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P_1

Pure Maths_1

Trigonometry
Notes_1

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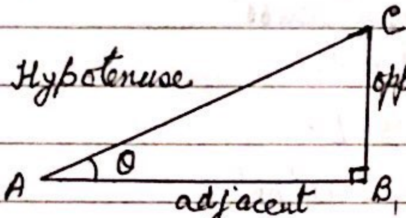
P.1

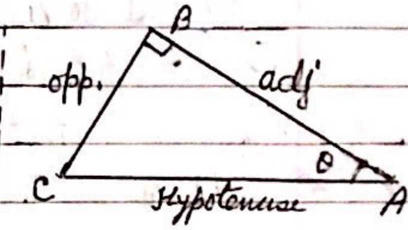
Trigonometry.

§ Trigonometry is the study of the relations between the sides and angles of a triangle.

§ **Trigonometric Ratios:** $0 \leq \theta \leq 90^\circ$

In a right triangle ABC, right angled at B, and let angle $BAC = \theta^\circ$



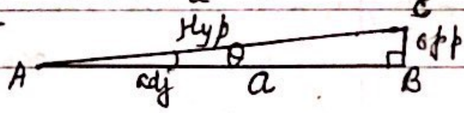


(i) $\sin\theta = \frac{opp}{Hyp} = \frac{BC}{AC}$

(ii) $\cos\theta = \frac{adj}{Hyp} = \frac{AB}{AC}$

(iii) $\tan\theta = \frac{opp}{adj} = \frac{BC}{AB}$ // $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\left\{ \begin{array}{l} \sin 0^\circ = \frac{0}{a} = 0 \\ \cos 0^\circ = \frac{a}{a} = 1 \\ \tan 0 = \frac{0}{a} = 0 \end{array} \right.$ $\because \left\{ \begin{array}{l} \theta \to 0^\circ \\ BC \to 0 \\ AC \to a \end{array} \right.$

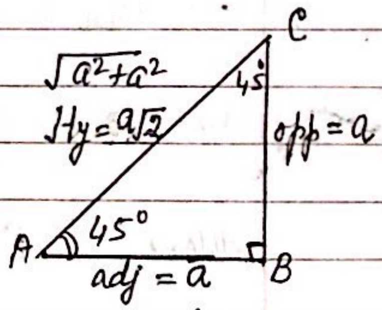


§ **Values of trig. ratios of some particular angles:**

π rad $= 180^\circ$ $1^\circ = \frac{\pi}{180}$ rad.

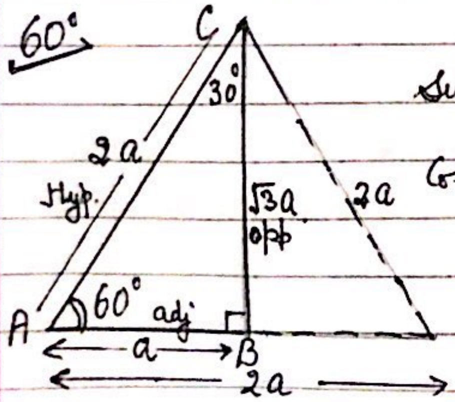
θ	0	$30^\circ; \frac{\pi}{6}$	$45^\circ; \frac{\pi}{4}$	$60^\circ; \frac{\pi}{3}$	$90^\circ; \frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞ not def.

45°



$\left. \begin{array}{l} \sin 45^\circ = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}} \\ \cos 45^\circ = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}} \\ \tan 45^\circ = \frac{a}{a} = 1 \end{array} \right\}$

60°



$\sin 60^\circ = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$

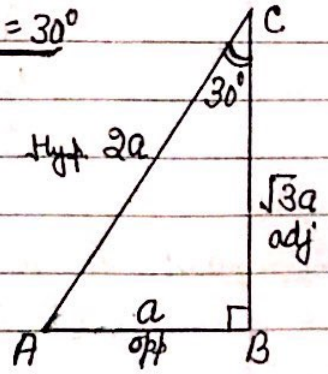
$\cos 60 = \frac{a}{2a} = \frac{1}{2}$

$\tan 60^\circ = \frac{\sqrt{3}a}{a} = \sqrt{3}$

In rt Δ ABC, $BC = \sqrt{AC^2 - AB^2}$

$= \sqrt{(2a)^2 - a^2} = \sqrt{3}a$

for angle $C = 30^\circ$



$\left. \begin{array}{l} \sin 30^\circ = \frac{a}{2a} = \frac{1}{2} \\ \cos 30^\circ = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2} \end{array} \right\}$ $\tan 30^\circ = \frac{a}{\sqrt{3}a}$

$= \frac{1}{\sqrt{3}}$ ✓

Example 1: Find the exact value of:

(i) $2\sin 30° \cdot \cos 30°$

$2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$ ✓

(ii) $\sin 30° \cos 60° + \cos 30° \sin 60°$

$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{4} + \frac{3}{4} = 1$ ✓

(iii) $\tan^2 30° + \tan^2 45° + \tan^2 60°$

$= \left(\frac{1}{\sqrt{3}}\right)^2 + 1^2 + (\sqrt{3})^2$

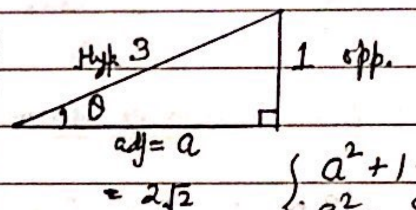
$= \frac{1}{3} + 1 + 3 = 4\frac{1}{3}$ or $\left(\frac{13}{3}\right)$

Example 2: Given $\sin\theta = \frac{1}{3}$ $\quad 0 \le \theta \le 90°$

Find the exact value of:

(i) $\cos\theta$

$\cos\theta = \frac{adj}{Hy} = \frac{2\sqrt{2}}{3}$ ✓



$$\begin{cases} a^2 + 1 = 3^2 \\ a^2 = 9 - 1 = 8 \\ a = \sqrt{8} = 2\sqrt{2} \end{cases}$$

(ii) $\tan\theta = \frac{opp}{adj} = \frac{1}{2\sqrt{2}}$ ✓

or $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}} = \frac{1}{3} \times \frac{3}{2\sqrt{2}} = \frac{1}{2\sqrt{2}}$ ✓

(iii) $2\sin\theta - \cos^2\theta + 3\tan^2\theta$

$= 2 \times \frac{1}{3} - \left(\frac{2\sqrt{2}}{3}\right)^2 + 3\left(\frac{1}{2\sqrt{2}}\right)^2$

$= \frac{2}{3} - \frac{8}{9} + 3 \times \frac{1}{8}$

$= \frac{11}{72}$ ✓

Example 3: Find the exact value of

$\sin^2\frac{\pi}{6} + \cos\frac{\pi}{3} + \tan^2\frac{\pi}{3}$

$= \left(\frac{1}{2}\right)^2 + \frac{1}{2} + (\sqrt{3})^2$

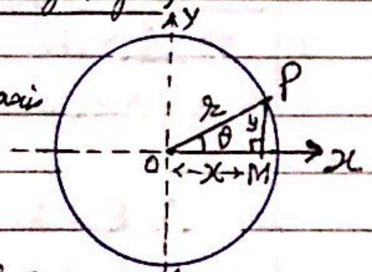
$= \frac{1}{4} + \frac{1}{2} + 3 = \frac{15}{4}$ (or $3\frac{3}{4}$)

§ Circular Functions (Trigonometric function of any angles):

Draw a circle with centre at O and radius r

Let P is any point on the circle, Draw $PM \perp$ X-axis

$\angle POM = \theta$, $PM = y$, $OM = x$, $OP = r$

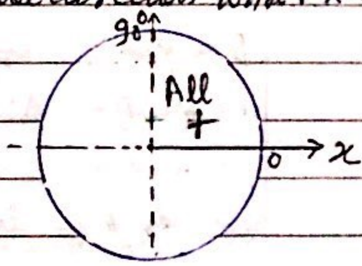


$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$, $x \neq 0$

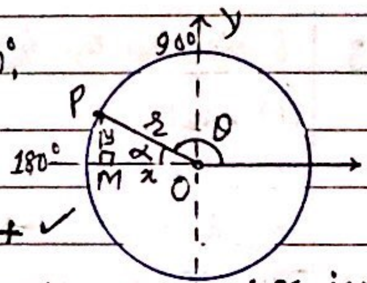
Note: { Angle θ is + if measured in anticlockwise direction with + X-axis.



