DATE 12.02.21

0606

Additional Maths.

Circular Measure Revision

SP-20	M-20	S-20	W-20

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Circular Measure

Revision

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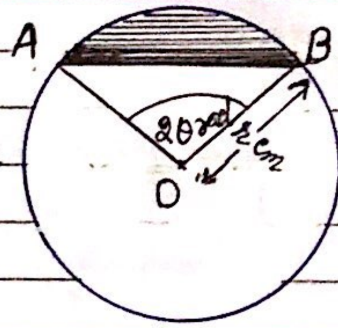
1 The diagram shows a circle, centre O, radius r cm. The points A and B lie on the circle such that angle $AOB = 2\theta$ radians.



(a) Given that the perimeter of the shaded region is 20 cm, show that $r = \frac{10}{\theta + \sin\theta}$ ---[3]

(b) Given that r and θ can vary, find the value of $\frac{dr}{d\theta}$ when $\theta = \frac{\pi}{6}$ ---[4]

[SP-20/02/Q8]

Solution (a) Perimeter $= l + AB$ — ①

$AB = 2r\sin\left(\frac{2\theta}{2}\right) = 2r\sin\theta$ — ②

∴ length of arc AB, $l = r\times(2\theta)$ — ③

Perimeter $= 2r\theta + 2r\sin\theta = 20$ (Given)

$\Rightarrow 2r(\theta + \sin\theta) = 20$

$r = \frac{20}{2\times(\theta+\sin\theta)}$

$\therefore r = \frac{10}{\theta+\sin\theta}$ ✓

(b) $r = \frac{10}{\theta+\sin\theta} = 10(\theta+\sin\theta)^{-1}$

$\frac{dr}{dr} = -10(\theta+\sin\theta)^{-2}\times\frac{d}{d\theta}(\theta+\sin\theta)$

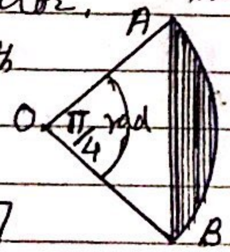
$= \frac{-10}{(\theta+\sin\theta)^2}\times(1+\cos\theta)$

$\left(\frac{dr}{d\theta}\right)_{\theta=\frac{\pi}{6}} = \frac{-10}{\left(\frac{\pi}{6}+\sin\frac{\pi}{6}\right)}\times\left(1+\cos\frac{\pi}{6}\right)$

$= \underline{-17.8}$ ✓

2 (a) A circle has a radius of 6 cm. A sector of this circle has a perimeter of $2(6+5\pi)$ cm. Find the area of this sector. ---[4]

(b) The diagram shows the sector AOB of a circle with centre O and radius 7 cm, angle $AOB = \frac{\pi}{4}$ radians. Find the perimeter of the shaded region. ---[3]



[M-20/22/Q6]

Solution (a) Perimeter of sector $= 2r + r\theta$

$\Rightarrow 2\times6 + 6\theta = 2(6+5\pi)$ (given)

$\Rightarrow \theta = \frac{5}{3}\pi$

∴ Area of sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}\times6^2\times\frac{5\pi}{3}$

$= \underline{94.2}$ (or 30π)

(b) Perimeter of shaded $=$ Chord $AB + l$ — ①

Chord $AB = 2r\sin\theta/2 = 2\times7\times\sin\frac{\pi}{4}/2$

$= 14\sin\frac{\pi}{8}$ — ②

and length of arc $l = r\theta$

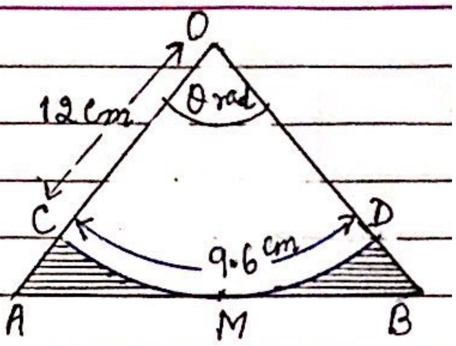
$= 7\times\frac{\pi}{4}$ — ③

∴ from ② & ③ in ①

$P = 14\sin\frac{\pi}{8} + \frac{7\pi}{4}$

$= \underline{10.9}$ (10.85 to 10.86)

3. The diagram shows an isosceles triangle OAB such that OA = OB and angle AOB = θ radians. The points C and D lie on OA and OB respectively. CD is an arc of length 9.6 cm of the circle, centre O, radius 12 cm. The arc CD touches the line AB at the point M



(a) Find the value of θ. ---[1]

(b) Find the total area of the shaded regions ---[4]

(c) Find the total perimeter of the shaded regions. ---[3]

