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Additional Maths

Factors of Polynomials Revision

SP-20	M-20	S-20	W-20

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Factors of Polynomials

Revision

1. The polynomial $p(x) = 2x^3 - 3x^2 + qx + 56$ has a factor $x-2$

(a) Show that $q = -30$ ---[1]

(b) Factorise $p(x)$ completely and hence state all the solutions of $p(x) = 0$ [SP-20/01/Q1]---[4]

Solution: $p(x) = 2x^3 - 3x^2 + qx + 56$ —— ①

(a) $(x-2)$ is a factor of $p(x) \Rightarrow p(2) = 0$

from ① $2 \cdot 2^3 - 3 \cdot 2^2 + q \times 2 + 56 = 0$

$\Rightarrow 2q = -60 \Rightarrow \underline{q = -30}$ ✓

(b) hence from ① and $q = -30$

$p(x) = 2x^3 - 3x^2 - 30x + 56$

$= (x-2)(2x^2 + x - 28)$

$= (x-2)[2x^2 + 8x - 7x - 28]$

$= (x-2)[2x(x+4) - 7(x+4)]$

$= (x-2)(x+4)(2x-7)$

Now $p(x) = 0$

$\Rightarrow (x-2)(x+4)(2x-7) = 0$

$\therefore$ Solutions are: $\underline{2, -4 \text{ and } \frac{7}{2}}$ ✓

$(x-2)$ is a factor of $p(x)$

$$
\begin{array}{l}
(x-2)\overline{)\,2x^3 - 3x^2 - 30x + 56\,}(2x^2 + x - 28 \\
\quad\ \ \underline{\overset{-}{2x^3} \overset{+}{-} 4x^2} \\
\qquad\qquad x^2 - 30x \\
\qquad\qquad \underline{\overset{-}{x^2} \overset{+}{-} 2x} \\
\qquad\qquad\quad -28x + 56 \\
\qquad\qquad\quad \underline{\overset{+}{-}28x \overset{-}{+} 56} \\
\qquad\qquad\qquad\quad \times
\end{array}
$$

2. $p(x) = ax^3 + 3x^2 + bx - 12$ has a factor of $(2x+1)$. When $p(x)$ is divided by $(x-3)$ the remainder is 105

(a) Find the value of a and of b. ---[5]

(b) Using your values of a and b, write $p(x)$ as a product of $(2x+1)$ and a quadratic factor. [M-20/12/Q7]-- [2]

Solution (a) $p(x) = ax^3 + 3x^2 + bx - 12$ —— ①

$(2x+1)$ is a factor of $p(x) \Rightarrow p(-\frac{1}{2}) = 0$

$\Rightarrow a(-\frac{1}{2})^3 + 3(-\frac{1}{2})^2 + b(-\frac{1}{2}) - 12 = 0$

$\Rightarrow -\frac{a}{8} + \frac{3}{4} - \frac{b}{2} - 12 = 0$

$\Rightarrow a + 4b = -90$ —— ②

when $p(x)$ is divided by $(x-3)$ remainder is $105 \Rightarrow R = p(3) = 105$

from ① $p(3) = a \cdot 3^3 + 3 \times 3^2 + b \times 3 - 12 = 105$

$\Rightarrow 27a + 3b = 90$

$\Rightarrow 9a + b = 30$ —— ③

Solving ② & ③ $\underline{a = 6,\ b = -24}$ ✓

(b) for $a = 6$ and $b = -24$ from ①

$p(x) = 6x^3 + 3x^2 - 24x - 12$ —— ④

$(2x+1)$ is a factor of $p(x)$

$$
\begin{array}{l}
2x+1\overline{)\,6x^3 + 3x^2 - 24x - 12\,}(3x^2 - 12 \\
\quad\ \ \underline{\overset{-}{6x^3} \overset{-}{+} 3x^2} \\
\qquad\qquad -24x - 12 \\
\qquad\qquad \underline{\overset{+}{-}24x \overset{+}{-} 12} \\
\qquad\qquad\qquad \times
\end{array}
$$

$\therefore p(x) = \underline{(2x+1)(3x^2 - 12)}$ ✓

3. $p(x) = 15x^3 + 22x^2 - 15x + 2$

(a) Find the remainder when $p(x)$ is divided by $(x+1)$ --[2]

(b) (i) Show that $x+2$ is a factor of $p(x)$ --[1]

(ii) Write $p(x)$ as a product of linear factors. S-20/21/Q3 --[3]