Case I: If θ lies in the first quadrant $0° < \theta < 90°$

$\sin\theta$, $\cos\theta$ and $\tan\theta$

All has + values. ✓



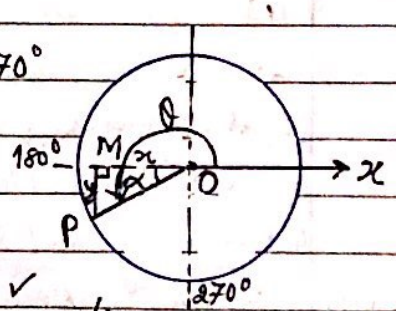
Case II θ lies in the second quadrant. $90° < \theta < 180°$.

then angle 'α' which OP makes with X-axis is defined as basic angle. $(\theta = 180 - \alpha)$

$\sin\theta = \sin(180-\alpha) = \sin\alpha = \frac{PM}{r} = +$ ✓

$\cos\theta = \cos(180-\alpha) = -\cos\alpha = -\frac{OM}{r} = -ve$ (x is -ve)

$\tan\theta = \tan(180-\alpha) = -\tan\alpha = -\frac{PM}{OM} = -ve$



Case III θ lies in the third quadrant. $180° < \theta < 270°$

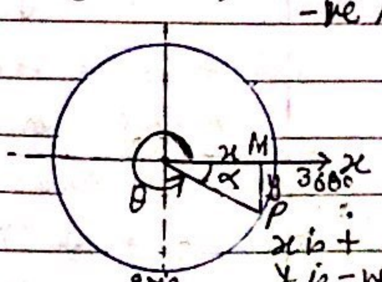
Basic angle $POM = \alpha$; $\theta = 180 + \alpha$

$\sin\theta = \sin(180+\alpha) = -\sin\alpha = -\frac{PM}{r} = -ve$

$\cos\theta = \cos(180+\alpha) = -\cos\alpha = -\frac{OM}{r} = -ve$

$\tan\theta = \tan(180+\alpha) = \frac{-\sin\alpha}{-\cos\alpha} = +\tan\alpha = \frac{OM}{PM} = +$ ✓

(here x & y both are -ve)



Case IV θ lies in the fourth quadrant: $270° < \theta < 360°$

basic angle $\alpha = 360 - \theta$

$\sin\theta = \sin(360-\alpha) = -\sin\alpha = -\frac{PM}{r} = -ve$

$\cos\theta = \cos(360-\alpha) = +\cos\alpha = +OM/r = +$ ✓

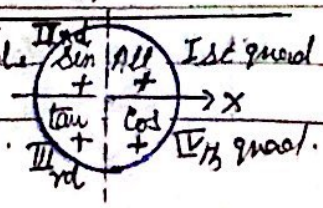
$\tan\theta = \tan(360-\alpha) = -\tan\alpha = -\frac{y}{x} = -ve$

x is +
y is -ve

§ Note: Signs of t-ratios; (i) $\sin\theta$ is + in Ist and IInd quads.

(ii) $\cos\theta$ is +ve in Ist & IVth quads.

(iii) $\tan\theta$ is +ve in Ist & IIIrd quads



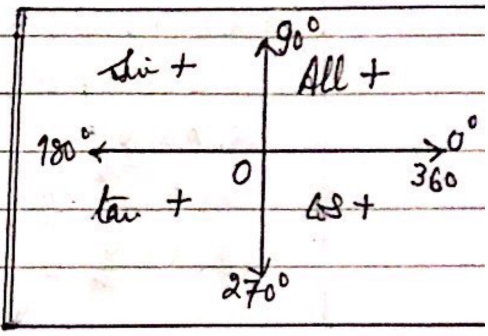
§ Formulae to find the values of t-ratios, $0 \leq \theta < 360°$ [or $\theta = (360 \cdot n + \alpha)$] $n \in I$

1. $90° < \theta < 180°$ (in second quad), the Basic angle $\alpha = 180 - \theta$

(i) $\sin\theta = \sin(180-\alpha) = \sin\alpha$

(ii) $\cos\theta = \cos(180-\alpha) = -\cos\alpha$

(iii) $\tan\theta = \tan(180-\alpha) = -\tan\alpha$



2. In 3rd quad. $180° < \theta < 270$;

$\theta = 180 + \alpha \Rightarrow$ Basic angle $\alpha = (\theta - 180°)$

(i) $\sin\theta = \sin(180+\alpha) = -\sin\alpha$

(ii) $\cos\theta = \cos(180+\alpha) = -\cos\alpha$

(iii) $\tan\theta = \tan(180+\alpha) = +\tan\alpha$

3. θ lies in the 4th quad. $270° < \theta < 360°$; Basic angle $\alpha = (360 - \theta)$ or $\theta = (360 - \alpha)$

(i) $\sin\theta = \sin(360-\alpha) = -\sin\alpha$

(ii) $\cos\theta = \cos(360-\alpha) = +\cos\alpha$

(iii) $\tan\theta = \tan(360-\alpha) = -\tan\alpha$

4. (i) $\sin(360 \times n + \alpha) = \sin\alpha$ (ii) $\cos(360 \times n + \alpha) = \cos\alpha$ (iii) $\tan(360 \times n + \alpha) = \tan\alpha$; $n \in I$

Example 3: Find the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ for the following angles;

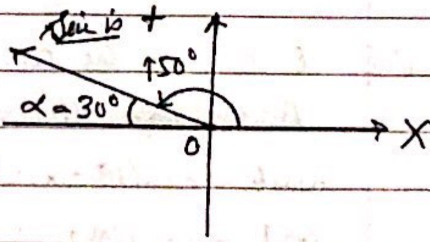
(i) $150°$ (ii) $225°$ (iii) $300°$

(i) for $150°$, Basic angle $\alpha = 180 - 150 = 30°$

(a) $\sin 150° = \sin(180 - 30°) = +\sin 30° = \frac{1}{2}$ ✓

(b) $\cos 150° = \cos(180 - 30°) = -\cos 30° = \frac{-\sqrt{3}}{2}$ ✓

(c) $\tan 150° = \tan(180 - 30°) = -\tan 30 = \frac{-1}{\sqrt{3}}$ ✓

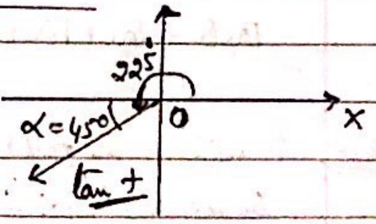


(ii) for $225°$, Basic angle $\alpha = 225 - 180° = 45°$

(a) $\sin 225° = \sin(180 + 45°) = -\sin 45° = -\frac{1}{\sqrt{2}}$ ✓

(b) $\cos 225 = \cos(180 + 45°) = -\cos 45° = -\frac{1}{\sqrt{2}}$

$\tan 225° = \tan(180 + 45°) = +\tan 45° = 1$

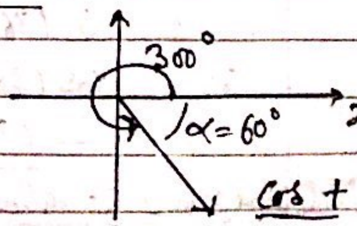


(iii) for $300°$, Basic angle $\alpha = 360 - 300 = 60°$

(a) $\sin 300° = \sin(360 - 60) = -\sin 60 = -\frac{\sqrt{3}}{2}$ ✓

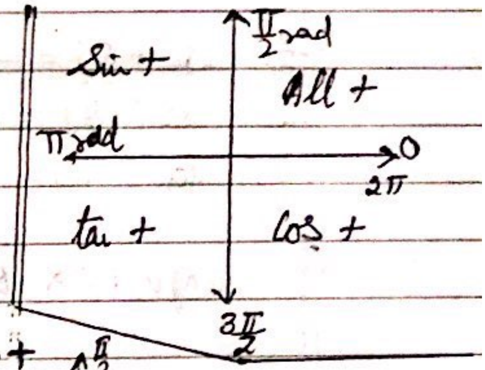
(b) $\cos 300° = \cos(360 - 60) = +\cos 60° = \frac{1}{2}$ ✓

(c) $\tan 300° = \tan(360 - 60) = -\tan 60° = -\sqrt{3}$



Example 4: Find the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ for the following angles:

(i) $\frac{2\pi}{3}$ (ii) $\frac{7\pi}{6}$ (iii) $\frac{7\pi}{4}$



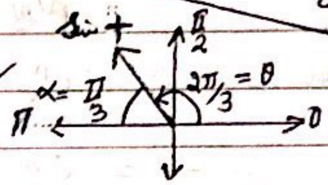
(i) for $\theta = \frac{2\pi}{3}$ in second quad.

