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Additional Maths

Quadratic Functions Revisions

SP-20	M-20	S-20	W-20

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Quadratic Functions

Revision

1(a) Express $12x^2-6x+5$ in the form $p(x-q)^2+r$, where p, q and r are constant to be found. ---[3]

(b) Hence find the greatest value of $(12x^2-6x+5)^{-1}$ and state the value of x at which this occurs. [SP-20/01/Q3] ---[2]

Solution (a) $12x^2-6x+5 = 12\left(x^2-\frac{1}{2}x\right)+5$

$$=12\left[x^2-\frac{1}{2}x+\left(\frac{1}{4}\right)^2-\frac{1}{16}\right]+5$$

$$=12\times\left(x-\frac{1}{4}\right)^2-\frac{12}{16}+5$$

$$=12\left(x-\frac{1}{4}\right)^2+\frac{17}{4}$$ ✓

$p=12$, $q=\frac{1}{4}$, $r=\frac{17}{4}$

(b) $(12x^2-6x+5)^{-1} = \dfrac{1}{12x^2-6x+5}$

$$=\frac{1}{12\left(x-\frac{1}{4}\right)^2+\frac{17}{4}}$$

→ Expression will be greatest when denominator is minimum. for $x=\frac{1}{4}$

greatest value $=\dfrac{1}{0+\frac{17}{4}}=\dfrac{4}{17}$ at $x=\frac{1}{4}$ ✓

2. Find the value of k for which the line $y=kx+3$ is a tangent to the curve $y=2x^2+4x+k-1$ ---[5]

[M-20/12/Q2]

Solution: Line, $y=kx+3$ --- ①

Curve, $y=2x^2+4x+(k-1)$ -- ②

from ① & ②

$2x^2+4x+(k-1)=kx+3$

$\Rightarrow 2x^2+(4-k)x+(k-4)=0$ — ③

$b^2-4ac=(4-k)^2-4\times2(k-4)=0$

$\Rightarrow 16+k^2-8k-8k+32$

$\Rightarrow k^2-16k+48=0$

$k=12$ or $k=4$ ✓

for line ① be tangent to the curve ②, it should intersect the curve at exactly one point

③ should have only one

$b^2-4ac=0$

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3. Find the value of x for which $12x^2-20x+5<(2x+1)(x-1)$ --[4]

[M-20/22/Q1]

Solution: $12x^2-20x+5<(2x+1)(x-1)$

$\Rightarrow 12x^2-20x+5<2x^2-2x+x-1$

$\Rightarrow 10x^2-19x+6<0$

$(5x-2)(2x-3)<0$

$5\left(x-\frac{2}{5}\right)\cdot 2\left(x-\frac{3}{2}\right)<0$

$\left(x-\frac{2}{5}\right)\left(x-\frac{3}{2}\right)<0$

$\Rightarrow \quad \frac{2}{5}<x<\frac{3}{2}$ ✓

$10x^2-19x+6$

$10x^2-15x-4x+6$

$5x(2x-3)-2(2x-3)$

$(5x-2)(2x-3)$

Critical values

$\frac{2}{5}, \frac{3}{2}$

$\frac{2}{5}<3/2$

4.(a) Write $9x^2-12x+5$ in the form $p(x-q)^2+r$, where p, q and r are constants. --[3]

(b) Hence write down the coordinates of the minimum point of the curve $y=9x^2-12x+5$ --[1]

[S-20/21/Q2]

Solution (a) $9x^2-12x+5=9\left[x^2-\frac{12}{9}x\right]+5$

$=9\left[x^2-\frac{4}{3}x+\left(\frac{2}{3}\right)^2-\frac{4}{9}\right]+5$

$=9\left(x-\frac{2}{3}\right)^2-9\times\frac{4}{9}+5$

$=9\left(x-\frac{2}{3}\right)^2+1$ ✓

(b) $y=9x^2-12x+5$

from Part (a), $y=9\left(x-\frac{2}{3}\right)^2+1$ is minimum for $\left(x-\frac{2}{3}\right)=0$.

or $x=\frac{2}{3}$

$\therefore y=9x^2-12x+5$

$=9(x-2/3)^2+1$

$x=2/3,\ y=1$

$\left(\frac{2}{3}, 1\right)$ ✓

Minimum point is $\left(\frac{2}{3}, 1\right)$ ✓

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5. Find the value of k for which the line $y = kx - 7$ and the curve $y = 3x^2 + 8x + 5$ do not intersect. ---[6]

[S-20/21/Q6]

Solution: Line: $y = kx - 7$ —— ①
Curve: $y = 3x^2 + 8x + 5$ —— ②
} ① & ② do not intersect if there is no common root.

from ① & ②

$3x^2 + 8x + 5 = kx - 7$

$\Rightarrow 3x^2 + (8-k)x + 12 = 0$ [for no solution. $b^2 - 4ac < 0$

$b^2 - 4ac = (8-k)^2 - 4 \times 3 \times 12$

$= 64 + k^2 - 16k - 144 < 0$

$\Rightarrow k^2 - 16k - 80 < 0$

$(k-20)(k+4) < 0$

$\Rightarrow \underline{-4 < k < 20}$ ✓ ($\because$ Critical values are $-4, 20$)

