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0606

Addtional Maths

Trigonometry Revision

SP-20	M-20	S-20	W-20

Suresh Goel
(Former Director)
Alliance World School
Noida. Delhi. NCR
INDIA.
(+91 9810444804)

Trigonometry Revision

1(a) (i) Show that $\dfrac{\operatorname{Cosec}\theta}{\operatorname{Cosec}\theta - \operatorname{Sin}\theta} = \sec^2\theta$ ---[3]

(ii) Hence solve $\dfrac{2\operatorname{Cosec}\phi}{\operatorname{Cosec}\phi - \operatorname{Sin}\phi} = 8$ for $0^\circ < \phi < 360^\circ$ ---[3]

(b) Solve $\sqrt{3}\tan\left(x + \dfrac{\pi}{4}\right) = 1$ for $0 < x < 2\pi$, giving your answer in terms of π.

[SP-20 | 02 | Q11] --[3]

Solution (a) (i) L.H.S. $\dfrac{\operatorname{Cosec}\theta}{\operatorname{Cosec}\theta - \operatorname{Sin}\theta}$

$$= \frac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta} - \sin\theta}$$

$$= \frac{\frac{1}{\sin\theta}}{\frac{1 - \sin^2\theta}{\sin\theta}}$$

$$= \frac{\frac{1}{\sin\theta}}{\frac{\cos^2\theta}{\sin\theta}}$$

$$= \frac{1}{\cancel{\sin\theta}} \times \frac{\cancel{\sin\theta}}{\cos^2\theta}$$

$$= \sec^2\theta = R.H.S.$$

(ii) To Solve, $0^\circ < \phi < 360^\circ$

$$\frac{2\operatorname{Cosec}\phi}{\operatorname{Cosec}\phi - \sin\phi} = 8$$

$\Rightarrow 2 \times \sec^2\phi = 8$ [from Part (i)]

$\Rightarrow \sec^2\phi = 4$

$\Rightarrow \sec\phi = \pm 2$

$\Rightarrow \cos\phi = \frac{1}{2}$ or $\cos\phi = -\frac{1}{2}$

$= \cos 60^\circ$; $= -\cos 60^\circ$

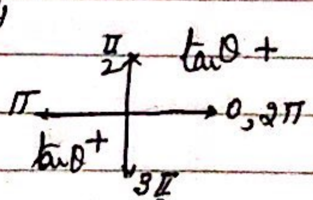
$\phi = 60^\circ, 360 - 60^\circ$; $180 - 60^\circ$, $180 + 60$

$= 60, 300, 120^\circ, 240^\circ$

$\therefore \phi = \underline{60^\circ, 120^\circ, 240^\circ, 300^\circ}$ ✓

(b) To Solve $\sqrt{3}\tan\left(x + \frac{\pi}{4}\right) = 1$ for $0 < x < 2\pi$

$\Rightarrow \tan\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}}$ $\Rightarrow \frac{\pi}{4} < x + \frac{\pi}{4} < 2\pi + \frac{\pi}{4}$



$\Rightarrow x + \frac{\pi}{4} = \tan\frac{\pi}{6}$

$\Rightarrow x + \frac{\pi}{4} = \frac{\pi}{6}^{\times}, \left(\pi + \frac{\pi}{6}\right), \left(2\pi + \frac{\pi}{6}\right)$

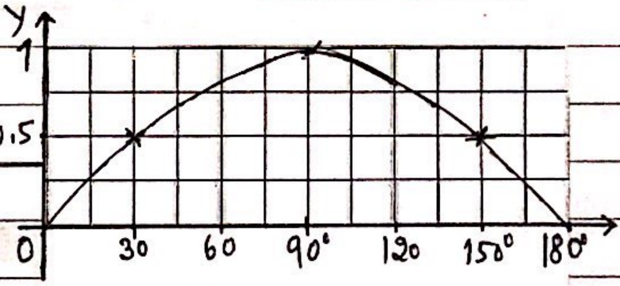
$x = \left(\pi + \frac{\pi}{6} - \frac{\pi}{4}\right)$; $\left(2\pi + \frac{\pi}{6} - \frac{\pi}{4}\right)$

$= \underline{\frac{11}{12}\pi \; ; \; \frac{23}{12}\pi}$ ✓

2. On the axis below, sketch the graph of $y = 2\sin\frac{3}{2}x - 1$ for $0 \le x \le 180°$, showing the coordinates of the points where the graph meets the axes. [SP-20/02/Q4] --[4]

Solution: Note 1. $y = \sin x$, $0 \le x \le 180$

Period $= 360°$, A.