Basic angle $\alpha = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

(a) $\sin\frac{2\pi}{3} = \sin\left(\pi - \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ ✓

(b) $\cos\frac{2\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = -\cos\frac{\pi}{3} = -\frac{1}{2}$ ✓

(c) $\tan\frac{2\pi}{3} = \tan\left(\pi - \frac{\pi}{3}\right) = -\tan\frac{\pi}{3} = -\sqrt{3}$ ✓

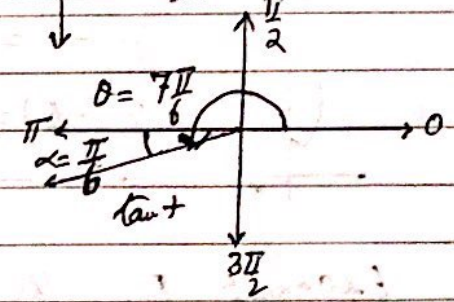


(ii) $\frac{7\pi}{6}$ lies in the 3rd quad, $\alpha = \frac{7\pi}{6} - \pi = \frac{\pi}{6}$

(a) $\sin\frac{7\pi}{6} = \sin\left(\pi + \frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$ ✓

(b) $\cos\frac{7\pi}{6} = \cos\left(\pi + \frac{\pi}{6}\right) = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$ ✓

(c) $\tan\frac{7\pi}{6} = \tan\left(\pi + \frac{\pi}{6}\right) = \tan\frac{\pi}{6} = +\frac{1}{\sqrt{3}}$ ✓



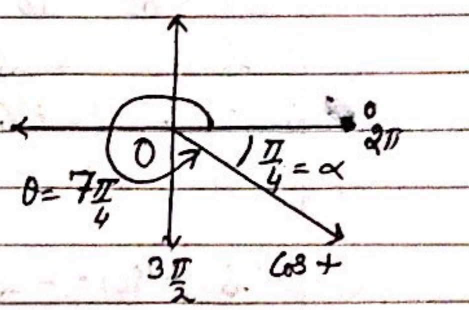
(iii) $\frac{7\pi}{4}$ lies in the 4th quad.

basic angle $\alpha = 2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$

(a) $\sin\frac{7\pi}{4} = \sin\left(2\pi - \frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$ ✓

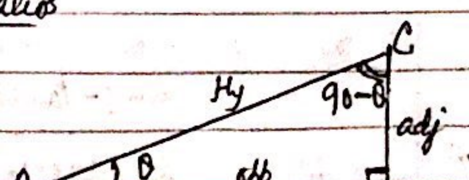
(b) $\cos\frac{7\pi}{4} = \cos\left(2\pi - \frac{\pi}{4}\right) = +\cos\frac{\pi}{4} = +\frac{1}{\sqrt{2}}$ ✓

(c) $\tan\frac{7\pi}{4} = \tan\left(2\pi - \frac{\pi}{4}\right) = -\tan\frac{\pi}{4} = -1$ ✓



§ Complementary angles - trigonometric Ratios

1. $\sin(90 - \theta) = \cos\theta$
2. $\cos(90 - \theta) = \sin\theta$



for angle $C = (90 - \theta)$

$\sin(90 - \theta) = \frac{AB}{AC} = \cos\theta$

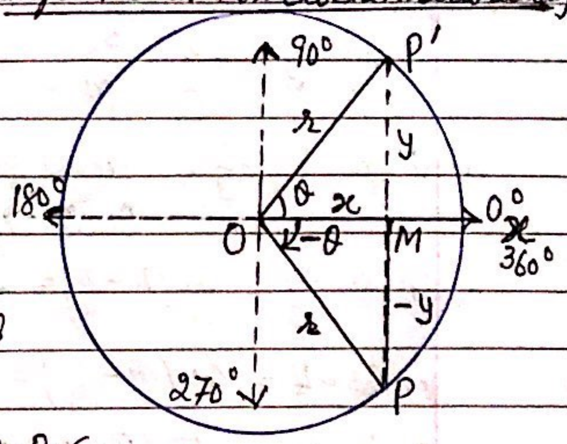
$\cos(90 - \theta) = \frac{BC}{AB} = \sin\theta$

Example: (i) $\sin 20° = \sin(90 - 70) = \cos 70°$

(ii) $\cos 20 = \cos(90 - 70) = \sin 70°$

(iii) $\sin 30° = \cos 60° = \frac{1}{2}$ and $\cos 30° = \sin 60 = \sqrt{3}/2$

§ Trigonometri Ratios of negative angle (angle measured in clock-wise direction)



1. $\sin(-\theta) = -\sin\theta$
2. $\cos(-\theta) = +\cos\theta$
3. $\tan(-\theta) = -\tan\theta$

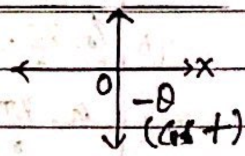
angle $POX = -\theta$ for angle $P'OX = \theta$

$P'M = y \rightarrow PM = y$

Now $\sin(-\theta) = \frac{-PM}{r} = = -\frac{y}{r} = -\sin\theta$ ✓

$\cos(-\theta) = \frac{OM}{r} = \cos\theta$ ✓

$\tan(-\theta) = -\frac{PM}{OM} = -\frac{y}{x} = -\tan\theta$ ✓



Example 5: Find the values of the following. $\sin\theta$, $\cos\theta$, $\tan\theta$.

(i) $-60°$ (ii) $-150°$ (iii) $-225°$ (iv) $-300°$

(i) (a) $\sin(-60°) = -\sin 60 = -\frac{\sqrt{3}}{2}$ ✓

(b) $\cos(-60°) = +\cos 60 = \frac{1}{2}$

(c) $\tan(-60) = -\tan 60 = -\sqrt{3}$ ✓

(ii) (a) $\sin(-150) = -\sin 150 = -\sin(180-30)$

$= -\sin 30° = -\frac{1}{2}$ ✓

(b) $\cos(-150) = +\cos 150 = \cos(180-30)$

$= -\cos 30 = -\frac{\sqrt{3}}{2}$ ✓

(c) $\tan(-150°) = -\tan 150 = -\tan(180-30)$

$= -(-\tan 30) = +\frac{1}{\sqrt{3}}$ ✓

(iv) (a) $\tan(-300) = -\tan 300° = -\tan(360-60)$

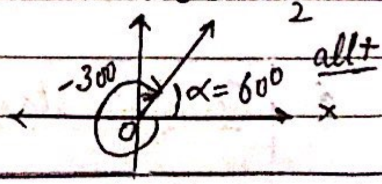
$= -(-\tan 60) = \tan 60 = \sqrt{3}$

(b) $\sin(-300) = -\sin 300 = -\sin(360-60)$

$= -(-\sin 60) = \sin 60 = \frac{\sqrt{3}}{2}$

(c) $\cos(-300) = +\cos 300 = \cos(360-60)$

$= \cos 60 = \frac{1}{2}$ ✓



(iii) (a) $\sin(-225°) = -\sin(225°) = -\sin(180+45°)$

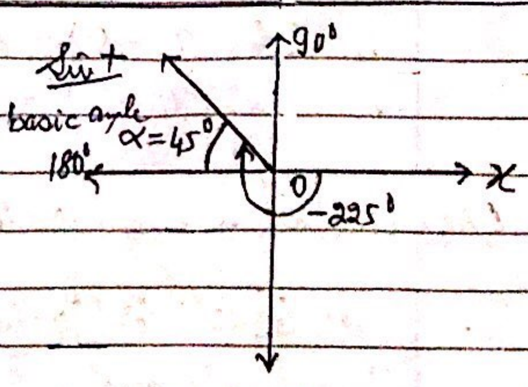
$= -(-\sin 45) = \frac{1}{\sqrt{2}}$ ✓

(b) $\cos(-225) = +\cos 225° = \cos(180+45°)$

$= -\cos 45° = -\frac{1}{\sqrt{2}}$

(c) $\tan(-225) = -\tan 225° = -\tan(180+45°)$

$= -\tan 45 = -1$ ✓



Example 6. Given that $\cos x = p$, where x is an acute angle in degrees, find in terms of p.

