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Additional Maths.

Series - 1
Binomial Theorem
Revision.

SP-20	M-20	S-20	W-20

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1. In the expansion of $(1+2x)^n$, the coefficient of x^4 is ten times the coefficient of x^2. Find the value of positive integer n.
[SP-20/01/Q9] - [6]

Solution: $(1+2x)^n = {}^nC_0 + {}^nC_1(2x) + {}^nC_2(2x)^2 + \cdots + {}^nC_r(2x)^r + \cdots + {}^nC_n(2x)^n$

term containing $x^r = {}^nC_r(2x)^r = {}^nC_r \cdot 2^r \cdot x^r$

$\therefore$ Coefficient of $x^r = {}^nC_r \cdot 2^r$

$\therefore$ Coefficient of $x^4 = {}^nC_4 \cdot 2^4$

Coefficient of $x^2 = {}^nC_2\, x^2$

Given Coeff of $x^4 = 10\times$ Coeff of x^2

$$\Rightarrow {}^nC_4 \cdot 2^4 = {}^nC_2 \times 2^2 \times 10$$

$$\Rightarrow 4\times \frac{n(n-1)(n-2)(n-3)}{4\times3\times2\times1} = \frac{n(n-1)}{2\times1}\times 10$$

$$\Rightarrow (n-2)(n-3) = 30$$

$$n^2-5n+6=30$$

$$n^2-5n-24=0$$

$$(n-8)(n+3)=0$$

$$\underline{n=8}\ ✓, \quad n=-3^{\times} \quad \because n>0$$

2. The first three terms in the expansion of $(3-ax)^5$, in ascending powers of x, can be written in the form $b-81x+cx^2$. Find the value of each of a, b and c.
[M-20/12/Q3] --[5]

Solution: $(3-ax)^5 = {}^5C_0\, 3^5 + {}^5C_1\, 3^4(-ax)^1 + {}^5C_2\, 3^3(-ax)^2 + \cdots$

Given $= b-81x+cx^2$

Comparing the respective terms,

$b = {}^5C_0\, 3^5 = 1\times243 = 243 \Rightarrow \underline{b=243}$ ✓

${}^5C_1\times3^4\times(-ax) = -81x \Rightarrow -5\times81a = -81 \Rightarrow a=\frac{1}{5}$ ✓

and ${}^5C_2\cdot3^3(-ax)^2 = cx^2 \Rightarrow 10\times27a^2 = c \Rightarrow c = 10\times27\times\left(\frac{1}{5}\right)^2 = \frac{54}{5}$ ✓

$\therefore$ $\underline{b=243}$, $\underline{a=\frac{1}{5}}$ and $\underline{c=\frac{54}{5}}$ (or 10.8) ✓

3 (a) Expand $(2-x)^5$, simplifying each coefficient. --[3]

(b) Hence solve; $\frac{e^{(2-x)^5} \times e^{80x}}{e^{10x^4+32}} = e^{-x^5}$ --[4]

S-20/21/Q8

Solution (a) $(2-x)^5 = {}^5C_0 2^5 + {}^5C_1 2^4(-x) + {}^5C_2 2^3(-x)^2 + {}^5C_3 2^2(-x)^3 +$
$+ {}^5C_4 2 \times (-x)^4 + {}^5C_5 (-x)^5$
$= 32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5$ ✓ —— (1)

(b)
$$\frac{e^{(2-x)^5} \cdot e^{80x}}{e^{10x^4+32}} = e^{-x^5}$$
$$\Rightarrow e^{(2-x)^5 + 80x - (10x^4+32)} = e^{-x^5}$$
$$\Rightarrow (2-x)^5 + 80x - 10x^4 - 32 = -x^5$$
$$\Rightarrow \cancel{32} - \cancel{80x} + 80x^2 - 40x^3 + \cancel{10x^4} - x^5 + \cancel{80x} - \cancel{10x^4} - \cancel{32} = -x^5 \text{ (from 1)}$$
$$\Rightarrow 80x^2 - 40x^3 = 0$$
$$\Rightarrow 40x^2(2-x) = 0 \Rightarrow \underline{x=0}; \quad \underline{x=2} ✓$$

4. (a) Find the first three terms in the expansion of $\left(4 - \frac{x}{16}\right)^6$ in ascending powers of x. Give each term in its simplest form. --[3]

(b) Hence find the term independent of x in the expansion of,
$$\left(4 - \frac{x}{16}\right)^6 \cdot \left(x - \frac{1}{x}\right)^2$$ --[3]

S-20/12/Q3

Solution (a) $\left(4 - \frac{x}{16}\right)^6 = {}^6C_0 4^6 + {}^6C_1 4^5 \cdot \left(-\frac{x}{16}\right) + {}^6C_2 \cdot 4^4 \cdot \left(-\frac{x}{16}\right)^2 + \cdots$
$= 4096 - 384x + 15x^2 \cdots$ (1)

(b) $\left(4 - \frac{x}{16}\right)^6 \cdot \left(x - \frac{1}{x}\right)^2$

from (1)
$= (4096 - 384x + 15x^2)\left(x^2 - 2 + \frac{1}{x^2}\right)$

Term independent of x
$= (-2) \times 4096 + 15 \times 1$
$= 15 - 8192$
$= \underline{-8177}$ ✓

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5 (a) Find the term independent of x in the binomial expansion of $\left(3x-\frac{1}{x}\right)^6$ --[2]

(b) In the exansion of $\left(1+\frac{x}{2}\right)^n$ the coefficient of x^4 is half the coefficient of x^6. Find the value of the positive constant n. --[6]

