Statistics -2

9709

Poisson Distribution

Exercise (march and June series 2019 – 2002)

With marking scheme

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Poisson distribution

1) S/2003/7/Q6

Computer breakdowns occur randomly on average once every 48 hours of use.

(i) Calculate the probability that there will be fewer than 4 breakdowns in 60 hours of use. [3]

(ii) Find the probability that the number of breakdowns in one year (8760 hours) of use is more than 200. [4]

(iii) Independently of the computer breaking down, the computer operator receives phone calls randomly on average twice in every 24-hour period. Find the probability that the total number of phone calls and computer breakdowns in a 60-hour period is exactly 4. [3]

2) S/2004/7/Q6

At a certain airfield planes land at random times at a constant average rate of one every 10 minutes.

(i) Find the probability that exactly 5 planes will land in a period of one hour. [2]

(ii) Find the probability that at least 2 planes will land in a period of 16 minutes. [3]

(iii) Given that 5 planes landed in an hour, calculate the conditional probability that 1 plane landed in the first half hour and 4 in the second half hour. [3]

3) S/2005/7/Q6

At a petrol station cars arrive independently and at random times at constant average rates of 8 cars per hour travelling east and 5 cars per hour travelling west.

(i) Find the probability that, in a quarter-hour period,

(a) one or more cars travelling east and one or more cars travelling west will arrive, [4]

(b) a total of 2 or more cars will arrive. [2]

(ii) Find the approximate probability that, in a 12-hour period, a total of more than 175 cars will arrive. [3]

4) S/2008/7/Q6

People arrive randomly and independently at the elevator in a block of flats at an average rate of 4 people every 5 minutes.

(i) Find the probability that exactly two people arrive in a 1-minute period. [2]

(ii) Find the probability that nobody arrives in a 15-second period. [2]

(iii) The probability that at least one person arrives in the next t minutes is 0.9. Find the value of t. [4]

5) S/2006/7/Q6

A dressmaker makes dresses for Easifit Fashions. Each dress requires 2.5 m^2 of material. Faults occur randomly in the material at an average rate of 4.8 per 20 m^2.

(i) Find the probability that a randomly chosen dress contains at least 2 faults. [3]

Each dress has a belt attached to it to make an outfit. Independently of faults in the material, the probability that a belt is faulty is 0.03. Find the probability that, in an outfit,

(ii) neither the dress nor its belt is faulty, [2]

(iii) the dress has at least one fault and its belt is faulty. [2]

The dressmaker attaches 300 randomly chosen belts to 300 randomly chosen dresses. An outfit in which the dress has at least one fault and its belt is faulty is rejected.

(iv) Use a suitable approximation to find the probability that fewer than 3 outfits are rejected. [3]

6) S/2007/7/Q5

It is proposed to model the number of people per hour calling a car breakdown service between the times 09 00 and 21 00 by a Poisson distribution.

(i) Explain why a Poisson distribution may be appropriate for this situation. [2]

People call the car breakdown service at an average rate of 20 per hour, and a Poisson distribution may be assumed to be a suitable model.

(ii) Find the probability that exactly 8 people call in any half hour. [2]

(iii) By using a suitable approximation, find the probability that exactly 250 people call in the 12 hours between 09 00 and 21 00. [4]

7) S/2009/7/Q3

Major avalanches can be regarded as randomly occurring events. They occur at a uniform average rate of 8 per year.

(i) Find the probability that more than 3 major avalanches occur in a 3-month period. [3]

(ii) Find the probability that any two separate 4-month periods have a total of 7 major avalanches. [3]

(iii) Find the probability that a total of fewer than 137 major avalanches occur in a 20-year period. [4]

8) S/2010/71/Q6

In restaurant A an average of 2.2% of tablecloths are stained and, independently, in restaurant B an average of 5.8% of tablecloths are stained.

(i) Random samples of 55 tablecloths are taken from each restaurant. Use a suitable Poisson approximation to find the probability that a total of more than 2 tablecloths are stained. [4]

(ii) Random samples of n tablecloths are taken from each restaurant. The probability that at least one tablecloth is stained is greater than 0.99. Find the least possible value of n. [4]

9) S/2010/73/Q7

A clinic deals only with flu vaccinations. The number of patients arriving every 15 minutes is modelled by the random variable X with distribution Po(4.2).

(i) State two assumptions required for the Poisson model to be valid. [2]

(ii) Find the probability that

(a) at least 1 patient will arrive in a 15-minute period, [2]

(b) fewer than 4 patients will arrive in a 10-minute period. [3]

(iii) The clinic is open for 20 hours each week. At the beginning of one week the clinic has enough vaccine for 370 patients. Use a suitable approximation to find the probability that this will not be enough vaccine for that week. [4]

10) S/2011/71/Q1

On average, 2 people in every 10 000 in the UK have a particular gene. A random sample of 6000 people in the UK is chosen. The random variable X denotes the number of people in the sample who have the gene. Use an approximating distribution to calculate the probability that there will be more than 2 people in the sample who have the gene. [4]

11) S/2011/72/Q3

The number of goals scored per match by Everly Rovers is represented by the random variable X which has mean 1.8.

(i) State two conditions for X to be modelled by a Poisson distribution. [2]

Assume now that $X \sim \text{Po}(1.8)$.

(ii) Find $P(2 < X < 6)$. [2]

(iii) The manager promises the team a bonus if they score at least 1 goal in each of the next 10 matches. Find the probability that they win the bonus. [3]

12) S/2011/73/Q4

On average, 1 in 2500 people have a particular gene.

(i) Use a suitable approximation to find the probability that, in a random sample of 10 000 people, more than 3 people have this gene. [4]

(ii) The probability that, in a random sample of n people, none of them has the gene is less than 0.01. Find the smallest possible value of n. [3]

13) S/2012/71/Q7

At work Jerry receives emails randomly at a constant average rate of 15 emails per hour.

(i) Find the probability that Jerry receives more than 2 emails during a 20-minute period at work. [3]

(ii) Jerry's working day is 8 hours long. Find the probability that Jerry receives fewer than 110 emails per day on each of 2 working days. [4]

(iii) At work Jerry also receives texts randomly and independently at a constant average rate of 1 text every 10 minutes. Find the probability that the total number of emails and texts that Jerry receives during a 5-minute period at work is more than 2 and less than 6. [4]

14) S/2012/72/Q4

Bacteria of a certain type are randomly distributed in the water in two ponds, A and B. The average numbers of bacteria per cm^3 in A and B are 0.32 and 0.45 respectively.