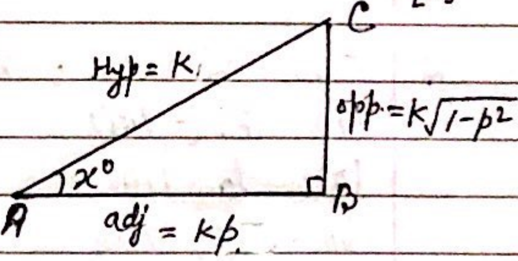
(i) $\sin x$ --[1]

(ii) $\tan x$ --[1]

(iii) $\tan(90^\circ - x)$ --[1]

[W-13/12/Q1]

Solution:



Given $\cos x = \frac{p}{1} = \frac{adj}{hyp.} = \frac{pk}{k}$

adj $AB = pk$, Hyp $AC = K$

opp $= \sqrt{K^2 - (pk)^2} = K\sqrt{1-p^2}$ (using Pythagoras theorem)

(i) $\sin x = \frac{opp}{Hy} = \frac{BC}{AC} = \frac{K\sqrt{1-p^2}}{K} = \sqrt{1-p^2}$ ✓

(ii) $\tan x = \frac{opp}{adj} = \frac{K\sqrt{1-p^2}}{Kp} = \frac{\sqrt{1-p^2}}{p}$

(iii) $\tan(90-x) = \frac{\sin(90-x)}{\cos(90-x)} = \frac{\cos x}{\sin x} = \frac{p}{\sqrt{1-p^2}}$ (Using t-ratio of complementary angles)

Example 7: The acute angle x radians is such that $\tan x = k$, where k is a positive constant. Express, in terms of k,

(i) $\tan(\pi - x)$ ---[1]

(ii) $\tan(\frac{1}{2}\pi - x)$ ---[1]

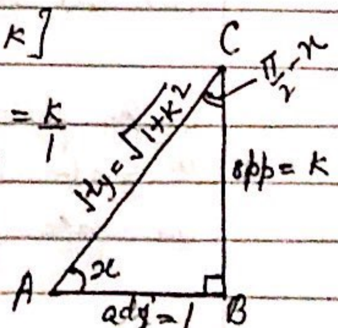
(iii) $\sin x$ --[2]

Solution: (i) $\tan(\pi - x) = -\tan x = -K$ [Given $\tan x = K$]

(ii) $\tan\left(\frac{\pi}{2} - x\right) = \frac{opp}{adj} = \frac{AB}{BC} = \frac{1}{K}$

(iii) $\sin x = \frac{opp}{Hyp} = \frac{BC}{AC} = \frac{K}{\sqrt{1+K^2}}$

$\tan x = \frac{BC}{AB} = \frac{K}{1}$



In rt ΔABC, using Pythagoras Theo.

$AC = \sqrt{AB^2 + BC^2} = \sqrt{1+K^2}$

Exercise_1

1. Find the exact value of the following.
   (i) $\sin 135^\circ$
   (ii) $\cos 180^\circ$
   (iii) $\tan(-210^\circ)$
   (iv) $\sin 180^\circ$
   (v) $\cos \frac{3\pi}{2}$
   (vi) $\tan \pi$

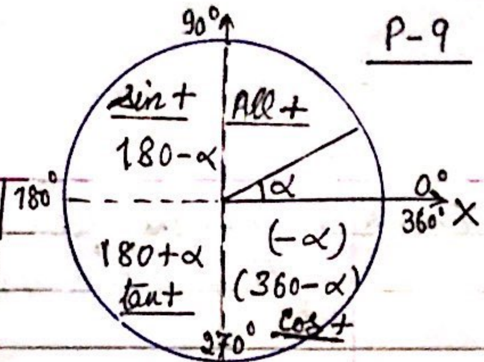
2. Find the value of the following. (using calculator)
   (i) $\cos(-40^\circ)$
   (ii) $\tan 110^\circ$
   (iii) $\sin 132^\circ$
   (iv) $\tan(-310^\circ)$
   (v) $\sin 220^\circ$
   (vi) $\sin 270^\circ$

3. Find the values of other two trigonometric function: ($\sin\theta, \cos\theta, \tan\theta$)
   (Without using a calculator)
   (i) $\cos\theta = -\frac{1}{2}$, θ lies in the third quadrant.
   (ii) $\sin\theta = \frac{3}{5}$; θ lies in the second quadrant.
   (iii) $\tan x = \frac{4}{3}$, θ lies in the third quadrant.
   (iv) $\cos x = \frac{5}{13}$, θ lies in the fourth quadrant.
   (v) $\tan x = -\frac{5}{12}$, θ lies in the second quadrant.

§ [Note: $\sin(n \cdot 360 + \theta) = \sin\theta$; $\cos(2n\pi + \theta) = \cos\theta$
and $\tan(360n + \theta) = \tan\theta$]

4. Find the values of the following trig. ratios.
   (i) $\sin 765^\circ$
   (ii) $\tan \frac{19\pi}{3}$
   (iii) $\sin\left(-\frac{11\pi}{3}\right)$
   (iv) $\tan\left(-\frac{15\pi}{4}\right)$

5. In the following the basic angle α is given, the angle θ lies in the quadrant and the range of θ lies in, find the value of θ.
   (i) $\alpha = \frac{\pi}{6}$, fourth quadrant, $0 < \theta \leq 4\pi$
   (ii) $\alpha = 50^\circ$, second quadrant, $0 < \theta < 360^\circ$
   (iii) $\alpha = 42^\circ$, third quadrant. $-180^\circ < \theta < 180^\circ$



§ Values of Inverse trigonometric function:
(Given the value of a trig. fun, to find the angle)

Given (i) $\sin\theta = a$; $-1 \le a \le 1$,
$\theta = \sin^{-1} a$

Given $\sin 30° = \frac{1}{2}$
$\Rightarrow \sin^{-1}\frac{1}{2} = 30°$

(ii) $\cos\theta = a$; $-1 \le a \le 1$
$\theta = \cos^{-1} a$

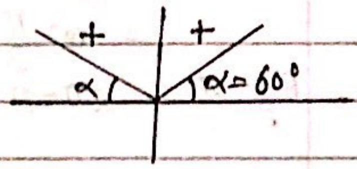
Given $\cos 45° = \frac{1}{\sqrt{2}}$
$\cos^{-1}\frac{1}{\sqrt{2}} = 45°$

(iii) $\tan\theta = a$ $\quad -\infty < a < \infty$
$\theta = \tan^{-1} a$

Given $\tan\frac{\pi}{3} = \sqrt{3}$
$\tan^{-1}\sqrt{3} = \frac{\pi}{3}$.

Example 8: Find the values of θ.

(i) $\sin\theta = \frac{\sqrt{3}}{2}$, $\quad 0 \le \theta < 360°$
$= \sin 60°$

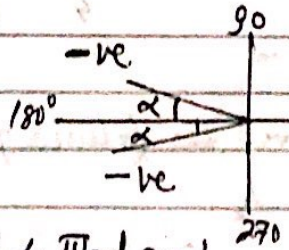


Basic angle $\alpha = 60°$

$\sin\theta$ is +ve in Ist and IInd quadrants

$\therefore \theta = \alpha$ or $180 - \alpha$
$= 60°$ or $180 - 60$
$= 60°$ or $120°$

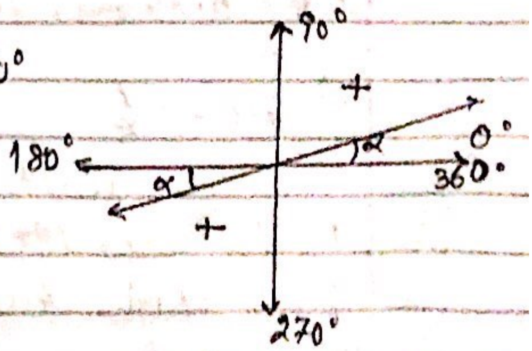
(ii) $\cos\theta = -\frac{1}{\sqrt{2}}$
$= -\cos 45°$



Basic angle $\alpha = 45°$
$\cos\theta$ is negative in IInd & IIIrd quad.

