DATE

0580

IGCSE Maths

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IGCSE 0580

Algebra Revision

Finding Value; Expand and Simplify, Solution of Linear Equations

1. $y = mx + c$ [SP-20/02/Q5]

Find the value of y when $m = -2$, $x = -7$ and $c = -3$ --[2]

Solution: $y = mx + c \Rightarrow y = -2 \times (-7) + (-3) = 14 - 3 = 11$ ✓

2. $y = mx + c$ [S-20/22/Q5]

Find the value of y when $m = -3$, $x = -2$ and $c = -8$

Solution: $y = mx + c \Rightarrow y = -3 \times (-2) + (-8) = 6 - 8 = -2$ ✓ ---[2]

3. Expand and simplify. $(x+3)(x-5)(3x-1)$ ---[3]

[S-20/21/Q21]

Solution: $(x+3)(x-5)(3x-1) = (x+3)[x(3x-1) - 5(3x-1)]$

$= (x+3)[3x^2 - x - 15x + 5]$

$= (x+3)[3x^2 - 16x + 5]$

$= x(3x^2 - 16x + 5) + 3(3x^2 - 16x + 5)$

$= 3x^3 - 16x^2 + 5x + 9x^2 - 48x + 15$

$= \underline{3x^3 - 7x^2 - 43x + 15}$ ✓

4. Simplify $5c - d - 3d - 2c$ ---[2]

[W-19/21/Q5]

Solution: $5c - d - 3d - 2c$

$= 5c - 2c - d - 3d$

$= \underline{3c - 4d}$ ✓

5. Expand $a(a^3+3)$ [W-19/22/Q3] ---[1]

Solution: $a(a^3+3) = a^{3+1} + 3a$

$= \underline{a^4 + 3a}$ ✓

6. Expand and simplify; $(x+3)(x+5)$ ---[2]

[W-19/23/Q6]

Solution: $(x+3)(x+5) = x(x+5) + 3(x+5)$

$= x^2 + 5x + 3x + 15$

$= \underline{x^2 + 8x + 15}$ ✓

7. Solve the equation: $\frac{1-x}{3} = 5$ [S-20/22/Q14] ---[2]

Solution: $\frac{1-x}{3} = \frac{5}{1} \Rightarrow (1-x)\times 1 = 5\times 3$

$\Rightarrow 1-x = 15$

$\Rightarrow -x = 15-1 \Rightarrow -x = 14$

$\Rightarrow x = -14$ ✓

8. Solve $\frac{x-2}{3} = 3$ ---[2]

[W-19/21/Q6]

Solution: $\frac{x-2}{3} = 3 \Rightarrow x-2 = 3\times 3$

$\Rightarrow x = 9+2 = 11$

$\therefore x = 11$ ✓

9. Write $\frac{x}{2} - \frac{2x+4}{x+1}$ as a single fraction, in its simplest form.

[W-19/21/Q18] ---[3]

Solution: $\frac{x}{2} - \frac{2x+4}{x+1} = \frac{x(x+1)-2(2x+4)}{2(x+1)}$

$= \frac{x^2+x-4x-8}{2(x+1)}$

$= \frac{x^2-3x-8}{2(x+1)}$ ✓

10. Write as a single fraction in its simplest form:

$\frac{x+3}{x-3} - \frac{x-2}{x+2}$ ---[4]

[M-20/42/Q5(a)]

Solution: $\frac{x+3}{x-3} - \frac{x-2}{x+2}$

$= \frac{(x+3)(x+2)-(x-2)(x-3)}{(x-3)(x+2)}$

$= \frac{x^2+2x+3x+6-(x^2-3x-2x+6)}{(x-3)(x+2)}$

$= \frac{\cancel{x^2}+5x+\cancel{6}-\cancel{x^2}+5x-\cancel{6}}{(x-3)(x+2)}$

$= \frac{10x}{(x-3)(x+2)}$ ✓

11. Expand and simplify: $(y+3)(y-4)(2y-1)$ ---[3]

[M-20/42/Q5(c)]

Solution: $(y+3)(y-4)(2y-1) = (y+3)[y(2y-1)-4(2y-1)]$

$= (y+3)[2y^2-y-8y+4]$

$= (y+3)(2y^2-9y+4)$

$= y(2y^2-9y+4)+3(2y^2-9y+4)$

$= 2y^3-9y^2+4y+6y^2-27y+12$

$= \underline{2y^3-3y^2-23y+12}$ ✓

12.(a) $S = ut + \frac{1}{2}at^2$ ---[2]

Find the value of S when $u=5.2$, $t=7$ and $a=1.6$

Solution: $S = ut + \frac{1}{2}at^2$

$= 5.2\times7 + \frac{1}{2}\times1.6\times7^2$

$= 36.4 + 0.8\times49$

$= 36.4 + 39.2 = \underline{75.6}$ ✓

(c) Solve (i) $\frac{15}{x} = -3$

(ii) $4(5-3x) = 23$ ---[3]

Solution: (i) $\frac{15}{x} = -3 \Rightarrow -3x = 15$

$\Rightarrow x = \frac{15}{-3} = -5 \Rightarrow \underline{x=-5}$ ✓

(ii) $4(5-3x) = 23$

$\Rightarrow 20-12x = 23$

$\Rightarrow -12x = 23-20$

$\Rightarrow -12x = 3$

$\Rightarrow x = -\frac{3}{12} = -\frac{1}{4}$ $\therefore$ $x = \underline{-\frac{1}{4}\ (\text{or } -0.25)}$ ✓

(e) Expand and simplify: $(3x-5y)(2x+y)$ ---[2]

[S-20/43/Q3]

Solution: $(3x-5y)(2x+y)$

$= 3x(2x+y) - 5y(2x+y)$

$= 6x^2+3xy-10xy-5y^2$

$= \underline{6x^2-7xy-5y^2}$ ✓

12. Solve the equation $6x-3=-12$ --[2]

[W-19/43/Q2(b)]

Solution: $6x-3=-12$

$$\Rightarrow 6x=-12+3$$

$$\Rightarrow 6x=-9$$

$$\Rightarrow x=-\frac{9}{6}=-\frac{3}{2}$$

$\therefore$ $\underline{x=-\frac{3}{2}}$ (or -1.5) ✓

13. Solve the equation:

$$\frac{2x+5}{3-x}=\frac{14}{15}$$ --[3]

[S-20/43/Q4(b)]

Solution: $\frac{2x+5}{3-x}=\frac{14}{15}$ $\Rightarrow 15(2x+5)=14(3-x)$

$$\Rightarrow 30x+75=42-14x$$

$$\Rightarrow 30x+14x=42-75$$

$$\Rightarrow 44x=-33$$

$$\Rightarrow x=-\frac{33}{44}=-\frac{3}{4}=-0.75$$

$\therefore$ $\underline{x=-0.75}$ ✓

14. Oranges cost 21 cents each.
Alex buys x oranges and Bobbie buys $(x+2)$ oranges.
The total cost of these oranges is \$4.20
Find the value of x. --[3]

[W-19/41/Q7(a)]

