



0606

Additional Maths

Equations, inequations
and Graphs.

Revision

SP-20	M-20	S-20	W-20
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- 1(a) Sketch the graph of $y = |2x+5|$ and the graph of $y = |2-x|$, stating the coordinates of the points where each graph meets the coordinate axes. ---[4]

(b) Solve $|2x+5| \leq |2-x|$ [SP-20/01/05] ---[3]

Solution: $y = |2x+5|$ ——— ①

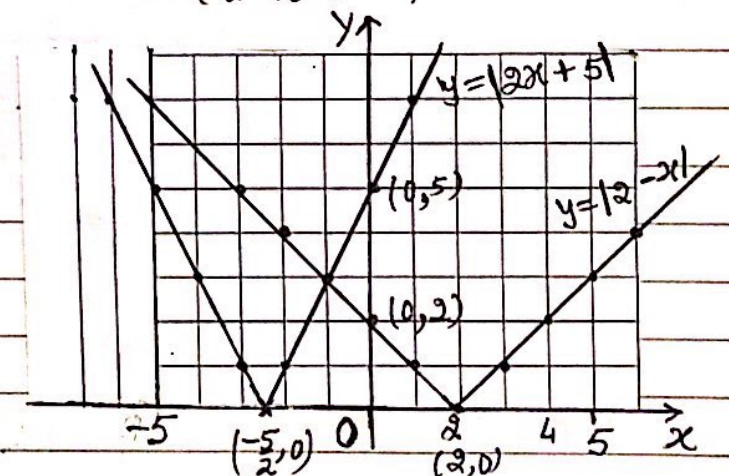
x	-4	-3	$-\frac{5}{2}$	-2	-1	0	1
y	3	1	0	1	3	5	7

Intersects x-axis at $(-\frac{5}{2}, 0)$ and y-axis at $(0, 5)$

$y = |2-x|$ ——— ②

x	-1	0	1	2	3	4	5
y	3	2	1	0	1	2	3

Intersects x-axis at $(2, 0)$
and y-axis at $(0, 2)$



(b) Solve: $|2x+5| \leq |2-x|$

$$\Rightarrow (2x+5)^2 \leq (2-x)^2$$

$$4x^2 + 20x + 25 \leq 4 + x^2 - 4x$$

$$\Rightarrow 3x^2 + 24x + 21 \leq 0$$

$$x^2 + 8x + 7 \leq 0$$

$$(x+7)(x+1) \leq 0$$

(critical values are
 $x = -7, -1$)

$$\therefore -7 \leq x \leq -1$$

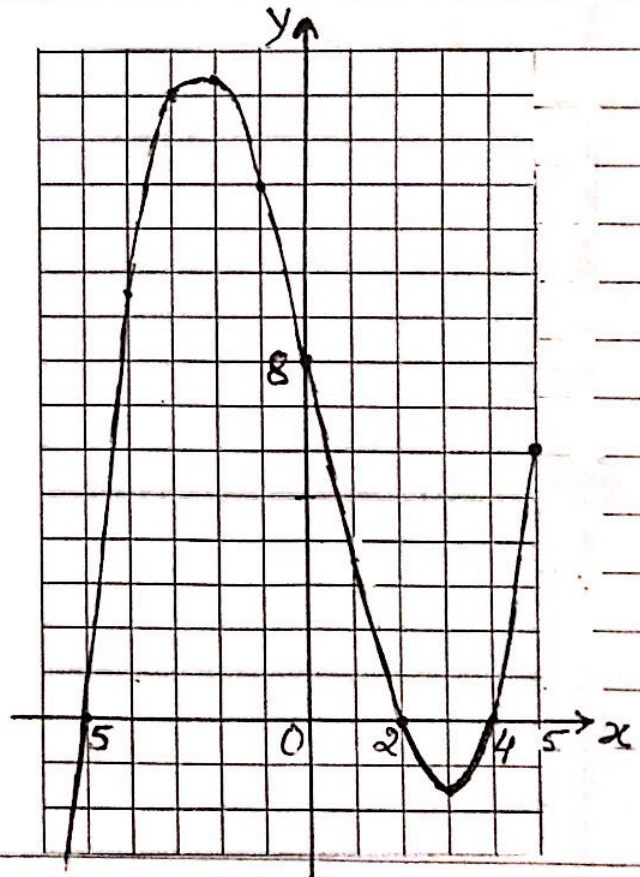
2. (a) Sketch the graph of $y = \frac{1}{5}(x-2)(x-4)(x+5)$, showing the coordinates of the points where the graph meets the coordinate axes. -- [2]
 (b) Explain why your sketch in part (a) can be used to solve $(x-2)(x-4)(x+5) \leq 0$ -- [1]
 (c) Hence solve $(x-2)(x-4)(x+5) \leq 0$ -- [1]

