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0606

Additional Maths

Indices and Surds
Revision

SP-20	M-20	S-20	W-20

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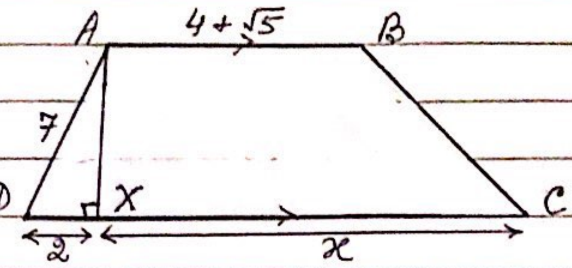
Indices and Surds

Revision

1 The diagram shows a trapezium ABCD in which $AD = 7$ cm and $AB = (4+\sqrt{5})$ cm. AX is perpendicular to DC with $DX = 2$ cm and $XC = x$ cm. Given that the area of trapezium ABCD is $15(\sqrt{5}+2)$ cm², obtain an expression for x in the form $a+b\sqrt{5}$, where a & b are integers.



SP-20/01/Q4 --- [6]

Solution: In rt $\triangle ADX$; $AX^2 + 2^2 = 7^2$ (∵ Pythagoras Theorem)

$\Rightarrow AX^2 = 49 - 4 = 45$

$AX = \sqrt{45} = 3\sqrt{5}$ — ①

Area of Trapezium $= \frac{1}{2}(AB + DC) \times AX$

$= \frac{1}{2}(4+\sqrt{5} + 2 + x) \times 3\sqrt{5}$ (from ①)

$= \frac{1}{2}(6+\sqrt{5}+x) \cdot 3\sqrt{5}$

or $= \frac{1}{2}[18\sqrt{5} + 15 + 3\sqrt{5}x] = 15(\sqrt{5}+2)$ (Given)

$18\sqrt{5} + 15 + 3\sqrt{5}x = 2 \times 15(\sqrt{5}+2)$

$\Rightarrow 18\sqrt{5} + 15 + 3\sqrt{5}x = 30\sqrt{5} + 60$

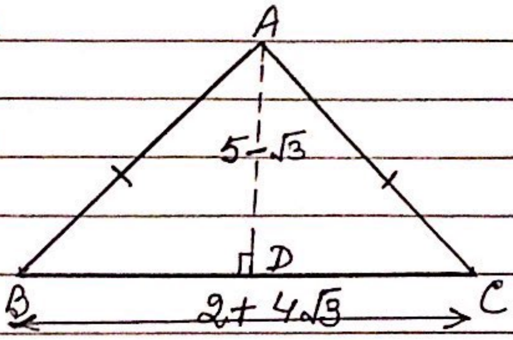
$\Rightarrow 3\sqrt{5}x = 12\sqrt{5} + 45$

$\Rightarrow x = \frac{12\sqrt{5}}{3\sqrt{5}} + \frac{45}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$ [∵ $\sqrt{5} \cdot \sqrt{5} = 5$]

$= 4 + \frac{45\sqrt{5}}{15}$

$x = 4 + 3\sqrt{5}$ ✓

2 In this question all lengths are in centimetres.
The diagram shows the isosceles triangle ABC, where AB = AC and $BC = 2+4\sqrt{3}$. The height AD, of the triangle is $5-\sqrt{3}$.



(a) Find the area of the triangle ABC, giving your answer in the form $a+b\sqrt{3}$, where a and b are integers. ---[2]

(b) Find tan ABC, giving your answer in the form $c+d\sqrt{3}$, where c and d are integers. ---[3]

(c) Find $\sec^2 ABC$, giving your answer in the form $e+f\sqrt{3}$, where e and f are integers. ---[2]

$\boxed{M\text{-}20/12/Q5}$

Solution (a) Area of Triangle $ABC = \frac{1}{2} BC \times AD$

$$= \frac{1}{2}(2+4\sqrt{3})(5-\sqrt{3})$$
$$= \frac{1}{2}[10-2\sqrt{3}+20\sqrt{3}-12]$$
$$= \frac{1}{2}(18\sqrt{3}-2)$$
$$= (9\sqrt{3}-1)$$