Solution: $21x+21(x+2)=420$ ($\because$ \$4.20 = 420 cents)

$$\Rightarrow 21x+21x+42=420$$

$$\Rightarrow 42x=420-42=378$$

$$x=\frac{378}{42}=9$$

$\therefore$ $\underline{x=9}$ ✓

15. Simplify: $3a + 7b - 4a + b$ [W-20/21/Q1] ---[2]

Solution: $3a + 7b - 4a + b$

$= 3a - 4a + 7b + b = -a + 8b$ ✓

16. Solve $\frac{2}{x} = \frac{6}{2-x}$ [W-20/41/Q8(d)] ---[3]

Solution: $\frac{2}{x} = \frac{6}{2-x} \Rightarrow 6x = 2(2-x)$

$6x = 4 - 2x \Rightarrow 6x + 2x = 4$

$\Rightarrow 8x = 4 \Rightarrow x = \frac{4}{8} = \frac{1}{2}$ ✓

or $x = \frac{1}{2}$ (or 0.5) ✓

17. Expand and simplify, $(x-2)(x+5)(2x-1)$ [W-20/41/Q8(e)] ---[3]

Solution: $(x-2)(x+5)(2x-1)$

$= (x-2)[x(2x-1) + 5(2x-1)]$

$= (x-2)[2x^2 - x + 10x - 5]$

$= (x-2)[2x^2 + 9x - 5]$

$= x(2x^2 + 9x - 5) - 2(2x^2 + 9x - 5)$

$= 2x^3 + 9x^2 - 5x - 4x^2 - 18x + 10$

$= 2x^3 + 5x^2 - 23x + 10$ ✓

18. Solve the equation: $6 - 2x = 3x$ [W-20/22/Q3] ---[2]

Solution: $6 - 2x = 3x$

$\Rightarrow -2x - 3x = -6 \Rightarrow -5x = -6 \Rightarrow x = -\frac{6}{-5} = \frac{6}{5}$

$x = \frac{6}{5}$ (or $1\frac{1}{5}$ or 1.2) ✓

19. Write a single fraction in its simplest form, $2 - \frac{2x-1}{x+1}$. ---[3]

[W-20/22/Q23]

Solution: $2 - \frac{2x-1}{x+1} = \frac{2(x+1) - (2x-1)}{(x+1)}$

$= \frac{2x + 2 - 2x + 1}{x+1}$

$= \frac{3}{x+1}$ ✓

20. Ahmed sells different types of cake in his shop.
The cost of each cake depends on its type and its size.
Every small cake costs \$$x$ and every large cake costs \$$(2x+1)$

(a) The cost of 3 small lemon cakes and 2 large lemon cakes is \$ 12.36. Find the cost of a small lemon cake ---[3]

(b) The cost of 18 small chocolate cakes is the same as the cost of 7 large chocolate cakes. Find the cost of a small chocolate cake. --[3]

(c) The number of small cherry cakes that can be bought for \$4 is the same as the the number of large cherry cakes that can be bought for \$13. Find the cost of a small cherry cake. --[3]

(d) Petra spends \$ 20 on small coffee cakes and \$ 10 on large coffee cakes. The total number of cakes is 45.
Write an equation in terms of x. Solve the equation to to find the cost of small coffee cake. --[7]

[W-20/42/Q5]

Solution: 3 small + 2 large = \$ 12.36

(a) $\Rightarrow 3x + 2(2x+1) = 12.36 \Rightarrow 3x + 4x + 2 = 12.36$
$\Rightarrow 7x = 10.36 \Rightarrow x = \frac{10.36}{7}$
Cost of small lemon cake $= x =$ \$1.48 ✓

(b) 18 small = 7 large
$\Rightarrow 18x = 7(2x+1) \Rightarrow 18x = 14x + 7 \Rightarrow 18x - 14x = 7$
$\Rightarrow 4x = 7 \Rightarrow x = \frac{7}{4} =$ \$1.75 ✓

(c) Cost of small = 4 $\Rightarrow$ No of small cakes $= \frac{4}{x}$ —— (1)
Total cost of large = 13 $\Rightarrow$ No of large $= \frac{13}{(2x+1)}$ —— (2)
Given for (1) and (2) $\frac{4}{x} = \frac{13}{2x+1} \Rightarrow 13x = 4(2x+1)$
$\Rightarrow 13x = 8x + 4 \Rightarrow 5x = 4$
$\Rightarrow x = \frac{4}{5}$ (or \$0.8 ✓) ✓

(d) $\frac{20}{x} + \frac{10}{2x+1} = 45$

$\Rightarrow \frac{20(2x+1) + 10x}{x(2x+1)} = 45$

$\Rightarrow 40x + 20 + 10x = 45(2x^2 + x)$

$\Rightarrow 90x^2 - 5x - 20 = 0$ ✓

$\Rightarrow 18x^2 - x - 4 = 0$

$(9x+4)(2x-1) = 0$

$x = \frac{1}{2}$ or $x = -\frac{4}{9}$ ✗

$x =$ \$ $\frac{1}{2}$ (or 0.5) ✓

21. Simplify: $\frac{3}{x-1} - \frac{2}{2x+1}$ [W-20/43/Q9(c)] --[3]

Solution: $\frac{3}{x-1} - \frac{2}{2x+1} = \frac{3(2x+1)-2(x-1)}{(x-1)(2x+1)}$

$= \frac{6x+3-2x+2}{2x^2+x-2x-1} = \frac{4x+5}{2x^2-x-1}$ ✓

Linear Inequaties

1. Solve the inequality $n+7 < 5n-8$ ---[2]

[SP-20/02/Q13]

Solution: $n+7 < 5n-8$

$\Rightarrow n-5n < -8-7$

$\Rightarrow -4n < -15$

$\Rightarrow n > \frac{-15}{-4}$ (Dividing both sides by a -ve number will reverse the inequality

$\Rightarrow n > \frac{15}{4}$

$\therefore n > 3.75$ ✓

Alternate:

$n+7 < 5n-8$ $[\because a<b \Rightarrow b>a]$

$\Rightarrow 5n-8 > n+7$

$\Rightarrow 5n-n > 7+8$

$\Rightarrow 4n > 15 \Rightarrow n > 3.75$ ✓

---

2. Solve $\frac{x}{2} - 13 > 12+3x$ ---[2]

Solution: $\frac{x}{2} - 13 > 12+3x$

$\Rightarrow \frac{x}{2} - 3x > 12+13$

$\Rightarrow -\frac{5}{2}x > 25$

Dividing both sides by $-\frac{5}{2}$, will reverse the inequality

$\Rightarrow x < 25 \times \left(-\frac{2}{5}\right)$

$\Rightarrow x < -10$ ✓

[W-19/22/Q9]

Alternate Method:

$\frac{x}{2} - 13 > 12+3x$ $\because [a>b \Rightarrow b<a]$

$\Rightarrow 12+3x < \frac{x}{2} - 13$

$\Rightarrow 3x - \frac{x}{2} < -13-12$

$\Rightarrow \frac{5}{2}x < -25$

$\Rightarrow x < \frac{-25}{5/2}$

$x < -25 \times \frac{2}{5}$

$x < -10$ ✓

3. Naga has n marbles. Panav has three times as many marbles as Naga. Naga loses 5 marbles and Panav buys 10 marbles. Together they now have more than 105 marbles. Write down and solve an inequality in n. ---[3]

[M-20/42/Q7(a)]

Solution: Naga has marbles $= n$

$\therefore$ Panav has marbles $= 3n$

In new situation:

Naga has left with $= (n-5)$

and Panav will have $= (3n+10)$

Given total no. $(n-5)+(3n+10) > 105$

$\Rightarrow 4n+5 > 105$

$\Rightarrow 4n > 105-5$

$n > \frac{100}{4} \Rightarrow n > 25$ ✓

4. Solve the inequality $3m+12 \leq 8m-5$ [S-20/43/Q4(a)]

Solution: $3m+12 \leq 8m-5$

$\Rightarrow 3m-8m \leq -5-12$

$-5m \leq -17$

$m \geq \frac{-17}{-5}$

Dividing both sides by -5 will reverse the inequality.