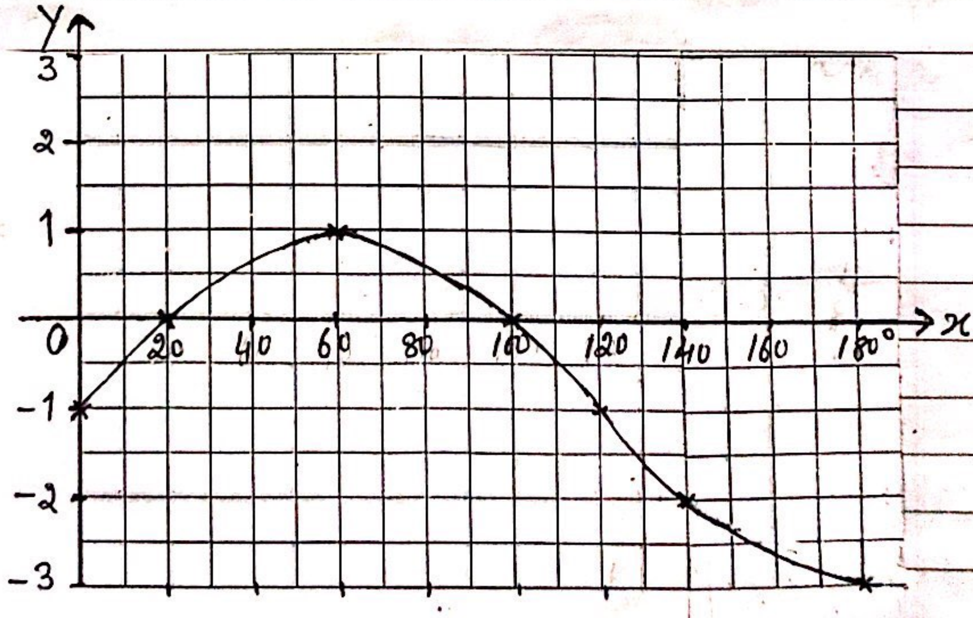
Note 2. $y = a\sin bx + c$

Amplitude $= a$, Period $= \frac{360°}{b}$, Translation along y-axis $= c$

Now given $y = 2\sin\frac{3}{2}x - 1$

$\Rightarrow$ Amplitude $= 2$; Period $= \frac{360}{3/2} = 240°$

Translation along y-axis $= -1$.



$y = 2\sin\frac{3}{2}x - 1$

$x°$	0	20	60	100	120	140	180
$\frac{3}{2}x°$	0	30	90°	150	180	210	270
y	-1	0	1	0	-1	-2	-3

3.(a) Solve $\tan(\alpha + 45^\circ) = -\frac{1}{\sqrt{2}}$ for $0^\circ \le \alpha \le 360^\circ$ ---[3]

(b) (i) Show that: $\frac{1}{\sin\theta - 1} - \frac{1}{\sin\theta + 1} = a\sec^2\theta$, where a is a constant ---[3] to be found.

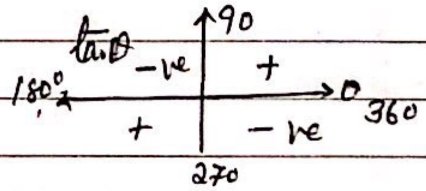
(ii) Hence solve; $\frac{1}{\sin 3\phi - 1} - \frac{1}{\sin 3\phi + 1} = -8$ for $-\frac{\pi}{3} \le \phi \le \frac{\pi}{3}$ radians ---[5]

Solution (a) To Solve $\tan(\alpha + 45^\circ) = -\frac{1}{\sqrt{2}}$ for $0^\circ \le \alpha \le 360^\circ$ $\boxed{M\text{-}20 / 12 / Q10}$

$\Rightarrow \tan(\alpha + 45^\circ) = -\tan 35.3^\circ$ | $\Rightarrow 45^\circ \le \alpha + 45^\circ \le 360 + 45^\circ$

$\Rightarrow \alpha + 45^\circ = (180 - 35.3), (360 - 35.3)$

$= 144.7 ; 324.7$



$\alpha = (144.7 - 45), (324.7 - 45)$

$\alpha = 99.7^\circ, 279.7^\circ$

(b)(i) L.H.S $\frac{1}{\sin\theta - 1} - \frac{1}{\sin\theta + 1}$

$= \frac{(\sin\theta + 1) - (\sin\theta - 1)}{(\sin\theta - 1)(\sin\theta + 1)}$

$= \frac{2}{\sin^2\theta - 1}$

$= \frac{2}{-(1 - \sin^2\theta)}$

$= \frac{-2}{\cos^2\theta}$

$= -2\sec^2\theta$

and $a = -2$ [$\because$ RHS $= a\sec^2\theta$]

(ii) To Solve $-\frac{\pi}{3} \le \phi \le \frac{\pi}{3}$

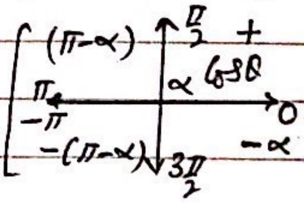
$\frac{1}{\sin 3\phi - 1} - \frac{1}{\sin 3\phi + 1} = -8$, $(-\pi \le 3\phi \le \pi)$

$\Rightarrow -2\sec^2 3\phi = -8$ [from (b)(i)]

$\Rightarrow \sec^2 3\phi = 4$

$\cos^2 3\phi = \frac{1}{4}$

$\cos 3\phi = \pm\frac{1}{2}$



$\Rightarrow \cos 3\phi = \frac{1}{2}$; $\cos 3\phi = -\frac{1}{2}$

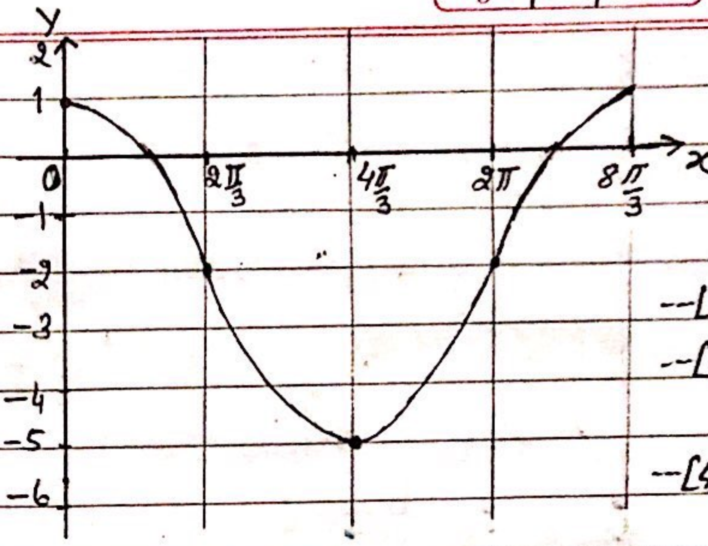
$= \cos\frac{\pi}{3}$; $= -\cos\frac{\pi}{3}$

$3\phi = \frac{\pi}{3}, -\frac{\pi}{3}$; $(\pi - \frac{\pi}{3}), -(\pi - \frac{\pi}{3})$

