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0606

Additional Maths

Permutations and Combinations

Revision

SP-20 | M-20 | S-20 | W-20

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Permutations and Combinations Revision

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1(a) A 6-character password is to be chosen from the following 9 characters:

letters	A	B	E	F
numbers	5	8	9	
symbols	*	$		

Each character may be used only once in any password.
Find the number of different 6-character passwords that may be chosen if

(i) there are no restrictions, ---[1]

(ii) the password consists of 2 letters, 2 numbers and 2 symbols in that order, ---[2]

(iii) the password must start and finish with a symbol. --[2]

SP-20|02|Q5

Solution (a) (i) ${}^9P_6 = 9\times8\times7\times6\times5\times4 = \underline{61480}$ ✓

(ii) ${}^4P_2 \times {}^3P_2 \times {}^2P_2 = (4\times3)\times(3\times2)\times(2\times1) = \underline{144}$ ✓

(iii) ⊗ - - - - ⊗ $= {}^7C_4 \times 2 = (7\times6\times5\times4)\times2 = \underline{1680}$ ✓

1(b) An examination consists of a section A, containing 10 short answered questions and a section B containing 5 long questions. Candidates are required to answer 6 questions from section A and 3 questions from section B.
Find the number of different selections of questions that can be made if,

(i) there are no further restrictions. --[2]

(ii) Candidates must answer the first 2 questions in section A and the first question in section B. --[2]

Solution:

(b)(i) ${}^{10}C_6 \times {}^5C_3 = 210\times10 = \underline{2100}$ ✓

(ii) ${}^8C_4 \times {}^4C_2 = 70\times6 = \underline{420}$ ✓

2.(a) (i) Find how many different 4-digit numbers can be formed using the digits 2, 3, 5, 7, 8 and 9, if each digit may be used only once in any number. --[1]

(ii) How many numbers found in part (i) are divisible by 5. --[1]

(iii) How many of the numbers found in part (i) are odd and greater than 7000? M-20|12|09 --[4]

Solution (a) Total number digits = 6 (2, 3, 5, 7, 8 and 9)

(i) each number has 4 diff. digits

∴ Number of such cases = $^6P_4$ = $6 \times 5 \times 4 \times 3 = 360$ ✓

(ii) For divisible by 5, digit 5 will be in the unit place

Rest 3 out of 5, $\underset{Th}{5} \times \underset{H}{4} \times \underset{T}{3} \times \underset{U}{1}$ = 60 ✓

(iii) Case I In thousand place digit = 8 → Unit place 3, 5, 7, 9 → $1 \times 4 \times 3 \times 4$ (Th H T U odd) = 48 ✓

(and Rest two place by remain 4 digit)

Case II Thousands place (7 or 9) → $2 \times 4 \times 3 \times 3$ (Th H T U) = 72 ✓

∴ Total = 48 + 72

= 120 ✓

2(b) The number of combinations of n items taken 3 at a time is $92n$. Find the value of n. --[4]

Solution: Combination of n items taken 3 at a time = $^nC_3$

$^nC_3 = 92n$ (given)

$\Rightarrow \frac{n(n-1)(n-2)}{3!} = 92n \Rightarrow (n-1)(n-2) = 92 \times 6$

$\Rightarrow n^2 - 3n + 2 = 552$

$\Rightarrow n^2 - 3n - 550 = 0$

$n^2 - 25n + 22n - 550 = 0$

$(n-25)(n+22) = 0$

$n = 25$, $n = -22$ ✗

∴ $n = 25$ ✓

3 (a) In an examination, candidates must select 2 questions from 5 questions in section A and select 4 questions from the 8 questions in section B. Find the number of ways in which this can be done. --[2]

(b) The digits of the number 6378129 are to be arranged so that the resulting 7-digit number is even. Find the number of ways in which this can be done. [S-20 | 21 | Q4] --[2]

Solution(a)

	Section A	Section B	
	5	8	
Selected	2	4	$= {}^5C_2 \times {}^8C_4 = 10 \times 70 = \underline{700}$ ✓

(b) 7 digits → 6, 3, 7, 8, 1, 2, 9

even digits → 6, 8, 2, (one of 3 in Unit Place)

∴ 7 digit even numbers = $\underbrace{6!}_{\text{Unit Pl}} \times 3 = 720 \times 3 = \underline{2160}$

4(a) (i) Find how many different 5 digit numbers can be formed using the digits 1, 2, 3, 5, 7 and 8, if each digit may be used only once in any number, --[1]

(ii) How many of the numbers found in part (i) are not divisible by 5. -[1]

(iii) How many numbers found in part (i) are even and greater than 30,000. [4]

(b) The number of combinations of n items taken 3 at a time is 6 times the number of combinations of n item taken 2 at a time. Find the value of the constant n. [S-20 | 12 | Q4] --[4]

Solution (a) (i) ${}^6P_5 = 6 \times 5 \times 4 \times 3 \times 2 = \underline{720}$ ✓

(ii) for divisible by 5, unit place digit is 5,

$\underbrace{5 \times 4 \times 3 \times 2}_{\text{(4 out of 5)}} \times \underbrace{1}_{U_5} = 120$

∴ Not divisible by 5 = 720 − 120 = 600 ✓

(iii) Starting with 8, $1 \times \underline{4} \times \underline{3} \times \underline{2} \times \underbrace{1}_{\text{Unit-(2)}} = 24$

Case I

Case II — Starting with 3, 5 or 7, $3 \times \underline{4} \times \underline{3} \times \underline{2} \times \underbrace{2}_{U_{(2 \text{ or } 8)}} = 144$

