 and 5 inclusive. The digit 5 appeared 180 times. Without carrying out the test, state the distribution that Max should use, including the values of any parameters. [2]

(iii) State what is meant by a Type II error in this context. [1]

25) june 2013/71/Q2

The times taken by students to complete a task are normally distributed with standard deviation 2.4 minutes. A lecturer claims that the mean time is 17.0 minutes. The times taken by a random sample of 5 students were 17.8, 22.4, 16.3, 23.1 and 11.4 minutes. Carry out a hypothesis test at the 5% significance level to determine whether the lecturer's claim should be accepted. [5]

26) june 2013/72/Q3

The heights of a certain variety of plant have been found to be normally distributed with mean 75.2 cm and standard deviation 5.7 cm. A biologist suspects that pollution in a certain region is causing the plants to be shorter than usual. He takes a random sample of n plants of this variety from this region and finds that their mean height is 73.1 cm. He then carries out an appropriate hypothesis test.

(i) He finds that the value of the test statistic z is -1.563, correct to 3 decimal places. Calculate the value of n. State an assumption necessary for your calculation. [4]

(ii) Use this value of the test statistic to carry out the hypothesis test at the 6% significance level. [3]

27) june 2013/73/Q2

A hockey player found that she scored a goal on 82% of her penalty shots. After attending a coaching course, she scored a goal on 19 out of 20 penalty shots. Making an assumption that should be stated, test at the 10% significance level whether she has improved. [5]

28) june 2012/71/Q3

When the council published a plan for a new road, only 15% of local residents approved the plan. The council then published a revised plan and, out of a random sample of 300 local residents, 60 approved the revised plan. Is there evidence, at the 2.5% significance level, that the proportion of local residents who approve the revised plan is greater than for the original plan? [5]

29) june 2012/72/Q7

The weights, X kilograms, of bags of carrots are normally distributed. The mean of X is μ. An inspector wishes to test whether $\mu = 2.0$. He weighs a random sample of 200 bags and his results are summarised as follows.

$$\Sigma x = 430 \qquad \Sigma x^2 = 1290$$

(i) Carry out the test, at the 10% significance level. [6]

(ii) You may now assume that the population variance of X is 1.85. The inspector weighs another random sample of 200 bags and carries out the same test at the 10% significance level.

(a) State the meaning of a Type II error in this context. [1]

(b) Given that $\mu = 2.12$, show that the probability of a Type II error is 0.652, correct to 3 significant figures. [7]

30) june 2012/73/Q6

Last year Samir found that the time for his journey to work had mean 45.7 minutes and standard deviation 3.2 minutes. Samir wishes to test whether his journey times have increased this year. He notes the times, in minutes, for a random sample of 8 journeys this year with the following results.

46.2 41.7 49.2 47.1 47.2 48.4 53.7 45.5

It may be assumed that the population of this year's journey times is normally distributed with standard deviation 3.2 minutes.

(i) State, with a reason, whether Samir should use a one-tail or a two-tail test. [2]

(ii) Show that there is no evidence at the 5% significance level that Samir's mean journey time has increased. [5]

(iii) State, with a reason, which one of the errors, Type I or Type II, might have been made in carrying out the test in part **(ii)**. [2]

31) june 2011/72/Q6

Jeevan thinks that a six-sided die is biased in favour of six. In order to test this, Jeevan throws the die 10 times. If the die shows a six on at least 4 throws out of 10, she will conclude that she is correct.

(i) State appropriate null and alternative hypotheses. [1]

(ii) Calculate the probability of a Type I error. [3]

(iii) Explain what is meant by a Type II error in this situation. [1]

(iv) If the die is actually biased so that the probability of throwing a six is $\frac{1}{2}$, calculate the probability of a Type II error. [3]

32) june 2011/73/Q3

At an election in 2010, 15% of voters in Bratfield voted for the Renewal Party. One year later, a researcher asked 30 randomly selected voters in Bratfield whether they would vote for the Renewal Party if there were an election next week. 2 of these 30 voters said that they would.