$= \frac{\pi}{3}, -\frac{\pi}{3}, \frac{2\pi}{3}, -\frac{2\pi}{3}$

$\phi = \frac{\pi}{9}, -\frac{\pi}{9}, \frac{2\pi}{9}, -\frac{2}{9}\pi$

4 The diagram shows the graph of $f(x) = a\cos bx + C$ for $0 \leq x \leq \frac{8\pi}{3}$ radians



(a) Explain why f is a function --[1]

(b) Write down the range of f. --[1]

(c) Find the value of a, b, and c. --[4]

M-20/22/Q8

<u>Solution</u> (a) f is a function as each value of x corresponds (mapped) to a unique value of y. ✓

(b) $-5 \leq f \leq 1$

(c) (i) amplitude $= \frac{1-(-5)}{2} = \frac{6}{2} = 3$

$\therefore \underline{a = 3}$ ✓

(ii) Period $= \frac{2\pi}{b} = \frac{8\pi}{3}$

$\Rightarrow b = \frac{2\pi \times 3}{8\pi}$

$= \frac{3}{4}$

$\therefore \underline{b = \frac{3}{4}}$ ✓ (or 0.75)

(iii) Translation along Y-axis $\longrightarrow$ $\underline{c = -2}$ ✓

$f(x) = a\cos bx + c$

a = amplitude of $f(x)$

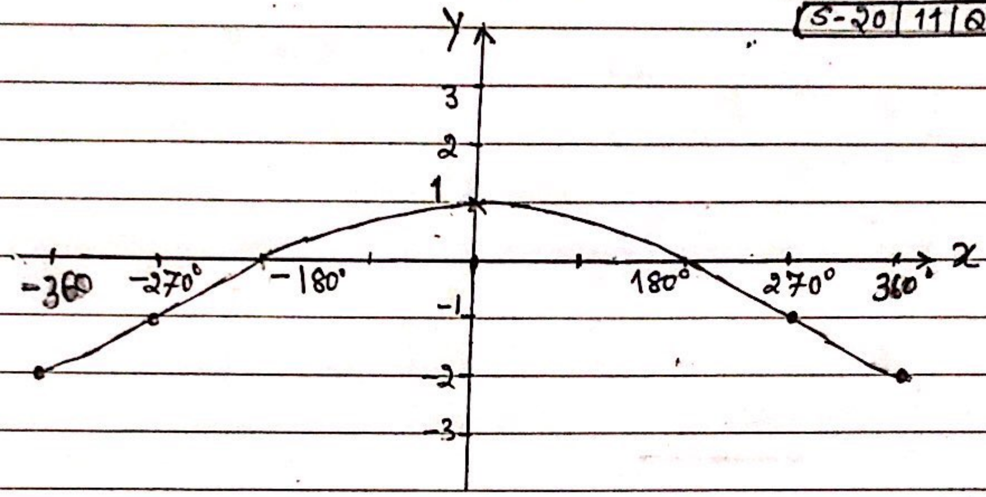
Period of $f(x) = \frac{\quad}{b}$

c = Translation along Y-axis

5 (a) Write down the period of $2\cos\frac{x}{3}-1$ --[1]

(b) On the axes below, sketch the graph of $y=2\cos\frac{x}{3}-1$ for $-360^\circ \leq x \leq 360^\circ$ --[3]

[S-20 | 11 | Q2]



x	0	180	360	−180	−360	270	−270
$x/3$	0	60	120	−60	−120	90	−90
y	1	0	−2	0	−2	−1	−1

(a) Period of $2\cos x/3 - 1 = \frac{360}{1/3} = 1080^\circ$ $\left\{ \text{Period of } y = \cos bx, \ \text{Period} = \frac{360}{b} \right.$

6.(a)(i) Show that $\frac{1}{\sec\theta - 1} - \frac{1}{\sec\theta + 1} = 2\cot^2\theta$ --[3]

[S-20 / 11 / Q10]

(ii) Hence solve, $\frac{1}{\sec 2x - 1} - \frac{1}{\sec 2x + 1} = 6$ $\quad -90^\circ < x < 90^\circ$ --[5]

Solution (a)(i) L.H.S

$$\frac{1}{\sec\theta-1} - \frac{1}{\sec\theta+1}$$

$$= \frac{(\sec\theta+1)-(\sec\theta-1)}{(\sec\theta-1)(\sec\theta+1)}$$

$$= \frac{2}{\sec^2\theta - 1}$$

$$= \frac{2}{\tan^2\theta} = 2\cot^2\theta$$

$= R.H.S$

(ii) Solve,

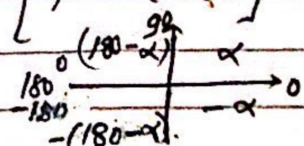
$-90 < x < 90^\circ$

$\Rightarrow -180^\circ < 2x < 180^\circ$

$$\frac{1}{\sec 2x - 1} - \frac{1}{\sec 2x + 1} = 6$$

$\Rightarrow 2\cot^2 2x = 6$ [from (a)(i)]

$\Rightarrow \tan^2 2x = \frac{1}{3}$



$\Rightarrow \tan 2x = \pm\frac{1}{\sqrt{3}}$

$\Rightarrow \quad = \pm \tan 30^\circ \quad (\alpha = 30^\circ)$

$2x = 30^\circ, -(180-30^\circ), -30^\circ, 180-30^\circ$

$= 30, -150, -30, 150$

$\Rightarrow x = 15^\circ, 75^\circ, -15^\circ, -75^\circ$ ✓

(Continued →)

(Continued→)

