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Additional Maths

Straight Line Graph
Revision

SP-20	M-20	S-20	W-20

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Straight line graph

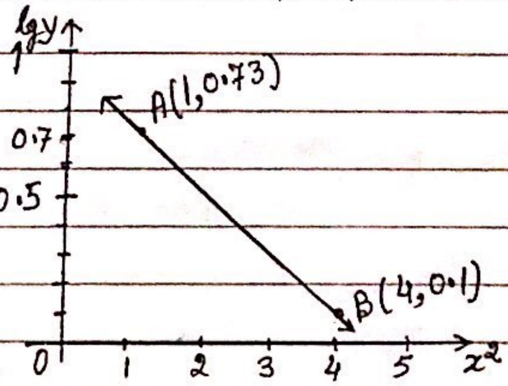
Revision

1 Variables x and y are such that when $\lg y$ is ploted against x^2, a straight line graph passing through the points (1, 0·73) and (4, 0·10) is obtained.

(a) Given that $y = Ab^{x^2}$, find the value of each of the constants A and b. --[4]

(b) Find the value of y when $x = 1·5$ --[2]

(c) Find the positive value of x when $y = 2$ $\boxed{SP\text{-}20/02/Q7}$ --[2]



Solution: Straight line graph, A(1, 0.73), B(4, 0.1)

Gradient of line $= \frac{0.1 - 0.73}{4-1} = -0.21$ ✓ —①

(a) Given $y = Ab^{x^2}$ ——— ②

$\Rightarrow \lg y = \lg(Ab^{x^2})$

$\Rightarrow \lg y = \lg b \cdot x^2 + \lg A$ —③ Represents line AB on Graph.

($y = mx + c$ form)

Gradient of line ③ from ① $m = \lg b = -0.21 \Rightarrow b = 10^{-0.21}$ [as $\lg b = \log_{10} b$]

$\Rightarrow b = \underline{0.617}$ ✓ (or 0.62) ✓

Now Y-int of line ③ $C = \lg A$

from ④ $\lg A = 0.94$

$\Rightarrow A = 10^{0.94} = \underline{8.91}$ ✓

Let the eqn of line AB is

$y = mx + c$

$y = -0.21x + c$

Passes through B(4, 0·1)

$\Rightarrow 0.1 = -0.21 \times 4 + c \Rightarrow c = 0.94$ —④ or Y-Int

(b) from ② $y = A \cdot b^{x^2}$

$\Rightarrow y = 8.71(0.617)^{x^2}$

when $x = 1·5 \Rightarrow y = 8.71 \times (0.617)^{2.25}$ (for $x = 1.5 \Rightarrow x^2 = 2.25$)

$y = 8.71 \times 0.3374 = \underline{2.93}$ ✓

(c) when $y = 2$, $x = ?$

from ③ $\lg y = \lg b \cdot x^2 + \lg A$

$\lg 2 = -0.21x^2 + 0.94$

$\Rightarrow 0.301 = -0.21x^2 + 0.94$

$\Rightarrow 0.21x^2 = 0.94 - 0.301$

$x^2 = \frac{0.639}{0.21} = 3.042$

$x = \sqrt{3.042}$

$x = \underline{1.74}$

2. The points A and B have coordinates $(-2,4)$ and $(6,10)$ respectively.

(a) Find the equation of the perpendicular bisector of the line AB, giving your answer in the form $ax+b+c=0$; a,b,c are constant. ---[4]

The point $C(5,p)$ lies on the perpendicular bisector of AB,

(b) Find the value of p. ---[1]

It is given that the line AB bisects the line CD.