Solution: $p(x) = 15x^3 + 22x^2 - 15x + 2$ —— ①

when $p(x)$ is divided by $(x+1)$

Remainder $= p(-1)$

from ① $R = 15\times(-1)^3 + 22(-1)^2 - 15(-1) + 2$

$= -15 + 22 + 15 + 2$

$\therefore R = 24$ ✓

b(i) for $(x+2)$ a factor of $p(x)$ consider

$p(-2) = 15(-2)^3 + 22(-2)^2 - 15(-2) + 2$

$= -120 + 88 + 30 + 2 = 0$

$\therefore (x+2)$ is a factor of $p(x)$ ✓

(b)(ii)

$p(x) = 15x^3 + 22x^2 - 15x + 2$ —— ①

as $(x+2)$ is a factor of $p(x)$

$$
\begin{array}{r|l}
x+2 & 15x^3 + 22x^2 - 15x + 2 \;\big(\; 15x^2 - 8x + 1 \\
 & \underline{15x^3 + 30x^2} \\
 & -8x^2 - 15x \\
 & \underline{\mp 8x^2 \mp 16x} \\
 & x + 2 \\
 & \underline{x+2} \\
 & \times
\end{array}
$$

$\therefore p(x) = (x+2)(15x^2 - 8x + 1)$

$= (x+2)(3x-1)(5x-1)$ ✓

4. $p(x) = 6x^3 + ax^2 + 12x + b$, where a and b are integers.

$p(x)$ has a remainder of 11 when divided by $x-3$ and a remainder of -21 when divided by $x+1$.

(a) Given that $p(x) = (x-2)Q(x)$, find $Q(x)$, a quadratic factor with numerical coefficients. --[6]

(b) Hence solve $p(x) = 0$ S-20/13/Q5 --[2]

Solution: $p(x) = 6x^3 + ax^2 + 12x + b$ --①

when divided by $(x-3)$, remainder

$R = P(3) = 6\times3^3 + a\times3^2 + 12\times3 + b = 11$

$\Rightarrow 9a + b = -187$ —— ②

and when $p(x)$ divided by $(x+1)$,

remainder $R = P(-1) = 6(-1)^3 + a(-1)^2 + 12\cdot(-1) + b = -21$

$\Rightarrow a + b = -3$ —— ③

Solving ② & ③ $a = -23$, $b = 20$

hence from ① $p(x) = 6x^3 - 23x^2 + 12x + 20$ —— ④

(a) $(x-2)$ is a factor of $P(x)$

$p(x) = (x-2)(6x^2 - 11x - 10)$

$$
\begin{array}{r|l}
(x-2) & 6x^3 - 23x^2 + 12x + 20 \;\big(\; 6x^2 - 11x - 10 \\
 & \underline{6x^3 - 12x^2} \\
 & -11x^2 + 12x \\
 & \underline{-11x^2 + 22x} \\
 & -10x + 20 \\
 & \underline{-10x + 20} \\
 & \times
\end{array}
$$

6(b) $p(x) = (x-2)(6x^2 - 11x - 10) = 0$

$= (x-2)(3x+2)(2x-5) = 0$

$\therefore x = 2, -\frac{2}{3}, \frac{5}{2}$ ✓

5. $p(x) = 2x^3 - 3x^2 - 23x + 12$

(a) Find the value of $p\left(\frac{1}{2}\right)$ ---[1]

(b) Write $p(x)$ as the product of three linear factors and hence solve $p(x) = 0$ [W-20/21/Q7] --[5]

Solution: $p(x) = 2x^3 - 3x^2 - 23x + 12$ —①

(a) $p\left(\frac{1}{2}\right) = 2\times\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)^2 - 23\times\frac{1}{2} + 12$

$= \frac{1}{4} - \frac{3}{4} - \frac{23}{2} + 12 = 0$ ✓

(b) $p\left(\frac{1}{2}\right) = 0 \Rightarrow \left(x - \frac{1}{2}\right)$ is a factor of $p(x)$

or $(2x-1)$ is a factor $p(x)$

$2x-1\overline{)2x^3 - 3x^2 - 23x + 12}\left(x^2 - x - 12\right.$

$\underline{-2x^3 \mp x^2}$

$-2x^2 - 23x$

$\underline{\pm 2x^2 \mp x}$

$-24x + 12$

$\underline{\mp 24x \pm 12}$

$\times\times\times$

Hence $p(x) = (2x-1)(x^2 - x - 12)$

$= (2x-1)(x-4)(x+3)$

Now $p(x) = (2x-1)(x-4)(x+3) = 0$

$\Rightarrow x = \frac{1}{2}, 4, -3$ ✓

---

6. The polynomial $p(x) = 6x^3 + ax^2 + bx + 2$, where a and b are integers, has a factor of $(x-2)$.

(a) Given that $p(1) = -2p(0)$, find the value of a and of b ---[4]

(b) Using your values of a and b.