(i) Use a binomial distribution to test, at the 4% significance level, the null hypothesis that there has been no change in the support for the Renewal Party in Bratfield against the alternative hypothesis that there has been a decrease in support since the 2010 election. [4]

(ii) **(a)** Explain why the conclusion in part **(i)** cannot involve a Type I error. [1]

(b) State the circumstances in which the conclusion in part **(i)** would involve a Type II error. [1]

33) june 2010/73/Q1

At the 2009 election, $\frac{1}{3}$ of the voters in Chington voted for the Citizens Party. One year later, a researcher questioned 20 randomly selected voters in Chington. Exactly 3 of these 20 voters said that if there were an election next week they would vote for the Citizens Party. Test at the 2.5% significance level whether there is evidence of a decrease in support for the Citizens Party in Chington, since the 2009 election. [5]

34) june 2009/7/Q1

In Europe the diameters of women's rings have mean 18.5 mm. Researchers claim that women in Jakarta have smaller fingers than women in Europe. The researchers took a random sample of 20 women in Jakarta and measured the diameters of their rings. The mean diameter was found to be 18.1 mm. Assuming that the diameters of women's rings in Jakarta have a normal distribution with standard deviation 1.1 mm, carry out a hypothesis test at the $2\frac{1}{2}\%$ level to determine whether the researchers' claim is justified. [5]

35) june 2009/7/Q4

In a certain city it is necessary to pass a driving test in order to be allowed to drive a car. The probability of passing the driving test at the first attempt is 0.36 on average. A particular driving instructor claims that the probability of his pupils passing at the first attempt is higher than 0.36. A random sample of 8 of his pupils showed that 7 passed at the first attempt.

(i) Carry out an appropriate hypothesis test to test the driving instructor's claim, using a significance level of 5%. [5]

(ii) In fact, most of this random sample happened to be careful and sensible drivers. State which type of error in the hypothesis test (Type I or Type II) could have been made in these circumstances and find the probability of this type of error when a sample of size 8 is used for the test. [4]

36) june 2008/7/Q4

People who diet can expect to lose an average of 3 kg in a month. In a book, the authors claim that people who follow a new diet will lose an average of more than 3 kg in a month. The weight losses of the 180 people in a random sample who had followed the new diet for a month were noted. The mean was 3.3 kg and the standard deviation was 2.8 kg.

(i) Test the authors' claim at the 5% significance level, stating your null and alternative hypotheses. [5]

(ii) State what is meant by a Type II error in words relating to the context of the test in part **(i)**. [2]

37) june 2007/7/Q4

At a certain airport 20% of people take longer than an hour to check in. A new computer system is installed, and it is claimed that this will reduce the time to check in. It is decided to accept the claim if, from a random sample of 22 people, the number taking longer than an hour to check in is either 0 or 1.

(i) Calculate the significance level of the test. [3]

(ii) State the probability that a Type I error occurs. [1]

(iii) Calculate the probability that a Type II error occurs if the probability that a person takes longer than an hour to check in is now 0.09. [3]

38) june 2005/7/Q4

A study of a large sample of books by a particular author shows that the number of words per sentence can be modelled by a normal distribution with mean 21.2 and standard deviation 7.3. A researcher claims to have discovered a previously unknown book by this author. The mean length of 90 sentences chosen at random in this book is found to be 19.4 words.

(i) Assuming the population standard deviation of sentence lengths in this book is also 7.3, test at the 5% level of significance whether the mean sentence length is the same as the author's. State your null and alternative hypotheses. [5]

(ii) State in words relating to the context of the test what is meant by a Type I error and state the probability of a Type I error in the test in part **(i)**. [2]

39) june 2004/7/Q1

Each multiple choice question in a test has 4 suggested answers, exactly one of which is correct. Rehka knows nothing about the subject of the test, but claims that she has a special method for answering the questions that is better than just guessing. There are 60 questions in the test, and Rehka gets 22 correct.