$\Rightarrow m \geq \frac{17}{5}$ or $m \geq 3.4$ ✓

5. Solve the inequality; $3x+12 < 5x-3$ [W-20/41/Q8(b)] ...[2]

Solution: $3x+12 < 5x-3 \Rightarrow 3x-5x < -3-12$

$-2x < -15$

dividing both sides by -2; $\Rightarrow x > \frac{-15}{-2}$

(Change the inequality) $\Rightarrow x > \frac{15}{2} \Rightarrow x > 7.5$ ✓

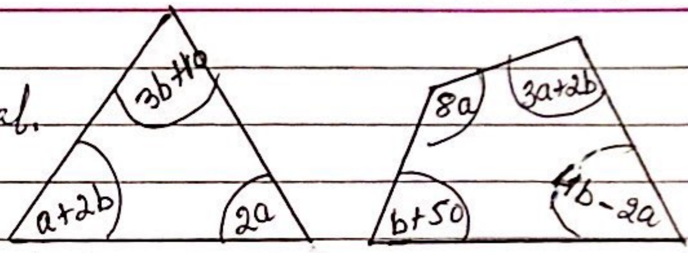
6. Find the integer values that satisfy the inequality. $2 < 2x \leq 10$. [W-20/43/Q9(a)] -- [2]

Solution: $2 < 2x \leq 10$

$\Rightarrow 1 < x \leq 5$ (Dividing each term by 2)

$\therefore$ The integer values of x are, 2, 3, 4, 5 ✓

1. The diagram shows a triangle and a quadrilateral. All angles are in degrees.



(i) For the triangle show that $3a+5b=170°$ ---[1]

(ii) For the quadrilateral show that $9a+7b=310°$ --[1]

(iii) Solve the simultaneous equations. --[3]

(IV) Find the size of the smallest angle in the triangle. --[1]

W-19/43/Q2(a)

Solution (i) In the triangle $3b+10°+a+2b+2a=180°$

$\Rightarrow 3a+5b=180-10 \Rightarrow 3a+5b=170°$ ✓

(ii) In the quadrilateral, $b+50+4b-2a+3a+2b+8a=360°$

$\Rightarrow 9a+7b=360-50$

$\Rightarrow 9a+7b=310°$ ✓

(iii) To Solve $3a+5b=170$ —— ①

$9a+7b=310°$ —— ②

multiplying equation ① by 3

$9a+15b=510$ —— ③

Subtract equation ③ from ②

$-8b=-200$

$\Rightarrow b=\frac{-200}{-8}=25$ ✓

Put $b=25$ in equation ①

$3a+5\times25=170$

$\Rightarrow 3a=170-125=45$

$\therefore a=\frac{45}{3}=15°$

$\therefore$ $\underline{a=15°}$ and $\underline{b=25°}$

(IV) Minimum angle in the triangle $=2a=2\times15°$

$=\underline{30°}$

2. The cost of one ruler is 'r' cents. The cost of one protractor is 'p' cents. The total cost of 5 rulers and 1 protractor is 245 cents. The total cost of 2 rulers and 3 protractors is 215 cents.

Write down two equations in terms of r and p, and solve these equations to find the cost of one protractor. --[5]

W-19/41/Q7(b)

(Elimination method)

Solution: $5r+p=245$ ---(1)

$2r+3p=215$ ---(2)

multiplying equation (1) by 3,

$15r+3p=735$ —(3)

$2r+3p=215$ —(2)

(3)-(2) $\Rightarrow$ $13r=520 \Rightarrow r=\frac{520}{13}=40$ ✓

Put $r=40$ in (1)

$5\times40+p=245$

$\Rightarrow p=245-200=45$ ✓

$\therefore$ The cost of one protractor $= 45$ Cents.

Alternate method

(Substitution method)

from equation (1) $p=(245-5r)$

Put in (2)

$2r+3(245-5r)=215$

$2r+735-15r=215$

$\Rightarrow -13r=215-735$
$=-520$

$\Rightarrow r=\frac{-520}{-13}=40$ ✓

from (1) $5\times40+p=245$

$p=245-200=45$ ✓

(Elimination method)

3. Solve the simultaneous equations: W-20/22/Q9 --[2]

$2x+y=7$ ---(1)

$3x-y=8$ ---(2)

add eqn (1) and (2)

$5x=15$

$x=\frac{15}{5}=3$

Put $x=3$ in eqn (1)

$2\times3+y=7$

$y=7-6=1$

$\therefore x=3$; $y=1$ ✓

(Alternate method)
{Substitution method}

from equation (1) $y=(7-2x)$ —(3)

Put the value of y in eqn (2)

$3x-(7-2x)=8$

$3x+2x-7=8$

$5x=15$

$x=3$ ✓

from (3) $y=7-2\times3=1$

$\therefore x=3,\ y=1$

4. Solve the simultaneous equations: $3x-8y=22$
$x+4y=4$ --[3]

[W-20/23/Q 10]

(Elimination method ↓)

Solution: $3x-8y=22$ ---①
$x+4y=4$ ---②

multiplying eqn② by 3
$3x+12y=12$ ---③

Subtract ① from ③ ⇒
$3x - 8y = 22$ --①

$20y=-10$
$y=-\frac{10}{20}=-\frac{1}{2}$ ✓

Put $y=-\frac{1}{2}$ in ②
$x+4\times\left(-\frac{1}{2}\right)=4$
$x-2=4 \Rightarrow x=6$

∴ $x=6$, $y=-\frac{1}{2}$ (or -0.5) ✓

(Alternate method)
(Substitution method ↓)

from eqn② $x=4-4y$ ——③

Put $x=4-4y$ in eqn ①
$3(4-4y)-8y=22$
$12-12y-8y=22$
$-20y=22-12$
$y=\frac{10}{-20}=-\frac{1}{2}$

Put $y=-\frac{1}{2}$ in ③
$x=4-4\left(-\frac{1}{2}\right)$
$=4+2=6$

∴ $x=6$ and $y=-\frac{1}{2}$ (or -0.5)

Change of Subject of a formula:

1. Make y the subject of the formula: $h^2 = x^2 + 2y^2$ --[3]

[S-20/22/Q19]

Solution: $h^2 = x^2 + 2y^2 \Rightarrow 2y^2 = h^2 - x^2$

$$\Rightarrow y^2 = \frac{h^2 - x^2}{2}$$

$$\Rightarrow y = \pm\sqrt{\frac{h^2 - x^2}{2}} \checkmark$$

2. $P = 2r + \pi r$ --[2]

Rearrange the formula to write r in terms of P and π.