$\theta = 180 - \alpha, \; 180 + \alpha$
$= 180 - 45, \; 180 + 45 \Rightarrow 135°$ or $225°$ ✓

(iii)
$\tan\theta = \frac{1}{\sqrt{3}} = \tan 30°$ Basic angle $\alpha = 30°$



$\tan\theta$ is positive in Ist and IIIrd quad.

$\therefore \theta = \alpha$ or $180 + \alpha$
$= 30°$ or $180 + 30°$

$\theta = 30°$ or $210°$

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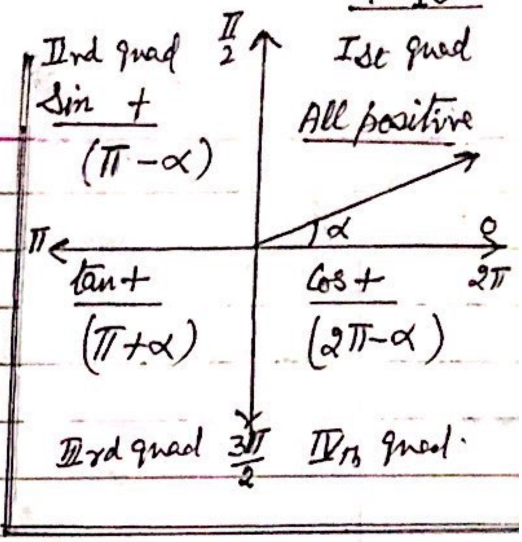


Example 9: Solve the following trig. equations; (Find the value of θ) $\quad 0 \le \theta < 2\pi$

(i) $\sin\theta = -\frac{1}{2}$

$= -\sin\frac{\pi}{6}$ $\quad$ (Basic angle $\alpha = \frac{\pi}{6}$; $\sin\theta$ is $-$ve in IIIrd & IVth quad.)

$\theta = \pi + \frac{\pi}{6},\ 2\pi - \frac{\pi}{6}$

$\therefore \theta = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$ ✓

(ii) $\cos\theta = \frac{1}{2}$

$= \cos\frac{\pi}{3}$ $\quad$ (Basic angle $\alpha = \frac{\pi}{3}$; $\cos\theta$ is + in Ist and (IV)th quadrants)

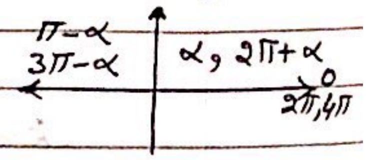
$\theta = \frac{\pi}{3},\ 2\pi - \frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{3}$ or $\frac{5\pi}{3}$ ✓

(iii) $\tan\theta = -1$

$= -\tan\frac{\pi}{4}$ $\quad$ (Basic angle $\alpha = \frac{\pi}{4}$; $\tan\theta$ is $-$ve in IInd and (IV)th quad.)

$\theta = \pi - \frac{\pi}{4}$ or $2\pi - \frac{\pi}{4}$ $\quad \therefore \theta = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$ ✓

Example 10: Solve the following, $\quad 0 \le \theta < 4\pi$.



(i) $\sin\theta = \frac{1}{3}$

$= \sin 0.34$; (Basic angle $\alpha = 0.34$ radian, and $\sin\theta$ is + in Ist and IInd quad.)

$\theta = 0.34,\ (\pi - 0.34),\ (2\pi + 0.34),\ (3\pi - 0.34)$

$\therefore \theta = 0.34$ rad, 2.8 rad, 6.623 rad and 9.0847 rad. ✓

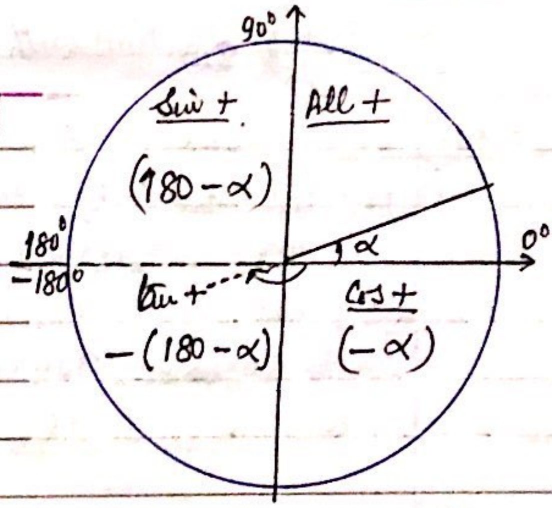
(ii) $\cos\theta = -\frac{1}{5}$ $\quad$ (Basic angle $\alpha = 1.37$, and $\cos\theta$ is $-$ve in IInd and IIIrd quadrants)

$= -\cos 1.37$

$\theta = (\pi - 1.37),\ (\pi + 1.37),\ (3\pi - 1.37),\ (3\pi + 1.37)$ rad.

$\theta = 1.77$ rad, 4.51 rad, 8.054 rad, 10.79 rad ✓

(iii) $\sin\theta = 0$
$= \sin 0 \Rightarrow$ $\quad \theta = 0,\ \pi,\ 2\pi,\ 3\pi$, ✓



Example 11. Solve the following trig. equations (Find the value of θ). $-180^\circ < \theta \leq 180^\circ$

(i) $\sin\theta = \frac{\sqrt{3}}{2}$
$= \sin 60^\circ$ {Basic angle $\alpha = 60^\circ$; $\sin\theta$ is + in Ist and IInd quad.}
$\theta = 60^\circ,\ 180 - 60^\circ$
$\therefore \theta = 60^\circ,\ 120^\circ$ ✓

(ii) $\cos\theta = -\frac{1}{2}$ {$\cos\theta$ is −ve in IInd and IIIrd quadrants
$= -\cos 60^\circ$ (Basic angle $\alpha = 60^\circ$)
$\theta = 180 - 60,\ -(180-60)$
$\therefore \theta = 120^\circ$ or -120° ✓

(iii) $\tan\theta = 1$ {Basic angle $\alpha = 45^\circ$
$= \tan 45^\circ$ [$\tan\theta$ is +ve in Ist and IIIrd quadrants)
$\theta = 45^\circ$ or $-(180-45)$
$\therefore \theta = 45^\circ$ or -135° ✓

(iv) $\sin\theta = -1$ ($\alpha = 90^\circ$
$= -\sin 90^\circ$ $\sin\theta$ is −ve in IIIrd & IVth quad.)
$\theta = -(180-90)$ or (-90°) [$-(180-\alpha)$ or $-\alpha$]
$\theta = -90^\circ$ or -90° $\therefore$ Only one angle $\theta = -90^\circ$ ✓

(v) $\tan\theta = -\frac{1}{4}$ [Basic angle $\alpha = 14^\circ$
$= -\tan 14^\circ$ [$\tan\theta$ is −ve in IInd & IV quadrants)
$\theta = (180-14)^\circ,\ -14^\circ$
$\theta = +166^\circ$ or -14° ✓

VI $\cos\theta = \frac{1}{3}$ [Basic angle $\alpha = 70.5$
$= \cos 70.5^\circ$ [$\cos\theta$ is + in Ist and IVth quadrants]
$\theta = 70.5^\circ$ or -70.5° ✓

Trig- equation with angles in multiples of x.