(i) State null and alternative hypotheses for a test of Rehka's claim. [1]

(ii) Using a normal approximation, test at the 5% significance level whether Rehka's claim is justified. [4]

40) june 2003/7/Q2

Before attending a basketball course, a player found that 60% of his shots made a score. After attending the course the player claimed he had improved. In his next game he tried 12 shots and scored in 10 of them. Assuming shots to be independent, test this claim at the 10% significance level. [5]

Answers

1)

(i)	Test is for "difference" oe
(ii)	0.05
	Conclude mean time is different when it is not
(iii)	Assume $\sigma = 6.4$
	H_0: pop mean = 91.4 H_1: pop mean $\neq$ 91.4
	$\bar{x} = \frac{568.5}{6}$ (= 94.75)
	$\frac{\text{'}94.75\text{'} - 91.4}{\frac{6.4}{\sqrt{6}}}$
	= 1.282 cv of z = 1.96
	'1.282' < 1.96

2)

(i) H_0: P(orange)=0.17 H_1: P (orange)< 0.17

(ii) wrongly concluding that % age is less than 17%

(iii) B(30, 0.17) stated or implied

$(1 - 0.17)^{30} + 30(1 - 0.17)^{29} \times 0.17 + {}^{30}C_2(1 - 0.17)^{28} \times 0.17^2$

= 0.0949 (3 sf)

(iv)	P($\geqslant$ 3 orange | p = 0.05)
	$= 1 - [(0.95)^{30} + 30(0.95)^{29} \times 0.05 + {}^{30}C_2(0.95)^{28} \times 0.05^2]$
	= 0.188 (3 sfs)

3)

(i)	(H_1): $\mu \neq 6.4$
(ii)	comp 2.43 with a z-value z = 2.576 AND
	No evidence that μ is not 6.4 or do not reject μ = 6.4
(iii)	Testing for an increase in μ, or for a decrease in μ, rather than a change

4)

H_0: P(hit target) = 0.65 H_1: P(hit target) > 0.65	**B1**
${}^{20}C_2 \times 0.35^2 \times 0.65^{18} + 19 \times 0.35 \times 0.65^{19} + 0.65^{20}$	**M1**
= 0.0121 (3 sf)	**A1**
Comp 0.01	**M1**
There is no evidence (at the 1% level) that she has improved	**A1**

5)

(i)	H_0: P(6) = $\frac{1}{6}$ H_1: P(6) < $\frac{1}{6}$
	$(\frac{5}{6})^{30} + 30(\frac{1}{6}) \times (\frac{5}{6})^{29} + {}^{30}C_2(\frac{1}{6})^2 \times (\frac{5}{6})^{28}$
	= 0.103
	'0.103' > 0.05
	No evidence (at 5% level) that die biased
(ii)	$(\frac{5}{6})^{30} + 30(\frac{1}{6}) \times (\frac{5}{6})^{29}$
	P(Type I) = 0.0295

6)

(i)

H_0: P(6) = $\frac{1}{6}$
H_1: P(6) < $\frac{1}{6}$

$(\frac{5}{6})^{30} + 30(\frac{1}{6}) \times (\frac{5}{6})^{29} + {}^{30}C_2(\frac{1}{6})^2 \times (\frac{5}{6})^{28}$

= 0.103

'0.103' > 0.05

No evidence (at 5% level) that die biased

(ii)

$(\frac{5}{6})^{30} + 30(\frac{1}{6}) \times (\frac{5}{6})^{29}$

P(Type I) = 0.0295

H_0: $\mu = 14.2$
H_1: $\mu < 14.2$

$$\frac{13.5 - 14.2}{\frac{3.1}{\sqrt{100}}}$$

= –2.258

comp –2.054 (or –2.055)

There is evidence (at 2% level) that mean mass in this area < 14.2

8) (i)

H_0: P(10) = 0.1
H_1: P(10) > 0.1

B(9,0.1)
$P(X \geqslant 3) =$
$1 - (0.9^9 + 9 \times 0.9^8 \times 0.1 + {}^9C_2 \times 0.9^7 \times 0.1^2)$

= 0.05297... or 0.053(0)

comp 0.01

No evidence (at 1% level) to reject H_0
Claim not justified

8)(ii)

H_0 not rejected

8)(iii)