(b) $\tan ABC = \frac{AD}{BD} = \frac{5-\sqrt{3}}{1+2\sqrt{3}}$ $\left[\because BD = \frac{1}{2}BC\right]$

$$= \frac{(5-\sqrt{3})}{(1+2\sqrt{3})} \times \frac{(1-2\sqrt{3})}{(1-2\sqrt{3})}$$
$$= \frac{5-10\sqrt{3}-\sqrt{3}+6}{1-12}$$
$$= \frac{11-11\sqrt{3}}{-11} = (\sqrt{3}-1)$$

(c) $\sec^2 ABC = 1+\tan^2 ABC$

$$= 1+(\sqrt{3}-1)^2$$
$$= 1+3-2\sqrt{3}+1$$
$$= (5-2\sqrt{3})$$

3. Find the exact solution of $3^{2x} - 3^{x+1} - 4 = 0$ --[4]

[M-20/22/Q3]

Solution: $3^{2x} - 3^{x+1} - 4 = 0$

$\Rightarrow (3^x)^2 - 3 \cdot 3^x - 4 = 0$

$y^2 - 3y - 4 = 0$ (Let $y = 3^x$)

$y^2 - 4y + y - 4 = 0$

$y(y-4) + 1(y-4) = 0$

$(y-4)(y+1) = 0$

$y = 4$, $y = -1$

$\Rightarrow 3^x = 4$; $3^x = -1$ ✗

$x = \log_3 4$ ✓

4. Find the positive solution of the equation:

$(5+4\sqrt{7})x^2 + (4-2\sqrt{7})x - 1 = 0$, giving your answer in the form $a+b\sqrt{7}$, where a and b are fractions in simplest form.

[S-20/11/Q4] --[5]

Solution: $(5+4\sqrt{7})x^2 + (4-2\sqrt{7})x - 1 = 0$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$b^2 - 4ac$
$= (4-2\sqrt{7})^2 - 4 \times (5+4\sqrt{7}) \times (-1)$
$= 16 + 28 - \cancel{16\sqrt{7}} + 20 + \cancel{16\sqrt{7}}$
$= 64$

$x = \dfrac{-(4-2\sqrt{7}) \pm \sqrt{64}}{2(5+4\sqrt{7})}$

$= \dfrac{-4 + 2\sqrt{7} + 8}{2(5+4\sqrt{7})}$; $x > 0$

$= \dfrac{4+2\sqrt{7}}{2(5+4\sqrt{7})} = \dfrac{(2+\sqrt{7})}{(5+4\sqrt{7})} \times \dfrac{(5-4\sqrt{7})}{(5-4\sqrt{7})}$

$= \dfrac{10 - 8\sqrt{7} + 5\sqrt{7} - 28}{25 - 112}$

$= \dfrac{-18 - 3\sqrt{7}}{-87}$

$= \dfrac{6+\sqrt{7}}{29}$

$= \dfrac{6}{29} + \dfrac{\sqrt{7}}{29}$ ✓

5. The point $(1-\sqrt{5}, p)$ lies on the curve $y = \frac{10+2\sqrt{5}}{x^2}$. Find the exact value of p, simplifying your answer ---[5]

S-20 | 22 | Q2

Solution: Curve; $y = \frac{10+2\sqrt{5}}{x^2}$ —①

Point $(1-\sqrt{5}, p)$ lies on ①

$$\Rightarrow p = \frac{10+2\sqrt{5}}{(1-\sqrt{5})^2} = 10+2\sqrt{5}$$

$$= \frac{10+2\sqrt{5}}{1+5-2\sqrt{5}} = \frac{10+2\sqrt{5}}{6-2\sqrt{5}}$$

$$= \frac{5+\sqrt{5}}{(3-\sqrt{5})} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$$

$$= \frac{15+5\sqrt{5}+3\sqrt{5}+5}{9-5}$$

$$= \frac{20+8\sqrt{5}}{4} = 5+2\sqrt{5} \checkmark$$

6. (a) Simplify $\frac{\sqrt{128}}{\sqrt{72}}$ ---[2]

(b) $\frac{1}{1+\sqrt{3}} - \frac{\sqrt{3}}{3+2\sqrt{3}}$, giving your answer as a fraction with an integer denominator. ---[4]