[W-19/21/Q11]

Solution: $P = 2r + \pi r \Rightarrow r(2 + \pi) = P$

$$\Rightarrow r = \frac{P}{(2 + \pi)} \checkmark$$

3. Make x the subject of the formula: $x = \frac{3 + x}{y}$ --[3]

[M-20/42/Q5(d)]

Solution: $x = \frac{3 + x}{y} \Rightarrow xy = 3 + x$

$$\Rightarrow xy - x = 3$$

$$\Rightarrow x(y - 1) = 3$$

$$\Rightarrow x = \frac{3}{(y - 1)} \checkmark$$

4. Rearrange $2(4x - y) = 5x - 3$ to make y the subject. --[3]

[W-19/43/Q2(c)]

Solution: $2(4x - y) = 5x - 3 \Rightarrow 8x - 2y = 5x - 3$

$$\Rightarrow -2y = 5x - 3 - 8x$$

$$\Rightarrow -2y = -3 - 3x$$

$$\Rightarrow 2y = 3 + 3x \Rightarrow y = \frac{3x + 3}{2} \checkmark$$

5. Make p the subject of: (i) $5p + 7 = m$ --[2]

(ii) $y^2 - 2p^2 = h$ [W-19/42/8(a)] --[3

Solution (i) $5p + 7 = m$

$$\Rightarrow 5p = m - 7$$

$$\rightarrow p = \frac{m - 7}{5} \checkmark$$

(ii) $y^2 - 2p^2 = h$

$$\Rightarrow -2p^2 = -y^2 + h \Rightarrow 2p^2 = y^2 - h$$

$$\Rightarrow p^2 = \frac{y^2 - h}{2}$$

$$\Rightarrow p = \pm\sqrt{\frac{y^2 - h}{2}} \checkmark$$

6. Make x the subject of this formula. $2y = 5x - 7$ ---[2]

[W-20/21/Q7]

Solution: $2y = 5x - 7 \Rightarrow 5x = 2y + 7$

$\Rightarrow \underline{x = \frac{2y+7}{5}}$ $\left[\text{or } x = \frac{2y}{5} + \frac{7}{5}\right]$ ✓

7. $m = 2p + \sqrt{\frac{x}{y}}$, Make x the subject of this formula. ---[3]

[W-20/23/Q15]

Solution: $m = 2p + \sqrt{\frac{x}{y}} \Rightarrow \sqrt{\frac{x}{y}} = m - 2p$

$\Rightarrow \frac{x}{y} = (m-2p)^2$

$\Rightarrow x = y($

or $\underline{x = y(m^2 - 4mp + 4p^2)}$ ✓

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Direct and inverse variation.

1. p is directly proportional to $(q+2)^2$ ---[3]
when $q=1$, $p=1$. Find p when $q=10$. [S-20/22/Q22]

Solution: Given $p \propto (q+2)^2$

$$\Rightarrow p = k(q+2)^2 \quad \text{—— (1)}$$

when $q=1, p=1 \Rightarrow 1 = k(1+2)^2 \Rightarrow 1 = 9k \Rightarrow k = \frac{1}{9}$

$$\therefore \text{ from (1)} \quad p = \frac{1}{9}(q+2)^2 \quad \text{—— (2)}$$

for $q=10 \Rightarrow \quad p = \frac{1}{9}(10+2)^2 = \frac{1}{9} \times 144 = 16$ ✓

$$\therefore p = 16 ✓$$

2. y is directly proportional to the cube root of $(x+3)$. ---[3]
when $x=5$, $y=2/3$; Find y when $x=24$. [S-20/23/Q11]

Solution: Given, $y \propto (x+3)^{1/3}$

$$\Rightarrow y = k(x+3)^{1/3} \quad \text{—— (1)}$$

when $x=5, y=2/3 \Rightarrow \frac{2}{3} = k(5+3)^{1/3}$

$$\Rightarrow \frac{2}{3} = k \cdot 8^{1/3} \Rightarrow 2k = 2/3$$

$$\Rightarrow k = \frac{1}{3} ✓$$

$$\therefore \text{ from (1)} \quad y = \frac{1}{3}(x+3)^{1/3}$$

Now when $x=24 \Rightarrow y = \frac{1}{3}(24+3)^{1/3} = \frac{1}{3} \times 27^{1/3}$

$$\Rightarrow y = \frac{1}{3} \times 3 = 1$$

$$\therefore y = 1 ✓$$

3. y is inversely proportional to x^2 ---[3]
when $x=4$, $y=2$; Find y when $x=\frac{1}{2}$ [W-19/21/Q15]

Solution: Given. $y \propto \frac{1}{x^2} \Rightarrow y = \frac{k}{x^2}$ —— (1)

when $x=4, y=2 \Rightarrow 2 = \frac{k}{4^2} \Rightarrow k = 16 \times 2 = 32$

$$\therefore \text{ from (1)} \quad y = \frac{32}{x^2} \quad \text{—— (2)}$$

Now when $x=\frac{1}{2} \Rightarrow y = \frac{32}{(\frac{1}{2})^2} = 32 \times 4 = 128$

$$\therefore y = 128 ✓$$

4. t is inversely proportional to the square of $(x+1)$
when $x=2$, $t=5$ [W-19/23/Q20]
(a) Write t in terms of x. ---[2]
(b) when $t=1.8$, find the positive value of x. ---[2]

Solution (a) Given $t \propto \frac{1}{(x+1)^2} \Rightarrow t = \frac{k}{(x+1)^2}$ —— ①

when $x=2, t=5 \Rightarrow 5 = \frac{k}{(2+1)^2} \Rightarrow k = 5\times 9 = 45$

$\therefore$ from ① $t = \frac{45}{(x+1)^2}$ —— ②

(b) Now when $t=1.8$
from ② $1.8 = \frac{45}{(x+1)^2} \Rightarrow (x+1)^2 = \frac{45}{1.8} = 25$

$\Rightarrow x+1 = \pm\sqrt{25}$

$x = 5-1$ [Only + Value)

$\therefore x = 4$ ✓

5. y is inversely proportional to x^2.
when $x=4$, $y=7.5$; Find y when $x=5$. ---[3]
[M-20/42/Q7(b)]

Solution: Given $y \propto \frac{1}{x^2} \Rightarrow y = \frac{k}{x^2}$ —— ①

for $x=4$, $y=7.5 \Rightarrow 7.5 = \frac{k}{4^2} \Rightarrow k = 16\times 7.5 = 120$

$\therefore$ from ① $y = \frac{120}{x^2}$ —— ②

Now for $x=5 \Rightarrow y = \frac{120}{5^2} = \frac{120}{25} = 4.8$

$\therefore y = 4.8$ ✓

6. y is inversely proportial to the square root of x. ---[2]
when $y=7$, $x=2.25$, write y in terms of x. [W-20/23/Q23]

Solution: $y = \frac{k}{\sqrt{x}}$ —①, $y=7$, $x=2.25 \Rightarrow 7 = \frac{k}{\sqrt{2.25}} \Rightarrow 7 = \frac{k}{1.5}$

$\Rightarrow k = 7\times 1.5 = 10.5$

from $y = \frac{10.5}{\sqrt{x}}$ ✓

Algebric Indices

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Date

1. (a) Simplify $(27x^6)^{1/3}$ ---[2]