6(a) Write $2x^2 + 3x - 4$ in the form $a(x+b)^2 + c$, where a, b and c are constants. ---[3]

(b) Hence write down the coordinates of the stationary point on the curve $y = 2x^2 + 3x - 4$ ---[2]

(c) Sketch the graph of $y = |2x^2 + 3x - 4|$. Showing the exact values of the intercepts of the curve with the coordinate axes. ---[3]

(d) Find the value of k for which $|2x^2 + 3x - 4| = k$ has exactly 3 values of x. ---[1]

[S-20/13/Q4]

Solution (a) $2x^2 + 3x - 4 = 2[x^2 + \frac{3}{2}x] - 4$

$= 2[x^2 + \frac{3}{2}x + (\frac{3}{4})^2 - \frac{9}{16}] - 4$

$= 2(x + \frac{3}{4})^2 - \frac{9}{8} - 4$

$= \underline{2(x + \frac{3}{4})^2 - \frac{41}{8}}$ ✓

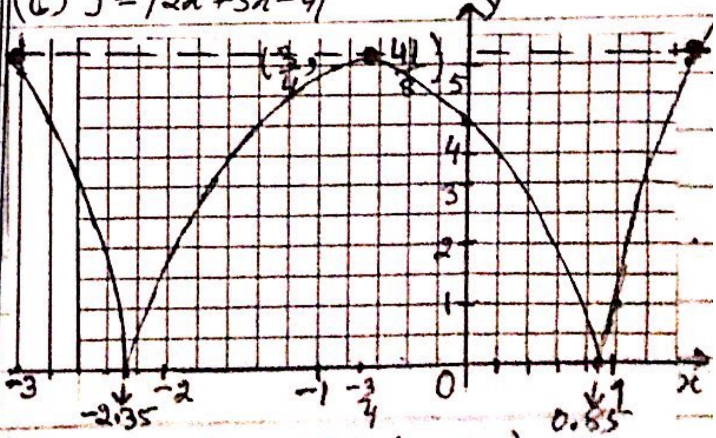
(b) Stationary point $(-\frac{3}{4}, -\frac{41}{8})$

(c) $2x^2 + 3x - 4 = 0$ for x-intercepts

$\frac{-3 \pm \sqrt{41}}{4} = 0.85, -2.35$

and y-int $= |-4| = 4$

(c) $y = |2x^2 + 3x - 4|$



(d) $k = \frac{41}{8}$ (3 Value of x)

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7. Find the set of real values of k for which $4x^2 - 4kx + 2k + 3 = 0$ has no real roots. ---[5]

[S-20 | 23 | Q2]

Solution: $4x^2 - 4kx + (2k+3) = 0$

has no real roots if $b^2 - 4ac < 0$

$(-4k)^2 - 4 \times 4 \times (2k+3) < 0$

$\therefore k^2 - 32k - 48 < 0$

$k^2 - 2k - 3 < 0$

$(k+1)(k-3) < 0$

$\underline{-1 < k < 3}$ ✓ [Critical values are $-1, 3$]

8. The curve $y = 2x^2 + k + 4$ intersects the straight line $y = (k+4)x$ at two distinct points. Find the possible values of k. ---[4]

[W-20 | 12 | Q1]

Solution: line : $y = (k+4)x$ —— ①

curve : $y = 2x^2 + (k+4)$ —— ②

for ① & ② to intersect

$2x^2 + k + 4 = (k+4)x$

$\Rightarrow 2x^2 - (k+4)x + (k+4) = 0$ —— ③

for ① & ② to intersect at 2 distinct points $b^2 - 4ac > 0$

$b^2 - 4ac = (-(k+4))^2 - 4 \times 2 \times (k+4) > 0$

$k^2 + 8k + 16 - 8k - 32 > 0$

$k^2 - 16 > 0$

$(k+4)(k-4) > 0$ (Critical Values are $-4, 4$)

$\therefore \underline{k < -4 \text{ or } k > 4}$ ✓

9. Solve the inequality $(x-8)(x-10) > 35$ --[4]

[W-20 | 22 | Q1]

Solution: $(x-8)(x-10) > 35$

$\Rightarrow x^2 - 18x + 80 > 35$

$\Rightarrow x^2 - 18x + 45 > 0$

$(x-3)(x-15) > 0$

$\underline{x < 3 \text{ or } x > 15}$ ✓ (Critical Values are $3, 15$)

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10. Find the values of k for which the equation $x^2+(k+9)x+9=0$ has two real distict roots. --[4]

[W-20/23| Q3]

Solution: $x^2+(k+9)x+9=0$

(for two distinct real roots, $b^2-4ac>0$)

$b^2-4ac = (k+9)^2-4\times1\times9>0$

$k^2+18k+81-36>0$

$k^2+18x+45>0$

$(k+15)(k+3)>0$ (Critical Value are $-3, -15$

$k<-15$ or $k>-3$ ✓