$P(X \geqslant 4)$
$= "0.05297" - {}^9C_3 \times 0.9^6 \times 0.1^3$

= 0.00833

Hence crit value is 4

B(9,0.5)
$P(X < 4)$

$= 0.5^9 + 9 \times 0.5^8 \times 0.5 + {}^9C_2 \times 0.5^7 \times 0.5^2 + {}^9C_3 \times 0.5^6 \times 0.5^3$

P(Type II) = 0.254 (3 sf)

9)(i)

Assume (pop) sd same (0.3)
H_0: Pop mean = 2.4

H_1: Pop mean ≠ 2.4

$$\pm\frac{2.3 - 2.4}{\frac{0.3}{\sqrt{30}}}$$

= ±1.826

comp $z = \pm 1.96$

No evidence that mean time changed

(ii) 0.05

(iii) Concluding mean time has not changed when it has.

10)

Assume sd still = 3.8

$H_0: \mu = 64.0 \quad H_1: \mu < 64.0$

$\frac{63.3-64.0}{\frac{3.8}{\sqrt{100}}}$

$= -1.842$

comp "1.842" with z-value
"1.842" < 1.96

No evidence that heights are shorter

11)

(i)	$0.75^{20} + 20 \times 0.75^{19} \times 0.25 + {}^{20}C_2 \times 0.75^{18} \times 0.25^2$
	$= 0.0913$
ii)	H_0: Pop proportion=0.25 H_1: Pop proportion<0.25
	$0.75^{25} + 25 \times 0.75^{24} \times 0.25$
	$= 0.00702$
	comp 0.01
	There is evidence that the claim is not justified

12)

(i)	H_0: Pop mean no. accidents = 5.64 H_1: Pop mean no. accidents < 5.64
	Use of $\lambda = 5.64$
	$= e^{-5.64}(1 + 5.64 + \frac{5.64^2}{2})$
	= 0.08(0)
	Comp with 0.05
	No evidence to believe mean no. of accidents has decreased; accept H_0 correctly defined)
	To
ii)	Mean < 0.47 but conclude that this is not so
	To
iii)	(Need greatest x such that $P(X \leqslant x) < 0.05$) $P(X \leqslant 1) = e^{-5.64}(1 + 5.64) = 0.024$ $P(X \leqslant 2) = 0.08$
	Hence rejection region is $X \leqslant 1$
	With $\lambda = 12 \times 0.05 = 0.6$, $1 - P(X \leqslant 1) = 1 - e^{-0.6}(1 + 0.6)$
	= 0.122 (3 sf)

13)

(i)	$H_0: \mu = 12.5$ $H_1: \mu \neq 12.5$ $\frac{13.5-12.5}{4.2 \div \sqrt{50}}$ $= 1.68(4)$ '1.684' < 1.96 No evidence that mean time has changed
(ii)	0.05

14)

$H_0: \lambda$ (or μ) = 42
$H_1: \lambda$ (or μ) $\neq$ 42
Po(42) ~ N(42, 42) stated or implied

$\frac{53.5-42}{\sqrt{42}}$
= 1.77(4) (or 0.038 for area comparison)

comp 1.96

No evidence that mean has changed

15)

H_o: Pop mean yield = 8.2
H_1: Pop mean yield > 8.2
$(\pm)\frac{8.7-8.2}{1.2/\sqrt{16}}$

$= (\pm)1.667$
Comp $z = 1.645$ Or Area comparison
0.0475-0.0478)
Reject H_0
Evidence that mean yield has increased

16)

(i) H_0: Pop mean = 2.5 (or 7.5)
H_0: Pop mean < 2.5 (or 7.5)

$\lambda = 7.5$
$P(X \leqslant 2) = e^{-7.5}(1+7.5+\frac{7.5^2}{2}) = 0.0203$
$P(X \leqslant 3) = 0.0203 + e^{-7.5} \times \frac{7.5^3}{3!} = 0.0591$

CR is $X \leqslant 2$

Reject H_0
Evidence that no of sightings fewer

(ii) P(Type I) = 0.0203 (3 sf)

(iii) H_0 was rejected oe

17)