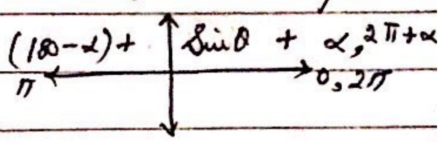
6(b) Solve $\operatorname{cosec}\left(y+\frac{\pi}{3}\right)=2$ for $0 \le y \le 2\pi$ radians, giving your answer in terms of π. [S-20 | 11 | Q10] --[4]

Solution: $\operatorname{cosec}\left(y+\frac{\pi}{3}\right)=2$

$\left(0 \le y \le 2\pi \Rightarrow \frac{\pi}{3} \le y+\frac{\pi}{3} \le 2\pi+\frac{\pi}{3} \right)$

$\Rightarrow \sin\left(y+\frac{\pi}{3}\right)=\frac{1}{2}=$

$= \sin\frac{\pi}{6}$



$\Rightarrow y+\frac{\pi}{3}=\frac{\pi}{6}^{\times}, \ \pi-\frac{\pi}{6}, \ 2\pi+\frac{\pi}{6}$

$\Rightarrow y+\frac{\pi}{3}=\frac{5\pi}{6}, \frac{13\pi}{6} \Rightarrow y=\frac{5\pi}{6}-\frac{\pi}{3}, \ \frac{13\pi}{6}-\frac{\pi}{3}$

$\therefore y=\frac{\pi}{2}; \ \frac{11}{6}\pi$ ✓

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7 (a) Solve $\tan 3x=-1$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ radians, giving answer in terms of π. --[4]

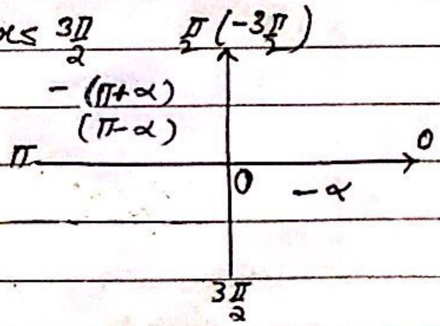
(b) Use your answer of part (a) to sketch the graph of $y=4\tan 3x+4$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ radians. Show the coordinates of the points where the curve meets the axes. [S-20 | 12 | Q10] --[3]

Solution(a) Solve $\tan 3x=-1$ $\quad -\frac{\pi}{2} \le x \le \frac{\pi}{2} \Rightarrow -\frac{3\pi}{2} \le 3x \le \frac{3\pi}{2}$

$\Rightarrow \tan 3x = -\tan\frac{\pi}{4}$

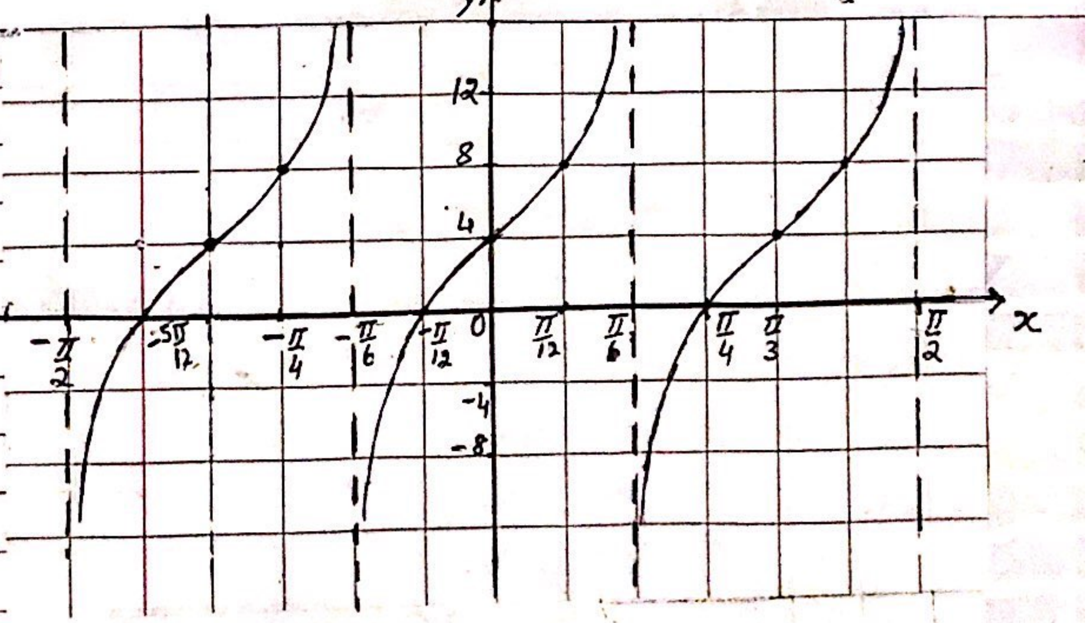
$\Rightarrow 3x = -\frac{5\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}$

$\Rightarrow x=\frac{\pi}{4}, -\frac{\pi}{12}, -\frac{5\pi}{12}$ ✓



(b)

$y=4\tan 3x+4$
$=4[\tan 3x+1]$



(i) Curve meets y-axis at (0, 4)

(ii) Curve meets x-axis at $\left(-\frac{\pi}{12},0\right)$, $\left(-\frac{5\pi}{12},0\right)$ and $\left(\frac{\pi}{4},0\right)$

8.(a) Solve $3\cot^2 x - 14\operatorname{cosec} x - 2 = 0$ for $0° < x < 360°$ ---[5]

(b) Show that $\dfrac{\sin^4 y - \cos^4 y}{\cot y} = \tan y - 2\cos y \sin y$ ---[4]