S-20 | 23 | Q5

Solution (a) $\frac{\sqrt{128}}{\sqrt{72}} = \frac{\sqrt{64\times2}}{\sqrt{36\times2}}$

$$= \frac{8\sqrt{2}}{6\sqrt{2}} = \frac{8}{6}$$

$$= 4/3 \checkmark$$

(b) $\frac{1}{1+\sqrt{3}} - \frac{\sqrt{3}}{3+2\sqrt{3}}$

$$= \frac{3+2\sqrt{3}-\sqrt{3}(1+\sqrt{3})}{(1+\sqrt{3})(3+2\sqrt{3})}$$

$$= \frac{\cancel{3}+2\sqrt{3}-\sqrt{3}-\cancel{3}}{3+2\sqrt{3}+3\sqrt{3}+6}$$

$$= \frac{\sqrt{3}}{9+5\sqrt{3}} \times \frac{9-5\sqrt{3}}{9-5\sqrt{3}} = \frac{9\sqrt{3}-15}{81-75} = \frac{9\sqrt{3}-15}{6} \checkmark$$

7(a) Write $\frac{\sqrt{p}\,(qr^2)^{1/3}}{(q^3p)^{-1}r^3}$ in the form $p^a q^b r^c$, where a, b, c are constants --[3]

(b) Solve $6x^{2/3} - 5x^{1/3} + 1 = 0$ | W-20 | 11 | Q3 | --[3]

Solution (a) $\frac{\sqrt{p}\,(qr^2)^{1/3}}{(q^3p)^{-1}r^3}$

$= p^{1/2} \cdot q^{1/3} \cdot r^{2/3} \times q^3 p^1 \cdot r^{-3}$

$= p^{\frac{1}{2}+1} q^{\frac{1}{3}+3} \cdot r^{2/3-3}$

$= \underline{p^{3/2} \cdot q^{10/3} \cdot r^{-7/3}}$ ✓

(b) Solve $6x^{2/3} - 5x^{1/3} + 1 = 0$

$6(x^{1/3})^2 - 5x^{1/3} + 1 = 0$

$= 6y^2 - 5y + 1 = 0$ Let $x^{1/3} = y$

$6y^2 - 3y - 2y + 1 = 0$

$3y(2y-1) - 1(2y-1) = 0$

$(2y-1)(3y-1) = 0$

$y = \frac{1}{2}$; $y = +\frac{1}{3}$

$\therefore x^{1/3} = \frac{1}{2}$, $x^{1/3} = +\frac{1}{3}$

$\Rightarrow \underline{x = 1/8 \; ; \; x = +\frac{1}{27}}$ ✓

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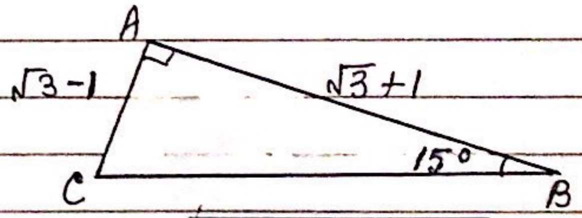
8 In the diagram $AC = \sqrt{3} - 1$, $AB = \sqrt{3} + 1$, angle $ABC = 15°$ and angle $CAB = 90°$,

(a) Show that $\tan 15° = 2 - \sqrt{3}$ ---[3]

(b) Find the exact length of BC, ---[2]



| W-20 | 21 | Q6 |

Solution (a) $\tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{3 - \sqrt{3} - \sqrt{3} + 1}{3-1}$

$= \frac{4 - 2\sqrt{3}}{2}$

$= \underline{(2 - \sqrt{3})}$ ✓

(b) In rt triangle CAB, using Pythagoras Theo.

$BC^2 = (\sqrt{3}-1)^2 + (\sqrt{3}+1)^2$

$= 3 + 1 - \cancel{2\sqrt{3}} + 3 + 1 + \cancel{2\sqrt{3}}$

$= 8$

$BC = \sqrt{8} = \underline{2\sqrt{2}}$ ✓

9. Find the value of x such that $\frac{4^{x+1}}{2^{x-1}} = 32^{x/3} \times 8^{1/3}$ ---[4]

[W-20/22/Q2]