(c) Find the coordinates of D. [M-20/12/Q6] ---[2]

Solution: Given $A(-2,4)$, $B(6,10)$

Mid point of AB, $M\left(\frac{-2+6}{2}, \frac{4+10}{2}\right) = (2,7)$

Gradient of AB, $m = \frac{10-4}{6+2} = \frac{6}{8}$

Grad. of line perp to AB $= -\frac{1}{m} = -\frac{8}{6} = -\frac{4}{3}$

∴ Equation of the perp. bisector of AB

$$y-7 = -\frac{4}{3}(x-2)$$

$$\Rightarrow \underline{4x+3y-29=0} \text{ — (1)}$$

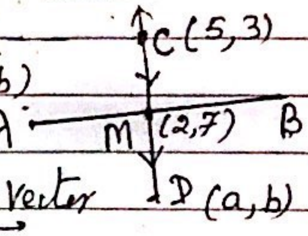
(b) $(5,p)$ lies on (1) perp. bisector

$4\times5+3p=29$

$\Rightarrow p=3$ ✓

∴ $C(5,p)$

(c) Let $D(a,b)$



Displacement Vector

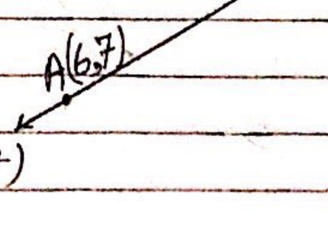
$\vec{CM} = \vec{MD}$

$$\Rightarrow \begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} a-2 \\ b-7 \end{pmatrix} \Rightarrow \begin{matrix} a=-1 \\ b=11 \end{matrix}$$

∴ $D(-1, 11)$ ✓

3. Variables x and y are such that, when $\lg y$ is plotted against x^3, a straight line graph through the points $(6,7)$ and $(10,9)$ is obtained. Find y as a function of x. [M-20/22/Q2] ---[4]

Solution: Gradient of line AB, $m = \frac{9-7}{10-6} = \frac{2}{4} = \frac{1}{2}$ ✓



Let Equ^n of line AB: $Y = mX + C$ — (1)

Passes through the point $A(6,7)$

$m=\frac{1}{2} \to$ from (1) $7 = \frac{1}{2}\times6 + C \Rightarrow C=4$ (Y-Intercept)

from (1) $Y = \frac{1}{2}X + 4$

Now for $\lg y$ and $x^3 \Rightarrow$ $\lg y = \frac{1}{2}x^3 + 4$

$$\Rightarrow y = 10^{\left(\frac{1}{2}x+4\right)} ✓$$

4\. The line $y = 5x + 6$ meets the curve $xy = 8$ at points A and B.

(a) Find the coordinates of A and B. --[3]

(b) Find the coordinates of the point where the perpendicular bisector of the line AB meets the line $y = x$. [S-20|11|Q6] --[5]

Solution (a) line: $y = 5x + 6$ —— ①

Curve: $xy = 8$ —— ②

from ① & ② $x(5x+6) = 8$

$5x^2 + 6x - 8 = 0$

$(x+2)(5x-4) = 0$

$x = -2, \quad x = \frac{4}{5}$

for ① $y = -4, \quad y = 10$

$\therefore$ $A(-2,-4)$ and $B(\frac{4}{5}, 10)$

(b) Mid point of AB, $M(-\frac{3}{5}, 3)$

Gradient of AB, $m = \frac{10+4}{\frac{4}{5}+2} = 5$

$\therefore$ Eqn of the perp. bisector of AB,

$y - 3 = -\frac{1}{5}(x + \frac{3}{5})$ —— ③ ✓

Given a line $y = x$ —— ④

Solving ③ & 4.

$x - 3 = -\frac{1}{5}(x + 3/5)$

$\Rightarrow x = \frac{12}{5}$, from ④ $y = \frac{12}{5}$

$\therefore$ The Point of intersection of the perp. bisector of AB and $y = x$ is $(\frac{12}{5}, \frac{12}{5})$

5\. Variables x and y are such that, when $\sqrt[4]{y}$ is plotted against $\frac{1}{x}$, a straight line graph passing through the points $(0.5, 9)$ and $(3, 34)$ is obtained. Find y as a function of x. --[4] [S-20|21|Q1]