S-20/22/Q8

Solution (a) To solve: $3\cot^2 x - 14\operatorname{cosec} x - 2 = 0$

$$\Rightarrow 3(\operatorname{cosec}^2 x - 1) - 14\operatorname{cosec} x - 2 = 0$$

$$\Rightarrow 3\operatorname{cosec}^2 x - 14\operatorname{cosec} x - 5 = 0$$

$$\Rightarrow 3\operatorname{cosec}^2 x - 15\operatorname{cosec} x + \operatorname{cosec} x - 5 = 0$$

$$(\operatorname{cosec} x - 5)(3\operatorname{cosec} x + 1) = 0$$

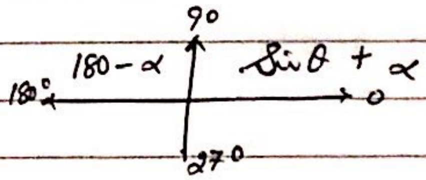
$$\Rightarrow \operatorname{cosec} x = 5 \quad \text{or} \quad \operatorname{cosec} x = -\frac{1}{3}$$

$$\Rightarrow \sin x = \frac{1}{5} \quad \text{or} \quad \sin x = -3^{\times} \qquad (\because -1 \leq \sin x \leq 1)$$

$$= \sin 11.5°$$

$$\therefore x = 11.5°,\ 180 - 11.5$$

$$= \underline{11.5°,\ 168.5°} \checkmark$$



(b) L.H.S. $\dfrac{\sin^4 y - \cos^4 y}{\cot y}$

$$= \frac{(\sin^2 y)^2 - (\cos^2 y)^2}{\frac{\cos y}{\sin y}} = \frac{\sin y}{\cos y} \times (\sin^2 y - \cos^2 y)(\sin^2 y + \cos^2 y)$$

$[\because \sin^2 y + \cos^2 y = 1]$

$$= \frac{\sin y}{\cos y} \times (1 - \cos^2 y - \cos^2 y) \times 1$$

$$= \frac{\sin y}{\cos y} \times (1 - 2\cos^2 y)$$

$$= \frac{\sin y}{\cos y} - \frac{\sin y}{\cos y} \times 2\cos^2 y$$

$$= \tan y - 2\cos y \sin y \checkmark$$

$$= \underline{R.H.S.}$$

9 (a) The curve $y = a \sin bx + c$ has a period of 180°, an amplitude of 20 and passes through the point $(90^\circ, -3)$. Find the value of each of the constant a, b and c. ---[3]

[S-20/23/Q6]

Solution: $y = a \sin bx + c$ —① $\quad a =$ amplitude $= 20$

$$\text{Period} = \frac{360}{b} = 180^\circ \Rightarrow b = 2$$

$$\therefore y = 20 \sin 2x + c \qquad [a = 20,\ b = 2 \text{ in ①}]$$

Passes through $(90, -3)$

$$\Rightarrow -3 = 20 \sin 2 \times 90 + c$$

$$\Rightarrow -3 = 20 \sin 180 + c$$

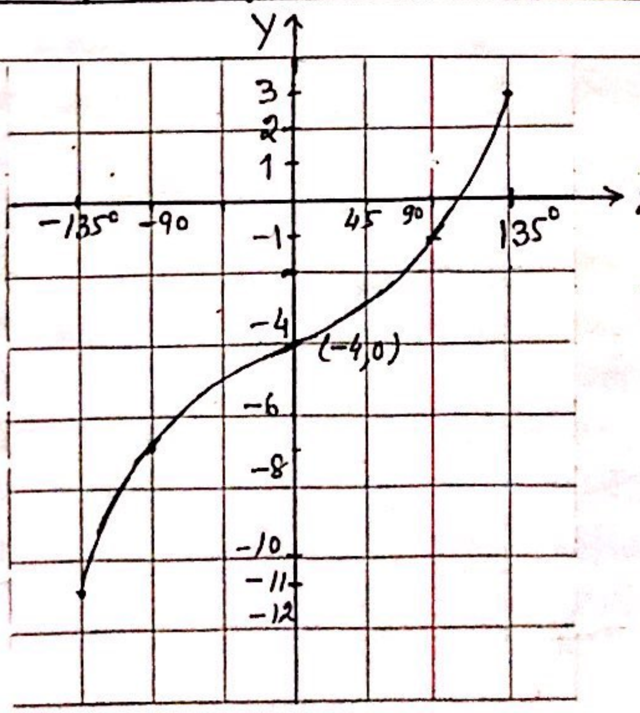
$$\Rightarrow -3 = 20 \times 0 + c \Rightarrow c = -3$$

$\therefore$ $a = 20$, $b = 2$, and $c = -3$

(b) The function g is defined, for $-135^\circ \le x \le 135^\circ$, by $g(x) = 3 \tan \frac{x}{2} - 4$. Sketch the graph of $y = g(x)$ on the axes below, stating the coordinates of the point where the graph crosses the axes. --[2]

Solution: $y = 3 \tan x/2 - 4$, $-135^\circ \le x \le 135^\circ \Rightarrow \frac{-135^\circ}{2} \le \frac{x}{2} \le \frac{135^\circ}{2}$

x	-135	-90	0	90	135°	106
$x/2$	$\frac{-135}{2}$	-45	0	45	$\frac{135}{2}$	53
y	-11	-7	-4	-1	3	0



The graph intersect the y-axis at $(0, -4)$

10 Solve the equation.

(a) $5\sec^2 A + 14\tan A - 8 = 0$ for $0 \le A \le 180°$ ---[4]

(b) $5\sin\left(4B - \frac{\pi}{8}\right) + 2 = 0$ for $-\frac{\pi}{4} \le B \le \frac{\pi}{4}$ --[4]

S-20/23/Q10

Solution (a) To Solve: $5\sec^2 A + 14\tan A - 8 = 0$ for $0 \le A \le 180°$

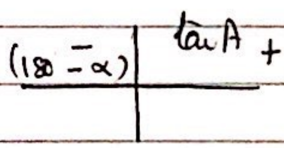
$\Rightarrow 5(1+\tan^2 A) + 14\tan A - 8 = 0$ $(\because 1+\tan^2 A = \sec^2 A)$

