Date 27.02.21

0580

IGCSE Maths

Contents. Pages

Algebra - 2.

Revision

SP-20	M-20	S-20	W-19	W-20

Suresh Goel
(Former Director)
Alliance World School,
Noida, Delhi - N.C.R.
INDIA.

(+91 9810444804)

Sequences

1(a) Complete the table for 5_{th} and the n_{th} term of each sequence:

1st term	2nd term	3rd term	4th term	5th term	n_{th} term
9	5	1	-3		
4	9	16	25		
1	8	27	64		
8	16	32	64		

---[11]

(b) 0, 1, 1, 2, 3, 5, 8, 13, 21, ---

This is a Fibonacci sequence.

After first two terms, rule to find the next term is "add the two previous terms". For example, $5+8=13$

Use this rule to complete each of the following Fibonacci sequence.

2   4   ---   ---   ---

1   —   ---   ----   11

---- -1 ----   ---   1 --[3]

(c) $\frac{1}{3}, \frac{3}{4}, \frac{4}{7}, \frac{7}{11}, \frac{11}{18}, \ldots$

(i) One term of this sequence is $\frac{p}{q}$

Find, in terms of p and q, the next term in this sequence. --[1]

(ii) Find the 6_{th} term of this sequence. --[1]

W-19/41/Q10

Solution (a)

5_{th} term	n_{th} term
-7	$13-4n$
36	$(n+1)^2$
125	n^3
128	2^{n+2}

(b) ---, ---, 6, 10, 16.

---, 3, 4, 7, ----

2, --, 1, 0, ---

(c) (i) $\frac{p}{p+q}$

(ii) $\frac{18}{29}$

2. Find the n_{th} term in each sequence.

(i) 4, 2, 0, −2, −4 ---[2]

(ii) 1, 17, 17, 31, 49 ---[2]

M-20/42/7(C)

Solution (i) 4, 2, 0, −2, −4

It is an arithmetic sequence,

with common difference $d = 2-4 = -2$

n_{th} term $= a+nd$ —① (Linear function in n)

for $n=1$, first term $4 = a + 1\times(-2)$ ($\because d=-2$)

$\Rightarrow a = 6$

$\therefore$ from ① n_{th} term $= \underline{(6-2n)}$ $\left[\begin{matrix} a=6 \\ d=-2 \end{matrix}\right]$

(ii) 1 7 17 31 49

Ist diff. 6 10 14 18

IInd difference 4 4 4 is constant } ①

$\therefore n_{th}$ term $= an^2+bn+c$ (quad in n)

$n = 1, 2, 3, 4$ with terms $(a+b+c)$ $(4a+2b+c)$ $(9a+3b+c)$, $(16a+4b+c)$

② Ist diff 3a+b 5a+b 7a+b

IInd difference 2a 2a -- is constant.

from:

① & ② Comparing $2a = 4 \Rightarrow a = 2$

$3a+b = 6 \Rightarrow 3\times2+b=6 \Rightarrow b=0$

$a+b+c = 1 \Rightarrow 2+0+c \Rightarrow c=-1$

$\therefore$ Required n_{th} term an^2+bn+c

$= 2n^2+0-1$

$\therefore n_{th}$ term $= 2n^2-1$

---

3. Find the n_{th} term of each sequence.

(a) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \ldots$ --[1]

(b) 1, 5, 25, 125, 625, -- W-19/23/Q14 --[1]

Solution (a) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}$ --- n_{th} term $= \underline{\frac{1}{2n}}$

(b) 1, 5, 25, 125, 625, --- or $5^0, 5^1, 5^2, 5^3$, -- n_{th} term $= \underline{5^{n-1}}$

4. Here is a sequence of numbers.

$7, 5, 3, 1, -1, ---$

(a) Find the next term in the sequence. --[1]

(b) Find an expression for the n_{th} term of this sequence. --[2]

SP.20|02|Q15

Solution (a) $7, 5, 3, 1, -1, --$

It is an arithmetic sequence

Next term $= -1-2 = -3$ ✓ (d = Common difference $= 5-7 = -2$ ✓ (is constant))

(b) Let n_{th} term $= a + nd$ —— ①

for $n=1$, 1st term $7 = a + 1\times(-2)$ as $d=-2$

$\Rightarrow a = 9$ ✓

$\therefore$ from ① n_{th} term $= \underline{9-2n}$ ✓ $\left\{ \begin{array}{l} \text{as } d=-2 \\ a=9 \end{array} \right.$

---

5. (a) n_{th} term of a sequence is $60-8n$

Find the largest number in this sequence. ---[1]

Solution: the largest number will be for $n=1$

$= 60-8\times1 = 52$ ✓

(b) Here are the first five terms of a different sequence;

$12 \quad 19 \quad 26 \quad 33 \quad 40$

--[2]

Find an expression for the n_{th} term. S-20|21|Q5

Solution: $12, 19, 26, 33, 40$ is an arithmetic sequence

with common difference $d = 19-12 = 7$ ✓

and first term $= 12$

Now the n_{th} term $= a + nd$

$= a + 7\times n$ —— ① ($\because d = 12$)

for $n=1$, first term $12 = a + 7\times1 \Rightarrow a = 5$

$\therefore$ from ① Required n_{th} term

$= a + nd$

$= \underline{5+7n}$ ✓ $\left(\because \begin{array}{l} a=5 \\ d=7 \end{array}\right)$

6.

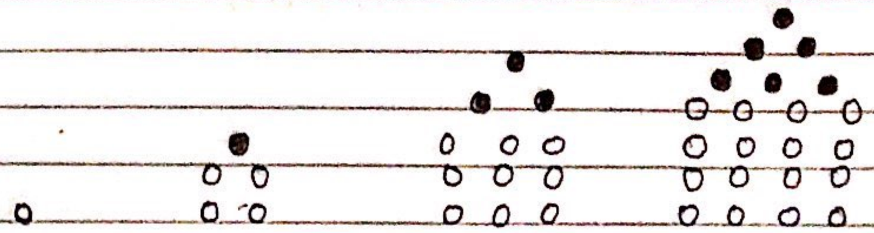


Diagram 1 Diagram 2 Diagram 3 Diagram 4

These are first four diagrams of a sequence.
The diagrams are made from white dots and black dots.

