Date 08.02.21

0606

Additional Maths.

Simultaneous Equations Revision

| SP-20 | M-20 | S-20 | W-20 |

Suresh Goel
(Former Director)
Alliance World School,
Noida - Delhi. NCR.
INDIA

(+91 9810 444 804)

IG 0606

Simultaneous Equations

Revision

1 Solve $xy = 3$ and $x^4y^5 = 486$ --[3]

[SP-20/02/Q1]

Solution: $xy = 3$

$\Rightarrow x = \frac{3}{y}$ —— ①

$x^4 \cdot y^5 = 486$ —— ②

from ① and ② $\left(\frac{3}{y}\right)^4 \cdot y^5 = 486$

$\Rightarrow \frac{81}{y^4} \times y^5 = 486$

$\Rightarrow y = \frac{486}{81} = 6$

for $y=6$, from ① $x = \frac{3}{6} = \frac{1}{2}$

$\therefore$ $x = \frac{1}{2}$ and $y = 6$ ✓

---

2. Find the coordinates of the points of intersection of the curves $x^2 = 5y - 1$ and $y = x^2 - 2x + 1$ [M-20/22/Q7] --[5]

Solution: Curve (i) $x^2 = 5y - 1$ —— ①

Curve (ii) $y = x^2 - 2x + 1$ —— ②

from ① and ② $x^2 = 5(x^2 - 2x + 1) - 1$

$\Rightarrow x^2 = 5x^2 - 10x + 5 - 1$

$\Rightarrow 4x^2 - 10x + 4 = 0$

$2(2x^2 - 5x + 2) = 0$

$(2x-1)(x-2) = 0$

$\Rightarrow x = \frac{1}{2}$; $x = 2$

from ② $y = \frac{1}{4}$, $y = 1$

$\therefore$ Points of intersection are

$(0.5, 0.25)$; $(2, 1)$ ✓

3(a) Solve the simultaneous equations,

$10^{x+2y} = 5$ and $10^{3x+4y} = 50$ ---[4]

giving x and y in exact simplified form. [S-20/21/Q7]

Solution (a) $10^{x+2y} = 5 \Rightarrow x + 2y = \log 5$ —— ①

$10^{3x+4y} = 50 \Rightarrow 3x + 4y = \log 50$ —— ②

multiply eqn ① by 2, $2x + 4y = 2\log 5$ —— ③

subtract eqn ③ from ② $x = \log 50 - 2\log 5$

$\Rightarrow x = \log \frac{50}{25} = \log 2$ ✓

Put $x = \log 2$ in ①

$\log 2 + 2y = \log 5$

$\Rightarrow 2y = \log 5 - \log 2 = \log \frac{5}{2}$

$y = \frac{1}{2} \log \frac{5}{2}$

$\therefore$ Req. Soln. $x = \log 2$ & $y = \frac{1}{2}\log\frac{5}{2}$

(b) Solve $2x^{2/3} - x^{1/3} - 10 = 0$ —— ④

Let $x^{1/3} = y \Rightarrow x^{2/3} = y^2$

from eqn ④

$2y^2 - y - 10 = 0$

$\Rightarrow (y+2)(2y-5) = 0$

$y = -2$, $y = \frac{5}{2}$

$\Rightarrow x^{1/3} = -2$, $x^{1/3} = \frac{5}{2}$

$x = -8$; $x = \frac{125}{8}$ ✓

4 Find the value of k for which the line $y = x - 3$ intersects the curve $y = k^2x^2 + 5kx + 1$ at two distinct points. --[6]

[S-20/22/Q3]

Solution: $y = x - 3$ —— (1)

$y = k^2x^2 + 5kx + 1$ —— (2)

from (1) and (2)

$k^2x^2 + 5kx + 1 = x - 3$

$\Rightarrow k^2x^2 + (5k-1)x + 4 = 0$ —— (3)