(i) Samples of $8\,\text{cm}^3$ of water from A and $12\,\text{cm}^3$ of water from B are taken at random. Find the probability that the total number of bacteria in these samples is at least 3. [3]

(ii) Find the probability that in a random sample of $155\,\text{cm}^3$ of water from A, the number of bacteria is less than 35. [5]

15) S/2012/73/Q4

The number of lions seen per day during a standard safari has the distribution Po(0.8). The number of lions seen per day during an off-road safari has the distribution Po(2.7). The two distributions are independent.

(i) Susan goes on a standard safari for one day. Find the probability that she sees at least 2 lions. [2]

(ii) Deena goes on a standard safari for 3 days and then on an off-road safari for 2 days. Find the probability that she sees a total of fewer than 5 lions. [3]

(iii) Khaled goes on a standard safari for n days, where n is an integer. He wants to ensure that his chance of not seeing any lions is less than 10%. Find the smallest possible value of n. [3]

16) S/2013/71/Q5

The probability that a new car of a certain type has faulty brakes is 0.008. A random sample of 520 new cars of this type is chosen, and the number, X, having faulty brakes is noted.

(i) Describe fully the distribution of X and describe also a suitable approximating distribution. Justify this approximating distribution. [4]

(ii) Use your approximating distribution to find

(a) $\text{P}(X > 3)$, [2]

(b) the smallest value of n such that $\text{P}(X = n) > \text{P}(X = n + 1)$. [3]

17) S/2013/72/Q1

It is known that 1.2% of rods made by a certain machine are bent. The random variable X denotes the number of bent rods in a random sample of 400 rods.

(i) State the distribution of X. [2]

(ii) State, with a reason, a suitable approximate distribution for X. [2]

(iii) Use your approximate distribution to find the probability that the sample will include more than 2 bent rods. [2]

18) S/2013/73/Q4

The independent random variables X and Y have the distributions Po(2) and Po(3) respectively.

(i) Given that $X + Y = 5$, find the probability that $X = 1$ and $Y = 4$. [4]

(ii) Given that $P(X = r) = \frac{2}{3}P(X = 0)$, show that $3 \times 2^{r-1} = r!$ and verify that $r = 4$ satisfies this equation. [2]

19) S/2014/71/Q4

The proportion of people who have a particular gene, on average, is 1 in 1000. A random sample of 3500 people in a certain country is chosen and the number of people, X, having the gene is found.

(i) State the distribution of X and state also an appropriate approximating distribution. Give the values of any parameters in each case. Justify your choice of the approximating distribution. [3]

(ii) Use the approximating distribution to find $P(X \leqslant 3)$. [2]

20) S/2014/72/Q4

(i) The random variable W has the distribution Po(1.5). Find the probability that the sum of 3 independent values of W is greater than 2. [3]

(ii) The random variable X has the distribution Po(λ). Given that $P(X = 0) = 0.523$, find the value of λ correct to 3 significant figures. [2]

(iii) The random variable Y has the distribution Po(μ), where $\mu \neq 0$. Given that

$$P(Y = 3) = 24 \times P(Y = 1),$$

find μ. [3]

21) S/2014/73/Q1

On average 1 in 25 000 people have a rare blood condition. Use a suitable approximating distribution to find the probability that fewer than 2 people in a random sample of 100 000 have the condition. [3]

22) S/2014/73/Q7

A Lost Property office is open 7 days a week. It may be assumed that items are handed in to the office randomly, singly and independently.

(i) State another condition for the number of items handed in to have a Poisson distribution. [1]

It is now given that the number of items handed in per week has the distribution Po(4.0).

(ii) Find the probability that exactly 2 items are handed in on a particular day. [2]

(iii) Find the probability that at least 4 items are handed in during a 10-day period. [3]

(iv) Find the probability that, during a certain week, 5 items are handed in altogether, but no items are handed in on the first day of the week. [3]

23) S/2015/71/Q6

A publishing firm has found that errors in the first draft of a new book occur at random and that, on average, there is 1 error in every 3 pages of a first draft. Find the probability that in a particular first draft there are

(i) exactly 2 errors in 10 pages, [2]

(ii) at least 3 errors in 6 pages, [3]

(iii) fewer than 50 errors in 200 pages. [4]

24) S/2015/72/Q7

In a certain lottery, 10 500 tickets have been sold altogether and each ticket has a probability of 0.0002 of winning a prize. The random variable X denotes the number of prize-winning tickets that have been sold.

(i) State, with a justification, an approximating distribution for X. [3]

(ii) Use your approximating distribution to find $P(X < 4)$. [3]

(iii) Use your approximating distribution to find the conditional probability that $X < 4$, given that $X \geqslant 1$. [4]

25) S/2015/73/Q7

People arrive at a checkout in a store at random, and at a constant mean rate of 0.7 per minute. Find the probability that

(i) exactly 3 people arrive at the checkout during a 5-minute period, [2]

(ii) at least 30 people arrive at the checkout during a 1-hour period. [4]

People arrive independently at another checkout in the store at random, and at a constant mean rate of 0.5 per minute.

(iii) Find the probability that a total of more than 3 people arrive at this pair of checkouts during a 2-minute period. [4]

26) S/2016/71/Q7

(a) A large number of spoons and forks made in a factory are inspected. It is found that 1% of the spoons and 1.5% of the forks are defective. A random sample of 140 items, consisting of 80 spoons and 60 forks, is chosen. Use the Poisson approximation to the binomial distribution to find the probability that the sample contains

(i) at least 1 defective spoon and at least 1 defective fork, [3]

(ii) fewer than 3 defective items. [3]

(b) The random variable X has the distribution $\text{Po}(\lambda)$. It is given that

$$P(X = 1) = p \quad \text{and} \quad P(X = 2) = 1.5p,$$

where p is a non-zero constant. Find the value of λ and hence find the value of p. [4]

27) S/2016/72/Q6

At a certain shop the demand for hair dryers has a Poisson distribution with mean 3.4 per week.