(a) Complete the table for Diagram 5 and Diagram 6.

Diagram	1	2	3	4	5	6
Number of White dots	1	4	9	16		
Number of black dots	0	1	3	6		
Total number of dots	1	5	12	22		

---[2]

(b) Write an expression, in terms of n, for the number of white dots in Diagram n. [1]

(c) The expression for the total number of dots in Diagram n is $\frac{1}{2}(3n^2 - n)$

(i) Find the total number of dots in Diagram 8. ---[1]

(ii) Find an expression for the number of black dots in Diagram n --[2]

(d) T is the total number of dots used to make **all** of the first n diagrams.

$T = an^3 + bn^2$; find the value of a and value of b. ---[5]

W-20/41/Q7

Solution (a)

25	36
10	15
35	51

(b) n^2

(c) (i) $\frac{1}{2}(3\times8^2-8) = 92$ ✓

(ii) $\frac{1}{2}(3n^2 - n) - n^2$

$= \frac{1}{2}[3n^2 - n - 2n^2]$

$= \frac{1}{2}[n^2 - n]$ ✓

$T = an^3 + bn^2$

for $n = 1$ for Diagram 1

Total: $a + b = 1$ —— (1)

for $n = 2$, $8a + 4b = (5+1)$

$8a + 4b = 6$ —— (2)

Solving (1) & (2)

$a = \frac{1}{2}$ and $b = \frac{1}{2}$ ✓

6

Sequence	1st term	2nd term	3rd term	4th term	5th term	nth term
A	13	9	5	1		
B	0	7	26	63		
C	$\frac{7}{8}$	$\frac{8}{16}$	$\frac{9}{32}$	$\frac{10}{64}$		

(a) Complete the table for the three sequences. ---[10]

(b) One term in sequence C is $\frac{p}{q}$.

Write down the next term in sequence C in terms of p and q

[W-20/42/Q11] ---[2]

Solution: (a) A: 13, 9, 5, 1, is an A.P

Common diff $= 9-13 = -4 \Rightarrow 5_{th}$ term $= 1-4 = \underline{-3}$ ✓

n_{th} term $= a+(n-1)d = 13+(n-1)\times(-4) = \underline{17-4n}$ ✓

B: 0, 7, 26, 63, --

5_{th} term $= 5^3-1 = 124$ ✓

n_{th} term $= \underline{n^3-1}$ ✓

C: $\frac{7}{8}, \frac{8}{16}, \frac{9}{32}, \frac{10}{64}$, ---

5_{th} term $= \underline{\frac{11}{128}}$ ✓

$\begin{cases} N^r \text{ is an A.P, common diff } 1;\ N^r = n+6 \\ D^r \text{ is a G.P, common ratio } 2;\ D^r = 8\times 2^{n-1} \end{cases}$

$= 2^{n+2}$

n_{th} term $= \frac{n+6}{2^{n+2}}$

(b) $\frac{n+6}{2^{n+2}} = \frac{p}{q}$ Given.

Next term $= \underline{\frac{p+1}{2q}}$

Scanned with CamScanner

7. The table shows the first four terms in sequences A, B and C.

Sequence	1st term	2nd term	3rd term	4th term	5th term	nth term
A	4	9	14	19		
B	3	10	29	66		
C	1	4	16	64		

complete the table. --- [9]

[W-20/43/Q11]

Solution

A: 4, 9, 14, 19 is an A.P with common diff = 5 ✓

$\therefore$ 5th term $= 19+5 = \underline{24}$ ✓

nth term $= 5n - k$ $\left[\begin{array}{l} \text{for } n=1 \\ t_1 = 5-k = 4 \\ k=1 \end{array}\right.$

$= \underline{5}$[illegible] - [illegible] ✓

B: 3, 10, 29, 66

$(1^3+2,\ 2^3+2,\ 3^3+2,\ 4^3+2)$ $\therefore$ 5th term $= 5^3+2 = \underline{127}$ ✓

nth term $= \underline{n^3+2}$ ✓

C: 1, 4, 16, 64 -- G.P with common ratio 4.

$\therefore$ 5th term $= 64 \times 4 = \underline{256}$ ✓

$(a \cdot r^{n-1}) =$ nth term $= 1 \times 4^{n-1} = \underline{4^{n-1}}$ ✓

Linear Programming

Page No. B-1
Date

1.

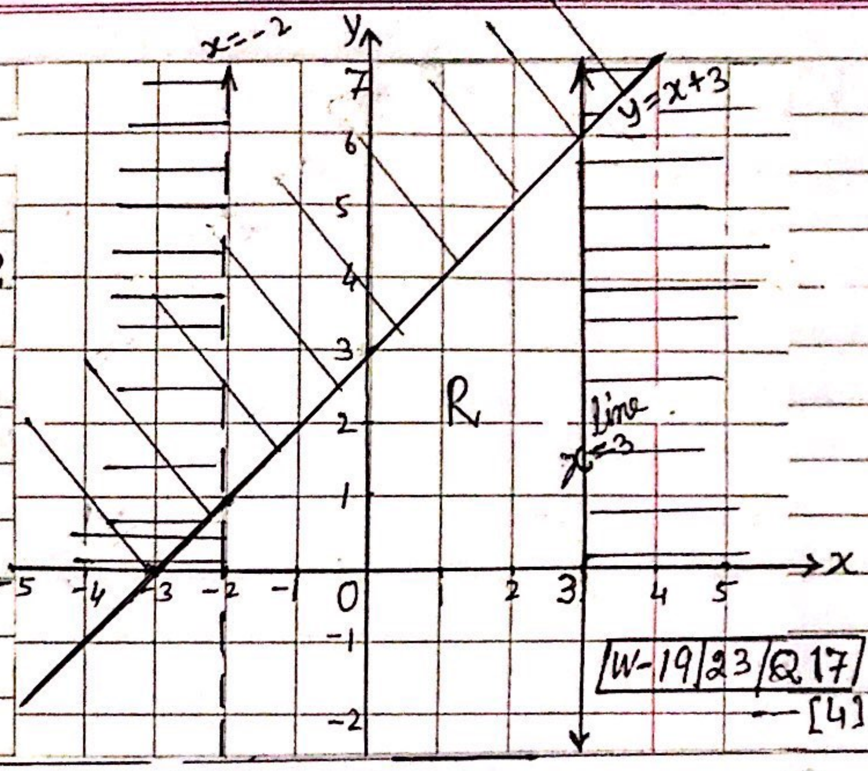
By shading the unwanted regions on the grid, draw and label the region R that satisfies the following inequalities:

$-2 < x \leq 3$

and $y \leq x+3$.