(i) H_0: $p = 0.2$ or $\mu = 10$
H_1: $p > 0.2$ or $\mu > 10$

(ii) N(10, 8) seen or implied
$\frac{125-10}{\sqrt{8}}$ or $\frac{\frac{125}{50}-02}{\sqrt{\frac{0.2\times0.8}{50}}}$

$= 0.884$

comp 1.282

Claim not justified
or No evidence to support claim

18)

(i) H_0: pop mean (or μ) = 12.4
H_1: pop mean (or μ) > 12.4

$\frac{12.9-12.4}{2.1+\sqrt{50}}$
1.684

comp cv $z = 1.96$
No evidence that pop mean time has increased

(ii) Not reject (or accept) that mean time is unchanged (or is 12.4) oe

although mean time has increased (or is more than 12.4) oe

(iii) True (or new) mean

19)

(i) H_0: $\lambda = 0.5$

H_1: $\lambda > 0.5$

(ii) $1 - e^{-0.5}(1 + 0.5)$
$= 0.0902$ (3 sf)
comp 0.1
Claim justified or there is evidence to support claim

20)

(i) Conclude flight times affected when in fact they have not been.

(ii) H_0: Pop mean (or μ) = 6.2
H_0: Pop mean (or μ) $\neq$ 6.2

$$\frac{5.98 - 6.2}{\frac{0.8}{\sqrt{40}}}$$

= −1.739 (±) Accept (±)1.74
comp z = 1.96

No evidence that flight times affected

(iii) H_0 was not rejected oe
Type II

21)

(i) H_0: P(correct) = $\frac{1}{8}$

H_1: P(correct) > $\frac{1}{8}$

(ii) $$1 - \left(\left(\frac{1}{8}\right)^{10} + 10\left(\frac{1}{8}\right)^{9}\left(\frac{7}{8}\right) + {}^{10}C_2\left(\frac{1}{8}\right)^{8}\left(\frac{7}{8}\right)^{2}\right)$$

= 0.120 (3 sf) or 0.119

(iii) 12%

22)

H_0: μ = 250
H_1: μ > 250

$$\frac{250.06 - 250}{0.2 \div \sqrt{40}}$$

= 1.90
comp with z = 1.645
Claim is justified
or There is evidence that claim is true

23)

H_0: Pop mean (or μ or λ) = 50 (or 5)

H_1: Pop mean (or μ or λ) $\neq$ 50 (or 5)

$$\frac{60.5 - 50}{\sqrt{50}} \ (\pm)$$

= (±)1.485 OR 0.0687 OR C.V

1.485 < 1.645 or 0.0687 > 0.05
No evidence that mean changed

24)

(i) H_0: p = 0.2
H_1: p < 0.2

P(0 or 1 5s in 25 | H_0)

= 0.0274 (3 s.f.)

Comp with 0.025

No evidence (at 2.5% level) to support claim

(ii) Normal

μ = 200, σ^2 = 160 or $\sigma = \sqrt{160}$

(iii) Concluding that the machine produces the right proportion of 5s, although it doesn't.

25)

H_0: Pop mean = 17
H_1: Pop mean ≠ 17

$$\frac{18.2 - 17}{\frac{2.4}{\sqrt{5}}}$$

= 1.12 (3 sf)

'1.12' < 1.96 oe

Claim can be accepted

26)

(i) $\dfrac{73.1-75.2}{\frac{5.7}{\sqrt{n}}} = -1.563$

$n = \{-1.563 \times 5.7 \div (-2.1)\}^2$

$n = 18$

Assume s.d. for the region is 5.7

(ii) H_0: pop mean (or μ) = 75.2

H_0: pop mean (or μ) < 75.2

1.563 comp 1.555

Evidence that plants shorter

27)

Assume shots independent OR prob of scoring constant

H_0: P(score) = 0.82

H_1: P(score) > 0.82

$20 \times 0.82^{19} \times 0.18 + 0.82^{20}$

= 0.102 (3 sf)

No evidence that improved

28)

H_o: $p = 0.15$

H_1: $p > 0.15$

$(N(300 \times 0.15, 300 \times 0.15 \times 0.85))$

= N(45, 38.25)

$\dfrac{59.5-'45'}{\sqrt{'38.25'}}$ (= 2.345)