$\Rightarrow 5\tan^2 A + 14\tan A - 3 = 0$

$5\tan^2 A + 15\tan A - \tan A - 3 = 0$

$5\tan A(\tan A + 3) - 1(\tan A + 3) = 0$

$(\tan A + 3)(5\tan A - 1) = 0$



$\Rightarrow \tan A = \frac{1}{5}$, $\tan A = -3$

$= \tan 11.3°$; $= -\tan 71.6$

$\Rightarrow A = 11.3°$ $A = 180 - 71.6 = 108.4°$

$\therefore A = 11.3° \, ; \, 108.4°$ ✓

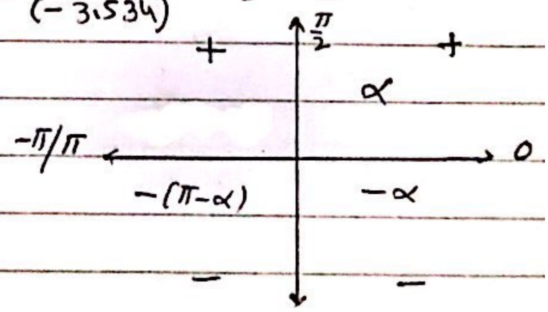
(b) To Solve: $5\sin\left(4B - \frac{\pi}{8}\right) + 2 = 0$

$$\begin{cases} -\frac{\pi}{4} \le B \le \frac{\pi}{4} \\ -\pi \le 4B \le \pi \\ -\frac{9}{8}\pi \le 4B - \frac{\pi}{8} \le \frac{7}{8}\pi \end{cases}$$

(-3.534) (2.75) rad

$\Rightarrow \sin\left(4B - \frac{\pi}{8}\right) = -\frac{2}{5}$

$= -\sin 0.411$



$\Rightarrow 4B - \frac{\pi}{8} = -0.4115, -(\pi - 0.411)$

$= -0.4115, -2.73,$

$4B = -0.4115 + \frac{\pi}{8}, -2.73 + \frac{\pi}{8}$

$= -0.0188, -2.337$

$B = \underline{-0.00470}, \underline{-0.584}$ ✓

11 (a) Write down the amplitude of $1+4\cos x/3$ ---[1]

(b) Write down the period of $1+4\cos x/3$ ---[1]

(c) Sketch the graph of $y=1+4\cos x/3$ for $-180^\circ \le x \le 180^\circ$ ---[3]

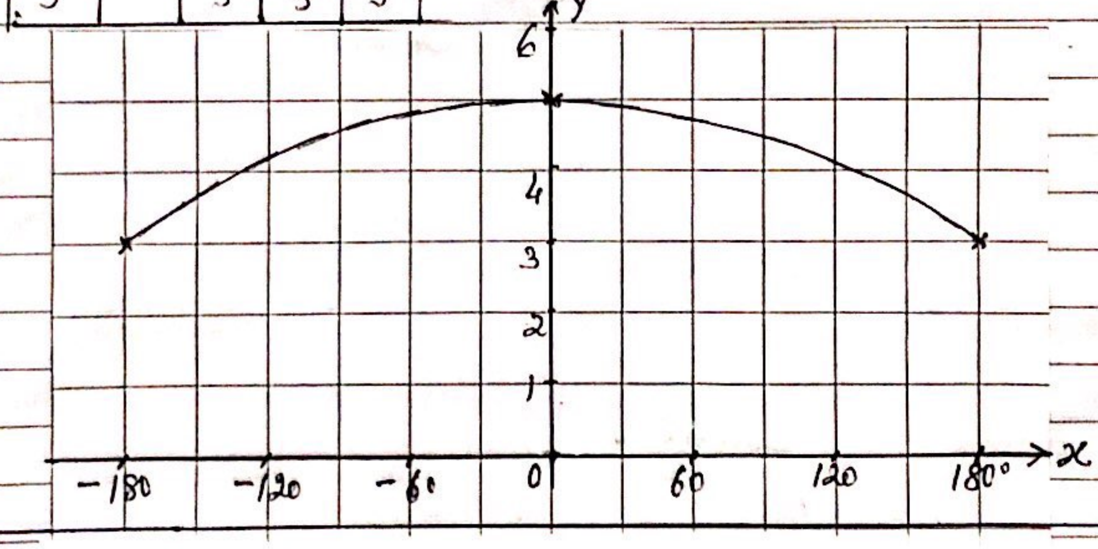
W-20/11/Q2

Solution (a) Amplitude of $1+4\cos x/3$ is 4 ✓

(b) Period of $1+4\cos x/3$ is $\frac{360}{1/3} = 1080^\circ$ ✓ { Period of $y=\cos bx$, Period $=\frac{360^\circ}{b}$

(c) $y = 1+4\cos x/3 \quad -180 \le x \le 180 \Rightarrow -60 \le \frac{x}{3} \le 60$

x		-180	0	180
$x/3$		-60	0	60
y		3	5	3



12 (a) (i) Show that $\frac{1}{(1+\operatorname{cosec}\theta)(\sin\theta-\sin^2\theta)} = \sec^2\theta$ --[4]

(ii) Hence solve $(1+\operatorname{cosec}\theta)(\sin\theta-\sin^2\theta) = \frac{3}{4}$ for $-180^\circ \le \theta \le 180^\circ$ --[4]

W-20/11/Q8

Solution (a)(i)

L.H.S. $\frac{1}{(1+\operatorname{cosec}\theta)(\sin\theta-\sin^2\theta)}$

$= \frac{1}{(1+\frac{1}{\sin\theta})\sin\theta(1-\sin\theta)}$

$= \frac{1\cancel{\sin\theta}}{(1+\sin\theta)\cancel{\sin\theta}(1-\sin\theta)}$

$= \frac{1}{(1-\sin^2\theta)}$

$= \frac{1}{\cos^2\theta} = \sec^2\theta = \text{R.H.S}$

(ii) To solve

$(1+\operatorname{cosec}\theta)(\sin\theta-\sin^2\theta) = \frac{3}{4}$

$\Rightarrow \cos^2\theta = \frac{3}{4}$ [from Part (a)(i)]