$y = x+3$

$(0,3), (-3,0)$



W-19/23/Q17

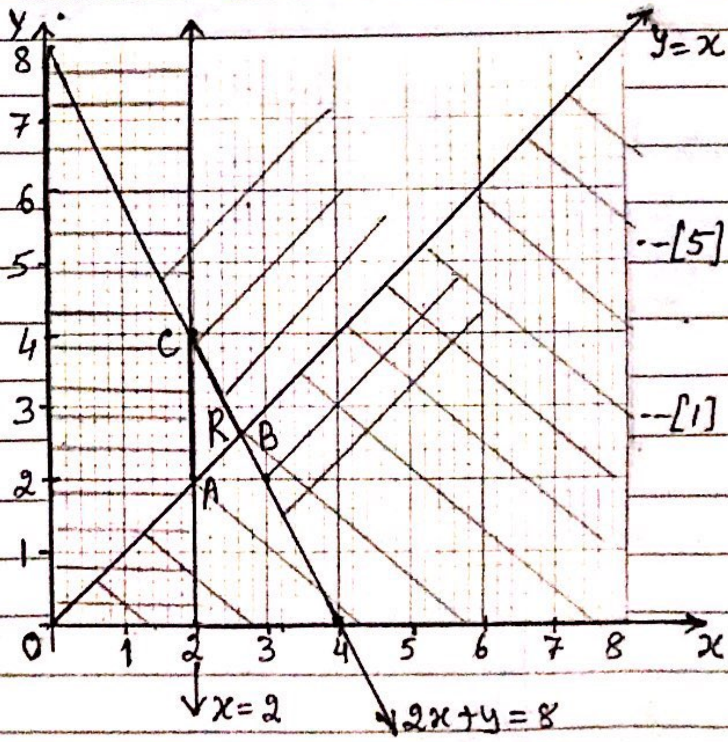
---[4]

2. (a) By drawing suitable lines and shading unwanted region, ...

$x \geq 2$, $y \geq x$ and $2x+y \leq 8$. --[5]

(b) Find the largest value of $(x+y)$ in the region R. --[1]

S-20/22/Q23



Solution: $2x+y \leq 8$

for $2x+y = 8$, $(0,8)$, $(4,0)$, $(2,4)$, $(3,2)$

Region R is bounded by triangle ABC. $A(2,2)$, $B(\frac{8}{3}, \frac{8}{3})$, $C(2,4)$

To find the max value of $P = (x+y)$

at $A(2,2)$ $P = 2+2 = 4$

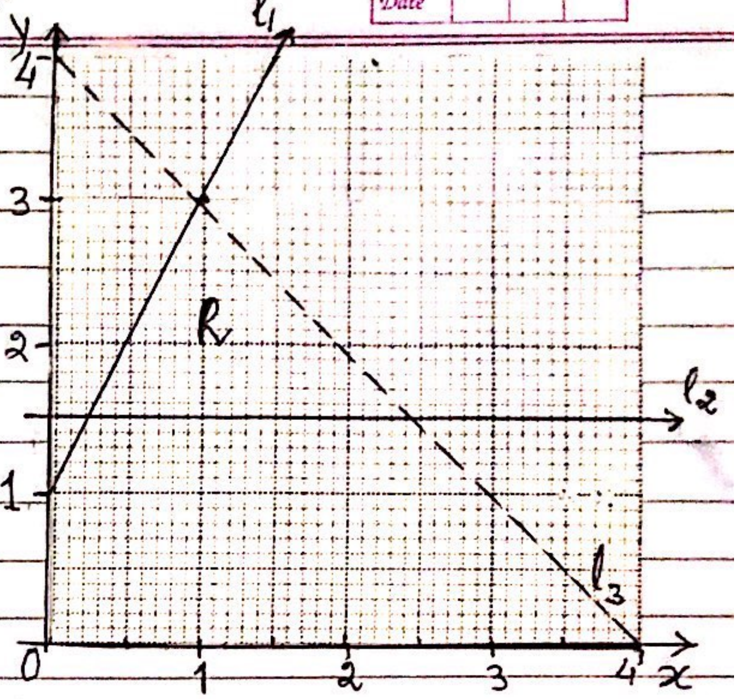
$B(\frac{8}{3}, \frac{8}{3})$ $P = \frac{8}{3} + \frac{8}{3} = \frac{16}{3} = 5\frac{1}{3}$

$C(2,4)$ $P = 2+4 = \underline{6}$ ✓

$\therefore$ Maximum value of $x+y = \underline{6}$ ✓

Linear Programming

3. Write down the three inequalities that define the region R.



[S-20/23/Q13] ...[4]

Solution:

line l_1, Passes through (0,1) and (1,3), Eqn of l_1

$y-1 = \frac{3-1}{1-0}(x-0)$

$\Rightarrow l_1: y = 2x+1$ ✓

line l_2; $y = 1.5$

line l_3; $x+y=4$

∴ Three inequalities defining R are;