Solution: Gradient of line $\overset{A}{(0.5, 9)}, \overset{B}{(3, 34)}$

$m = \frac{34-9}{3-0.5} = \frac{25}{2.5} = 10$ ✓

Let Equation of line AB is

$Y = mX + C$ —— ① Passes through the point $(3, 34)$

$\Rightarrow 34 = 10 \times 3 + C \Rightarrow C = 4$ ✓

$\therefore$ Equation line with $\sqrt[4]{y}$ and $\frac{1}{x}$

for ① $\sqrt[4]{y} = 10 \times \frac{1}{x} + 4$

$\Rightarrow \quad y = \left(\frac{10}{x} + 4\right)^4$ ✓

6. The points A and B are (4, 3) and (12, −7) respectively.

(a) Find the equation of the line L, the perpendicular bisector of line AB. ...[4]

(b) The line parallel to AB which passes through the point (5, 12), intersects L at the point C. Find the coordinates of C. ...[4]

S-20/22/Q5

Solution (a) Mid point of A(4, 3) and B(12, −7)

$M(8,-2)$ ✓

Gradient of AB, $m_{AB} = \frac{3+7}{4-12} = -\frac{5}{4}$ ✓

Grad of perp. $m_{\perp} = \frac{4}{5}$

∴ Equation L, the perp bisector of AB,

$y+2 = \frac{4}{5}(x-8)$

L: $y = \frac{4}{5}x - \frac{42}{5}$ ——— ①

(b) Eqn of line parallel to line AB

Passing through (5, 12)

$y-12 = -\frac{5}{4}(x-5)$ ——— ②

Solving ① and ②

$\frac{4}{5}x - \frac{42}{5} - 12 = \frac{5}{4}(x-5)$

$\frac{4}{5}x + \frac{5}{4}x = \frac{25}{4} + \frac{42}{5} + 12$

$\Rightarrow x = 13$

from ① $y = 2$

∴ $\underline{C(13,2)}$

7. Find the equation of the perpendicular bisector of the line joining the points (4, −7) and (−8, 9). S-20/23/Q1 ...[4]

Solution: $A(4,-7), B(-8,9) \Rightarrow$ Mid point of AB, $\left(\frac{4-8}{2}, \frac{-7+9}{2}\right) = (-2,1)$

Gradient of AB, $m_{AB} = \frac{9+7}{-8-4} = \frac{-16}{12} = -\frac{4}{3}$ ✓

∴ Gradient of perp. $m = \frac{3}{4}$

∴ Equation of the perp. bisector AB, $M(-2,1)$

$y-1 = \frac{3}{4}(x+2)$

$4y-4 = 3x+6$

or $\underline{3x-4y-10=0}$ ✓

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8. Variables x and y are connected by the relationship $y = Ax^n$, where A and n are constants.

(a) Transform the relation $y = Ax^n$ to straight ---[2]

when $\ln y$ is plotted against $\ln x$ a straight line graph passing through the points $(0, 0.5)$ and $(3.2, 1.7)$ is obtained.

(b) Find the value of n and of A. ---[4]

(c) Find the value of y when $x = 11$. [S-20/23/Q7] ---[2]

Solution (a) Given $y = Ax^n \Rightarrow \ln y = \ln(Ax^n)$

$\Rightarrow \ln y = \ln x^n + \ln A$

$\Rightarrow \ln y = n.\ln x + \ln A$ —①

(b) Gradient of line $A(0,0.5), B(3.2, 1.7)$

$m_{AB} = \frac{1.7-0.5}{3.2-0} = \frac{1.2}{3.2} = 3/8$

Equ^n of AB

$Y = mX + C \Rightarrow Y = \frac{3}{8}X + C$

Passes through $(0, 0.5) \Rightarrow .5 = \frac{3}{8} \times 0 + C$

$\Rightarrow$ Y-int $C = 0.5$

$\therefore Y = \frac{3}{8}X + 0.5$ —②

Comparing ① & ② $n = \frac{3}{8}$ ✓

and $\ln A = 0.5 \Rightarrow A = e^{0.5} = 1.6$ ✓

(C) from ①

$\ln y = \frac{3}{8}\ln x + \ln e^{0.5}$

$\Rightarrow y = e^{0.5}.x^{3/8}$ ✓

for $x = 11$

$y = e^{0.5}.(11)^{3/8} = 4.05$ ✓

9. The population P, in millions, of a country is given by $P = A.b^t$, where t is the number of years after January 2000 and A and b are constants. In January 2010 the population was 40 millions and had increased to 45 millions by January 2013. [W-20/21/Q8]