$\Rightarrow \cos\theta = \frac{\sqrt3}{2}, \quad \cos\theta = -\frac{\sqrt3}{2}$

$= \cos 30^\circ, \quad = -\cos 30^\circ$

$\theta = 30^\circ, -30^\circ; \quad \theta = (180-30), -(180-30)$

$= 30, -30; \quad \theta = 150, -150^\circ$

$\therefore \theta = -150^\circ, -30^\circ$

$30^\circ, 150^\circ$ ✓

$(180-\alpha)$, α, $\cos\theta +$, 0, $-(180-\alpha)$, $-\alpha$, $(\cos\theta +)$

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(Continued →)

12(b) Solve, $\sin\left(3\phi+\frac{2\pi}{3}\right)=\cos\left(3\phi+\frac{2\pi}{3}\right)$ for $0\le\phi\le\frac{2\pi}{3}$ radians, ---[4]
giving use answers in terms of π. [W.20 / 11 / Q8]

Solution: To solve,

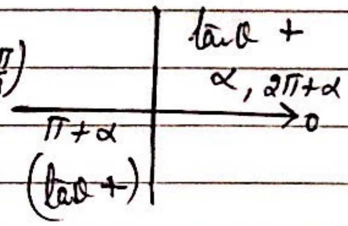
$$\sin\left(3\phi+\frac{2\pi}{3}\right)=\cos\left(3\phi+\frac{2\pi}{3}\right)$$

$$\Rightarrow \tan\left(3\phi+\frac{2\pi}{3}\right)=1$$
$$=\tan\frac{\pi}{4}$$

$$\left\{\begin{array}{l} 0\le\phi\le\frac{2\pi}{3} \\ 0\le 3\phi\le 2\pi \\ \frac{2\pi}{3}\le 3\phi+\frac{2\pi}{3}\le 2\pi+\frac{2\pi}{3}\end{array}\right.$$

$$\Rightarrow 3\phi+\frac{2\pi}{3}=\frac{\pi}{4},\ \pi+\frac{\pi}{4},\ 2\pi+\frac{\pi}{4}$$

$$3\phi=\left(\frac{\pi}{4}-\frac{2\pi}{3}\right)^{\times},\ \left(\frac{5\pi}{4}-\frac{2\pi}{3}\right),\ \left(\frac{9\pi}{4}-\frac{2\pi}{3}\right)$$



$$3\phi=\frac{7\pi}{12},\ \frac{19\pi}{12}$$

$$\Rightarrow \phi=\frac{7\pi}{36},\ \frac{19}{36}\pi \checkmark$$

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13 (a) Write down the amplitude of $2\cos x/3-1$ ---[1]
(b) Write down the period of $2\cos x/3-1$ ---[1]
(c) Sketch the graph of $y=2\cos x/3-1$ for $-\pi\le x\le 3\pi$ radians. ---[3]

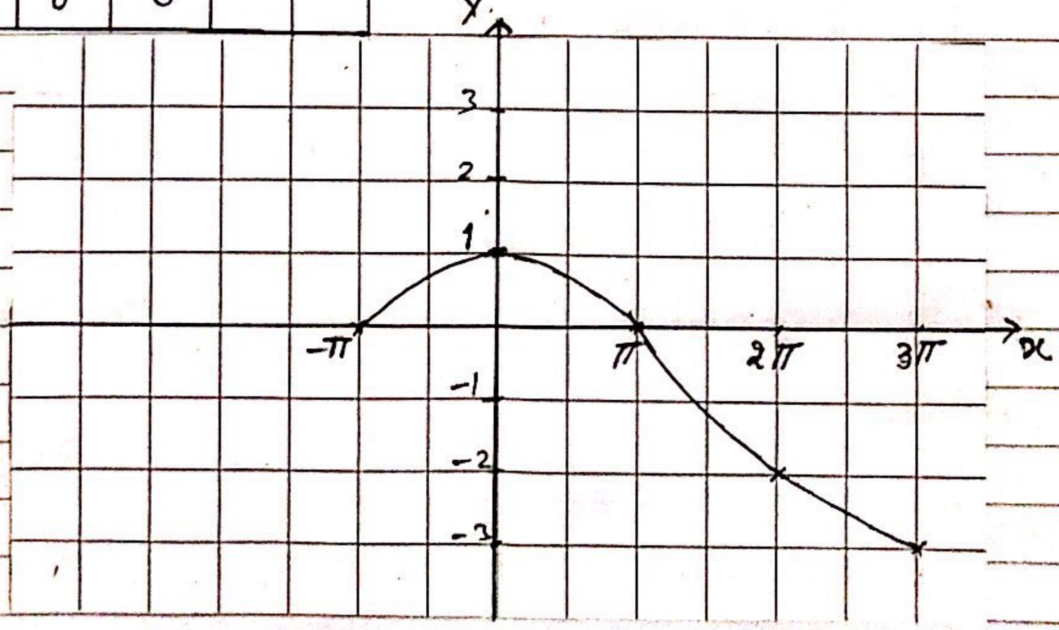
Solution: [W.20 / 12 / 03]

(a) Amplitude of $2\cos x/3-1$ is $\underline{2}$ ✓

(b) Period of $2\cos x/3-1$ is $\frac{360}{1/3}=\underline{1080^\circ}$ ✓ $\left(\because \text{Period of } \cos bx = \frac{360^\circ}{b}\right)$

(c) $y=2\cos x/3-1$

x	0	π	$-\pi$	2π	3π
y	1	0	0		



14 (a) Show that $\frac{\sin x \cdot \tan x}{1-\cos x} = 1 + \sec x$ ---[4]