$x+y < 4$ ✓

$y \geq 1.5$ ✓

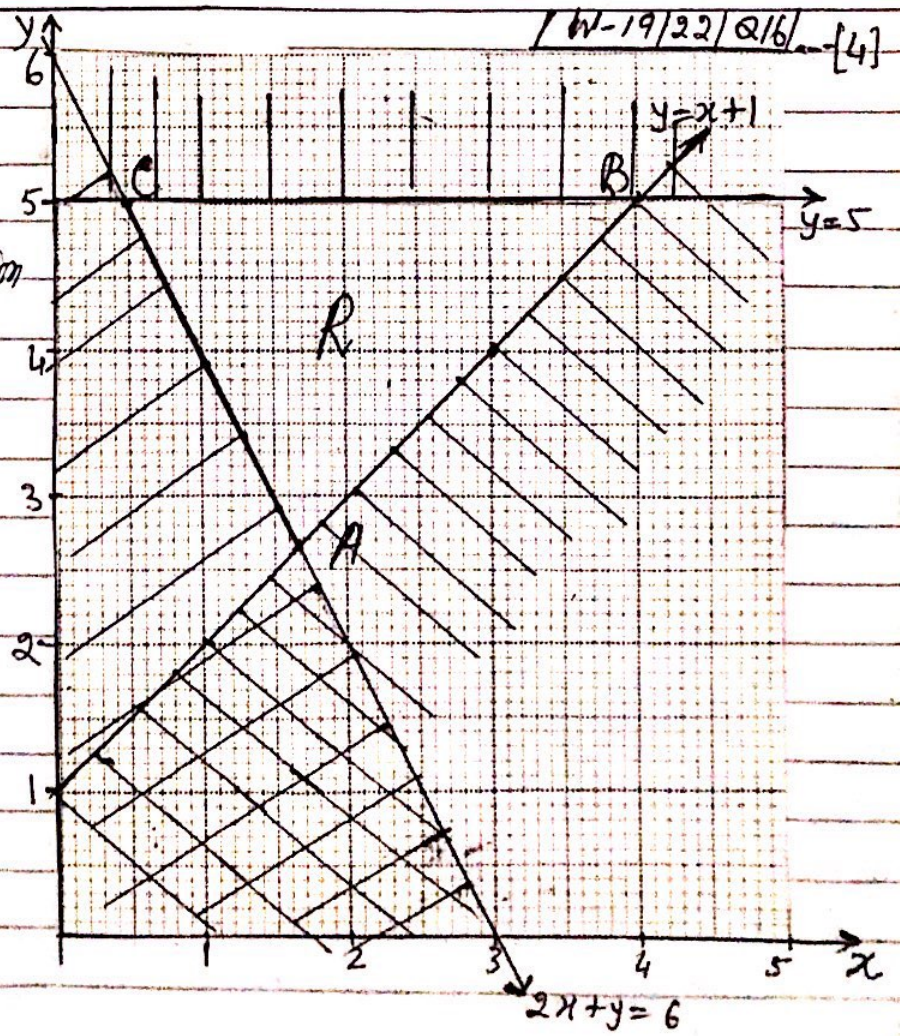
$y \leq 2x+1$ ✓

[W-19/22/Q16] ...[4]

4. By shading the unwanted region of the grid, find and label the region R, that satisfies the following inequalities:

$y \leq 5$, $2x+y \geq 6$, and $y \geq x+1$

$y = x+1 \to (0,1), (3,4), (4,5)$



Triangle ABC and inside represents → 'R'

5. Raheem makes baskets and mats.
Each week he makes x baskets and y mats.
He makes fewer than 10 mats.
The number of mats he makes is greater than or equal to the number of baskets he makes.

(a) One of the inequalities that shows this information is $y < 10$.
Write down the other inequality. ---[1]

(b) He takes $2\frac{1}{4}$ hours to make a basket and $1\frac{1}{2}$ hour to make a mat. Each week he works for a maximum of 22.5 hours.
Show that $3x + 2y \leq 30$ ---[2]

(c) On the grid, draw three straight lines and shade the unwanted regions to show these inequilities. ---[5]

(d) He makes \$40 profit on each basket he sells and \$28 on each mat he sells.
Calculate the maximum profit he can make each week. ---[2]

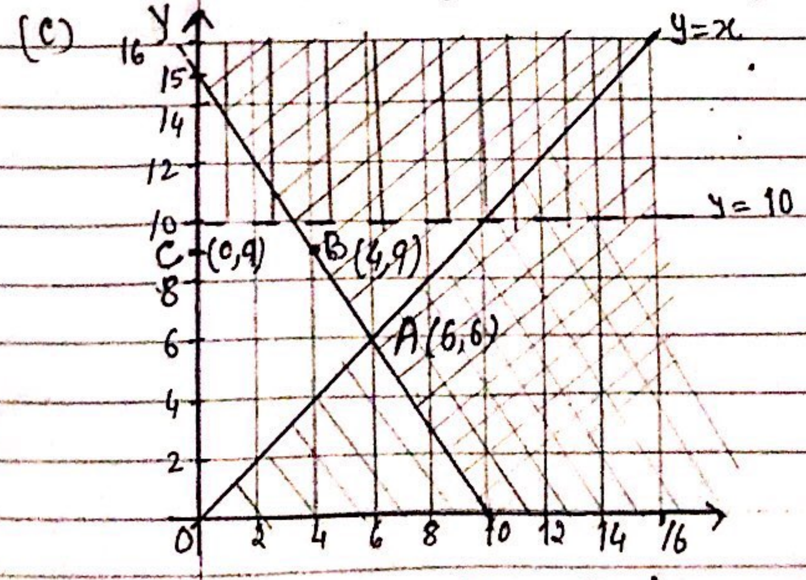
S-20/41/Q6

Solution: (a) $y \geq x$

(b) $\frac{9}{4}x + \frac{3}{2}y \leq 22.5$

$\Rightarrow 9x + 6y \leq 90 \Rightarrow 3x + 2y \leq 30$ ✓

(c)



(c) $3x+2y=30$ —① $(0,15), (10,0)$ ✓

$y<10$ —②

and $y \geq x$ —③

(d) Profit function $P = (40x + 28y)$

$O(0,0) \rightarrow P = 0$

$A(6,6) \rightarrow P = 40\times6 + 28\times6 = \$408$

$B(4,9) \rightarrow P = 40\times4 + 28\times9 = \$412$ ✓

$C(0,9) \rightarrow P = 40\times0 + 28\times9 = \$252$

Max Profit = \$412. ✓

Linear Programming

6. A car hire company has x small cars and y large cars.
The company has at least 6 cars in total.
The number of large cars is less than or equal to the number of small cars.
The largest number small cars is 8.