(i) Find the remainder when $p(x)$ is divided by $2x-1$ --[2]

(ii) Factorise $p(x)$. [W-20/12/Q10] --[2]

Solution: $p(x) = 6x^3 + ax^2 + bx + 2$ —①

$(x-2)$ is a factor of $p(x) \Rightarrow p(2) = 0$

$\Rightarrow 6\times 2^3 + a\times 2^2 + b\times 2 + 2 = 0$

$\rightarrow 2a + b = -25$ —②

(a) Now $p(1) = -2p(0)$

$\Rightarrow 6 + a + b + 2 = -2\times 2$

$\Rightarrow a + b = -12$ —③

Solving ② & ③ $a = -13, b = 1$

$\therefore p(x) = 6x^3 - 13x^2 + x + 2$ —④

(b)(i) when $p(x)$ is divided by $(2x-1)$.

Remainder $R = p\left(+\frac{1}{2}\right)$

from ④ $= 6\left(+\frac{1}{2}\right)^3 - 13\left(+\frac{1}{2}\right)^2 + \frac{1}{2} + 2$

$= 0$ ✓

b(ii) $(x-2)$ is a factor of $p(x)$

$x-2\overline{)6x^3 - 13x^2 + x + 2}\left(6x^2 - x - 1\right.$

$\underline{-6x^3 \mp 12x^2}$

$-x^2 + x$

$\underline{\pm x^2 \mp 2x}$

$-x + 2$

$\underline{\mp x \pm 2}$

$\times$

$\therefore p(x) = (x-2)(6x^2 - x - 1)$

$= (x-2)(2x-1)(3x+1)$

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7. The polynomial $p(x) = ax^3 + bx^2 - 19x + 4$, where a and b are constants, has a factor $x+4$ and is such that $2p(1) = 5 \cdot p(0)$

(a) Show that $p(x) = (x+4)(Ax^2+bx+C)$, where A, B and C are the integers to be found. --[6]

(b) Hence factorise $p(x)$ --[1]

(c) Find the remainder when $p'(x)$ is divided by x. ---[1]

W-20/13/Q7

Solution: $p(x) = ax^3 + bx^2 - 19x + 4$ —— ①

$(x+4)$ is a factor of $p(x) \Rightarrow p(-4) = 0$

$\Rightarrow a(-4)^3 + b(-4)^2 - 19(-4) + 4 = 0$

$-64a + 16b + 76 + 4 = 0$

$\Rightarrow -4a + b = -5$ —— ②

Also $2 \cdot p(1) = 5 \cdot p(0)$

$\Rightarrow 2[a + b - 19 + 4] = 5 \times 4$

$2a + 2b = 50$

$\Rightarrow a + b = +25$ —— ③

Sub ② from ③ $\Rightarrow 5a = 30 \Rightarrow \underline{a = 6}$ ✓

from ③ $6 + b = 25$

$\underline{b = 19}$ ✓

(a) Now $p(x) = 6x^3 + 19x^2 - 19x + 4$

$(x+4)$ is a factor of $p(x)$

$$
\begin{array}{r}
6x^2 - 5x + 1 \\
x+4 \,\big)\, 6x^3 + 19x^2 - 19x + 4 \\
-\;6x^3 + 24x^2 \\
\hline
-5x^2 - 19x \\
-5x^2 - 20x \\
\hline
x + 4 \\
x + 4 \\
\hline
\times
\end{array}
$$

$\therefore p(x) = (x+4)[6x^2 - 5x + 1]$

(b) $(x+4)[6x^2 - 3x - 2x + 1]$

$(x+4)[3x(2x-1) - 1(2x-1)]$

$(x+4)(2x-1)(3x-1)$ ✓

(c) $p(x) = 6x^3 + 19x^2 - 19x + 4$

$\Rightarrow p'(x) = 18x^2 + 38x - 19$ —— ③

when $p'(x)$ is divided by x

Remainder $= p'(0) = -19$ [from ③

$\underline{R = -19}$ ✓

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