(b) Find the value of $(64x^4)^{0.5} \times 4x^{-2}$ ---[3]

$\boxed{SP-20/02/Q27}$

Solution (a) Simplify $(27x^6)^{1/3} = [3^3 \cdot (x^2)^3]^{1/3}$

$= ((3x^2)^3)^{1/3}$

$= (3x^2)^{3\times\frac{1}{3}} = \underline{3x^2}$ ✓

(b) $(64x^4)^{0.5} \times 4x^{-2} = (8^2(x^2)^2)^{1/2} \times 4x^{-2}$

$= [(8x^2)^2]^{1/2} \times 4x^{-2}$

$= (8x^2)^{1/2\times 2} \times 4x^{-2}$

$= 8x^2 \cdot 4x^{-2} = 32x^{2-2} = 32x^0 = 32\times 1$

$= \underline{32}$ ✓

2. Simplify: $\frac{p}{2q} \times \frac{4pq}{t}$ --- [2]

$\boxed{S-20/22/Q10}$

$= \frac{p\times 2p\times \cancel{2q}}{\cancel{2q}\times t} = \underline{\frac{2p^2}{t}}$ ✓

3. Simplify $8t^8 \div 4t^4 = \frac{8t^8}{4t^4} = 2t^{8-4} = \underline{2t^4}$ ✓ ---[2]

$\boxed{S-20/22/Q13}$

4. Simplify (a) $(5x^4)^3$ --[2]

$= 5^3 \times (x^4)^3 = \underline{125 \times x^{12}}$ ✓ $\boxed{S-20/22/Q21}$

(b) $(256\, x^{256})^{3/8}$ --[2]

$= [2^8 \times (x^{32})^8]^{3/8}$

$[(2x^{32})^8]^{3/8} = (2x^{32})^{8\times\frac{3}{8}}$

$= (2x^{32})^3 = 2^3 \cdot (x^{32})^3$

$= \underline{8\, x^{96}}$ ✓

5. $\sqrt[3]{y^2} = \sqrt[6]{x}$ and $y = \sqrt[n]{x}$ find the value of n. ---[2]

$\boxed{S-20/22/Q26}$

Solution: Given $\sqrt[3]{y^2} = \sqrt[6]{x} \Rightarrow y^{2/3} = x^{1/6} \Rightarrow x = (y^{2/3})^6$

or $x = y^{2/3\times 6} = y^4$

Now $y^4 = x \Rightarrow y = \sqrt[4]{x}$ $(y = \sqrt[n]{x})$

$\Rightarrow \underline{n=4}$ ✓

Algebic Indices

6. Simplify: (a) $p^2 \times p^4$ ---[1]

$= p^{2+4} = p^6$ ✓

(b) $m^{15} \div m^5$ --[1]

$= m^{15-5} = m^{10}$ ✓

(c) $(k^3)^5 = k^{3\times5} = k^{15}$ ✓ [S-20/23/Q6] --[1]

---

7. Simplify: $2x^3 \times 3x^2$ ---[2]

$= 6x^{3+2} = 6x^5$ ✓ [W-19/21/Q7]

8. Simplify: $\left(\frac{x^3}{8}\right)^{-\frac{4}{3}}$ [W-19/21/Q10] ---[2]

$$= \left(\frac{x^3}{2^3}\right)^{-4/3} = \left(\left(\frac{x}{2}\right)^3\right)^{-\frac{4}{3}} = \left(\frac{x}{2}\right)^{3\times-\frac{4}{3}} = \left(\frac{x}{2}\right)^{-4}$$

$$= \frac{1}{\left(\frac{x}{2}\right)^4} = \frac{1}{\frac{x^4}{16}} = \frac{16}{x^4}$$ ✓

---

9. (a) $3^{-2} \times 3^x = 81$ find the value of x. ---[2]

$\Rightarrow 3^{x-2} = 3^4 \Rightarrow x-2=4 \Rightarrow x=6$ ✓

(b) $x^{-\frac{1}{3}} = 32x^{-2}$ find the value of x ---[3]

$\Rightarrow \frac{x^{-1/3}}{x^{-2}} = 32 \Rightarrow x^{2-\frac{1}{3}} = 32$ [W-19/22/Q21]

$\Rightarrow x^{5/3} = 2^5$

$\Rightarrow x = (2^5)^{3/5} = 2^{5\times\frac{3}{5}} = 2^3 = 8$ ✓

$\therefore x = 8$ ✓

---

10. $2^{12} \div 2^{\frac{k}{2}} = 32$ find the value of k, -- [2]

[M-20/42/Q5(b)]

Solution: $2^{12} \div 2^{k/2} = 32 \Rightarrow 2^{12-k/2} = 2^5$

$\Rightarrow 12 - k/2 = 5$

$\Rightarrow \frac{k}{2} = 12-5 = 7$

$\Rightarrow k = 7\times2 = 14$

$\therefore k = 14$ ✓

11. Simplify: $(27x^9)^{2/3}$ [S-20/41/Q3(d)] ...[2]

Solution: $(27x^9)^{2/3} = \left(3^3 \cdot (x^3)^3\right)^{2/3} = (3x^3)^{3\times\frac{2}{3}} = (3x^3)^2 = 3^2 \cdot (x^3)^2$
$= 9x^6$ ✓

12. Simplify. $(3x^2y^4)^3$ ---[2]
[W-20/41/Q8(c)]

Solution: $(3x^2y^4)^3 = 3^3 \cdot (x^2)^3 \cdot (y^4)^3$
$= 27x^{2\times3} \cdot y^{4\times3} = 27x^6y^{12}$ ✓

13. Simplify. $2x^2 \times 5x^5$ [W-20/22/Q11] ---[2]

Solution: $2x^2 \times 5x^5 = 2\times5\, x^{2+5} = 10x^7$ ✓

14. Simplify. $a^2 \div a^6$ [W-20/23/Q4] --[1]

Solution: $a^2 \div a^6 = a^{2-6} = a^{-4}$ (or $\frac{1}{a^4}$) ✓

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Factorise

1. Factorise Completely: $3x^2 - 12xy$ [M-20/22/Q9(a)] ---[2]

Solution: $3x^2 - 12xy = 3x(x-4y)$ ✓

2. Factorise Completely: (a) $21a^2 + 28ab$ ---[2]
$= 7a(3a+4b)$ ✓

(b) $20x^2 - 45y^2$ ---[3]
$= 5(4x^2 - 9y^2) = 5[(2x)^2 - (3y)^2]$ $[\because a^2 - b^2 = (a+b)(a-b)]$
$= 5(2x+3y)(2x-3y)$ ✓ [S-20/21/Q9]

3. Simplify: $\dfrac{2x^2 + x - 15}{ax + 3a - 2bx - 6b}$ —— ① ---[5] [S-20/22/Q25]

Numerator: $2x^2 + x - 15$
$= 2x^2 + 6x - 5x - 15$
$= 2x(x+3) - 5(x+3)$
$= (x+3)(2x-5)$ —— ②

