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0606

Additional Maths

Logarithmic and Exponential Functions Revision

SP-20	M-20	S-20	W-20

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Logarithmic and Exponential Functions
Revision

1(a)(i) Sketch the graph of $y = e^{x} - 5$, showing the exact coordinates of any points where the graph cuts the coordinates axes. ---[3]

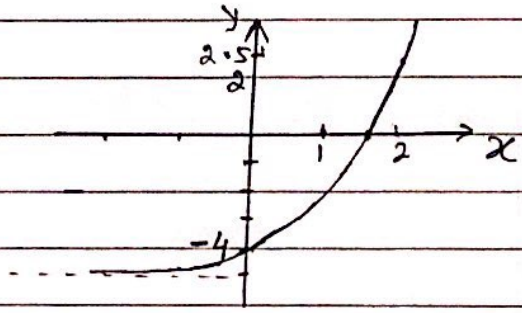
(ii) Find the range of values of k for which the equation $e^{x} - 5 = k$ has no solution. ---[1]

(b) Simplify $\log_a \sqrt{2} + \log_a 8 + \log_a \left(\frac{1}{2}\right)$, giving your answer in the form $p \log_a 2$, where p is a constant. ---[2]

(c) Solve the equation $\log_3 x - \log_9 4x = 1$ ---[4]

Solution (a) (i)

x		-2	0	1	$\ln 5$ (1.6)	2
y		-4.9	-4	1.7	0	2.4



(ii) $k \leq -5$ ✓

(b) $\log_a \sqrt{2} + \log_a 8 + \log_a \frac{1}{2}$

$= \log_a 2^{1/2} + \log_a 2^3 + \log_a 2^{-1} \quad = \frac{1}{2}\log_a 2 + 3\log_a 2 - \log_a 2$

$= 2\frac{1}{2} \cdot \log_a 2$ ✓

(c) Solve; $\log_3 x - \log_9 4x = 1$

$\Rightarrow \log_3 x - \dfrac{\log_3 4x}{\log_3 9} = \log_3 3$ $\quad [\log_3 9 = \log_3 3^2 = 2$

$\Rightarrow \log_3 x - \frac{1}{2}[\log_3 x + \log_3 4] = \log_3 3$

$\Rightarrow \log_3 x - \frac{1}{2}\log_3 x - \frac{1}{2} \cdot \log_3 2^2 = \log_3 3$

$\Rightarrow \frac{1}{2}\log_3 x = (\log_3 3 + \log_3 2)$

$\log_3 x = 2\log 6 = \log_3 6^2$

$\Rightarrow x = 36$ ✓

2(a) Solve the equation $\frac{9^{5x}}{27^{x-2}} = 243$ ---[3]

(b) $\log_a \sqrt{b} - \frac{1}{2} = \log_b a$, where $a>0$ and $b>0$ --[5]

Solve the equation for b, giving your answer interms of a

[S-20/22/Q9]

Solution (a) $\frac{9^{5x}}{27^{x-2}} = 243$

$\Rightarrow \frac{(3^2)^{5x}}{(3^3)^{x-2}} = 3^5$

$\Rightarrow \frac{3^{10x}}{3^{(3x-6)}} = 3^5$

$\Rightarrow 3^{10x-(3x-6)} = 3^5$

$\Rightarrow 10x - 3x + 6 = 5$

$7x = -1 \Rightarrow x = -\frac{1}{7}$ ✓

(b) $\log_a \sqrt{b} - \frac{1}{2} = \log_b a$

$\Rightarrow \log_a b^{1/2} - \frac{1}{2} = \frac{1}{\log_a b}$

$\Rightarrow \frac{1}{2}\log_a b - \frac{1}{2} = \frac{1}{\log_a b}$ [Let $\log_a b = x$

$\frac{1}{2}(x-1) = \frac{1}{x}$

$\Rightarrow x^2 - x = 2$

$\Rightarrow x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\Rightarrow x = 2$; $x = -1$

$\Rightarrow \log_a b = 2$; $\log_a b = -1$

$\Rightarrow b = a^2$ ✓ ; $b = a^{-1} = \frac{1}{a}$ ✓

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3 (a) Given that $\log_2 x + 2\log_4 y = 8$ Find the value of xy ---[3]

(b) Using substitution, $y = 2^x$, solve $2^{2x+1} - 2^{x+1} - 2^x + 1 = 0$ --[4]

[S-20/13/Q2]