(i) Find the probability that, in a randomly chosen two-week period, the demand is for exactly 5 hair dryers. [3]

(ii) At the beginning of a week the shop has a certain number of hair dryers for sale. Find the probability that the shop has enough hair dryers to satisfy the demand for the week if

(a) they have 4 hair dryers in the shop, [2]

(b) they have 5 hair dryers in the shop. [2]

(iii) Find the smallest number of hair dryers that the shop needs to have at the beginning of a week so that the probability of being able to satisfy the demand that week is at least 0.9. [3]

28) S/2016/73/Q6

X and Y are independent random variables with distributions Po(1.6) and Po(2.3) respectively.

(i) Find $P(X + Y = 4)$. [3]

A random sample of 75 values of X is taken.

(ii) State the approximate distribution of the sample mean, $\overline{X}$, including the values of the parameters. [2]

(iii) Hence find the probability that the sample mean is more than 1.7. [3]

(iv) Explain whether the Central Limit theorem was needed to answer part **(ii)**. [1]

29) S/2017/71/Q1

On average, 1 clover plant in 10 000 has four leaves instead of three.

(i) Use an approximating distribution to calculate the probability that, in a random sample of 2000 clover plants, more than 2 will have four leaves. [3]

(ii) Justify your approximating distribution. [2]

30) S/2017/71/Q6

The number of sports injuries per month at a certain college has a Poisson distribution. In the past the mean has been 1.1 injuries per month. The principal recently introduced new safety guidelines and she decides to test, at the 2% significance level, whether the mean number of sports injuries has been reduced. She notes the number of sports injuries during a 6-month period.

(i) Find the critical region for the test and state the probability of a Type I error. [6]

(ii) State what is meant by a Type I error in this context. [1]

(iii) During the 6-month period there are a total of 2 sports injuries. Carry out the test. [3]

31) S/2017/72/Q2

Javier writes an article containing 52 460 words. He plans to upload the article to his website, but he knows that this process sometimes introduces errors. He assumes that for each word in the uploaded version of his article, the probability that it contains an error is 0.000 08. The number of words containing an error is denoted by X.

(i) Find $E(X)$ and $Var(X)$, giving your answers correct to three decimal places. [2]

Javier wants to use the Poisson distribution as an approximating distribution to calculate the probability that there will be fewer than 5 words containing an error in his uploaded article.

(ii) Explain how your answers to part **(i)** are consistent with the use of the Poisson distribution as an approximating distribution. [1]

(iii) Use the Poisson distribution to calculate $P(X < 5)$. [2]

32) S/2017/Q6

Old televisions arrive randomly and independently at a recycling centre at an average rate of 1.2 per day.

(i) Find the probability that exactly 2 televisions arrive in a 2-day period. [2]

(ii) Use an appropriate approximating distribution to find the probability that at least 55 televisions arrive in a 50-day period. [4]

Independently of televisions, old computers arrive randomly and independently at the same recycling centre at an average rate of 4 per 7-day week.

(iii) Find the probability that the total number of televisions and computers that arrive at the recycling centre in a 3-day period is less than 4. [3]

33) S/2017/73/Q5

(i) A random variable X has the distribution Po(42).

(a) Use an appropriate approximating distribution to find $P(X \geqslant 40)$. [4]

(b) Justify your use of the approximating distribution. [1]

(ii) A random variable Y has the distribution B(60, 0.02).

(a) Use an appropriate approximating distribution to find $P(Y > 2)$. [3]

(b) Justify your use of the approximating distribution. [1]

34) S/2018/71/Q7

The number of absences by girls from a certain class on any day is modelled by a random variable with distribution Po(0.2). The number of absences by boys from the same class on any day is modelled by an independent random variable with distribution Po(0.3).

(i) Find the probability that, during a randomly chosen 2-day period, the total number of absences is less than 3. [3]

(ii) Find the probability that, during a randomly chosen 5-day period, the number of absences by boys is more than 3. [2]

(iii) The teacher claims that, during the football season, there are more absences by boys than usual. In order to test this claim at the 5% significance level, he notes the number of absences by boys during a randomly chosen 5-day period during the football season.

(a) State what is meant by a Type I error in this context. [1]

(b) State appropriate null and alternative hypotheses and find the probability of a Type I error. [3]

(c) In fact there were 4 absences by boys during this period. Test the teacher's claim at the 5% significance level. [3]

35) S/2018/72/Q1

The numbers of alpha, beta and gamma particles emitted per minute by a certain piece of rock have independent distributions Po(0.2), Po(0.3) and Po(0.6) respectively. Find the probability that the total number of particles emitted during a 4-minute period is less than 4. [3]

36) S/2018/72/Q6

Accidents on a particular road occur at a constant average rate of 1 every 4.8 weeks.

(i) State, in context, one condition for the number of accidents in a given period to be modelled by a Poisson distribution. [1]

Assume now that a Poisson distribution is a suitable model.

(ii) Find the probability that exactly 4 accidents will occur during a randomly chosen 12-week period. [2]

(iii) Find the probability that more than 3 accidents will occur during a randomly chosen 10-week period. [3]

(iv) Use a suitable approximating distribution to find the probability that fewer than 30 accidents will occur during a randomly chosen 2-year period ($104\frac{2}{7}$ weeks). [4]

37) S/2018/73/Q1

A random variable X has the distribution B(75, 0.03).

(i) Use the Poisson approximation to the binomial distribution to calculate $P(X < 3)$. [3]

(ii) Justify the use of the Poisson approximation. [1]

38) S/2018/73/Q4

The numbers, M and F, of male and female students who leave a particular school each year to study engineering have means 3.1 and 0.8 respectively.

(i) State, in context, one condition required for M to have a Poisson distribution. [1]

Assume that M and F can be modelled by independent Poisson distributions.

(ii) Find the probability that the total number of students who leave to study engineering in a particular year is more than 3. [3]

(iii) Given that the total number of students who leave to study engineering in a particular year is more than 3, find the probability that no female students leave to study engineering in that year. [3]

39) S/2019/71/Q5

(a) The random variable X has the distribution $\text{Po}(2.3)$.