Denominator: $ax + 3a - 2bx - 6b$
$= a(x+3) - 2b(x+3)$
$= (a-2b)(x+3)$ —— ③

from ② and ③ in ① $= \dfrac{\cancel{(x+3)}(2x-5)}{\cancel{(x+3)}(a-2b)} = \dfrac{2x-5}{a-2b}$ ✓

4. (a) Factorise: $18y - 3ay + 12x - 2ax$ ---[2] [W-19/22/Q20]
$= 3y(6-a) + 2x(6-a)$
$= (6-a)(3y+2x)$ ✓

(b) Factorise: $3x^2 - 48y^2$ ---[3]
$= 3(x^2 - 16y^2)$
$= 3(x^2 - (4y)^2)$
$= 3(x+4y)(x-4y)$ ✓

5. Factorise: $5p + pt$ ---[1] [W-19/21/Q2]
$= p(5+t)$ ✓

6. Factorise (a) $12x+15$ ---[1]

$= \underline{3(4x+5)}$ ✓

(b) $xy-2x+3y-6$ ---[2]

$= x(y-2)+3(y-2)$

$= \underline{(x+3)(y+2)}$ ✓

[W-19/23/Q11]

---

7. Simplify: $\dfrac{x^2+5x}{x^2-25}$ ---[3]

[W-19/43/Q2]

Solution: $\dfrac{x^2+5x}{x^2-25} = \dfrac{x(x+5)}{x^2-5^2}$

$= \dfrac{x\cancel{(x+5)}}{(x-5)\cancel{(x+5)}} = \underline{\dfrac{x}{(x-5)}}$ ✓

---

8. Factorise $6x^2+7x-20$ [W-20/21/Q16] ---[2]

Solution: $6x^2+7x-20$

$= 6x^2+15x-8x-20$

$= 3x(2x+5)-4(2x+5)$

$= \underline{(3x-4)(2x+5)}$ ✓

$6\times -20 = -120$
Take two fector of 120
whose diffence is 7
$6\times 20 = 3\times 2\times 4\times 5 = \underline{15\times 8}$.

---

9. Simplify: $\dfrac{x^2-5x}{2x^2-50}$ ---[4]

[W-20/21/Q22]

Solution: $\dfrac{x^2-5x}{2x^2-50} = \dfrac{x(x-5)}{2(x^2-25)}$

$= \dfrac{x(x-5)}{2[x^2-5^2]} = \dfrac{x\cancel{(x-5)}}{2(x+5)\cancel{(x-5)}}$

$= \underline{\dfrac{x}{2(x+5)}}$ ✓

---

10. Factorise completely: $3a^2b-ab^2$ ---[2]

[W-20/41/Q8(a)]

Solution: $3a^2b-ab^2 = \underline{ab(3a-b)}$ ✓

11. Factorise completely. $4-8x$ ---[1]

[W-20/22/Q6]

Solution $4-8x = \underline{4(1-2x)}$ ✓

12. Simplify $\dfrac{ux-2u-x+2}{u^2-1}$ ---[4]

[W-20/22/Q26]

Solution: $\dfrac{ux-2u-x+2}{u^2-1} = \dfrac{u(x-2)-1(x-2)}{(u-1)(u+1)}$

$= \dfrac{\cancel{(u-1)}(x-2)}{\cancel{(u-1)}(u+1)} = \underline{\dfrac{x-2}{u+1}}$ ✓

13. Factorise $3x+8y-6ax-16ay$ ---[2]

[W-20/23/Q20]

Solution: $3x+8y-6ax-16ay$

$(3x+8y)-2a(3x+8y) = \underline{(3x+8y)(1-2a)}$ ✓

14. Factorise completely: (i) $6y^2-15xy$ --[2]

(ii) y^2-9x^2 --[2]

[W-20/43/Q9(b)]

Solution: (i) $6y^2-15xy$

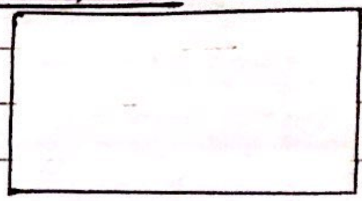
$= 3y(2y-5x)$ ✓

(ii) $y^2-9x^2 = (y)^2-(3x)^2$

$= \underline{(y-3x)(y+3x)}$ ✓

Solving Quadratic Equations → Factor method / Quadratic formula / Completing the square.

1. The perimeter of a rectangle is 80cm.
The area of the rectangle is A cm^2.



x Cm.

(i) Show that $x^2 - 40x + A = 0$ --- [3]

(ii) When A is 300, solve the equation $x^2 - 40x + A = 0$ by factorisation. --- [3]

(iii) When $A = 200$, solve the equation $x^2 - 40x + A = 0$ using quadratic formula, giving your answers to 2 d.p. --- [4]

|SP-20|04|Q5(a)|

Solution: Let the length of the rectangle = y cm.
Given breadth = x cm.

(i) Hence perimeter $= 2(x+y) = 80$

$$\Rightarrow x + y = 40 \Rightarrow y = 40 - x \quad \text{—— (1)}$$

Area of the rectangle $A = xy$

$$A = x(40 - x)$$

$$\Rightarrow A = 40x - x^2$$

$$\Rightarrow \underline{x^2 - 4x + A = 0} \quad \text{—— (2)} \checkmark$$

(ii) for $A = 300$, from (2)

$$x^2 - 40x + 300 = 0$$

$$x^2 - 30x - 10x + 300 = 0$$

$$x(x - 30) - 10(x - 30) = 0$$

$$(x - 30)(x - 10) = 0$$

$$x - 10 = 0 \text{ or } x - 30 = 0 \Rightarrow \underline{x = 10 \text{ cm or } x = 30 \text{ cm}} \checkmark$$

(iii) for $A = 200$ from (2) $x^2 - 40x + 200 = 0$

$\left\{ \begin{array}{l} \text{for quadratic equ}^n \\ ax^2 + bx + c = 0 \\ a = 1,\ b = -40,\ c = 200 \\ b^2 - 4ac = (-40)^2 - 4 \times 1 \times 200 \\ \quad = 1600 - 800 = 800 \end{array} \right.$

Using quad. formula:

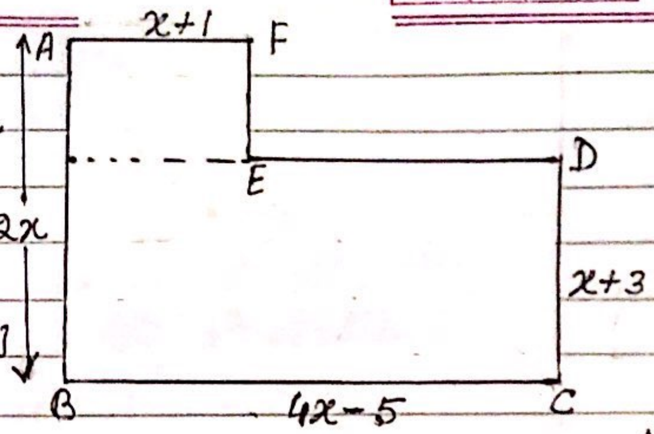
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{+40 \pm \sqrt{800}}{2 \times 1}$$

$$= \frac{40 \pm 28.2842}{2} = \frac{68.2842}{2} \text{ or } \frac{11.7158}{2}$$

$$\therefore x = \underline{34.14} \text{ or } x = \underline{5.86} \checkmark$$

2\. All the lengths in this question are in centimeters. The diagram shows a shape ABCDEF. The total area of the shape is 342cm.