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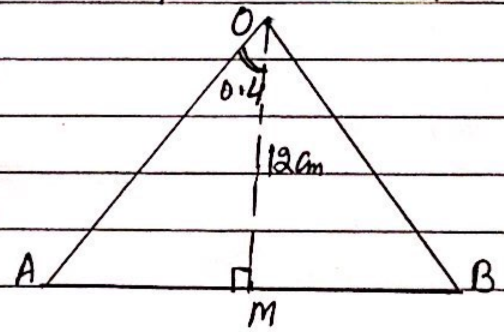
Solution (a) length of arc $CD = r\theta = 12\theta = 9.6$

$\Rightarrow \theta = \frac{9.6}{12} = \underline{0.8 \text{ radians}}$ ✓



(b) Area of shaded regions

$=$ ar of $\Delta OAB -$ ar of sector OCD — ①

area of sector $OCD = \frac{1}{2}r^2\theta = \frac{1}{2}\times 12^2 \times 0.8$

$= 57.6$ ✓ — ②

In rt ΔOAM, $\frac{AM}{OM} = \tan 0.4$

$\Rightarrow AM = 12 \times \tan 0.4 = 5.074$

$\therefore AB = 2\times AM = 2\times 5.074$

$= 10.148$ cm

$\therefore$ Area of $\Delta OAB = \frac{1}{2}\times AB \times OM$

$= \frac{1}{2}\times 10.148 \times 12$

$= 60.88$ — ③

from ② & ③ in ①

Area of shaded region $= 60.88 - 57.6$

$= \underline{3.28 \text{ cm}^2}$ ✓

(C) Perimeter of the shaded region

$= \text{arc } \widehat{CD} + 2AM + 2AC$ — ④

In ΔOAM, $\frac{AM}{OA} = \sin 0.4$

$OA = \frac{5.074}{\sin 0.4} = 13.03$

$\therefore AC = OA - OC$

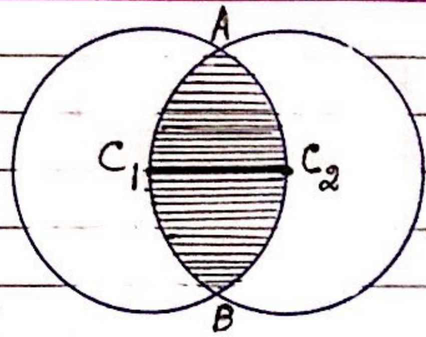
$= 13.03 - 12 = 1.03$

from ④

$P = 9.6 + 2\times 5.074 + 2\times 1.03$

$= \underline{21.8 \text{ cm}}$ ✓

4. The circles with centres C_1 and C_2 have equal radii of length r cm. The line C_1C_2 is a radius of both the circles. The two circles intersect at A and B.

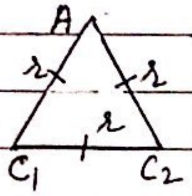


(a) Given that the perimeter of the shaded region is 4π cm. Find the value of r. --[4]

(b) Find the exact area of the shaded region. --[4]

[S-20/22/Q11]

Solution (a) ΔAC_1C_2 is an equilateral triangle.



$\therefore \angle AC_1C_2 = \frac{\pi}{3}$

angle $AC_1C_2 = 2\cdot\frac{\pi}{3}$

$\therefore$ Length of arc $= r\theta = \frac{2\pi}{3}\times r$

Given $l = \frac{2\pi}{3}r = \frac{4\pi}{2}$ $\left(\frac{\text{Peri}}{2}\right)$

$\therefore r = 3$ ✓

Area of the shaded region

$= 2[\text{area of sector } C_1AB - \text{ar } \Delta C_1AB]$

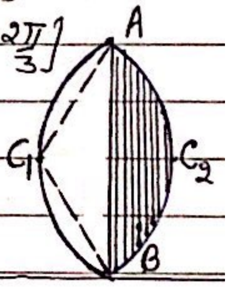
$= 2\left[\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta\right]$

$= 2\left[\frac{1}{2}\times 3^2\times\frac{2\pi}{3} - \frac{1}{2}\times 3^2\sin\frac{2\pi}{3}\right]$

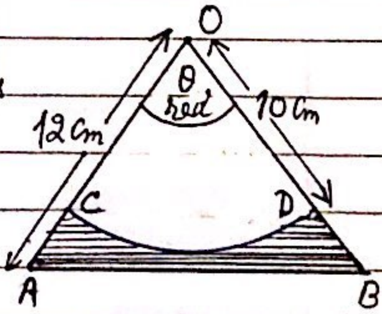
$= 9\left[\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right]$

$= 3\left[2\pi - 3\frac{\sqrt{3}}{2}\right]$

$= 6\pi - \frac{9\sqrt{3}}{2}$



5. The diagram shows an isosceles triangle OAB such that OA = OB = 12 cm and angle AOB = θ radians. Points C and D lie on OA and OB respectively such that CD is an arc of the circle, centre O, radius 10 cm. The area of sector OCD = 35 cm^2.