(i) Find $\text{P}(2 \leqslant X \leqslant 4)$. [2]

(ii) Find the probability that the sum of two independent values of X is greater than 2. [3]

(iii) The random variable S is the sum of 50 independent values of X. Use a suitable approximating distribution to find $\text{P}(S \leqslant 110)$. [4]

(b) The random variable Y has the distribution $\text{Po}(\lambda)$. Given that $\text{P}(Y = 3) = \text{P}(Y = 5)$, find λ. [3]

40) S/2019/72/Q1

The random variable X has the distribution $\text{Po}(5)$.

(i) Find $\text{P}(X = 2)$. [1]

It is given that $\text{P}(X = n) = \text{P}(X = n + 1)$.

(ii) Write down an equation in n. [1]

41) S/2019/72/Q7

All the seats on a certain daily flight are always sold. The number of passengers who have bought seats but fail to arrive for this flight on a particular day is modelled by the distribution $\text{B}(320, 0.005)$.

(i) Explain what the number 320 represents in this context. [1]

(ii) The total number of passengers who have bought seats but fail to arrive for this flight on 2 randomly chosen days is denoted by X. Use a suitable approximating distribution to find $\text{P}(2 < X < 6)$. [3]

(iii) Justify the use of your approximating distribution. [2]

After some changes, the airline wishes to test whether the mean number of passengers per day who fail to arrive for this flight has decreased.

(iv) During 5 randomly chosen days, a total of 2 passengers failed to arrive. Carry out the test at the 2.5% significance level. [5]

42) S/2019/73/Q7

Each day at a certain doctor's surgery there are 70 appointments available in the morning and 60 in the afternoon. All the appointments are filled every day. The probability that any patient misses a particular morning appointment is 0.04, and the probability that any patient misses a particular afternoon appointment is 0.05. All missed appointments are independent of each other.

Use suitable approximating distributions to answer the following.

(i) Find the probability that on a randomly chosen morning there are at least 3 missed appointments. [3]

(ii) Find the probability that on a randomly chosen day there are a total of exactly 6 missed appointments. [3]

(iii) Find the probability that in a randomly chosen 10-day period there are more than 50 missed appointments. [4]

43) M/2016/72/Q6

The battery in Sue's phone runs out at random moments. Over a long period, she has found that the battery runs out, on average, 3.3 times in a 30-day period.

(i) Find the probability that the battery runs out fewer than 3 times in a 25-day period. [3]

(ii) **(a)** Use an approximating distribution to find the probability that the battery runs out more than 50 times in a year (365 days). [4]

(b) Justify the approximating distribution used in part **(ii)(a)**. [1]

(iii) Independently of her phone battery, Sue's computer battery also runs out at random moments. On average, it runs out twice in a 15-day period. Find the probability that the total number of times that her phone battery and her computer battery run out in a 10-day period is at least 4. [3]

44) M/2017/72/Q7

The number of planes arriving at an airport every hour during daytime is modelled by the random variable X with distribution Po(5.2).

(i) State two assumptions required for the Poisson model to be valid in this context. [2]

(ii) **(a)** Find the probability that the number of planes arriving in a 15-minute period is greater than 1 and less than 4, [3]

(b) Find the probability that more than 3 planes will arrive in a 40-minute period. [2]

(iii) The airport has enough staff to deal with a maximum of 60 planes landing during a 10-hour day. Use a suitable approximation to find the probability that, on a randomly chosen 10-hour day, staff will be able to deal with all the planes that land. [4]

45) M/2018/72/Q4

A store sells two types of computer, laptops and tablets. The number of laptops sold per hour is modelled by a random variable with distribution Po(0.9). The number of tablets sold per hour is modelled by an independent random variable with distribution Po(1.5).

(i) Find the probability that, during a randomly chosen hour, the total number of laptops and tablets sold in the store is less than 4. [3]

(ii) The manager claims that on sunny Saturdays fewer laptops than usual are sold. In order to test this claim, an employee notes the number of laptops sold during a 4-hour period on a randomly chosen sunny Saturday. In fact only 1 laptop is sold during this period. Test the manager's claim at the 10% significance level. [5]

46) M/2019/72/Q5

The number of eagles seen per hour in a certain location has the distribution Po(1.8). The number of vultures seen per hour in the same location has the independent distribution Po(2.6).

(i) Find the probability that, in a randomly chosen hour, at least 2 eagles are seen. [2]

(ii) Find the probability that, in a randomly chosen half-hour period, the total number of eagles and vultures seen is less than 5. [3]

Alex wants to be at least 99% certain of seeing at least 1 eagle.

(iii) Find the minimum time for which she should watch for eagles. [3]

Answers

1)

(i) $\lambda = 1.25$

$P(X < 4) =$

$e^{-1.25}\left(1+1.25+\frac{1.25^2}{2}+\frac{1.25^3}{6}\right)$

$= 0.962$

(ii) $X \sim N(182.5, 182.5)$

$P(> 200 \text{ breakdowns}) =$

$1 - \Phi\left(\frac{200.5-182.5}{\sqrt{182.5}}\right)$

$= 1 - \Phi(1.332)$

$= 0.0915\ (0.0914)$

(iii) $\lambda = 5$ for phone calls

$\lambda = 6.25$ for total

$P(X = 4) = e^{-6.25}\left(\frac{6.25^4}{4!}\right)$

$= 0.123$

2)

(i) $P(5) = e^{-6} \times \frac{6^5}{5!} = 0.161$

(ii) $P(X \geq 2) = 1 - \{P(0) + P(1)\}$

$= 1 - e^{-1.6}(1+1.6)$

$= 0.475$

(iii)

$P(1 \text{ then } 4 \mid 5) = \frac{(e^{-3} \times 3) \times \left(e^{-3} \times \frac{3^4}{4!}\right)}{e^{-6} \times \frac{6^5}{5!}}$

$= 0.156$ or $5/32$

3)

(i) (a) East $P(\geq 1) = 1 - e^{-2} = 0.8647$

West $P(\geq 1) = 1 - e^{-1.25}$

$= 0.7135$

$P(\text{Both}) = 0.8647 \times 0.7135$

$= 0.617$

(b) $P(\text{total} \geq 2) = 1 - e^{-13/4}(1+13/4)$

$= 0.835$

or $P(\text{total} \geq 2) = P(2) + P(3) + \ldots P(13)$ etc.