[SP-20/02/Q2]

(a) $y = \frac{1}{5}(x-2)(x-4)(x+5)$

x	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	-16	0	9.6	14	14.4	12	8	3.6	0	-1.6	0	6

Curve intersects
X-axis at (2,0), (4,0) ✓
and
Y-axis at (0,8) ✓



- (b) Multiplying $\frac{1}{5}(x-2)(x+4)(x+5) \leq 0$ does not change the values of x as $\Rightarrow (x-2)(x+4)(x+5) \leq 0$ same.
 (c) Graph below x-axis (on axis)
 $x \leq -5$ or $2 \leq x \leq 4$

3.(a) Sketch the graph of $y = -3(x-2)(x-4)(x+1)$, showing the coordinates of the points where the curve intersects the coordinate axes. ---[3]

(b) Hence find the value of x for which $-3(x-2)(x-4)(x+1) > 0$ ---[2]

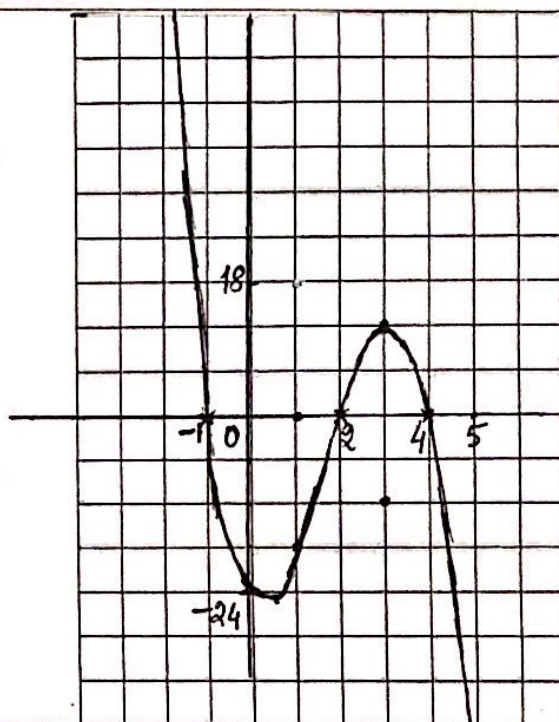
[M-20/12/Q1]

Solution(a) $y = -3(x-2)(x-4)(x+1)$

x	-2	-1	0	1	2	3	4	5
y	72	0	-24	-18	0	+12	0	-54

x-Intercepts (2,0), (4,0), (-1,0)

y-Intercept (0,-24)



(b) $-3(x-2)(x-4)(x+1) > 0$

for graph is above x-axis

$x < -1$ or $2 < x < 4$ ✓

4(a) Sketch the graph of $y = |5x-7|$, showing the coordinates of the points where the graph meets the coordinate axes. ---[3]

(b) Solve $5|5x-7|-1=14$

[M-20/22/Q5] ---[3]

Solution(a) $y = |5x-7| = \begin{cases} 5x-7, & x \geq \frac{7}{5} \\ -(5x-7), & x < \frac{7}{5} \end{cases}$