[S-20|23|Q9]

Solution (a) General Term in the expansion of $\left(3x-\frac{1}{x}\right)^6$

General Term in $(x+a)^n = {}^nC_r x^{n-r} a^r$

$$= {}^6C_r (3x)^{6-r} \cdot \left(-\frac{1}{x}\right)^r$$

$$= {}^6C_r 3^{6-r} \cdot x^{6-r} \cdot (-1)^r x^{-r}$$

$$= (-1)^r \, {}^6C_r \cdot 3^{6-r} \cdot x^{6-2r} \quad \text{—(1)}$$

for the term independent of x, $6-2r=0$
$\Rightarrow r=3$

$\therefore$ from (1) the term independent of $x = (-1)^3 \cdot {}^6C_3 \, 3^3$

$$= -\frac{6\times5\times4}{3\times2\times1} \times 27 = \underline{-540}$$

(b) $\left(1+\frac{x}{2}\right)^n$ Gen. term $= {}^nC_r \left(\frac{x}{2}\right)^r$

$$= {}^nC_r \cdot \left(\frac{1}{2}\right)^r \cdot x^r$$

Coeff of $x^4 = {}^nC_4 \left(\frac{1}{2}\right)^4$ —(1)

and Coeff of $x^6 = {}^nC_6 \left(\frac{1}{2}\right)^6$ —(2)

Given ${}^nC_4 \left(\frac{1}{2}\right)^4 = \frac{1}{2} \times {}^nC_6 \left(\frac{1}{2}\right)^6$

$$\Rightarrow \frac{n(n-1)(n-2)(n-3)}{4\times3\times2\times1} = \frac{1}{8} \times \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{6!}$$

$$\Rightarrow 1 = \frac{1}{8} \times \frac{(n-4)(n-5)}{30}$$

$$\Rightarrow (n-4)(n-5) = 240$$

$$\Rightarrow n^2 - 9n - 220 = 0$$

$$(n-20)(n-11) = 0$$

$$\Rightarrow n = 20 \; ; \; n = -11 \quad (n>0)$$

$$\therefore \; n = \underline{20}$$

6 The first three terms in the expansion of $(a+bx)^5(1+x)$ are $32-208x+cx^2$. Find the value of each of the integers a, b and c. ---[7]

[W-20|21|Q5]

Solution: $(1+x)(a+bx)^5 = (1+x)[a^5 + {}^5C_1 a^4 bx + {}^5C_2 a^3 (bx)^2 + \dots]$

$= a^5 + 5a^4bx + 10a^3b^2x^2 +$
$+ a^5x + 5a^4b\,x^2 + \dots$

$= a^5 + (a^5+5a^4b)x + (10a^3b^2+5a^4b)x^2 + \dots$

$= 32 - 208x + cx^2$ (Given)

$\Rightarrow a^5 = 32 \rightarrow a = 2$ ✓

$a^5 + 5a^4b = -208 \Rightarrow 32 + 80b = -208$ (for $a=2$)

$\Rightarrow b = -3$ ✓

and $c = 10a^3b^2 + 5a^4b$

$c = 10\times 2^3(-3)^2 + 5\times 2^4\times(-3)$ $[a=2,\ b=-3]$

$c = 720 - 240 = 480$ ✓ $\Rightarrow \therefore a = 2$ ✓, $b = -3$ ✓, $c = 480$ ✓

7 Find the coefficient of x^2 in the expansion of $\left(x-\frac{3}{x}\right)\left(x+\frac{2}{x}\right)^5$ --- [5]

[W-20|12|Q5]

Solution: $\left(x+\frac{2}{x}\right)^5 = x^5 + {}^5C_1 x^4\cdot\left(\frac{2}{x}\right) + {}^5C_2 x^3\cdot\left(\frac{2}{x}\right)^2 + \dots$

$= x^5 + 10x^3 + 40x + \dots$

Now $\left(x-\frac{3}{x}\right)\left(x+\frac{2}{x}\right)^5 = \left(x-\frac{3}{x}\right)\left(x^5+10x^3+40x+\dots\right)$

term in $x^2 = x\times 40x - \frac{3}{x}\times 10x^3$

$\therefore$ coeff of $x^2 = 40 - 30 = 10$ ✓

8. Given that the coefficient of x^2 in the expansion of $(1+x)\left(1-\frac{x}{2}\right)^n$ is $\frac{25}{4}$, find the value of the positive integer n. [W-20|13|Q5] --[5]

Solution: $\left(1-\frac{x}{2}\right)^n = 1 + {}^nC_1\left(-\frac{x}{2}\right) + {}^nC_2\left(-\frac{x}{2}\right)^2 + \dots$

$= 1 - \frac{n}{2}x + \frac{n(n-1)}{8}\cdot x^2 + \dots$

$\therefore (1+x)\left(1-\frac{x}{2}\right)^n = (1+x)\left(1-\frac{n}{2}x+\frac{n(n-1)}{8}x^2+\dots\right)$

the term in $x^2 = \frac{n(n-1)}{8}x^2 - \frac{n}{2}x^2$

$\therefore$ Coefficient of x^2; $\frac{n(n-1)}{8} - \frac{n}{2} = \frac{25}{4}$ (Given)

$\Rightarrow \frac{n^2-n-4n}{8} = \frac{25}{4}$

$\Rightarrow n^2 - 5n - 50 = 0$

$(n-10)(n+5) = 0$

$n = 10,\ -5^{\times}$ $(n>0)$

$\therefore n = 10$ ✓