Example 12: Solve the following t-equation;

(i) $\cos 2x = 0.6$; for $0 \le x \le 180^\circ \Rightarrow 0 \le 2x \le 360^\circ$;

$= \cos 53.13^\circ$ [basic angle $\alpha = 53.13$

$2x = 53.13$ or $360 - 53.13$ $\cos x$ is +ve in Ist & 4th quadrants]

$2x = 53.13$ or 306.87

$x = \underline{26.6^\circ \text{ or } 153.4^\circ}$ ✓

(ii) $\sin 2x = -0.5$; for $0 \le x \le 180^\circ \Rightarrow 0 \le 2x \le 360^\circ$

$= -\sin 30^\circ$ [$\alpha = 30^\circ$, $\sin\theta$ is −ve in IIIrd and

$2x = 180 + 30^\circ,\ 360 - 30^\circ$ IVth quadrants]

$2x = 210^\circ,\ 330^\circ$

$\therefore\ x = \underline{105^\circ\ ;\ 165^\circ}$ ✓

(iii) $2\tan\left(\frac{\theta}{2}\right) + \sqrt{3} = 0$; for $0 \le \theta \le 540^\circ \Rightarrow 0 \le \frac{\theta}{2} \le 270^\circ$

$\Rightarrow \tan \theta/_2 = -\frac{\sqrt{3}}{2}$ [$\alpha = 40.89$

$= -\tan 40.89^\circ$ [$\tan\theta$ is −ve in IInd and $\cancel{\text{IV}_{th}}$ quad]

will exceed 270°

$\frac{\theta}{2} = (180 - 40.89) = 139.11$

$\therefore \theta = 139.11 \times 2 = 278.22^\circ$

$\therefore \theta = \underline{278.2^\circ}$

(iv) $2\sin x - 3\cos x = 0$; $0 \le x \le 360^\circ$

$\rightarrow \tan x = \frac{3}{2}$ [basic angle $\alpha = 56.3^\circ$ and $\tan\theta$

$= \tan 56.3$ is +ve in Ist and IIIrd quad.]

$x = 56.3,\ 180 + 56.3$

$x = \underline{56.3^\circ\ ;\ 236.3^\circ}$ ✓

(v) $3\sin(2\theta - 4) = 2$ $0 < \theta < \pi \Rightarrow$ $0 < 2\theta < 2\pi$

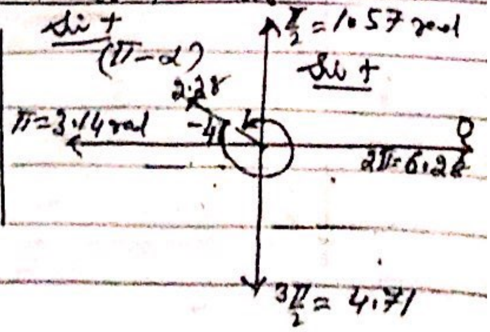
$\rightarrow \sin(2\theta - 4) = 2/3$ $-4 < 2\theta - 4 < 2\pi - 4$

$= \sin 0.73$ $-4 < 2\theta - 4 < 2.28$

$2\theta - 4 = 0.73,\ \pi - 0.73 = 2.41$ ✗, $-(\pi + 0.73) = -3.87$

$2\theta = 4.73,\ ,\ 0.1284,$

$\theta = \underline{2.36}$ ✓, $\underline{0.0642}$, ✓



Squared trig. equations:

Example 13: Solve the following t-equations:

(i) $3\sin^2 2x = \cos^2 2x$; $0 \leq x \leq 180^\circ \Rightarrow 0 \leq 2x \leq 360^\circ$

$\Rightarrow \tan^2 2x = \frac{1}{3}$

$\tan 2x = \pm \frac{1}{\sqrt{3}}$

$= \pm \tan 30^\circ$

Basic angle $\alpha = 30^\circ$
$\tan\theta$ is + in Ist & IIIrd quad.
$\tan\theta$ is −ve in IInd & IVth quad.

$\tan 2x = \tan 30^\circ$ or $\tan 2x = -\tan 30^\circ$

$2x^\circ = 30^\circ ; (180+30^\circ)$ or $2x = 180-30, 360-30$

$x = 15^\circ ; 105^\circ$ or $x = 75^\circ ; 165^\circ$

$\therefore$ $x = \underline{15^\circ, 75^\circ, 105^\circ \text{ and } 165^\circ}$ ✓ [S-15° / 13 / Q 4(ii) ... [4]]

(ii) Solve; $4\cos^2 x = 1$; $0 \leq x \leq 360^\circ$

$\cos x = \pm \frac{1}{2}$

$= \pm \cos 60$

basic angle $\alpha = 60^\circ$
($\cos\theta$ is + in Ist and ... quad.
−ve in IInd & IIIrd quad.)

$\cos x = \cos 60$ or $\cos x = -\cos 60$

$x = 60^\circ$ or $(360-60)$ or $(180-60), (180+60)$

$x = 60, 300,$ or $120^\circ, 240^\circ$

$x = \underline{60, 120^\circ, 240^\circ \text{ and } 300^\circ}$. ✓

Solve:

(iii) $\sin^2\theta = 1$; $0 \leq \theta \leq 360^\circ$

$\sin\theta = \pm 1$

$= \pm \sin 90^\circ$

$\alpha = 90^\circ$
$\sin\theta$ is + in Ist & IInd quad
and −ve in IIIrd & IVth

$\therefore$ $\sin\theta = \sin 90^\circ$ or $\sin\theta = -\sin 90^\circ$

$\theta = 90^\circ, 180-90^\circ, 180+90$ or $360-90$

$\theta = 90^\circ, 270^\circ$.

$\theta = \underline{90^\circ, 270^\circ}$

Example 14: Solve the equation, $4\sin^2\theta - 15\sin\theta - 4 = 0$; $0 \le \theta \le 360°$

$\Rightarrow 4\sin^2\theta - 16\sin\theta + \sin\theta - 4 = 0$

$4\sin\theta(\sin\theta - 4) + 1(\sin\theta - 4) = 0$

$(\sin\theta - 4)(4\sin\theta + 1) = 0$

$\Rightarrow \sin\theta = -\frac{1}{4}$ or $\sin\theta = +4^{\times}$ (Since $-1 \le \sin\theta \le 1$)

$= -\sin 14.5$ {Basic angle $\alpha = 14.5$ and $\sin\theta$ is -ve in third & fourth quad.)

$\theta = (180 + 14.5), (360 - 14.5)$

$\underline{\theta = 194.5 \; ; \; 345.5°}$ ✓

[SP-1/17/Q3] --[3]

Example 15: Solve the equation; $6\cos^2 x - \cos x - 1 = 0$; $0 \le x \le 180°$

[W-14/12/Q5]

$\Rightarrow 6\cos^2 x - 3\cos x + 2\cos x - 1 = 0$

$3\cos x(2\cos x - 1) + 1(2\cos x - 1) = 0$

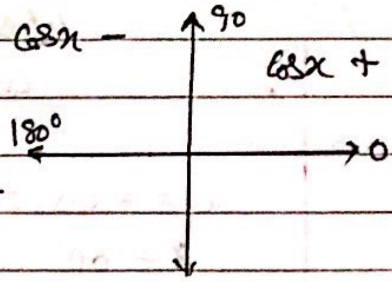
$(2\cos x - 1)(3\cos x + 1) = 0$

$\Rightarrow \cos x = \frac{1}{2}$ or $\cos x = -\frac{1}{3}$

$= \cos 60$ or $= -\cos 70.5$

$x = 60$ or $x = 180 - 70.5$

$\therefore \underline{x = 60°} \; ; \; \underline{x = 109.5°}$



Example 16: Solve the equation: $\tan^2\theta - 5\tan\theta + 6 = 0$ for $0 \le \theta \le 180°$

[S-15/12/Q5] --[4]

$\Rightarrow \tan^2\theta - 3\tan\theta - 2\tan\theta + 6 = 0$

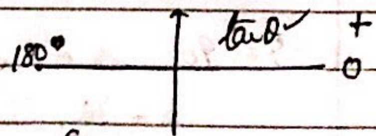
$\tan\theta(\tan\theta - 3) - 2(\tan\theta - 3) = 0$

$(\tan\theta - 3)(\tan\theta - 2) = 0$

$\tan\theta = 3$ or $\tan\theta = 2$

$= \tan 71.6$ or $\tan\theta = 63.4°$

$\underline{\theta = 71.6 \text{ or } 63.4°}$ ✓



($\tan\theta$ is +ve in Ist quad)