(ii) $T \sim N(156, 156)$

$P(>175) = 1 - \Phi\{(175.5-156)/\sqrt{156}\}$

$= 1 - \Phi(1.5612)$

$= 1 - 0.9407$

$= 0.0593/0.0592$

4)

(i) $\lambda = 0.8$

$P(2) = e^{-0.8}\frac{0.8^2}{2} = 0.144$

(ii) $\lambda = 0.2$

$P(0) = e^{-0.2} = 0.819$

(iii) $1 - e^{-0.8t} = 0.9$

$\ln 0.1 = -0.8t$

$t = 2.88$

5)

(i) $1 - e^{-0.6}(1+0.6)$

$= 1 - 0.878$

$= 0.122$

(ii) $(e^{-0.6})(0.97)$

$= 0.532$

(iii) $P(F,F) = (1-e^{-0.6})(0.03)$

$= 0.0135\ (0.01354)$

(iv) X approx $P(300 \times 0.01354) \sim P(4.062)$

$P(X<3) = e^{-4.062}\left(1+4.062+\frac{4.062^2}{2}\right)$

$= 0.229$

6)

(i) people call randomly, independently, at an average uniform rate

(ii) $P(8) = e^{-10}\ 10^8 / 8!$

$= 0.113$

(iii) $X \sim N(240, 240)$

$P(X=250) = \Phi\left(\frac{250.5-240}{\sqrt{240}}\right) - \Phi\left(\frac{249.5-240}{\sqrt{240}}\right)$

$= \Phi(0.678) - \Phi(0.613)$

$= 0.7512 - 0.7301$

$= 0.0211$ (accept 3sf or more rounding to 0.021)

7)

(i) $\lambda = 2$

$P(X > 3) = 1 - P(0, 1, 2, 3)$

$= 1 - e^{-2}\left(1+2+\frac{2^2}{2}+\frac{2^3}{3!}\right)$

$= 1 - 0.857 \quad = 0.143$

(ii) $\lambda = 16/3$

$P(7) = e^{-16/3}\left(\frac{(16/3)^7}{7!}\right)$

$= 0.118$

(iii) $X \sim N(160, 160)$

$P(X < 137) = P\left(z < \frac{136.5-160}{\sqrt{160}}\right)$

$= P(z < -1.858)$

$= 1 - 0.9684 = 0.0316$

8)

(i) $\lambda_A = np = 0.022 \times 55 = 1.21$

$\lambda_B = 0.058 \times 55 = 3.19$

total $\lambda = 4.4$

$P(\text{more than } 2) = 1 - P(0, 1, 2)$

$= 1 - e^{-4.4}\left(1+4.4+\frac{4.4^2}{2!}\right)$

$= 1 - 0.185$

$= 0.815$

(ii) $\lambda = 0.08n$

P(at least 1 stained tablecloth) = $1 - P(0)$

$1 - e^{-0.08n} > 0.99$

$0.01 > e^{-0.08n}$

$n > 57.6$

least value of $n = 58$

9)

(i) Patients arrive at constant mean rate

Patients arrive at random

Patients arrive independently

Patients arrive singly

(ii) (a) $1 - e^{-4.2}$

$= 0.985$

(b) $4.2 \times {}^{10}/_{15}$ oe

$e^{-2.8} \times (1 + 2.8 + \frac{2.8^2}{2!} + \frac{2.8^3}{3!})$

$= 0.692$

(iii) $N(336, 336)$ stated or implied

$\frac{370.5-336}{\sqrt{336}}$ $(= 1.882)$

$1 - \Phi(\text{"}1.882\text{"})$

$= 0.0300/0.0299$

10)

Poisson

$\lambda = 1.2$

$1 - e^{-1.2}(1 + 1.2 + \frac{1.2^2}{2})$

$= 0.121$

11)

(i) Constant average rate of goals scored
Goals random
Goals indep

(ii) $e^{-1.8}(\frac{1.8^3}{3!}+\frac{1.8^4}{4!}+\frac{1.8^5}{5!})$
$= 0.259$

(iii) $1 - e^{-1.8}$
$(1-e^{-1.8})^{10}$
$= 0.164$

12)

(i) Po(4)

$1 - e^{-4}(1 + 4 + \frac{4^2}{2!} + \frac{4^3}{3!})$
$= 1 - 0.43347..$
$= 0.567$ or 0.566

(ii) $\lambda = {}^{n}/_{2500}$

$e^{-\frac{n}{2500}} < 0.01$
$-\frac{n}{2500} < \ln 0.01$

$n > 11512.9\ldots$
Smallest $n = 11513$

$(\frac{2499}{2500})^n$
$(\frac{2499}{2500})^n < 0.01$
$n \times \ln(\frac{2499}{2500}) < \ln 0.01$

$n > 11510.6\ldots$
Smallest $n = 11511$

13)

(i) $\lambda = 5$
$1 - e^{-5}\left(1 + 5 + \frac{5^2}{2!}\right)$
$= 0.875$

(ii) $X \sim N(120, 120)$
$\frac{109.5-120}{\sqrt{120}}$ $(= -0.9585)$
$1 - \Phi(\text{“}0.9585\text{”})$
$(= 1 - 0.8312)$
$\text{“}0.1688\text{”}^2$
$= 0.0285$ to 0.0286

(iii) $\lambda = 15 \times \frac{5}{60} + 0.5$
$= 1.75$
$e^{-1.75}\left(\frac{1.75^3}{3!} + \frac{1.75^4}{4!} + \frac{1.75^5}{5!}\right)$
$= 0.247$ (3 sfs)

14)

(i) $\lambda = 8 \times 0.32 + 12 \times 0.45$ $(= 7.96)$
$1 - e^{-7.96}(1 + 7.96 + \frac{7.96^2}{2})$
$= 0.986$ (3 sfs)

(ii) $\lambda = 155 \times 0.32 = 49.6$
N(‘49.6’, ‘49.6’)
$\frac{34.5 - \text{‘}49.6\text{’}}{\sqrt{\text{‘}49.6\text{’}}}$ $(= -2.144)$
$\Phi(\text{‘}-2.144\text{’}) = 1 - \Phi(\text{‘}2.144\text{’})$
$= 0.016(0)$