Solution: $\frac{4^{x+1}}{2^{x-1}} = 32^{x/3} \cdot 8^{1/3}$

$\Rightarrow \frac{(2^2)^{x+1}}{2^{x-1}} = ((2)^5)^{x/3} \cdot (2^3)^{1/3}$

$\Rightarrow 2^{2x+2} \cdot 2^{-(x-1)} = 2^{5x/3} \cdot 2^1$

$\Rightarrow 2^{x+3} = 2^{5x/3+1}$

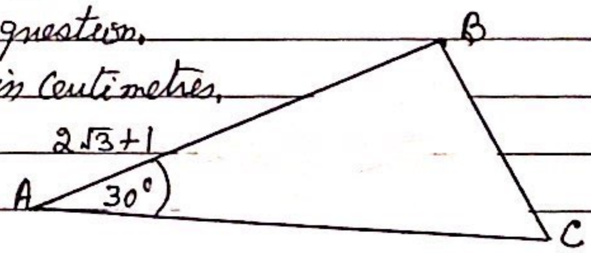
$\Rightarrow x+3 = \frac{5}{3}x+1 \Rightarrow \frac{5}{3}x - x = 2$

$\Rightarrow \frac{2}{3}x = 2 \Rightarrow \underline{x = 3}$ ✓

10. Do not use calculator in this question.

In this question all lengths are in centimetres.

(a) Given that the area of the triangle ABC is $5.5\,cm^2$, find the exact length of AC.



Write your answer in the form $a+b\sqrt{3}$, where a and b are integers. [4]

(b) Show that $BC^2 = c+d\sqrt{3}$, where c and d are the integers to be found. ---[4]

[W-20/22/Q8]

You may use:
$\sin 30° = \frac{1}{2}$
$\cos 30° = \frac{\sqrt{3}}{2}$
$\tan 30° = \frac{1}{\sqrt{3}}$

Solution (a) Area of triangle ABC:

$\frac{1}{2} AB \times AC \times \sin 30 = 5.5$ (Given)

$= \frac{1}{2} \times (2\sqrt{3}+1) \cdot AC \times \frac{1}{2} = 5.5$

$\Rightarrow AC = \frac{5.5 \times 2 \times 2}{(2\sqrt{3}+1)}$

$= \frac{22}{(2\sqrt{3}+1)} \times \frac{2\sqrt{3}-1}{2\sqrt{3}-1}$

$= \frac{22(2\sqrt{3}-1)}{12-1}$

$= 2(2\sqrt{3}-1)$

$AC = \underline{4\sqrt{3}-2}$ ✓

(b) Using cosine rule

$BC^2 = AB^2 + AC^2 - 2AB \times AC \cos 30$

$= (2\sqrt{3}+1)^2 + (4\sqrt{3}-2)^2 - \cancel{2} \times (2\sqrt{3}+1)(4\sqrt{3}-2) \times \frac{\sqrt{3}}{\cancel{2}}$

$= 12+1+4\sqrt{3}+48+4-16\sqrt{3} - \sqrt{3}(24-\cancel{4\sqrt{3}}+\cancel{4\sqrt{3}}-2)$

$= 13+4\sqrt{3}+52-16\sqrt{3}-22\sqrt{3}$

$= 65-34\sqrt{3}$

$\therefore BC^2 = \underline{65-34\sqrt{3}}$

11 Solve the quadratic equation $(\sqrt{7}-2)x^2-4x+(\sqrt{7}+2)=0$, giving each of your answers in the form $a+b\sqrt{7}$, where a and b are constants.

W-20|23|Q 11 --[7]

Solution: To solve: $(\sqrt{7}-2)x^2-4x+(\sqrt{7}+2)=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\left\{\begin{aligned} &b^2-4ac\\ &=(-4)^2-4\cdot(\sqrt{7}-2)(\sqrt{7}+2)\\ &=16-4\times 3=4\end{aligned}\right.$$

$$=\frac{4\pm\sqrt{4}}{2(\sqrt{7}-2)}$$

$$=\frac{4\pm 2}{2(\sqrt{7}-2)}\times\frac{(\sqrt{7}+2)}{(\sqrt{7}+2)}$$

$$=\frac{\cancel{2}(2\pm 1)(\sqrt{7}+2)}{\cancel{2}(7-4)}$$

$$=\frac{3(\sqrt{7}+2)}{3};\quad\frac{1\cdot(\sqrt{7}+2)}{3}$$

$$=(\sqrt{7}+2);\quad\frac{1}{3}(\sqrt{7}+2)$$

$$x=2+\sqrt{7},\quad\frac{2}{3}+\frac{1}{3}\sqrt{7}\ \checkmark$$