(a) Write down three inequalities, in terms of x and/or y, to show this information. ---[3]

(b) A small car can carry 4 people and a large car can carry 6 people. One day the largest number of people to be carried is 60, show that $2x+3y \leq 30$ ---[1]

(c) By shading the unwanted regions on the grid, show and label the region R that satisfy all four inequalities. ---[6]

(d) (i) Find the number of small cars and the number of large cars needed to carry exactly 60 people. ---[1]

(ii) When the company uses 7 cars, find the largest number of people that can be carried. W-19/41/Q9 ---[2]

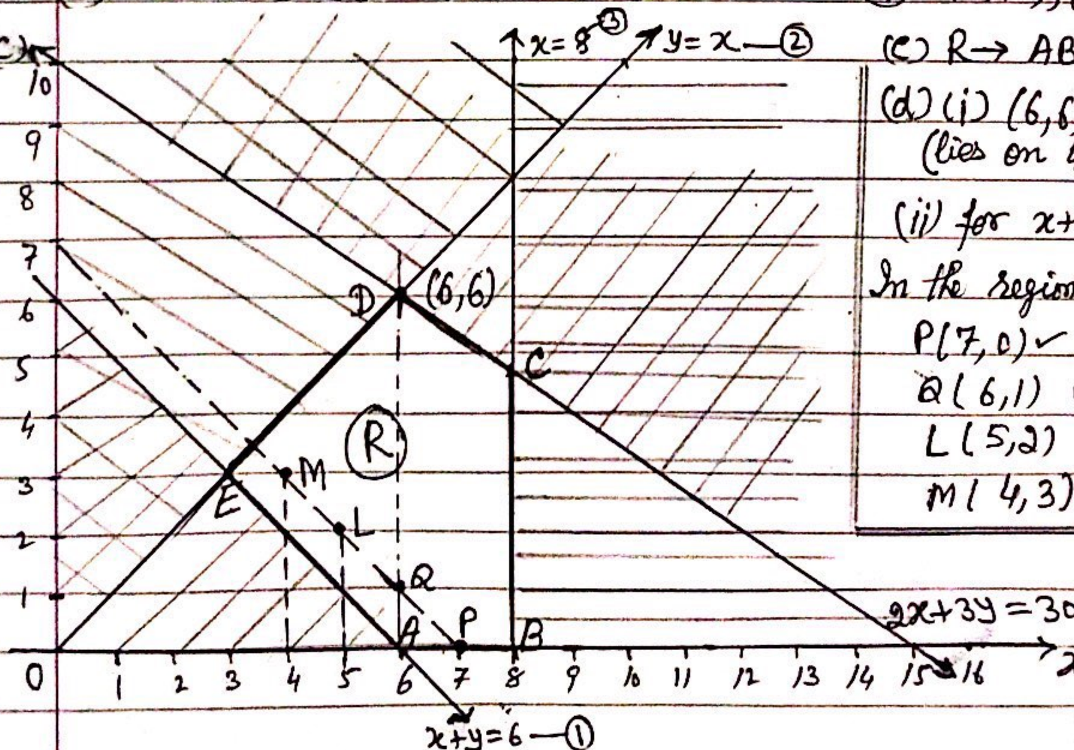
Solution (a) $x+y \geq 6$ ——— ①

$y \leq x$ ——— ②

$x \leq 8$ ——— ③

(b) $4x+6y \leq 60 \Rightarrow 2x+3y \leq 30$ ——— ④ $(0,10), (15,0)$

(c)