x	0	1	$\frac{7}{5}$	2	3
y	7	2	0	3	8

(b) $5|5x-7|$

$$\Rightarrow |5x-7| = 3$$

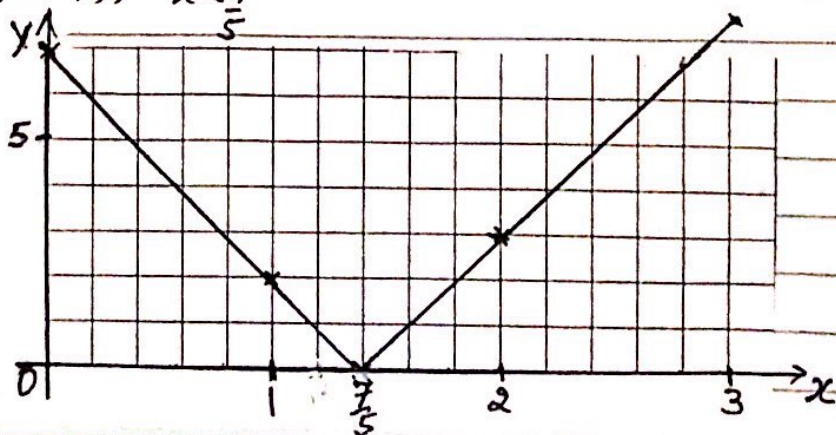
$$\Rightarrow (5x-7)^2 = 3^2$$

$$25x^2 - 70x + 49 = 9$$

$$5x^2 - 14x + 8 = 0$$

$$-(5x-4)(x-2) = 0$$

$$x = 2, \frac{4}{5} \checkmark$$



y-Int. (0, 7) ; x-Intercept ($\frac{7}{5}$, 0)

5. The diagram shows the graph of a cubic curve $y = f(x)$

(a) Find an expression for $f(x)$ --- [2]

(b) Solve $f(x) \leq 0$ --- [2]

[S-20/11/Q1]

Solution (a) The curve passes through

$(-2, 0)$, $(-1, 0)$ and $(5, 0)$

hence $y = k(x - \alpha)(x - \beta)(x - \gamma)$

$y = k(x - (-2))(x - (-1))(x - 5)$

$y = k(x + 2)(x + 1)(x - 5)$ --- (1)

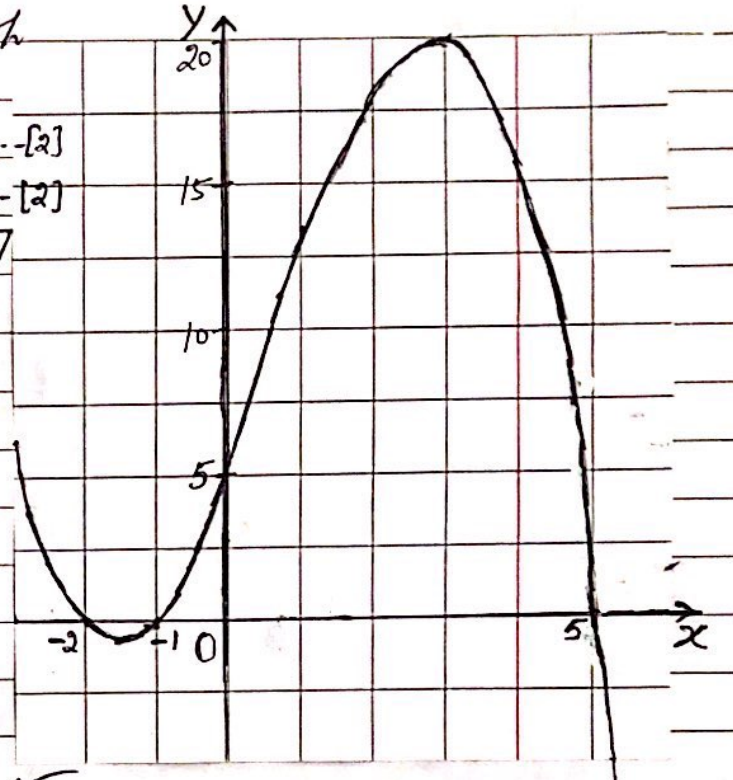
Passes through $(0, 5)$

for (1) $5 = k(2)(1)(-5) \Rightarrow k = -\frac{1}{2}$

for (1)

$y = -\frac{1}{2}(x + 2)(x + 1)(x - 5)$ ✓

(b) $f(x) \leq 0 \Rightarrow x \geq 5$ or $-2 \leq x \leq -1$ ✓



6. Sketch the graph of $y = |(x - 2)(x + 1)(x + 2)|$ showing the coordinates of the points where the curve meets the axes. --- [3]

[S-20/12/Q1]