Total = 24 + 144 = 168 ✓

(b) Given, ${}^nC_3 = 6 \cdot {}^nC_2$

$\Rightarrow \frac{n(n-1)(n-2)}{3!} = 6 \times \frac{n(n-1)}{2!}$

$n(n-1)[n-2-18] = 0 \Rightarrow \underline{n = 20}$ ✓ $[n \neq 0 \text{ or } 1]$

5 (a) (i) Find how many different 5-digit numbers can be formed using five of the eight digits 1, 2, 3, 4, 5, 6, 7, 8 if each digit is used only once. [2]

(ii) Find how many of these 5-digit numbers are greater than 60000. --[2]

(b) A team of 3 people is to be selected from 4 men and 5 women. Find the number of different teams that could be selected which include at least 2 women.

[S-20/23/Q4] --[2]

Solution (a) (i) ${}^8P_5 = 8 \times 7 \times 6 \times 5 \times 4 = 6720$ ✓

(ii) 5 digit numbers greater than 60000 = $\underbrace{3}_{\text{(starting with 6,7,8)}} \times \underbrace{7 \times 6 \times 5 \times 4}_{\text{(any 4 out of 7)}} = 2520$ ✓

(b) 4 Men and 5 Women

Case I: 1 & 2 = ${}^4C_1 \times {}^5C_2 = 4 \times 10 = 40$

Case II: 0 & 3 = ${}^4C_0 \times {}^5C_3 = 1 \times 10 = 10$

Total = $40 + 10 = 50$ ✓

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6 (a) (i) Find how many different 4-digit numbers can be formed using the digits 1, 3, 4, 6, 7 and 9. Each digit may be used once only in any 4-digit number. --[1]

(ii) How many of these 4-digit numbers are even and greater than 6000. [3]

(b) A committee of 5 people is to be formed from 6 doctors, 4 dentists and 3 nurses. Find the number of different committees that could be formed if,

(i) there are no restrictions. --[1]

(ii) the committee contains at least one doctor. --[2]

(iii) the committee contains all the nurses. --[1]

[W-20/11/Q5]

Solution (a) (i) ${}^6P_4 = 6 \times 5 \times 4 \times 3 = 360$ ✓

(ii) Start with six → $1 \times 4 \times 3 \times 1 = 12$

Start with 7 or 9 → $2 \times 4 \times 3 \times 2 = 48$

Total = $12 + 48 = 60$ ✓

(b) (i) Doctors - 6, Dentists - 4, Nurses - 3

${}^{13}C_5 = \frac{13 \times 12 \times 11 \times 10 \times 9}{5!} = 1287$ ✓

(ii) No doctor = ${}^7C_5$

With atleast one doctor = $1287 - {}^7C_5 = 1266$ ✓

(iii) With all 3 nurses

Rest 2 out of rest 10 = ${}^{10}C_2 = 45$ ✓

7 (a) Find the number of ways in which 12 people can be put into 3 groups containing 3, 4 and 5 people respectively. ---[3]

(b) 4 digit numbers are to be formed using four of the digits 2, 3, 7, 8 and 9. Each digit may be used once only in any 4-digit number. Find how many 4-digit numbers can be formed if,

(i) there are no restrictions, ---[1]

(ii) the number is even. ---[1]

(iii) the number is greater than 7000 and odd. W-20/12/Q8/ ---[3]

Solution (a) ${}^{12}C_3 \times {}^{9}C_4 = 220 \times 126 = \underline{27720}$ ✓ [or may be ${}^{12}C_5 \times {}^{7}C_4$ or ${}^{12}C_4 \times {}^{8}C_5$,]

(b)(i) ${}^{5}P_4 = 5 \times 4 \times 3 \times 2 = \underline{120}$ ✓

(ii) $\underbrace{4 \times 3 \times 2}_{\text{(3 out of 4)}} \times \underbrace{2}_{\text{U (2 or 8)}} = \underline{48}$ ✓

(iii) Start with 9 or 7 → $\underbrace{2}_{\text{Th}} \times \underbrace{(3 \times 2)}_{\text{(3,7,9) - (at TH. Place)}} \times \underbrace{2}_{\text{U (odd)}} = 24$

Start with 8 → $\underbrace{1}_{\text{Th}} \times (3 \times 2) \times \underbrace{3}_{\text{U}} = 18$

Total = $24 + 18 = \underline{42}$ ✓

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8. A 4-digit code is to be formed using 4 different numbers selected from 1, 2, 3, 4, 5, 6, 7, 8 and 9. Find how many different codes can be formed if,

(a) there are no restrictions. ---[1]

(b) Only prime numbers are used. ---[1]

(c) two even numbers are followed by two odd numbers ---[2]

(d) the code forms an even number. ---[2]

W-20/23/Q6

Solution: (a) ${}^{9}P_4 = 9 \times 8 \times 7 \times 6 = \underline{3024}$ ✓

(b) ${}^{4}P_4 = 4 \times 3 \times 2 \times 1 = \underline{24}$ ✓ (Four → prime numbers 2, 3, 5, 7)

(c) ${}^{4}P_2 \times {}^{5}P_2 = 12 \times 20 = \underline{240}$ ✓

(d) even digits → 2, 4, 6, 8 → four

${}^{8}P_3 \times {}^{4}P_1 = \underbrace{(8 \times 7 \times 6)}_{\text{(Four even digits)}} \times 4$

$= (8 \times 7 \times 6) \times 4 = \underline{1344}$ ✓