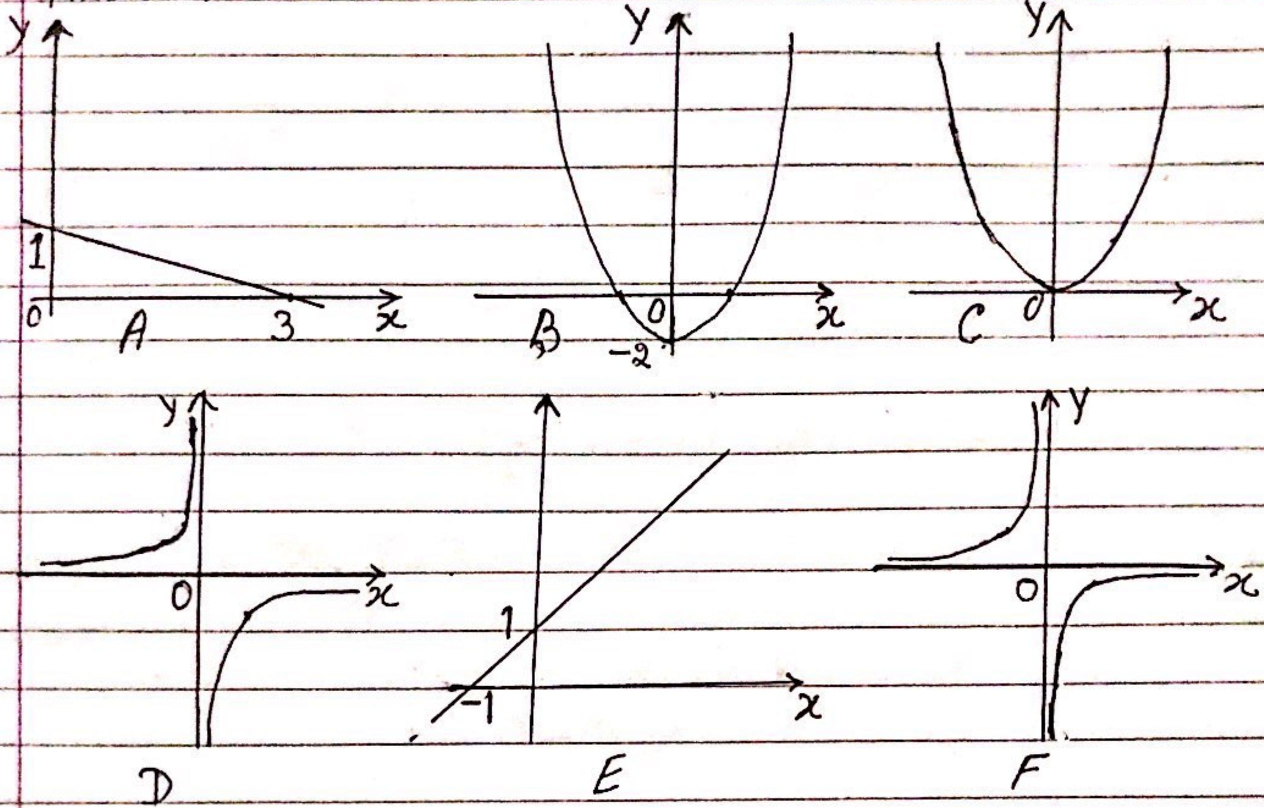
(c) R → ABCDE

(d) (i) (6,6) at D.
(lies on $4x+6y=60$)

(ii) for $x+y=7$
In the region R, line PM
P(7,0)
Q(6,1)
L(5,2)
M(4,3)

Graphs

1. The diagrams A, B, C, D, E and F are six graphs of different function:



Complete the table to identify the correct graph for each function. One has been done for you:

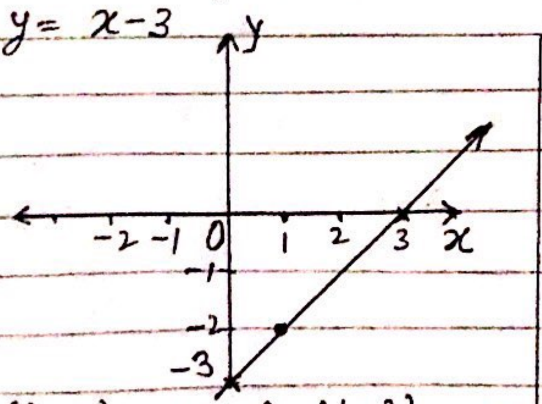
Function	$y = x+1$	$y = 1 - x/3$	$y = 2x^2$	$y = -\frac{4}{x}$
Diagram	E			

A ✓ C ✓ D ✓ ...[3]

[SP-20/02/Q21]

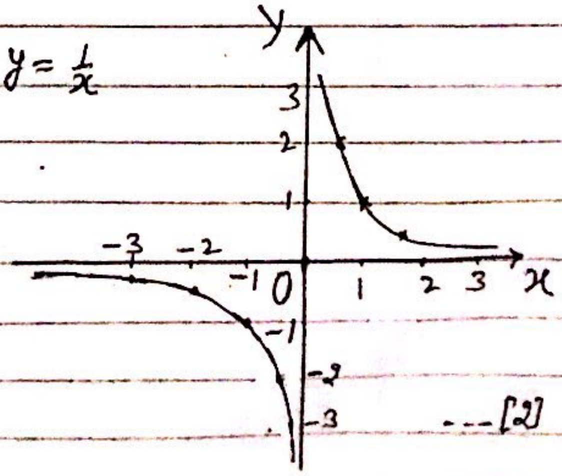
2. Sketch the graph of:

(a) $y = x - 3$



$(0,-3)$, $(3,0)$ $(1,-2)$ ...[1]

(b) $y = \frac{1}{x}$

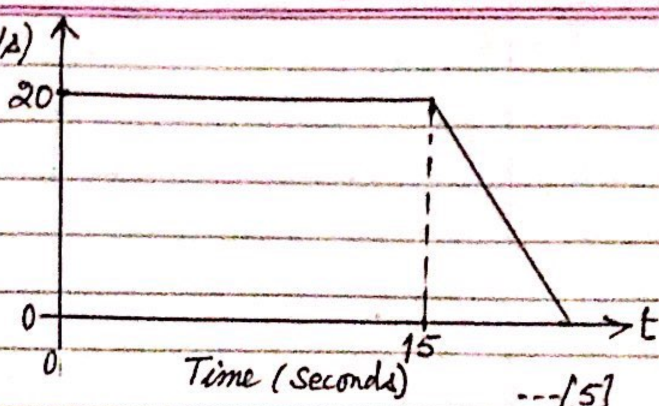


...[2]

[M-20/22/Q10]

Travel Graphs.

3. A car travels at 20 m/s for 15 seconds before it comes to rest by decelerating at $2.5\ m/s^2$.
Find the total distance travelled.



---[5]

[W-19/21/Q 26]

Solution: speed = 20 m/s

deceleration = $\frac{\text{speed}}{\text{time}} \Rightarrow 2.5 = \frac{20}{t} \Rightarrow t = \frac{20}{2.5} = 8$ sec.

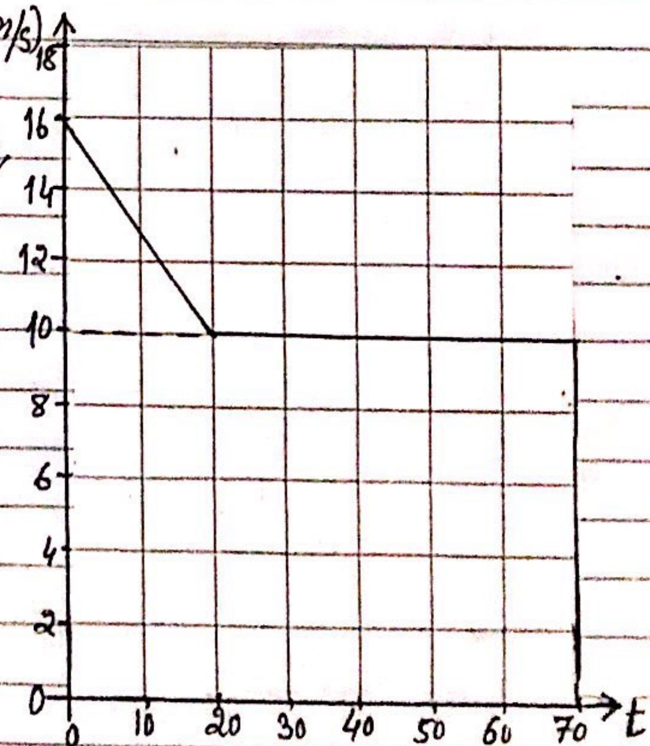
$\therefore$ Total distance travelled $= \frac{1}{2}(15 + (15+8)) \times 20$