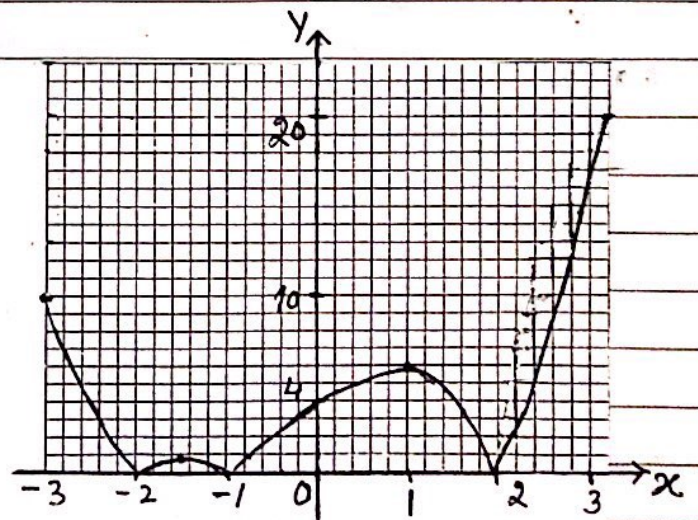
Solution: $y = |(x - 2)(x + 1)(x + 2)|$

x	-3	-2	-1	0	1	2	3	-1.5
y	10	0	0	4	6	0	20	$\frac{7}{8}$

Curve meets the x -axis

at $(2, 0)$, $(-1, 0)$, $(-2, 0)$

and y -axis at $(0, 4)$ ✓



- 7 The three roots of $p(x)=0$, where $p(x)=2x^3+ax^2+bx+c$ are $x=\frac{1}{2}$, $x=n$ and $x=-n$ where a, b, c and n are integers. The y-intercept of the graph of $y=p(x)$ is 4. Find $p(x)$, simplifying your coefficients. -- [5]
- [S-20/22/Q4]

Solution: $p(x) = 2x^3 + ax^2 + bx + c$ — (1)

Now given roots $x=\frac{1}{2}$, $x=n$ and $x=-n$

$$\Rightarrow p(x) = k(x - \frac{1}{2})(x - n)(x - (-n))$$

$$p(x) = \frac{k}{2}(2x-1)(x-n)(x+n) \text{ — (2)}$$

Comparing the coeff. of x^3 in (1) & (2)
 $k = 2$

$$\therefore \text{from (1)} \quad p(x) = (2x-1)(x-n)(x+n) \text{ — (3)}$$

$$\Rightarrow \begin{aligned} (0-1)(0-n)(0+n) &= 4 & \left[\begin{array}{l} y\text{-int} = 4 \\ y=4, x=0 \end{array} \right] \\ n^2 &= 4 \text{ — (4)} \end{aligned}$$

$$\text{from (3)} \quad p(x) = (2x-1)(x^2-n^2) = (2x-1)(x^2-4) \quad \text{from (4) } n^2=4$$

$$\Rightarrow p(x) = 2x^3 - x^2 - 8x + 4 \quad \checkmark \text{ [as in (1)]}$$

8. (a) Sketch the graph of $y = -(x+2)(x-1)(x-6)$, showing the coordinates of the points where the graph meets the coordinate axes. -- [2]

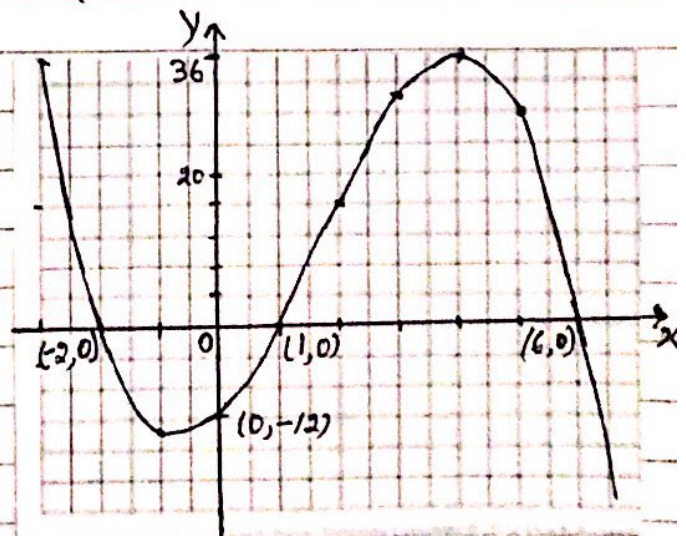
(b) Hence solve $-(x+2)(x-1)(x-6) \leq 0$ [S-20/23/Q3] -- [2]