15)

(i) $1 - e^{-0.8}(1 + 0.8)$
$= 0.191$ (3 sfs)

(ii) $\lambda = 3 \times 0.8 + 2 \times 2.7$ $(= 7.8)$
$e^{-7.8}(1 + 7.8 + \frac{7.8^2}{2} + \frac{7.8^3}{3!} + \frac{7.8^4}{4!})$
$= 0.112$ (3 sfs)

(iii) $e^{-0.8n} < 0.1$ Allow ‘=’
$-0.8n < \ln 0.1$ Allow ‘=’

min $n = 3$

16)

(i) B(520, 0.008)
Po(4.16)
$n = 500$ which is large,
$np = 4.16$ which is < 5 or p small <0.1

(ii) (a) $1 - e^{-4.16}(1 + 4.16 + \frac{4.16^2}{2} + \frac{4.16^3}{3!})$
$= 0.597$ (3 sf)

(b) $e^{-4.16} \times \frac{4.16^n}{n!} > e^{-4.16} \times \frac{4.16^{n+1}}{(n+1)!}$
$1 > \frac{4.16}{n+1}$
$n > 3.16$
Smallest n is 4

17)

(i) Binomial
$n = 400$, $p = 0.012$

(ii) Poisson
n large and mean $= 4.8$, which is < 5

(iii) $1 - e^{-4.8}(1 + 4.8 + \frac{4.8^2}{2})$
$= 0.857/0.858$

18)

(i) $e^{-2} \times 2$ $(\times)$ $e^{-3} \times \frac{3^4}{4!}$
$e^{-5} \times \frac{5^4}{5!}$
$\div$
$\frac{162}{625}$ or 0.259 (3 sf)

(ii) $(e^{-2} \times \frac{2^r}{r!} = \frac{2}{3}e^{-2} \Rightarrow)$
$3 \times 2^r = 2 \times r!$ OR $2^{r-1} = \frac{1}{3} \times r!$
$(\Rightarrow 3 \times 2^{r-1} = r!)$
$3 \times 2^3 = 24$ OR $3! = 24$ seen

19)

(i) B(3500, 0.001)
Poisson with mean = 3.5
$n > 50$ and $np < 5$

(ii) $e^{-3.5}(1 + 3.5 + \frac{3.5^2}{2} + \frac{3.5^3}{3!})$
$= 0.537$ (3 dp)

20)

(i) $\lambda = 4.5$

$1 - e^{-4.5}\left(1 + 4.5 + \frac{4.5^2}{2}\right)$

$= 0.826$ (3 s.f.)

(ii) $e^{-\lambda} = 0.523$

$(-\lambda = \ln 0.523)$

$\lambda = 0.648$ (3 s.f.)

(iii) $e^{-\mu} \times \frac{\mu^3}{3!} = 24 \times e^{-\mu} \times \mu$

$\frac{\mu^2}{6} = 24$

$\mu = 12$

21)

$e^{-4}(1 + 4)$

$= 0.0916$ (3 s.f.)

22)

(i) Constant mean (or average) rate

(ii) $e^{-\frac{4}{7}} \times \frac{\left(\frac{4}{7}\right)^2}{2!}$ or $e^{-0.571} \times \frac{0.571^2}{2!}$

$= 0.0922$ or 0.0921 (3 s.f.)

(iii) $\lambda = \frac{40}{7}$ or $5.71\ldots$

$1 - e^{-\frac{40}{7}}\left(1 + \frac{40}{7} + \frac{\left(\frac{40}{7}\right)^2}{2!} + \frac{\left(\frac{40}{7}\right)^3}{3!}\right)$

$= 0.821$ (3 s.f.)

(iv) $\frac{24}{7}$ o.e. 3 s.f. or better seen

$e^{-\frac{4}{7}} \times e^{-\frac{24}{7}} \times \frac{\left(\frac{24}{7}\right)^5}{5!}$

$= 0.0723$ (3 s.f.)

23)

(i) $e^{-\frac{10}{3}} \times \frac{\left(\frac{10}{3}\right)^2}{2}$

$= 0.198$ (3 sf)

(ii) $1 - e^{-2}\left(1 + 2 + \frac{2^2}{2}\right)$

$= 0.323$ (3 sf)

(iii) $N\left(\frac{200}{3}, \frac{200}{3}\right)$

$\frac{49.5 - \frac{200}{3}}{\sqrt{\frac{200}{3}}}$ $(= -2.102)$

$\Phi(\text{'}-2.102\text{'}) = 1 - \Phi(\text{'}2.102\text{'})$
$= 0.0178$ (3 sf)

24)

(i) Poisson

(Actually binomial with)
$n > 50$
and np (or λ) $(= 2.1)$ which is < 5

(ii) $\lambda = 2.1$
$e^{-2.1}\left(1 + 2.1 + \frac{2.1^2}{2} + \frac{2.1^3}{3!}\right)$

$= 0.839$ (3 sf)

(iii) $P(X \geqslant 1) = 1 - e^{-2.1}$ $(= 0.87754)$
$P(X = 1,2,3) = e^{-2.1}\left(2.1 + \frac{2.1^2}{2} + \frac{2.1^3}{3!}\right)$ $(= 0.71619)$

$\frac{P(X = 1,2,3)}{P(X \geqslant 1)}$
$\left(= \frac{0.71619}{0.87754}\right)$
$= 0.816$ (3 sf)

25)

(i) $e^{-3.5} \times \frac{3.5^3}{3!}$

$= 0.216$ (3 sf)

(ii) N(42, 42) stated or implied

$\frac{29.5 - 42}{\sqrt{42}}$ $(= -1.929)$

$P(z > \text{'}-1.929\text{'}) = \Phi(\text{'}1.929\text{'})$
$= 0.973$ (3 sf)

(iii) $(\lambda) = 2.4$

$1 - e^{-2.4}\left(1 + 2.4 + \frac{2.4^2}{2} + \frac{2.4^3}{3!}\right)$

$= 0.221$ (3 sf)

26)

(a) (i) 0.01×80 and 0.015×60
$(1 - e^{-0.8}) \times (1 - e^{-0.9})$
$= 0.327$ (3 sf)