Example 17: Solve the equation; $2\sin^4\theta + \sin^2\theta - 1 = 0$; $0° \le \theta \le 360°$

factorise $\Rightarrow (2\sin^2\theta - 1)(\sin^2\theta + 1) = 0$

$\Rightarrow \sin^2\theta = \frac{1}{2}$ or $\sin^2\theta = -1^{\times}$

$\sin\theta = \pm\frac{1}{\sqrt{2}}$

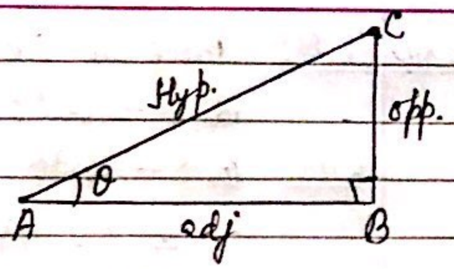
$\sin\theta = +\sin 45°$ or $\sin\theta = -\sin 45°$

$\theta = 45, 180 - 45°$, or $\theta = 180 + 45, 360 - 45$

$\theta = \underline{45°, 135°, 225°, 315°}$ ✓

Trigonometric Identities

§ Trigonometric Identities:



$\sin\theta = \frac{opp}{Hyp} = \frac{BC}{AC}$ —— (1)

$\cos\theta = \frac{adj}{Hyp} = \frac{AB}{AC}$ —— (2)

$\tan\theta = \frac{opp}{adj} = \frac{BC}{AB}$

Here. $\frac{\sin\theta}{\cos\theta} = \tan\theta$ ✓

Now in right angled triangle ABC,
Using Pythagoras theorem;

$$BC^2 + AB^2 = AC^2$$

dividing by AC^2:

$$\frac{BC^2}{AC^2} + \frac{AB^2}{AC^2} = \frac{AC^2}{AC^2}$$

$$\left(\frac{BC}{AC}\right)^2 + \left(\frac{AB}{AC}\right)^2 = 1$$

$$\Rightarrow (\sin\theta)^2 + (\cos\theta)^2 = 1 \quad \text{(from (1) and (2))}$$

$$\left\{\begin{array}{l} \sin^2\theta + \cos^2\theta = 1. \\ \text{or} \quad 1 - \sin^2\theta = \cos^2\theta \\ \text{or} \quad 1 - \cos^2\theta = \sin^2\theta \end{array}\right.$$

Identities are true for all values of θ.

Example 18: Prove the identity:

$$\frac{\cos^2 x}{1-\sin x} = 1+\sin x$$

Solution: L.H.S. $\frac{\cos^2 x}{1-\sin x} = \frac{1-\sin^2 x}{1-\sin x}$ $\quad \left[\because \sin^2 x + \cos^2 x = 1 \\ \cos^2 x = 1 - \sin^2 x \right]$

$$= \frac{(1+\sin x)(1-\sin x)}{(1-\sin x)}$$

$$= (1+\sin x) \quad = \text{R.H.S}$$

Example 19: Prove the identity.

$$\tan^2\theta - \sin^2\theta = \tan^2\theta \cdot \sin^2\theta$$

L.H.S. $\tan^2\theta - \sin^2\theta$

$$= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$$

$$= \frac{\sin^2\theta - \sin^2\theta \cdot \cos^2\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}$$

$$= \frac{\sin^2\theta}{\cos^2\theta} \times \sin^2\theta \qquad \left[\begin{array}{l}\sin^2\theta + \cos^2\theta = 1 \\ \Rightarrow 1-\cos^2\theta = \sin^2\theta\end{array}\right]$$

$$= \tan^2\theta \cdot \sin^2\theta = \text{R.H.S}$$

Example 20: Prove the identity:

$$\frac{\sin\theta}{\sin\theta+\cos\theta} + \frac{\cos\theta}{\sin\theta-\cos\theta} = \frac{1}{\sin^2\theta - \cos^2\theta}$$

L.H.S. $\dfrac{\sin\theta}{\sin\theta+\cos\theta} + \dfrac{\cos\theta}{\sin\theta-\cos\theta}$

$$= \frac{\sin\theta(\sin\theta-\cos\theta) + \cos\theta(\sin\theta+\cos\theta)}{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}$$

$$= \frac{\sin^2\theta - \cancel{\sin\theta\cdot\cos\theta} + \cancel{\sin\theta\cdot\cos\theta} + \cos^2\theta}{\sin^2\theta - \cos^2\theta} \qquad [(a+b)(a-b) = a^2-b^2] \quad [\sin^2\theta+\cos^2\theta = 1]$$

$$= \frac{1}{(\sin^2\theta - \cos^2\theta)} = \text{R.H.S.}$$

Example 21: Prove the identity: $\dfrac{\sin x \cdot \tan x}{1-\cos x} = 1 + \dfrac{1}{\cos x}$

L.H.S $\dfrac{\sin x \cdot \tan x}{1-\cos x}$

$$= \frac{\sin x \times \dfrac{\sin x}{\cos x}}{\dfrac{(1-\cos x)}{1}}$$

$$= \frac{\sin^2 x}{\cos x} \times \frac{1}{(1-\cos x)}$$

$$= \frac{(1-\cos^2 x)}{\cos x(1-\cos x)} \qquad [\sin^2 x = 1-\cos^2 x]$$

$$= \frac{(1+\cos x)\cancel{(1-\cos x)}}{\cos x \cdot \cancel{(1-\cos x)}}$$

$$= \frac{1}{\cos x} + \frac{\cos x}{\cos x}$$

$$= 1 + \frac{1}{\cos x} = \text{R.H.S}$$

Example 22(i) Prove the identity $\left(\frac{1}{\cos\theta} - \tan\theta\right)^2 = \frac{1-\sin\theta}{1+\sin\theta}$ . ...[3]

(ii) Hence solve the equation $\left(\frac{1}{\cos\theta} - \tan\theta\right)^2 = \frac{1}{2}$ for $0° \leq \theta \leq 360°$ ...[3]

$\boxed{S\text{-}17/12/Q3}$

Solution; (i) Prove; $\left(\frac{1}{\cos\theta} - \tan\theta\right)^2 = \frac{1-\sin\theta}{1+\sin\theta}$

L.H.S $\left(\frac{1}{\cos\theta} - \tan\theta\right)^2 = \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$

$$= \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$$

$$= \frac{(1-\sin\theta)^2}{\cos^2\theta}$$

$$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta} \qquad \left[\begin{array}{l}\sin^2\theta + \cos^2\theta = 1\\ \cos^2\theta = 1-\sin^2\theta\end{array}\right.$$

$$= \frac{\cancel{(1-\sin\theta)}(1-\sin\theta)}{\cancel{(1-\sin\theta)}(1+\sin\theta)} \qquad [a^2-b^2=(a-b)(a+b)$$

$$= \frac{1-\sin\theta}{1+\sin\theta} = R.H.S ✓$$

(ii) Solve the equation;

$$\left(\frac{1}{\cos\theta} - \tan\theta\right)^2 = \frac{1}{2}$$

$\Rightarrow \frac{1-\sin\theta}{1+\sin\theta} = \frac{1}{2}$ (using Part (i))

$\Rightarrow 2(1-\sin\theta) = 1+\sin\theta$

$\Rightarrow 3\sin\theta = 1 \Rightarrow \sin\theta = \frac{1}{3}$ $\qquad 0 \leq \theta \leq 360°$

$= \sin 19.5°$

$\therefore \theta = 19.5, (180-19.5)$

$\theta = \underline{19.5°; 160.5°}$ ✓

[Basic value $\alpha = 19.5$ and $\sin\theta$ is +ve in the Ist and IInd quadrants.