(a) Show that $\theta = 0.7$ --[1]

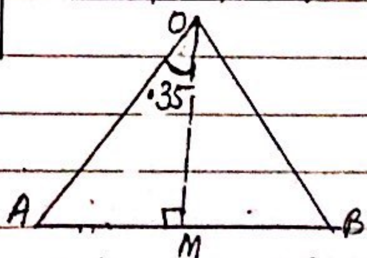
(b) Find the perimeter of the shaded region -[4]

(c) Find the area of the shaded region ---[3]

[S-20/13/Q7]

Solution (a) Area of sector OCD $= \frac{1}{2}r^2\theta = 35$

$\Rightarrow \frac{1}{2}\times 10^2\times\theta = 35 \Rightarrow \theta = 0.7$



(b) Perimeter of shaded = arc CD + AB + 2·AC —(1)

arc CD $= r\theta = 10\times 0.7 = 7$ ✓

$\frac{AM}{12} = \sin 0.35 \Rightarrow AM = 4.1147$ ($AM \perp AB$)

$AB = 2\cdot AM = 2\times 4.1147 = 8.23$ ✓

$AC = 12 - 10 = 2$

from (1) $P = 7 + 8.23 + 2\times 2 = 19.2$ cm

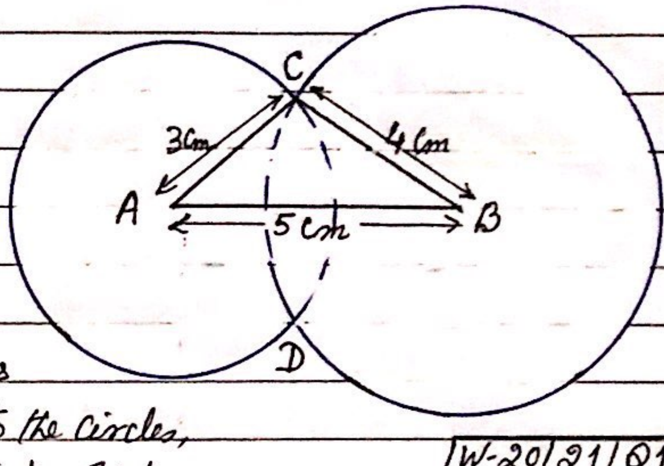
(c) Shaded area = ar ΔOAB − ar of Sect OCD

$= \frac{1}{2}r^2\cdot\sin\theta - 35$ ← (Given)

$= \frac{1}{2}\times 12^2\times\sin 0.7 - 35$

$= 46.4 - 35 = 11.4$ cm^2

6. The diagram shows a shape consisting of two circles of radius 3cm and 4cm with centres A and B, which are 5cm apart. The circles intersect at C and D as shown. The lines AC and BC are tangents to the circles, centres B and A respectively. Find



W-20/21/Q12

(a) the angle CAB in radians. ---[2]

(b) the perimeter of the whole shape. ---[4]

(c) The area of the whole shape. ---[4]

Solution (a) CAB is a right triangle, right angle at vertex C.

$\therefore \tan CAB = 4/3 \Rightarrow$ angle $CAB = \underline{0.927}$

(b) angle $CBD = 2\left(\frac{\pi}{2} - 0.927\right) = 1.287$

Perimeter $= 3(2\pi - 2\times 0.927)$
$+ 4(2\pi - 1.287)$
$= 13.287 + 19.985 = \underline{33.3}$

(c) Area of shape $= 2$ ar ΔACB + area of two sectors

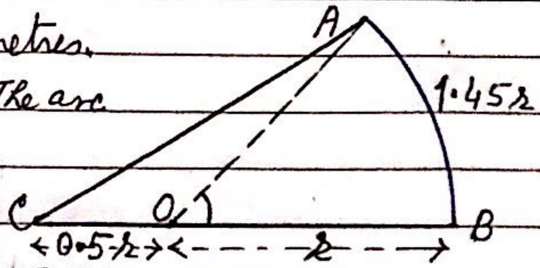
$= 2\times\left(\frac{1}{2}\times 3\times 4\right) + \frac{1}{2}\times 3^2(2\pi - 2\times 0.927)$
$+ \frac{1}{2}\times 4^2(2\pi - 1.287)$