(a) Show that $b = 1.04$ to 2 decimal places and find A to the nearest Integer. ---[4]

(b) Find the population in January 2020, giving your answers to nearest million. [1]

(c) In January of which year will the population be over 100 millions for first time. [3]

Solution (a) $A \cdot b^{10} = 40$ —①

and $Ab^{13} = 45$ —②

from ① & ② $b^3 = \frac{45}{40} \Rightarrow b = 1.04$ ✓

and $A = 27$ ✓

(b) Population in 2020 $= A.b^{20}$

$= 27 \times (1.04)^{20}$

$= 59$ ✓

(C) $P = Ab^t \Rightarrow 100 = 27(1.04)^t$

$\Rightarrow (1.04)^t = \frac{100}{27}$

$\Rightarrow t = \log\left(\frac{100}{27}\right) / \log 1.04$

$= 33.4 \rightarrow$ Year 2034

10. (a) Find the equation of the perp. bisector of the line joining the points (12,1) and (4,3), giving your answer in form $y = mx + c$. --[5]

(b) The perp. bisector cuts the axes at points A and B. Find the length AB. [W-20/22/Q3] ---[3]

Solution (a) Given points $P(12,1)$, $Q(4,3)$

Mid point of PQ, $M(8,2)$ ✓

Gradient of PQ, $m_{PQ} = \frac{3-1}{4-12} = \frac{2}{-8} = \frac{-1}{4}$

$\therefore$ Grad. of perp $= 4$ ✓

$\therefore$ Equation of perp bisector

$M(8,2) \Rightarrow y - 2 = 4(x-8)$

or $\underline{y = 4x - 30}$ ✓ — ①

(b) perp. bisector $y = 4x - 30$ — ①

cuts x-axis at A, $y = 0$

$0 = 4x - 30 \Rightarrow x = 7.5$

$\therefore A(7.5, 0)$ ✓

① Cuts Y-axis for $x = 0$

$\Rightarrow y = 0 - 30$, $B(0, -30)$

$\therefore$ Distance $AB = \sqrt{30^2 + (7.5)^2} = \underline{30.9}$ ✓

11. It is known that $y = A \times 10^{bx^2}$ where A and b are constants. When $\lg y$ is plotted against x^2, a straight line passing through the points (3.63, 5.25) and (4.83, 6.88) is obtained.

(a) Find the value of A and of b. ---[4]

Using your values of A and b, find

(b) the value of y when $x = 2$. ---[2]

(c) the positive value of x when $y = 4$ [W-20/13/Q6] --[2]

Solution: $y = A \cdot 10^{bx^2}$

$\Rightarrow \lg y = b \cdot x^2 + \lg A$ — ①

Now let the line pass through the points $(3.63, 5.25)$, $(4.83, 6.88)$ is of the form $Y = mX + C$ — ②

$m = \frac{6.88 - 5.25}{4.83 - 3.63} = \frac{1.63}{1.2} = 1.358$ ✓

from ② $Y = 1.358X + C$ passes through

$(3.63, 5.25) \Rightarrow 5.25 = 1.358 \times 3.63 + C$

$\Rightarrow C = 0.32$

from ② $Y = 1.358X + 0.32$ — ③

Comparing ① and ③

$\underline{b = 1.358}$ ✓ and $\lg A = 0.32$

$\Rightarrow A = 10^{0.32} = \underline{2.09}$ ✓

(b) from ① $\lg y = 1.358x^2 + 0.32$

for $x = 2$

$\lg y = 1.358 \times 2^2 + 0.32$

$= 5.752$

$y = 10^{5.752} = \underline{565000}$ ✓

(c) $y = A \cdot 10^{bx^2} = 2.09 \times 10^{1.358x^2}$

for $y = 4$ $\Rightarrow 4 = 2.09 \times 10^{1.358x^2}$

$10^{1.358x^2} = 1.91$

$\Rightarrow 1.358x^2 = \lg 1.91 = 0.281$

$\Rightarrow x^2 = \frac{0.281}{1.358} = .2069$

$\Rightarrow x = \underline{0.46}$ ✓