Solution: $y = -(x+2)(x-1)(x-6)$

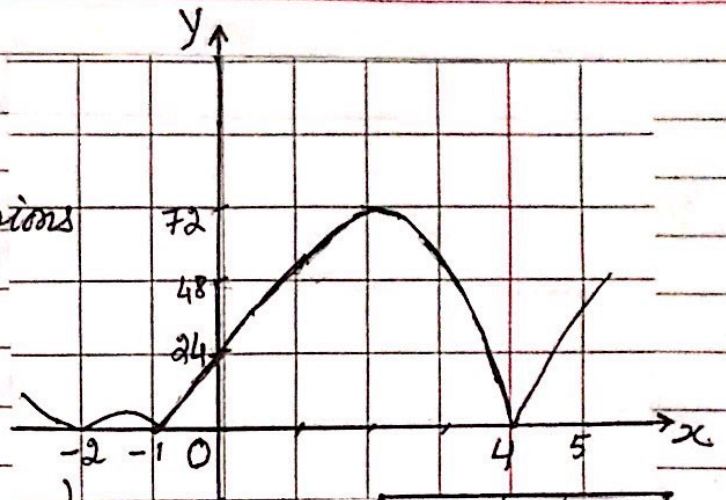
x	-3	-2	-1	0	1	2	3	4	5	6	7
y	36	0	-14	-12	0	16	30	36	28	0	-54

(b) Solve $-(x+2)(x-1)(x-6) \leq 0$

$$\Rightarrow \underline{-2 \leq x \leq 1} \quad \text{or} \quad \underline{x \geq 6} \quad \checkmark$$



- 9 The diagram shows the graph of $y = |p(x)|$, where $p(x)$ is a cubic function. Find two possible expressions for $p(x)$. --- [3]



Solutions Let the polynomial is $y = p(x)$

$$p(x) = k(x - (-2))(x - (-1))(x - 4)$$

[W-20/11/Q1]

$$\text{or } y = k(x+2)(x+1)(x-4) \quad \text{--- ①}$$

[$\because -2, -1$ and 4 are the roots of the polynomial]

Curve passes through $(0, 24)$ [Y-intercept = 24]

$$\text{from ① } 24 = k(+2)(+1)(-4) \Rightarrow -8k = 24$$

$$k = -3$$

\therefore The two possible equation of $p(x)$ are

$$y = \pm 3(x+2)(x+1)(x-4) \checkmark$$

10. Solve the inequality $|3x+2| > 8+x$ --- [3]

[W-20/21/Q1]

Solution: $|3x+2| > 8+x$

$$\begin{cases} |3x+2| = \begin{cases} 3x+2 & \text{for } 3x+2 \geq 0 \\ & \text{or } x \geq -2/3 \\ -(3x+2) & \text{for } x < -2/3 \end{cases} \end{cases}$$

Case I

$$3x+2 > 8+x \quad \text{for } x \geq -2/3$$

$$\Rightarrow 2x > 6 \Rightarrow x > 3 \checkmark \quad (\text{for } x \geq -2/3)$$

Case II

$$-(3x+2) > 8+x \quad \text{for } x < -2/3$$

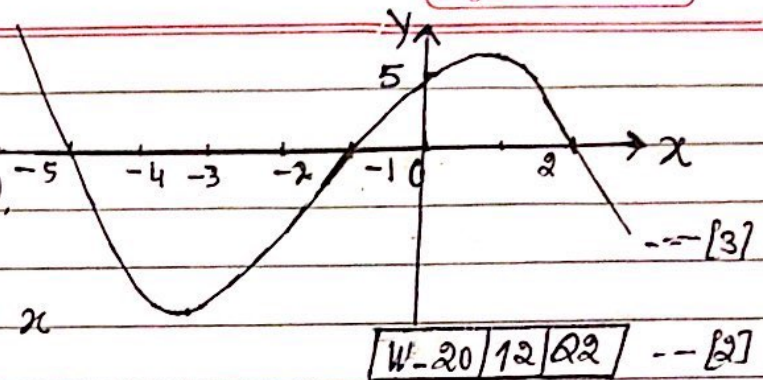
$$\Rightarrow -3x-2 > 8+x$$

$$\Rightarrow 4x < -10$$

$$\Rightarrow x < -2.5 \checkmark \quad [x < -2/3 \text{ or } (-0.666)]$$

$$\therefore \underline{x < -2.5} \quad \text{or} \quad \underline{x > 3} \checkmark$$