$= 12 + 19.93 + 39.97$

$= 12 + 71.9$

$= \underline{83.9 \text{ cm}^2}$

7. In this question all lengths are in centimetres. The diagram show the figure ABC. The arc AB is part of a circle, centre O, radius r, and is of length 1.45r. The point O lies on CB such that CO = 0.5r



(a) Find in radians angle AOB. ---[1]

(b) Find the area ABC, giving your answer in the form kr^2, k is constant. ---[3]

(c) Given perimeter of ABC is 12cm. Find the value of r. W-20/13/Q8 --[4]

Solution (a) length of arc $AB = 1.45r = r\cdot\theta$

$\Rightarrow \angle AOB = \theta = \underline{1.45}$ ✓

(b) Area of Sector $AOB = \frac{1}{2}r^2\theta = \frac{1}{2}r^2\times 1.45$ ✓

+ area of $\Delta COA = \frac{1}{2}\times 0.5\times r\times \sin(\pi - 1.45)$

Total area $= 0.973r^2$ ✓

(c) $AC^2 = r^2 + 0.25r^2 - 2r\times 0.5r\cos(\pi - 1.45)$

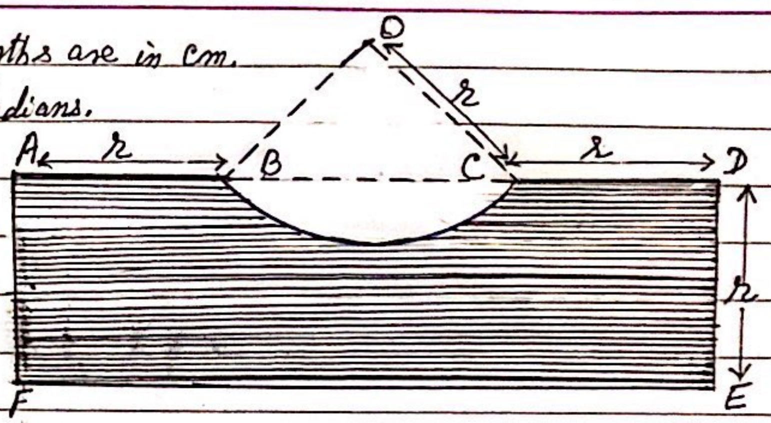
$\Rightarrow AC = 1.17r$

Perimeter $= 1.45r + 1.5r + 1.17r$

$\Rightarrow 4.12r = 12$ (Given)

$\Rightarrow r = \frac{12}{4.12} = \underline{2.91 \text{ cm}}$ ✓

8 In this question all lengths are in cm. and all angles are in radians.

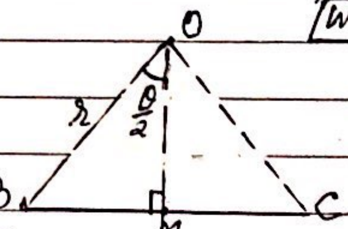


The diagram shows the rectangle ADEF, where AF = DF = r. The points B and C lie on AD such that AB = CD = r. The curve BC is an arc of the circle, centre O, radius r and has a length 1.5r.

(a) Show that the perimeter of the shaded region is $(7.5 + 2\sin 0.75)r$ --- [5]

(b) Find the area of the shaded region, giving your answer in the form kr^2, where k is a constant correct to 2 decimal places. --- [4]

W-20/12/Q11



Solution (a) Let angle BOC = θ radians

length of arc BC = $r\theta = 1.5r$ (Given)

$\Rightarrow$ angle BOC = $\theta = 1.5$ rad ✓

$OM \perp BC$, $OM = \frac{BC}{2}$, Angle BOM $= \frac{1.5}{2} = 0.75$

In rt Δ BOM, $\sin\frac{\theta}{2} = \frac{BM}{OB} = \frac{\frac{BC}{2}}{r}$

$\Rightarrow \frac{BC}{2} = r\sin 0.75$

$BC = 2r\sin 0.75$ ✓

Perimeter = (AB + CD) + arc BC + 2r + 2r + BC

$= 2r + 1.5r + 4r + 2r\sin 0.75$

$= 7.5r + 2r\sin 0.75$

$= \underline{r(7.5 + 2\sin 0.75)}$ ✓

(b) Area of shaded region

= Area of rectangle − area of segment

$= (2r + 2r\sin 0.75)r - \left[\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta\right]$

$= (2r + 2r\sin 0.75)\cdot r - \frac{1}{2}r^2[1.5 - \sin 1.5]$

$= 3.36r^2 - 0.75r^2 + 0.4987r^2$

$= \underline{3.11r^2}$ ✓