Example 23 (i) Prove the identity: $\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta} = \frac{4}{\sin\theta\tan\theta}$ --[4]

(ii) Hence solve for $0° < \theta < 360°$, the equation:

$$\sin\theta\left(\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta}\right) = 3$$ --[3]

[S-16 | 12 | Q7]

Solution: L.H.S

(i)
$$\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta} = \frac{(1+\cos\theta)^2-(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{(1+2\cos\theta+\cos^2\theta)-(1-2\cos\theta+\cos^2\theta)}{(1-\cos^2\theta)}$$

$$= \frac{4\cos\theta}{\sin^2\theta} \qquad [1-\cos^2\theta = \sin^2\theta]$$

$$= \frac{4}{\sin\theta \times \frac{\sin\theta}{\cos\theta}} \qquad \left[\frac{b}{a^2} = \frac{b\times\frac{1}{b}}{a^2\times\frac{1}{b}} = \frac{1}{a\times\frac{a}{b}}\right.$$

$$= \frac{4}{\sin\theta \cdot \tan\theta} = R.H.S.$$

(ii) Solve $\sin\theta\left(\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta}\right) = 3$ $\qquad 0 < \theta < 360°$

$$\Rightarrow \cancel{\sin\theta} \times \frac{4}{\cancel{\sin\theta}\cdot\tan\theta} = 3$$

$$\Rightarrow \tan\theta = \frac{4}{3} \qquad 0 < \theta < 360$$

$$= \tan 53.1$$

(Basic angle $\alpha = 53.1°$
$\tan\theta$ is + in 1_{st} & 3rd quad.)

$\therefore \theta = 53.1°,\ 180 + 53.1$

$\theta = \underline{53.1°,\ 233.1°}$ ✓

Example 24(i) Show that the equation, $\cos 2x(\tan^2 2x+3)+3=0$ can be expressed as, $2\cos^2 2x+3\cos 2x+1=0$ --[3]

(ii) Hence solve the equation, $\cos 2x(\tan^2 2x+3)+3=0$ for $0 \leq x \leq 180^\circ$ --[4]

[W-17/12/Q5]

Solution: $\cos 2x(\tan^2 2x+3)+3=0$

(i) $\Rightarrow \cos 2x\left(\dfrac{\sin^2 2x}{\cos^2 2x}+3\right)+3=0$

$\Rightarrow \cos 2x\left(\dfrac{\sin^2 2x+3\cos^2 2x}{\cos^2 2x}\right)+3=0$

$\Rightarrow \dfrac{\sin^2 2x+3\cos^2 2x}{\cos 2x}+3=0$ $\because$ $[\sin^2 2x = 1-\cos^2 2x]$

$\rightarrow (1-\cos^2 2x)+3\cos^2 2x+3\cos 2x=0$

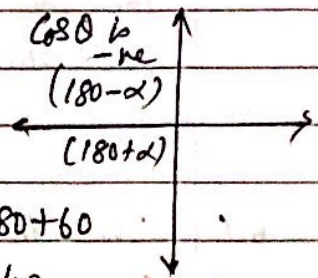
$\Rightarrow 2\cos^2 2x+3\cos 2x+1=0$ ✓

(ii) Factorise to solve, $2\cos^2 2x+3\cos 2x+1=0$; $0 \leq x \leq 180^\circ$

$2\cos^2 2x+2\cos 2x+\cos 2x+1=0$; $\Rightarrow 0 \leq 2x \leq 360^\circ$

$2\cos 2x(\cos 2x+1)+1(\cos 2x+1)=0$

$(\cos 2x+1)(2\cos 2x+1)=0$



$\cos 2x=-1$ or $\cos 2x=-\frac{1}{2}$

$\cos 2x=-\cos 0$; $\cos 2x=-\cos 60^\circ$

$2x=180-0$ or $180+0$; $2x=(180-60)$ or $180+60$

$2x=180$; $2x=120$; 240

$x=90^\circ$; $x=60^\circ$; 120°

$\therefore$ $x=\underline{60^\circ, 90^\circ, 120^\circ}$ ✓

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Exercise - 2

1(i) Prove the identity. $\left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)^2 = \frac{1-\cos\theta}{1+\cos\theta}$ --[3]

(ii) Hence solve the equation. $\left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)^2 = \frac{2}{5}$, for $0 \leq \theta \leq 360^\circ$ [4]

2 Solve the equation, $15\sin^2 x = 13 + \cos x$ for $0^\circ \leq x \leq 180^\circ$ --[4]

3(i) Given that, $3\sin^2 x - 8\cos x - 7 = 0$,
Show that, for the real value of x, $\cos x = -\frac{2}{3}$ ---[3]

(ii) Hence solve the equation;
$3\sin^2(\theta+70) - 8\cos(\theta+70^\circ) - 7 = 0$, for $0^\circ \leq \theta \leq 180^\circ$ --[4]

4.(i) Solve the equation: $4\sin^2 x + 8\cos x - 7 = 0$ for $0 \leq x \leq 360^\circ$ --[4]

(ii) Hence find the solution of the equation:
$4\sin^2(\frac{1}{2}\theta) + 8\cos(\frac{1}{2}\theta) - 7 = 0$ for $0^\circ \leq \theta \leq 360^\circ$ --[2]

5.(i) Show that the equation $2\cos x = 3\tan x$ can be written as a quadratic equation in $\sin x$. --[3]

(ii) Solve the equation $2\cos 2y = 3\tan 2y$, for $0 \leq y \leq 180^\circ$ --[4]

[W-12/12/Q6]

6.(i) Solve the equation, $\sin 2x + 3\cos 2x = 0$ for $0^\circ \leq x \leq 360^\circ$ --[5]

(ii) How many solutions has the equation:
$\sin 2x + 3\cos 2x = 0$ for $0^\circ \leq x \leq 1080^\circ$

[S-12/13/Q4]

7(i) Express $2\cos^2\theta = \tan^2\theta$ as a quadratic equation in $\cos^2\theta$. ---[2]

(ii) Solve the equation: $2\cos^2\theta = \tan^2\theta$ for $0 \leq \theta \leq \pi$
giving solutions in terms of π. --[3]

[S-13/13/Q3]

Answers

Exercise-1

1. (i) $\frac{1}{\sqrt{2}}$ (iv) 0
   (ii) -1 (v) 0
   (iii) $-\frac{1}{\sqrt{3}}$ (vi) 0

2. (i) 0.766 (iv) -1.191
   (ii) -2.747 (v) -0.643
   (iii) 0.743 (vi) -1

3. (i) $\sin\theta = -\frac{\sqrt{3}}{2}$; $\tan\theta = \sqrt{3}$
   (ii) $\cos\theta = -4/5$; $\tan\theta = -\frac{3}{4}$
   (iii) $\sin\theta = -4/5$; $\cos\theta = -3/5$
   (iv) $\sin\theta = \frac{-12}{13}$, $\tan\theta = -12/5$
   (v) $\sin\theta = 5/13$, $\cos\theta = -12/13$.

4. (i) $\frac{1}{\sqrt{2}}$ (iii) $\frac{\sqrt{3}}{2}$
   (ii) $\sqrt{3}$ (iv) 1

5. (i) $\frac{7\pi}{4}$, $\frac{15\pi}{4}$
   (ii) 130°
   (iii) -138°

Exercise-2

1(ii) $\cos\theta = 3/7$
$\theta = 64.6^\circ$ or 295.4° ✓

2. $(5\cos x + 2)(3\cos x - 1) = 0$
$x = 113.6^\circ,\ 70.5^\circ$ ✓

3.(ii) $\cos(\theta + 70^\circ) = -2/3$
$\theta + 70 = 131.8$ or 228.2 ✗
$\theta = 158.2$

4(i) $(2\cos x - 1)(2\cos x - 3) = 0$
$\Rightarrow x = 60^\circ$ or 300°
(ii) $\theta = 120^\circ$ (only)

Exercise-2

5.(i) $2\sin^2 x + 3\sin x - 2 = 0$
(ii) $\sin 2y = \frac{1}{2}$ or -2 ✗
$y = 15^\circ$ or 75°

6.(i) $x = 54.2^\circ, 144.2^\circ, 234.2^\circ, 324.2^\circ$
(ii) 12 solution.

7(i) $2\cos^4\theta + \cos^2\theta - 1 = 0$
(ii) $(2\cos^2\theta - 1)(\cos^2\theta + 1) = 0$
$\cos\theta = \pm\frac{1}{\sqrt{2}}$ or $\cos^2\theta = -1$ ✗
$\theta = \frac{\pi}{4},\ \frac{3\pi}{4}$

← X —— X →