(Now for line to intersect the quad. curve. $b^2 - 4AC > 0$)

for (3) $B^2 - 4AC$

$= (5k-1)^2 - 4k^2 \times 4$

$= 25k^2 - 10k - 16k^2 + 1$

$= 9k^2 - 10k + 1$ $(=0)$

$(9k-1)(k-1)$ $(=0)$ $\Rightarrow$

critical point $\frac{1}{9}, 1$

$\therefore$ $B^2 - 4AC > 0$

for $k < 1$ or $k > 0$ ✓

for two distinct points of intersection

5. Find the coordinates of the points of intersection of curve $x^2 + xy = 9$ and the line $y = \frac{2}{3}x - 2$ [W-20/21/Q2] --[5]

Solution: Line: $y = \frac{2}{3}x - 2$ —— (1)

Curve: $x^2 + xy = 9$ —— (2)

from (1) and (2)

$x^2 + x\left(\frac{2}{3}x - 2\right) = 9$

$\Rightarrow x^2 + \frac{2}{3}x^2 - 2x - 9 = 0$

$\Rightarrow 5x^2 - 6x - 27 = 0$

$(x-3)(5x+9 = 0$

$x = 3$; $x = -\frac{9}{5}$

fr (1) $y = 0$; $-\frac{16}{5}$

$\therefore$ Req. points $(3, 0)$, $\left(-\frac{9}{5}, -\frac{16}{5}\right)$ ✓

6. Solve the simultaneous equation.

$\log_3(x+y) = 2$ and $2\log_3(x+1) = \log_3(y+2)$ --[6]

[W-20/22/Q4]

Solution: $\log_3(x+y) = 2 \Rightarrow x+y = 3^2 \Rightarrow x+y = 9$ —— ①

$2\log_3(x+\ ) = \log_3(y+2) \Rightarrow \log_3(x+1)^2 = \log_3(y+2)$ —— ②

$\Rightarrow x^2+2x+1 = y+2$

$\Rightarrow x^2+2x+1 = 9-x+2$ [from ① $y = 9-x$]

$\Rightarrow x^2+3x-10=0$

$(x+5)(x-2)=0$

$x = -5 \times;\ 2$

from ① $y = \times;\ 7$

[for $x = -5$ In eqn ② $\log_3(x+1)$ is not defied]

$\therefore x = 2$ and $y = 7$ only.

---

7 Solve the simultaneous equations:

$x^2+3xy = 4$ —— ①

$2x+5y = 4$ —— ② --[5]

[W-20/23/Q2]

Solution: from ② $y = \frac{4-2x}{5}$ —— ③

from ① and ③

$x^2+3x\left(\frac{4-2x}{5}\right) = 4$

$\Rightarrow 5x^2+12x-6x^2 = 20$

$\Rightarrow x^2-12x+20=0$

$(x-2)(x-10)=0$

$x = 2\ ;\ x = 10$

from ③ $y = 0,\ y = -\frac{16}{5}$

$\therefore (2,0), (10, -\frac{16}{5})$

8. Solve the following equations:

$3^x \times 9^{y-1} = 243$ —— ① ....[5]

$8 \times 2^{y-\frac{1}{2}} = \frac{2^{2x+1}}{4\sqrt{2}}$ —— ②

[W-20/23/Q5]

Solution: from ① $3^x \cdot ((3)^2)^{y-1} = 3^5$

$\Rightarrow 3^{(x+2y-2)} = 3^5$

$\Rightarrow x+2y-2 = 5 \Rightarrow x+2y = 7$ —— ③

from Eqn ② $2^3 \cdot 2^{y-\frac{1}{2}} = \frac{2^{2x+1}}{2^{5/2}}$

$\Rightarrow 2^{3+y-\frac{1}{2}} = 2^{2x+1-5/2}$

$\Rightarrow y+5/2 = 2x-3/2 \Rightarrow 2x-y = 4$

$\Rightarrow 4x-2y = 8$ —— ④

add ③ & ④ $5x = 15 \Rightarrow x = 3$

$x = 3 \rightarrow$ from ③ $3+2y = 7 \Rightarrow y = 2$

$\therefore x = 3,\ y = 2$ ✓