(b) Solve the equation $5\tan x - 3\cot x = 2\sec x$ for $0^\circ \le x \le 360^\circ$ --[6]

[W-20/22/Q11]

Solution (a) L.H.S. $\frac{\sin x \cdot \tan x}{(1-\cos x)}$

$$= \frac{\sin x \cdot \frac{\sin x}{\cos x}}{(1-\cos x)}$$

$$= \frac{\sin^2 x}{\cos x(1-\cos x)}$$

$$= \frac{(1-\cos^2 x)}{\cos x(1-\cos x)}$$

$$= \frac{\cancel{(1-\cos x)}(1+\cos x)}{\cos x\,\cancel{(1-\cos x)}}$$

$$= \frac{1}{\cos x} + \frac{\cos x}{\cos x} = \underline{\sec x + 1 = R.H.S}$$

(b) To Solve:

$5\tan x - 3\cot x = 2\sec x$ $\qquad 0^\circ \le x \le 360^\circ$

$\Rightarrow 5\frac{\sin x}{\cos x} - 3\frac{\cos x}{\sin x} = \frac{2}{\cos x}$

$\Rightarrow 5\sin^2 x - 3\cos^2 x = 2\sin x$ (Multiply each term by $\sin x \cdot \cos x$)

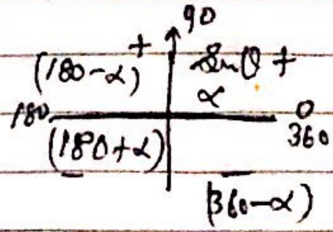
$\Rightarrow 5\sin^2 x - 3(1-\sin^2 x) - 2\sin x = 0$

$\Rightarrow 8\sin^2 x - 2\sin x - 3 = 0$

$(2\sin x + 1)(4\sin x - 3) = 0$

$\Rightarrow \sin x = -\frac{1}{2}$,	$\sin x = \frac{3}{4}$
$= -\sin 30^\circ$,	$= \sin 48.6^\circ$
$x = (180+30)^\circ, (360-30)^\circ$;	$x = 48.6^\circ, 180-48.6^\circ$
$= 210^\circ, 330^\circ$;	$= 48.6^\circ, 131.4^\circ$

$\therefore x = \underline{48.6^\circ, 131.4^\circ, 210^\circ, 330^\circ}$ ✓



15 (a) Given that $2\cos x = 3\tan x$, show that $2\sin^2 x + 3\sin x - 2 = 0$ ---[3]

(b) Hence solve $2\cos\left(2\alpha+\frac{\pi}{4}\right) = 3\tan\left(2\alpha+\frac{\pi}{4}\right)$ for $0<\alpha<\pi$ radians, giving your answers in terms of π. [W-20|13|Q 11] ---[4]

<u>Solution</u>(a) Given $2\cos x = 3\tan x$

$$\Rightarrow 2\cos x = 3\frac{\sin x}{\cos x}$$

$$\Rightarrow 2\cos^2 x = 3\sin x$$

$$\Rightarrow 2(1-\sin^2 x) - 3\sin x = 0$$

$$\Rightarrow 2 - 2\sin^2 x - 3\sin x = 0$$

$$\Rightarrow \underline{2\sin^2 x + 3\sin x - 2 = 0} \checkmark$$

(b) Solve $2\cos\left(2\alpha+\frac{\pi}{4}\right) = 3\tan\left(2\alpha+\frac{\pi}{4}\right)$ for

$$\left\{\begin{array}{l} 0<\alpha<\pi \\ 0<2\alpha<2\pi \\ \frac{\pi}{4}<2\alpha+\frac{\pi}{4}<2\pi+\frac{\pi}{4} \end{array}\right.$$

$$\Rightarrow 2\sin^2\left(2\alpha+\frac{\pi}{4}\right) + 3\sin\left(2\alpha+\frac{\pi}{4}\right) - 2 = 0$$

from part (a)

(Let $2\alpha+\frac{\pi}{4}=\theta$) $\qquad \frac{\pi}{4}\le\theta\le 2\pi+\frac{\pi}{4}$

$$\Rightarrow 2\sin^2\theta + 3\sin\theta - 2 = 0$$

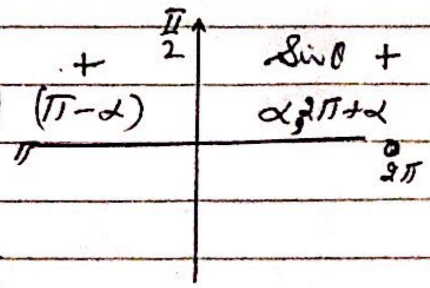
$$2\sin^2\theta + 4\sin\theta - \sin\theta - 2 = 0$$

$$2\sin\theta(\sin\theta+2) - 1(\sin\theta+2) = 0$$

$$(2\sin\theta - 1)(\sin\theta+2) = 0$$

$$\Rightarrow \sin\theta = \frac{1}{2}, \quad \sin\theta = -2^{\times} \quad (\because -1\le\sin\theta\le 1)$$

$$= \sin\frac{\pi}{6}$$



$$\therefore \theta = 2\alpha+\frac{\pi}{4} = \cancel{\frac{\pi}{6}}, \left(\pi-\frac{\pi}{6}\right), \left(2\pi+\frac{\pi}{6}\right)$$

$$\text{or} \quad 2\alpha+\frac{\pi}{4} = \frac{5\pi}{6}, \frac{13\pi}{6}$$

$$2\alpha = \frac{5\pi}{6}-\frac{\pi}{4}, \frac{13\pi}{6}-\frac{\pi}{4}$$

$$2\alpha = \frac{7}{12}\pi ; \frac{23\pi}{12}$$

$$\alpha = \frac{7\pi}{24} ; \alpha = \frac{23\pi}{24} \checkmark$$