Solution (a) $\log_2 x + 2\log_4 y = 8$

$\Rightarrow \log_2 x + 2 \cdot \frac{\log_2 y}{\log_2 4} = 8$

$\Rightarrow \log_2 x + \cancel{2} \cdot \frac{\log_2 y}{\cancel{2}} = 2^3$

$\Rightarrow \log_2 xy = 8$

$\Rightarrow xy = 2^8$

$xy = 256$ ✓

(b) $2^{2x+1} - 2^{x+1} - 2^x + 1 = 0$

$2^{2x} \times 2^1 - 2^x \cdot 2^1 - 2^x + 1 = 0$

$2 \cdot (2^x)^2 - 2 \cdot 2^x - 2^x + 1 = 0$

$2 \cdot (2^x)^2 - 3 \cdot 2^x + 1 = 0$

$2y^2 - 3y + 1 = 0$ [Put $2^x = y$

$2y^2 - 2y - y + 1 = 0$

$2y(y-1) - 1(y-1) = 0$

$(y-1)(2y-1) = 0 \Rightarrow y = 1,\ y = \frac{1}{2}$

$\Rightarrow 2^x = 1$, $2^x = 2^{-1}$ [$\because \frac{1}{2} = 2^{-1}$

$\Rightarrow x = 0$ ✓, $x = -1$ ✓ [$\because 2^0 = 1$

4 Write $3\lg x + 2 - \lg y$ as a single logarithm. ---[3]

[W-20/21/Q3]

Solution $3\lg x + 2 - \lg y$ $[\lg x = \log_{10} x$

$= \lg x^3 + \lg 100 - \lg y$ $[\because \quad 2 = 2\lg 10 = \lg 10^2 = \lg 100$

$= \lg \frac{100x^3}{y}$

5. The number, b, of bacteria in a sample is $b = P + Qe^{2t}$, where P and Q are constants and t is time in weeks. Initially there are 500 bacteria which increase to 600 after 1 weeks.

(a) Find the value of P and of Q. ---[4]

(b) Find the number of bacteria present after 2 weeks. ---[1]

(c) Find the first week in which the number of bacteria is greater than 1000000. --[3]

Solution (a) $b = P + Qe^{2t}$ —— ①

$t = 0,\ b = 500 \Rightarrow 500 = P + Qe^0$

$\Rightarrow P + Q = 500$ —— ②

also $t = 1,\ b = 600 \Rightarrow 600 = P + Qe^2$ —— ③

from ② $P = 500 - Q$ to ③

$\Rightarrow 600 = 500 - Q + Qe^2$

$\Rightarrow Q = \frac{100}{e^2 - 1} = 15.65$

$Q = 15.7$ ✓

$P = 500 - 15.7 = 484.3$ ✓

(b) $b = P + Qe^{2t}$

for $t = 2 \Rightarrow b = P + Qe^4$

$b = 484.3 + 15.7 \times 54.6$

$= 484.3 + 857 = 1341$ ✓

(c) $e^{2t} = \frac{b - P}{Q}$

$= \frac{1000000 - 484.3}{15.7}$

$e^{2t} = \frac{999515.7}{15.7} = 63663.42$

$\Rightarrow 2t = \ln 63663.42$

$2t = 11.06$

$t = 5.53$

$\therefore$ 6th week ✓

6 $\log_2(y+1) = 3 - 2\log_2 x$
and $\log_2(x+2) = 2 + \log_2 y$

(a) Show $x^3+6x^2-32=0$ ---[4]
(b) Find the roots of $x^3+6x^2-32=0$ ---[4]
(c) Give a reason why only one root is a valid solution of the logarithmic equations. Find the value of y corresponding to this root.
[W-20|23|Q8]--[2]

Solution: (a) Given $\log_2(y+1) = 3 - 2\log_2 x$ — (1)

$\Rightarrow \log_2(y+1) + \log_2 x^2 = 3$

$\Rightarrow \log_2(y+1)\cdot x^2 = 3$

$\Rightarrow x^2(y+1) = 2^3$

$x^2(y+1) = 8$ —— (2)

and $\log_2(x+2) = 2 + \log_2 y$ —— (3)

$\Rightarrow \log_2\frac{(x+2)}{y} = 2$

$\frac{x+2}{y} = 2^2$

$\Rightarrow x+2 = 4y$

$\Rightarrow y = \frac{(x+2)}{4}$ —— (4)

from (2) and (4)

$x^2\left(\frac{x+2}{4}+1\right) = 8$

$x^2(x+6) = 32$

$\Rightarrow x^3+6x^2-32=0$ ✓ —— (5)

(b) To Solve: $x^3+6x^2-32=0$ —— (5)

By hit and trial for $x=2$ in (5)

$2^3+6\times 2^2-32 = 8+24-32 = 0$

$\therefore$ 2 is a root of (5)

$\therefore (x-2)$ is a factor of

$p(x) = x^3+6x^2-32$

$$\begin{array}{l} x-2\,)\,x^3+6x^2-32\,(\,x^2+8x+16 \\ \quad -x^3 \mp 2x^2 \\ \hline \quad\quad 8x^2 \\ \quad\quad -8x^2 \mp 16x \\ \hline \quad\quad\quad 16x-32 \\ \quad\quad\quad 16x \mp 32 \\ \hline \end{array}$$

$\therefore$ (5) may be written as

$(x-2)(x^2+8x+16) = 0$

$(x-2)(x+4)^2 = 0$

$\therefore$ roots of (5) are

$2, -4$ and -4

(c) But $x=-4$ is not a valid root

as for $x=-4$, $\log_2(x+2)$ is not def.

as $\log_2(-4+2)$ is not defined

$\therefore$ only one root $x=2$ is valid

from (4) $y = \frac{2+2}{4} = 1 \Rightarrow y = 1$ ✓