(a) Show that $x^2+x-72=0$ --- [5]

(b) Solve by factorisation: $x^2+x-72=0$ --- [3]

(c) Work out the perimeter of the shape ABCDEF. --- [2]

(d) Calculate angle DBC. S-20/43/Q5 --- [2]

Solution: Height of the small rectangle $= 2x-(x+3) = (x-3)$

(a) Area of the shape $= (x-3)(x+1) + (x+3)(4x-5) = 342$

$$\Rightarrow x^2+x-3x-3+4x^2-5x+12x-15 = 342$$
$$\Rightarrow 5x^2+5x-360=0$$
$$\Rightarrow x^2+x-72=0$$

(b) To Solve: $x^2+x-72=0$

$$x^2+9x-8x-72=0$$
$$x(x+9)-8(x+9)=0$$
$$(x-8)(x+9)=0$$
$$x=8 \text{ or } x=-9$$

(c) Perimeter of the shape $= 2x+4x-5+x+3+4x-5+x-3$

$$= 12x-10$$
$$= 12\times 8-10$$
$$= 86$$

[Put $x=8$, $x=-9$ ✗]

(d) $\tan DBC = \dfrac{x+3}{4x-5} = \dfrac{8+3}{4\times 8-5} = \dfrac{11}{27}$

angle DBC $= \tan^{-1}\left(\frac{11}{27}\right) = 22.16$

$\therefore$ angle DBC $= 22.2°$

3. Carol walks 12 Km at x Km/h and then a further 6 Km at $(x-1)$ Km/h. The total time taken is 5 hours.

(i) Write an equation, in terms of x and show that it simplifies to $5x^2-23x+12=0$ --- [3]

(ii) Factorise: $5x^2-23x+12$ --- [2]

(iii) Solve the equation $5x^2-23x+12=0$ -- [1]

(iv) Write down Carol's walking speed during the final 6 Km. -- [1]

[W-19|41|Q7]

Solution:

(i) $\frac{12}{x}+\frac{6}{x-1}=5$

$\Rightarrow \frac{12(x-1)+6x}{x(x-1)}=5 \Rightarrow 12x-12+6x=5x(x-1)$

$\Rightarrow 18x-12=5x^2-5x$

$\Rightarrow 5x^2-5x-18x+12=0$

$\Rightarrow \underline{5x^2-23x+12=0}$ ✓

(ii) $5x^2-23x+12$ $\quad [\because 5\times 12=60=20\times 3]$

$5x^2-20x-3x+12$

$5x(x-4)-3(x-4)$

$\underline{(x-4)(5x-3)}$ ✓

(iii) To solve $5x^2-23x+12=0$

or $(x-4)(5x-3)=0$ (from Part (ii))

$x-4=0$ or $5x-3=0$

$\underline{x=4 \text{ or } x=3/5}$ ✓

(iv) Speed during final 6 Km $=(x-1)$ [for $x=4$

$=4-1$

$=\underline{3 \text{ Km/H}}$ ✓

4. Solve: $\frac{1}{x} - \frac{2}{x+1} = 3$

Show all your working and give your answers correct to 2 decimal places. ...[7]

[W-19/43/Q10]

<u>Solution:</u> $\frac{1}{x} - \frac{2}{x+1} = 3$

$$\Rightarrow \frac{x+1-2x}{x(x+1)} = 3$$

$$\Rightarrow 1-x = 3x(x+1)$$

$$\Rightarrow 1-x = 3x^2+3x$$

$$\Rightarrow 3x^2+4x-1=0$$

Quad. formula: $ax^2+bx+c=0$

$a=3,\ b=4,\ c=-1$

$b^2-4ac = 4^2-4\times3(-1)$

$= 16+12 = 28$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{28}}{2\times3}$$

$$= \frac{-4 \pm 5.291}{6}$$

$$= \frac{1.291}{6} \text{ or } \frac{-9.291}{6}$$

$$x = 0.215 \text{ or } -1.548$$

$$\therefore x = \underline{0.22} \text{ or } x = \underline{-1.55}$$

---

5. The solution of the equation $x^2+bx+c=0$ are $\frac{-7+\sqrt{61}}{2}$ and $\frac{-7-\sqrt{61}}{2}$. ...[3]

Find the value of b and the value of c. [S-20/42/Q9(b)]

<u>Solution:</u> $x^2+bx+c=0$

Solution are given $x = \frac{-7 \pm \sqrt{61}}{2}$ — ①

Quadratic formula for $ax^2+bx+c=0$

$a=1,\ b=b$ and $c=c$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-b \pm \sqrt{b^2-4c}}{2}$ — ②

Comparing ① and ②

$-b=-7 \Rightarrow \underline{b=7}$ and $b^2-4c=61$

$\Rightarrow 7^2-4c=61$

$\Rightarrow c = \frac{12}{-4} = \underline{-3}$

$\therefore \underline{b=7 \text{ and } c=-3}$

Completing the Square.

6. $x^2 - 12x + a = (x+b)^2$ --- [2]

Find the value of a and the value of b. [M-20/22/Q20]

Solution: $x^2 - 12x + a = (x+b)^2$

$\Rightarrow x^2 - 12x + a = x^2 + 2bx + b^2$ —— ①

Comparing the coeff of x and the constant terms:

$\Rightarrow 2b = -12 \Rightarrow b = -6$ ✓

and $a = b^2 = (-6)^2 = 36$ ✓

$\therefore$ $a = 36$ and $b = -6$ ✓

7. Find the turning point of $y = x^2 + 4x - 3$ by completing the square.

[SP-20/02/Q25] --- [4]

Solution: $y = x^2 + 4x - 3$

$y = x^2 + 4x + 2^2 - 2^2 - 3$

$= (x+2)^2 - 7$

Turning point $(-2, -7)$ ✓

[Note: $y = (x-h)^2 + k$

Turning point (or vertex is (h,k) ✓)]

Given $x^2 + bx + c$

To make complete square add and subtract. $\left(\frac{b}{2}\right)^2$

$x^2 + bx + \left(\frac{b}{2}\right)^2 - \frac{b^2}{4} + c$

$\left(x + \frac{b}{2}\right)^2 - \left(\frac{b^2 - 4c}{4}\right)$

8. (a) $x^2 - 18x - 27$ in the form $(x+k)^2 + h$ --- [2]

(b) Use your answer to part (a) to solve the eqn $x^2 - 18x - 27 = 0$ --- [2]

[S-20/23/Q18]

Solution (a) $x^2 - 18x - 27$

$= x^2 - 18x + 9^2 - 81 - 27$

$= (x-9)^2 - 108$ ✓

(b) To solve $x^2 - 18x - 27 = 0$

Part (a) $\Rightarrow$ $(x-9)^2 - 108 = 0 \Rightarrow (x-9)^2 = 108$

$x - 9 = \pm\sqrt{108} = \pm 10.39$

$\therefore x = 10.39 + 9$ or $-10.39 + 9$

$x = 19.39$ or -1.39 ✓

9 (i) Write x^2+8x-9 in the form $(x+k)^2+h$ ...[2]