Allow wrong or no cc

$z = 1.96$ 2.345>1.96

Evidence prop is higher for new plan

29)(i)

H_0: $\mu = 2.0$ H_1: $\mu \neq 2.0$

$\bar{x} = \frac{430}{200} = 2.15$

$s^2 = \frac{200}{199}\left(\frac{1290}{200} - \left(\frac{430}{200}\right)^2\right)$

= 1.8366834

$\dfrac{2.15-2.0}{\sqrt{\frac{'1.8366834'}{200}}}$ (= 1.565)

$z = 1.645$

No evidence that $\mu \neq 2.0$

29)

(ii) (a) Concluding $\mu = 2.0$ although not true

(b) $\dfrac{\bar{x}-2.0}{\sqrt{\frac{1.85}{200}}} = 1.645$

$\bar{x} = 2 + 0.1582$

Rejection region is

$\bar{x} < 1.8418$ and $\bar{x} > 2.1582$

$\dfrac{2.1582-2.12}{\sqrt{\frac{1.85}{200}}}$ (= 0.397)

$P(\bar{x} < 2.1582 \mid \mu = 2.12) = \Phi('0.397')$

= 0.6543

$\dfrac{1.8418-2.12}{\sqrt{\frac{1.85}{200}}}$ (= – 2.893)

$P(\bar{x} < -2.893 \mid \mu = 2.12) = 1 - \Phi('2.893')$

(= 0.0019)

$\Rightarrow P(1.8418 < \bar{x} < 2.1582 \mid \mu = 2.12)$

= 0.6543 – 0.0019

= 0.6524

P(Type II error) = 0.652 (3 sfs)

30)

(i) Test is for bias in one direction

One-tail

(ii) H_0: pop mean = 45.7

H_1: pop mean > 45.7

$\bar{x}$ = 47.375 or 47.4 or 379/8

$\dfrac{'47.375'-45.7}{\frac{3.2}{\sqrt{8}}}$

(= 1.481 to 1.503))

$z = 1.645$

'1.481' < 1.645

hence no evidence mean time increased

Not rejected H_0

Type II possible

31)

(i) $H_0: P(6) = \frac{1}{6}$ $H_1: P(6) > \frac{1}{6}$

(ii) $\left(\frac{5}{6}\right)^{10} + 10\times\left(\frac{5}{6}\right)^{9}\times\frac{1}{6} + \binom{10}{2}\times\left(\frac{5}{6}\right)^{8}\times\frac{1}{6}^{2} + \binom{10}{3}\times\left(\frac{5}{6}\right)^{7}\times\left(\frac{1}{6}\right)^{3}$

$$1-\left(\left(\frac{5}{6}\right)^{10} + 10\times\left(\frac{5}{6}\right)^{9}\times\frac{1}{6} + \binom{10}{2}\times\left(\frac{5}{6}\right)^{8}\times\left(\frac{1}{6}\right)^{2} + \binom{10}{3}\times\left(\frac{5}{6}\right)^{7}\times\left(\frac{1}{6}\right)^{3}\right)$$

= 0.0697 (3 sfs)

(iii) Die biased towards a six but result < 4 so no evidence of bias

(iv) P(0, 1, 2 or 3 sixes)

$\left(\left(\frac{1}{2}\right)^{10} + 10\times\left(\frac{1}{2}\right)^{9}\times\frac{1}{2} + \binom{10}{2}\times\left(\frac{1}{2}\right)^{8}\times\left(\frac{1}{2}\right)^{2} + \binom{10}{3}\times\left(\frac{1}{2}\right)^{7}\times\left(\frac{1}{2}\right)^{3}\right)$

= 0.172 or 11/64

32)

(i) $0.85^{30} + 30\times 0.85^{29}\times 0.15 + {}^{30}C_2\times 0.85^{28}\times 0.15^2$
= 0.151
> 0.04
No evidence decrease or Accept no decrease

(ii) **(a)** Not rejected Ho

(b) Has been decrease or π (or p) < 0.15

33)