(ii) $\lambda = 0.02 \times 40 + 0.015 \times 60$

$e^{-1.7} \times (1 + 1.7 + \frac{1.7^2}{2})$
$= 0.757$ (3 sf)

(b) $e^{-\lambda} \times \lambda = p$ and $e^{-\lambda} \times \frac{\lambda^2}{2} = 1.5p$

$\lambda = 3$

$p = e^{-3} \times 3$
$= 0.149$ (3 sf)

27)

(i) $\lambda = 6.8$
$e^{-6.8} \times \frac{6.8^5}{5!}$
$= 0.135$ (3 sf)

(ii) (a) $e^{-3.4}(1 + 3.4 + \frac{3.4^2}{2} + \frac{3.4^3}{3!} + \frac{3.4^4}{4!})$

$= 0.744$ (3 sf)

(b) '0.744' $+ e^{-3.4} \times \frac{3.4^5}{5!}$
$= 0.87(0)$ (3 sf) or 0.871

(iii) $P(X \leqslant 6) =$ '0.870' $+ e^{-3.4} \times \frac{3.4^6}{6!}$
$= 0.94$

Need 6 hair driers

28)

(i) $\lambda = 3.9$
$e^{-3.9} \times \frac{3.9^4}{4!}$

$= 0.195$

(ii) $\bar{X} \sim N(1.6, \frac{1.6}{75})$

(iii) $\frac{1.7 - 1.6}{\sqrt{\frac{1.6}{75}}}$ $(= 0.685)$
$1 - \Phi(\text{"}0.685\text{"})$
$= 0.247$ (3 sf)

(iv) X not normally distr. So CLT needed

29)

(i)	Poisson with $\lambda = 0.2$
	$1 - e^{-0.2}(1 + 0.2 + \frac{0.2^2}{2})$
	$= 0.00115$ (3 sf)
(ii)	n large $(n > 50)$
	$np = 0.2 < 5$ or p small

30)

(i) mean = 6.6

$P(X \leqslant 1) = e^{-6.6}(1 + 6.6) = 0.0103$

$P(X \leqslant 2) = e^{-6.6}(1 + 6.6 + \frac{6.6^2}{2}) = 0.0400$

CR is $X \leqslant 1$

P(Type I error) = $P(X \leqslant 1) = 0.0103$

(ii) Wrongly concluding that (mean) no of (sports) injuries has decreased

(iii) H_0: $\lambda = 6.6$ H_1: $\lambda < 6.6$

2 not in CR

No evidence mean no. of injuries has decreased

(iv) N(39.6, 39.6)

$\frac{29.5 - 39.6}{\sqrt{39.6}}$ (= −1.605)

$\Phi(\text{“}-1.605\text{”}) = 1 - \Phi(\text{“}1.605\text{”})$

= 0.0543 (3 sfs)

31)

(i) E(X) = 4.197

Var (X) = 4.196

(ii) $E(X) \approx Var(X)$

(iii) $e^{-4.1968}\left(1 + 4.1968 + \frac{4.1968^2}{2} + \frac{4.1968^3}{3!} + \frac{4.1968^4}{4!}\right)$

= 0.59(0) (3 sfs)

32)

(i) $e^{-2.4} \times \frac{2.4^2}{2!}$

= 0.261 (3 sfs)

(ii) N(60, 60)

$\frac{54.5 - 60}{\sqrt{60}}$ (= −0.710)

$1 - \phi(\text{"}-0.710\text{"}) = \phi(\text{"}0.710\text{"})$

= 0.761 (3 sf)

(iii) $\lambda = 3.6 + 12 \div 7$ (= 186/35) (= 5.314)

$e^{-5.314}\left(1 + 5.314 + \frac{5.314^2}{2} + \frac{5.314^3}{3!}\right)$

= 0.224 (3 sfs)

33)

(i)(a) $X \sim N(42, 42)$

$\frac{39.5 - \text{"}42\text{"}}{\sqrt{\text{"}42\text{"}}}$ (= −0.386)

$1 - \phi(\text{"}-0.386\text{"}) = \phi(\text{"}0.386\text{"})$

= 0.65(0) (3 sf)

(i)(b) 42 > (e.g. 15) or mean is large

(ii)(a) $Y \sim Po(1.2)$

$1 - e^{-1.2}(1 + 1.2 + \frac{1.2^2}{2})$

= 0.121 (3 sf)

(ii)(b) $60 \times 0.02 = 1.2 < 5$ or mean is small

34)

(i) Po(1.0)

$e^{-1}(1 + 1 + \frac{1^2}{2})$

= 0.920 (3 sfs)

(ii) $P(X > 3) = 1 - e^{-1.5}(1 + 1.5 + \frac{1.5^2}{2} + \frac{1.5^3}{3!}) = 0.0656$

(iii)(a) Incorrectly concluding that more absences than usual when there are not oe

(iii)(b) H_0: $\lambda = 1.5$ (or 0.3)
H_1: $\lambda > 1.5$ (or 0.3)

$P(X > 4) = \text{“}0.0656\text{”} - e^{-1.5} \times \frac{1.5^4}{4!}$
= 0.0186 (3 sf)

P(Type I) = 0.0186 or 0.0185

(iii)(c) $P(X > 3) = \text{"}0.0656\text{"}$

0.0656 > 0.05

No evidence of more than usual male absences

35)

$\lambda = 4.4$

$P(X<4) = e^{-4.4}(1 + 4.4 + \frac{4.4^2}{2} + \frac{4.4^3}{3!})$

= 0.359

36)

(i) Accidents occur independently or randomly

(ii) $e^{-2.5} \times \frac{2.5^4}{4!}$

$= 0.134$ (3 sfs)

(iii) $\lambda = \frac{25}{12}$ or 2.08(333)

$1 - e^{-\frac{25}{12}}(1 + \frac{25}{12} + \frac{\frac{25}{12}^2}{2!} + \frac{\frac{25}{12}^3}{3!})$

$= 0.158$ (3 sfs)

(iv) $N(\frac{1825}{84}, \frac{1825}{84})$ or N(21.7(26), 21.7(26))

$\frac{29.5 - \frac{1825}{84}}{\sqrt{\frac{1825}{84}}}$

Φ("1.668")

$= 0.952$ (3 sfs)