(Area of trapezium) $= \frac{1}{2} \times 38 \times 20 = \underline{380\ m}$ ✓

4. The diagram shows the speed-time graph of 70 seconds, of a car journey.

(a) Calculate the deceleration of the car during first 20 seconds. ---[1]

(b) Calculate the total distance travelled by the car during the 70 seconds. ---[3]



[W-19/23/Q 19]

Solution:

(a) deceleration $= \frac{\text{speed change}}{\text{time}} = \frac{6}{20} = \underline{0.3\ ms^{-2}}$ ✓

(b) Total distance.

Area $= \frac{1}{2} \times 20 \times 6 + 70 \times 10$

$= 60 + 700 = \underline{760\ m}$ ✓

5. $f(x) = \frac{20}{x} + x$, $x \neq 0$

(a) Complete the table:

x	-10	-8	-5	-2	-1.6	1.6	2	5	8	10
$f(x)$	-12	-10.5	-9	-12	-14.1	14.1	12			12

---[2]

(b) On the grid, draw the graph of $y = f(x)$ for $-10 \leq x \leq -1.6$ and $1.6 \leq x \leq 10$. ---[5]

(c) Using your graph solve the equation $f(x) = 11$. ---[2]

(d) k is a prime number and $f(x) = k$ has no solution. Find the possible value of k. ---[2]

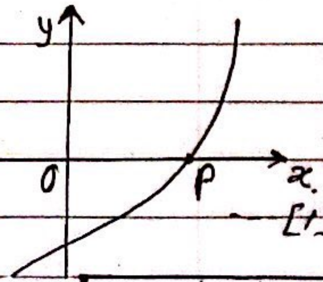
(e) The gradient of the graph of $y = f(x)$ at the point $(2, 12)$ is -4. Write down the coordinates of the other point on the graph of $y = f(x)$ where gradient is -4. ---[1]

(f) (i) The equation $f(x) = x^2$ can be written as $x^3 + px^2 + q = 0$. Show that $p = -1$ and $q = -20$ ---[2]

(ii) On the grid, draw the graph of $y = x^2$ for $-4 \leq x \leq 4$ ---[2]

(iii) Use your graphs, solve the equation $x^3 - x^2 - 20 = 0$ ---[1]

(iv) The diagram show a sketch of the graph of $y = x^3 - x^2 - 20$.
P is a point $(n, 0)$.
Write down the value of n. ---[1]



[SP-20/04/Q3]

Solution: (a) 9, 10.5

(b) Graph is on the next page →

Graph

5(b)

(c) $f(x) = 11$

$\Rightarrow x = 2.6$ or $x = 9$ ✓

(d) 2, 3, 5, 7

(e) $(-2, -12)$

(f) (i) $f(x) = \frac{20}{x} + x = x^2$

$\Rightarrow x^3 - x^2 - 20 = 0$

Comparing $x^3 + px^2 + q = 0$

$\Rightarrow p = -1,\ q = -20$ ✓

(ii) $y = x^2 \quad -4 \le x \le 4$

(iii) $y = x^2$ and $y = f(x)$

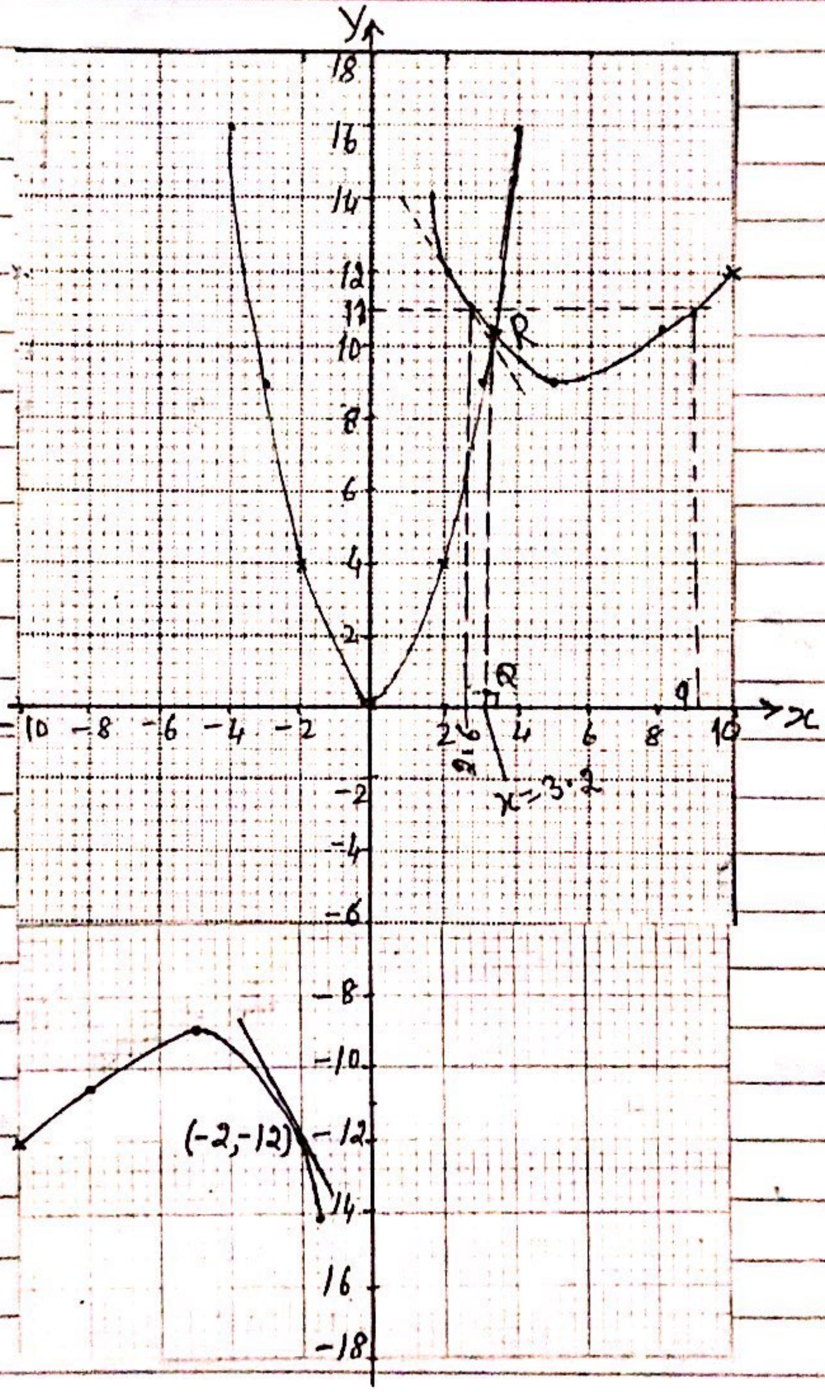
from Part f(i)