11. The diagram shows the graph of $y = f(x)$, where $f(x)$ is a cubic polynomial.



- (a) Find $f(x)$
(b) Write down the values of x such that $f(x) < 0$

Solution: x -intercepts of the curve are $-5, -1$ and 2

- (a) \therefore Equation of cubic polynomial:

$$y = k(x - (-5))(x - (-1))(x - 2)$$

$$\text{or } y = k(x + 5)(x + 1)(x - 2) \quad \text{--- (1)}$$

The curve passes through $(0, 5)$ [\because y -int. is 5]

from (1) $5 = k(0 + 5)(0 + 1)(0 - 2)$

$$\text{or } 5 = -10k \Rightarrow k = -\frac{1}{2}$$

\therefore Equation of polynomial $y = -\frac{1}{2}(x + 5)(x + 1)(x - 2)$ ✓

- (b) $f(x) < 0$ for $-5 < x < -1$ or $x > 2$ ✓

12. (a) Sketch the graph of $y = (x - 2)(x + 1)(3 - x)$, stating the intercepts on the coordinate axes. --- [3]

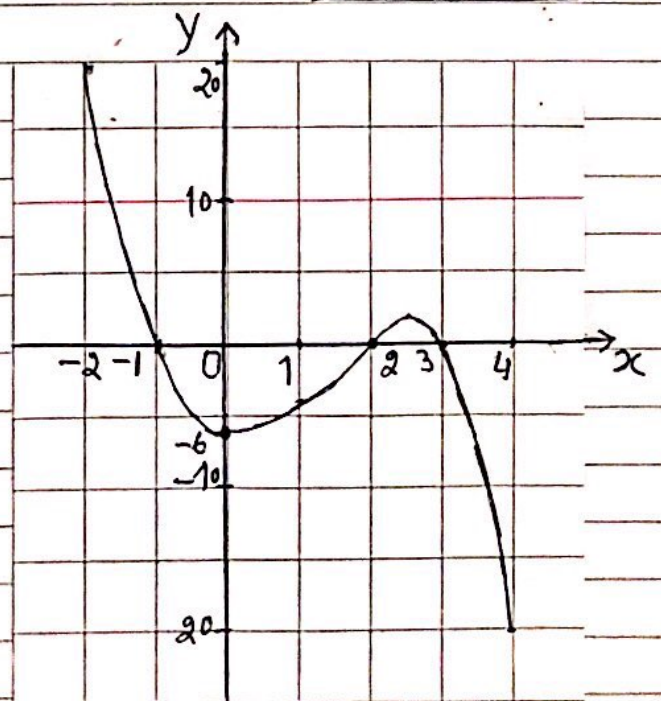
- (b) Hence the value of x such that $(x - 2)(x + 1)(3 - x) > 0$ --- [2]

[W-20/13/Q1]

Solution (a) $y = (x - 2)(x + 1)(3 - x)$

x	-2	-1	0	1	2	3	4
y	20	0	-6	-4	0	0	-20

- (b) $(x - 2)(x + 1)(3 - x) > 0$
Graph above x -axis for
 $x < -1$ or $2 < x < 3$ ✓



13 Solve $|3x-2| = 4+x$ ---[3]

Solution: Case I $3x-2 = 4+x$

$$\Rightarrow 2x = 6 \Rightarrow x = 3 \checkmark$$

Case II $3x-2 = -(4+x)$

$$3x-2 = -4-x$$

$$\Rightarrow 4x = -2 \Rightarrow x = -\frac{1}{2} (-0.5)$$

$\therefore \underline{x = 3, x = -0.5 \checkmark}$

$$\left\{ \begin{array}{l} |x| = a \quad a \in \mathbb{R} \\ \quad \quad a > 0 \\ \Rightarrow x = \pm a \end{array} \right.$$