37)

(i) Po(2.25)

$e^{-2.25}(1 + 2.25 + \frac{2.25^2}{2})$

$= 0.609$ (3 sf)

(ii) $\mu = 2.25$, which is less than 5; n large

38)

(i) No of males leaving (to do eng) each yr has const mean or

Males leave (to do eng) indep of other males leaving (to do eng) or

Males leave (to do eng) at random

(ii) $\lambda = 3.9$

$1 - e^{-3.9}(1 + 3.9 + \frac{3.9^2}{2!} + \frac{3.9^3}{3!})$

0.546753 or 0.547 (3 sf)

(iii) $P(F = 0 \text{ and } M > 3) =$

$e^{-0.8} \times \left[1 - e^{-3.1}\left(1 + 3.1 + \frac{3.1^2}{2!} + \frac{3.1^3}{3!}\right)\right]$

(= 0.16857)

$\frac{P(F=0 \text{ and } M>3)}{P(M+F>3)}$

$\frac{"0.16857"}{"0.54675"}$

$= 0.308$ (3 sf)

39)

(a)(i) $e^{-2.3}(\frac{2.3^2}{2} + \frac{2.3^3}{3!} + \frac{2.3^4}{4!})$

$= 0.585$

(a)(ii) $(\lambda) = 4.6$

$1 - e^{-4.6}(1 + 4.6 + \frac{4.6^2}{2})$

$= 0.837$ (3 sf)

(a)(iii) $S \sim N(115, 115)$

$\frac{110.5 - 115}{\sqrt{115}}$

$1 - \Phi$('0.420')

$= 0.337$

(b) $e^{-\lambda} \times \frac{\lambda^3}{3!} = e^{-\lambda} \times \frac{\lambda^5}{5!}$

$\lambda^3 = \frac{\lambda^5}{4 \times 5}$ or $\lambda^2 = 20$ oe

$\lambda = \sqrt{20}$ or $2\sqrt{5}$ or 4.47 (3 sf)

40)

(i) 0.0842 (3 sf)

(ii) $e^{-5} \text{ x } \frac{5^n}{n!} = e^{-5} \text{ x } \frac{5^{n+1}}{(n+1)!}$

(iii) $1 = \frac{5}{n+1}$

$n = 4$

41)

(i) Max no. of passengers plane can take c

(ii) $\lambda = 3.2$

$e^{-3.2}(\frac{3.2^3}{3!} + \frac{3.2^4}{4!} + \frac{3.2^5}{5!})$

$= 0.5146 = 0.515$ (3 sfs)

(iii) $n > 50$

$np = 1.6$, which is < 5 or p=0.005 which is <0.1

(iv) H_0: Pop mean (for 5 days) = 8
H_1: Pop mean (for 5 days) < 8

$e^{-8}(1 + 8 + \frac{8^2}{2!})$

$= 0.0138$

Comp 0.025

Evidence that mean no. failing to arrive has decreased

42)

(i) Use of Po(2.8)

$1 - e^{-2.8}(1 + 2.8 + \frac{2.8^2}{2}))$

$= 0.531$ or $0.53(0)$ (3 sf)

(ii) Use of Po(5.8)

$e^{-5.8} \times \frac{5.8^6}{6!}$

$= 0.16(0)$ (3 sf)

(iii) Use of N(58, 58)

$\frac{50.5 - '58'}{\sqrt{'58'}}$ $(= -0.985)$

$\Phi('0.985')$ $= 0.838$ (3 sf)

43)

(i) $\lambda = 3.3 \times \frac{25}{30} = 2.75$

$e^{-2.75}(1 + 2.75 + \frac{2.75^2}{2})$

$= 0.481$ (3 sf)

(ii) (a) $\lambda (= 3.3 \times \frac{365}{30}) = 40.15$

$(X \sim Po(40.15) \Rightarrow X \sim N(40.15, 40.15))$

$\frac{50.5 - "40.15"}{\sqrt{"40.15"}}$ $(= 1.633)$

$1 - \Phi("1.633")$

$= 0.0513$ (3 sf)

(b) $\lambda > 15$

(iii) $\lambda = \frac{73}{30}$ oe or $1.1+1.33 = 2.43$ (3 sf)

$1 - e^{-2.43}(1 + 2.43 + \frac{2.43^2}{2} + \frac{2.43^3}{3!})$

$= 0.228$ (3 sf)

44)

(i) Planes arrive at constant mean rate

Planes arrive at random

(ii)(a) $(\lambda =)\ 5.2 \div 4$

$e^{-1.3}(\frac{1.3^2}{2} + \frac{1.3^3}{3!})$

$= 0.330$ (3 sfs)

(ii)(b) $1 - e^{-3.467} \times (1 + 3.467 + \frac{3.467^2}{2!} + \frac{3.467^3}{3!})$

$= 0.456$ (3 sfs)

(iii) N(52, 52) stated or implied

$\frac{60.5 - 52}{\sqrt{52}}$ $(= 1.179)$

$\Phi("1.179")$

$= 0.881$ (3 sf)

45)

(i) (Po)(2.4)

$e^{-2.4}(1 + 2.4 + \frac{2.4^2}{2} + \frac{2.4^3}{3!})$

$= 0.779$ (3 sfs)

(ii) H_0: λ (or mean) $= 3.6$ (or 0.9)
H_1: λ (or mean) < 3.6 (or 0.9)

$e^{-3.6}(1 + 3.6)$

$= 0.126$

$0.126 > 0.1$

No evidence that fewer than usual sold

46)

(i) $1 - e^{-1.8}(1 + 1.8)$

$= 0.537$ (3 sf)

(ii) $\lambda = 2.2$

$e^{-2.2}(1 + 2.2 + \frac{2.2^2}{2!} + \frac{2.2^3}{3!} + \frac{2.2^4}{4!})$

$= 0.928$ (3 sf) or 0.927

(iii) $1 - e^{-1.8t} \geqslant 0.99$ or $1 - e^{-\lambda} \geqslant 0.99$

$e^{-1.8t} \leqslant 0.01$ or $e^{-\lambda} \leqslant 0.01$
$-1.8t \leqslant \ln 0.01$

$t \geqslant 2.56$
She must watch for at least 2.56 (hours)