(ii) Use your answer to part (i) to solve the equation $x^2+8x-9=0$

$\boxed{S\text{-}20 \mid 42 \mid Q9(a)}$ --[2]

Solution:

(i) $x^2+8x-9 = x^2+8x+\left(\frac{8}{2}\right)^2-\left(\frac{8}{2}\right)^2-9$

$= (x+4)^2-16-9$

$= (x+4)^2-25$ —— (1)

(ii) To solve $x^2+8x-9=0$

from Part(i), $\Rightarrow (x+4)^2-25=0$

$(x+4)^2=25$

$x+4=\pm\sqrt{25}$

$x+4=\pm 5$

$\Rightarrow x+4=5$; $x+4=-5$

$\underline{x=1}$ ✓ ; $\underline{x=-9}$ ✓

10. Alan invests \$200 at a rate of r% per year compound interest.
After 2 years the value of his investment is \$ 206.46
(i) Show that $r^2 + 200r - 323 = 0$ ---[3]
(ii) Solve the equation $r^2 + 200r - 323 = 0$ to find the rate of interest.
Give your answer correct to 2 decimal places. ---[3]
[W-20/41/Q8f]

Solution: Value after two years: Amount $= 200\left(1+\frac{r}{100}\right)^2 = 206.46$
(i)

$$\Rightarrow 200\left[1+\frac{2r}{100}+\frac{r^2}{10000}\right] = 206.46$$

$$\Rightarrow \frac{200}{10000}\left[10000+200r+r^2\right] = 206.46$$

$$\Rightarrow r^2+200r+10000 = 206.46 \times \frac{100}{2} = 10323$$

$$\text{or} \quad r^2+200r-323=0 \checkmark$$

(ii) To Solve $r^2+200r-323=0$

$$r = \frac{-200 \pm \sqrt{41292}}{2} \qquad \left\{ \begin{aligned} b^2-4ac &= (200)^2+4\times1\times323 \\ &= 40000+1292 \\ &= 41292 \end{aligned} \right.$$

$$= \frac{-200 \pm 203.2}{2}$$

$$= 1.6 \checkmark \quad \text{(-ve value is not valid)}$$

11. Petra spends \$20 on small coffee cakes and \$10 on large coffee cakes. The total number of cakes is 45.
Every small cake costs \$ x and every large cake costs \$ $(2x+1)$.
Write an equation in terms of x.
Solve this equation to find the cost of a small coffee cake. ---[7]
[W-20/42/Q5(d)]

Solution:

$$\frac{20}{x} + \frac{10}{2x+1} = 45$$

$$\frac{20(2x+1)+10x}{x(2x+1)} = 45$$

$$\Rightarrow 40x+20+10x = 45(2x^2+x)$$

$$\Rightarrow 90x^2-5x-20=0 \checkmark$$

$$90x^2-5x-20=0$$

$$\Rightarrow 18x^2-x-4=0$$

$$(9x+4)(2x-1)=0$$

$$x=\frac{1}{2} \quad \text{or} \quad x=-\frac{4}{9} \times$$

$$\therefore x = \$\tfrac{1}{2} \text{ (or } 0.5)$$

Solving Simultaneous Equations, one linear and one quadratic equation

1. Solve the simultaneous equations:

$$y = 5x^2 + 4x - 19$$
$$y = 4x + 1$$

[S-20/42/Q9(a)] ---[5]

Solution: $y = 5x^2 + 4x - 19$ --- ①

$y = 4x + 1$ --- ②

from ① and ② $5x^2 + 4x - 19 = 4x + 1$

$\Rightarrow 5x^2 = 20$

$\Rightarrow x^2 = 4 \Rightarrow x = \pm\sqrt{4} = \pm 2$

from ① $x = 2, y = 9$

and $x = -2, y = -7$ ∴ Solution: $x = 2, y = 9$ ✓ and $x = -2, y = -7$ ✓

2. Solve the simultaneous equations:

$$x = 7 - 3y$$
$$x^2 - y^2 = 39$$

[M-20/22/Q16] ---[6]

Solution: $x = 7 - 3y$ --- ①

$x^2 - y^2 = 39$ --- ②

from ① and ② $(7-3y)^2 - y^2 = 39$

$\Rightarrow 49 + 9y^2 - 42y - y^2 - 39 = 0$

$\Rightarrow 8y^2 - 42y + 10 = 0$

or $4y^2 - 21y + 5 = 0$

$4y^2 - 20y - y + 5 = 0$

$4y(y-5) - 1(y-5) = 0$

$(y-5)(4y-1) = 0$

$y = 5$; $y = \frac{1}{4}$

from ① $y = 5 \Rightarrow x = -8$ ✓

and $y = \frac{1}{4} \Rightarrow x = \frac{25}{4}$ ✓

$y = 5$, $x = -8$ ✓

and $y = \frac{1}{4}$, $x = \frac{25}{4}$ ✓

3. The straight line $y=3x+2$ intersects the curve $y=2x^2+7x-11$ at two points. Find the coordinates of these two points. Give your answers correct to 2 decimal places. --[6]
[W-20/43/Q9(d)]

Solution: Line: $y=3x+2$ ------ ①
Curve: $y=2x^2+7x-11$ --- ②
from ① and ② $2x^2+7x-11=3x+2$

$\Rightarrow 2x^2+4x-13=0$

$$\left\{\begin{aligned} &b^2-4ac \\ &=4^2-4\times 2\times(-13) \\ &=16+104=120 \end{aligned}\right.$$

$$x=\frac{-4\pm\sqrt{120}}{2\times 2}$$

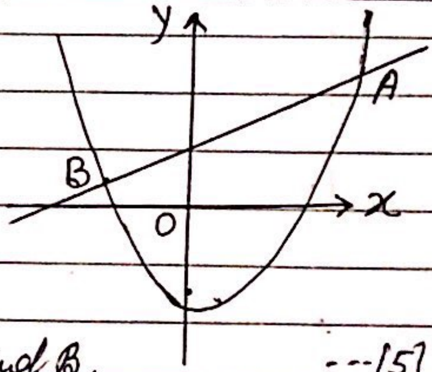
$$=\frac{-4\pm 10.954}{4}$$

from ① $= -3.734 \ ; \ 1.738$ or $x=1.74 \ ; \ -3.74$

for $x=1.74, y=7.22$ and for $x=-3.74, y=-9.22$

$\therefore$ Points of intersect $(1.74, 7.22)$, $(-3.74, -9.22)$

4. The diagram shows a curve $y=2x^2-2x-7$. The straight line with equation $y=3x+5$ intersect the curve at the points A and B. Find the coordinates of the points A and B. --[5]
[W-20/41/Q5(b)]



Solution: Line: $y=3x+5$ ---- ①
Curve: $y=2x^2-2x-7$ -- ②
from ① and ②

$2x^2-2x-7=3x+5$

$\Rightarrow 2x^2-5x-12=0$

$2x^2-8x+3x-12=0$

$2x(x-4)+3(x-4)$

$(x-\ )(2x+3)$

$\Rightarrow x=4, \ x=-3/2$

from ① at $x=4$, $y=17$
and at $x=-\frac{3}{2}$, $y=\frac{1}{2}$ (or 0.5)

$\therefore A(4,17)$, $B(-1.5, 0.5)$