H_0: Pop prop = $^1/_3$ (or unchanged)
H_1: Pop prop < $^1/_3$ (or decreased)

$$\left(\frac{2}{3}\right)^{20} + 30\left(\frac{2}{3}\right)^{19}\left(\frac{1}{3}\right) + {}^{20}C_2\left(\frac{2}{3}\right)^{18}\left(\frac{1}{3}\right)^2 + {}^{20}C_3\left(\frac{2}{3}\right)^{17}\left(\frac{1}{3}\right)^3$$

= 0.0604/0.0605

comp "0.0604" with 0.025
No evidence that support decreased
or support probably not decreased

SC Use Of Normal
Standardising with or without cc
Obtains z = –1.502
Valid Comparison with z = –1.96
Correct conclusion

34)

$H_0: \mu = 18.5$
$H_1: \mu < 18.5$

Test statistic $z = \frac{18.1-18.5}{(1.1/\sqrt{20})}$
$= -1.626$

CV $z = \pm 1.96$

Not enough evidence to support the claim that fingers are smaller.

35)

(i) $H_0: p = 0.36$
$H_1: p > 0.36$

$P(7) = {}^8C_7\times(0.36)^7(0.64)^1 = 0.00401$
$P(8) = (0.36)^8 = 0.000282$
$\Sigma P = 0.00429 < 0.05$

Accept driving instructor's claim

(ii) Type I error
$P(6) = {}^8C_6\times(0.36)^6(0.64)^2 = 0.02496$
$P(5) = {}^8C_5\times(0.36)^5(0.64)^3 = 0.08876,$
> 0.05

P(Type I error) = 0.0292 or 0.0293

36)

(i) $H_0: \mu = 3$
$H_1: \mu > 3$

Test statistic $z = \frac{3.3-3}{2.8/\sqrt{179}}$

= 1.43

critical value z = 1.645

not enough evidence to support the claim

(ii) Say no extra weight loss when there is.

37)

(i) $X \sim B(22, 0.2)$
$P(0, 1) = 0.8^{22} + 0.2\times 0.8^{21}\ {}_{22}C_1$

= 0.0480 (4.8%)

(ii) P(Type I error) = 0.0480

(iii) P(Type II error) = 1 – P(0, 1)
$= 1-(0.91^{22} + 0.09\times 0.91^{21}\times {}_{22}C_1)$
= 0.601

38)

(i) H_0: $\mu = 21.2$

H_1: $\mu \neq 21.2$

Test statistic $z = \dfrac{19.4 - 21.2}{(7.3/\sqrt{90})}$

$= -2.34$

CV $z = \pm 1.96$

In CR, reject H_0. Sig evidence to say not the same author

or $\Phi(-2.339) = 1 - 0.9903$

$= 0.0097/0.0096$

Compare with 0.025

say sig evidence to say not the same sentence length or author

or $x = 21.2 \pm 1.96x(7.3/\sqrt{90})$

$= 19.7(22.7)$

Compare with 19.4 etc.

(ii) Say it is not the same sentence length or author when it is

P (Type I error) = 5%

39)

(i) H_0: $\mu = 15$ or $p = 0.25$

H_1: $\mu > 15$ or $p > 0.25$

(ii) Test statistic

$z = \pm \dfrac{21.5 - 15}{\sqrt{60 \times 0.25 \times 0.75}} = 1.938$

OR test statistic

$z = \pm \dfrac{22/60 - 0.5/60 - 15/60}{\sqrt{\dfrac{0.25 \times 0.75}{60}}} = 1.938$

CV $z = 1.645$

In CR Claim justified

40)

$H_0 : p = 0.6 \quad H_1 : p > 0.6$

$P(X \geq 10) = {}_{12}C_{10}0.6^{10}0.4^2 + {}_{12}C_{11}0.6^{11}0.4^1 + 0.6^{12}$

$= 0.0834$

Reject H_0, i.e. accept claim at 10% level

S.R. Use of Normal scores 4/5 max

$z = \dfrac{9.5 - 7.2}{\sqrt{2.88}}$

(or equiv. Using N(0.6, 0.24/12))

$= 1.3552$

Pr(>9.5) = 1 – 0.9123 = 0.0877

Reject H_0, i.e. accept claim at 10% level