$\Rightarrow x^3 - x^2 - 20 = 0$

the two curves intersect at R,

Solution: $x = 3.2$ ✓

(iv) $n = 3.2$ ✓



Scanned with CamScanner

Graphs

6 (a) The table shows some values for $y = 2x^3 - 4x^2 + 3$

x	-1	-0.5	0	0.5	1	1.5	2
y	-3	1.75				0.75	3

(i) Complete the table. ---[3]

(ii) On the grid draw the graph of $y = 2x^3 - 4x^2 + 3$ for $-1 \leq x \leq 2$ --[4]

(iii) Use your graph to solve the equation, $2x^3 - 4x^2 + 3 = 1.5$ ---[3]

(iv) The equation $2x^3 - 4x^2 + 3 = k$ has only one solution for $-1 \leq x \leq 2$

Write down a possible integer value of k. ---[1]

a (i) 3, 2.25, 1,

(ii) graph

(0, 3), (0.5, 2.25), (1, 1)

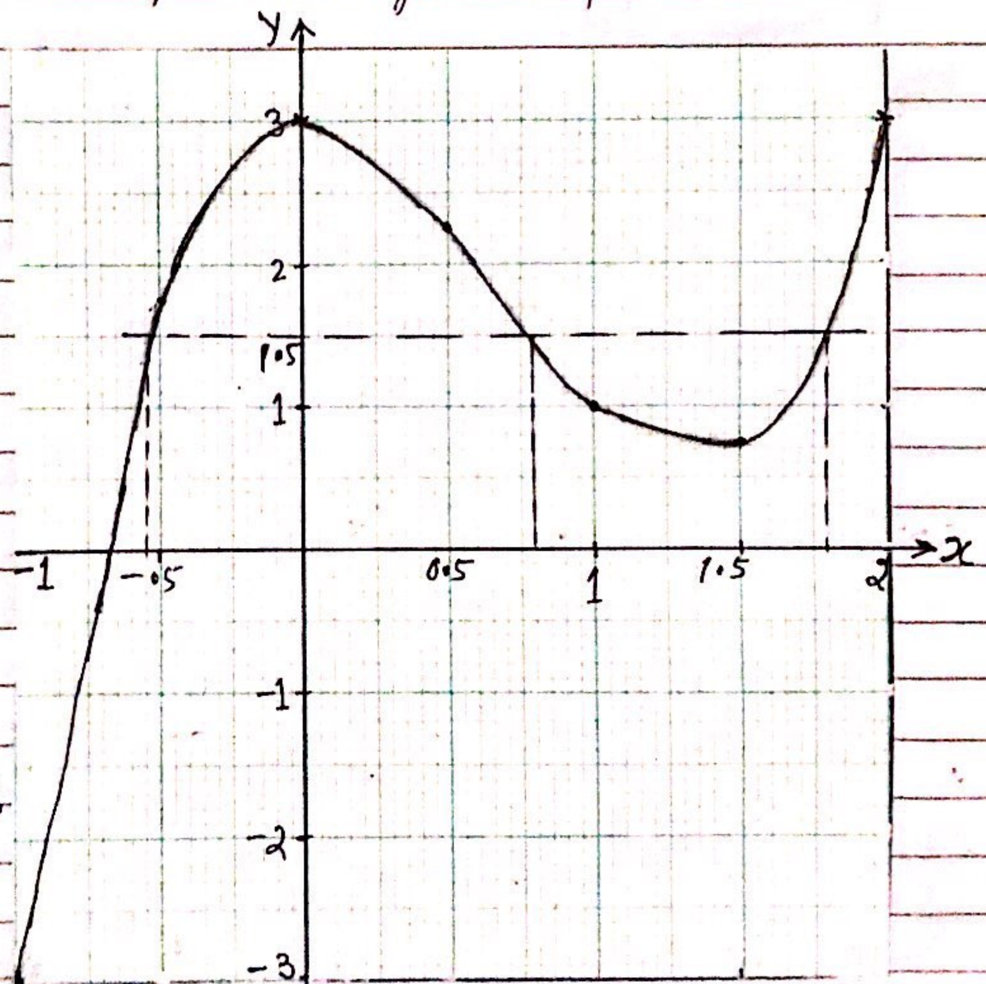
(a)(iii)

$y = 2x^3 - 4x^2 + 3 = 1.5$

$\Rightarrow x = -0.6, \checkmark$
$0.8, \checkmark$
$1.8 \checkmark$

a (iv)

$k = -3, -2, -1, 0$



(b)(i) On the grid, draw the tangent to the curve at $x=1$.

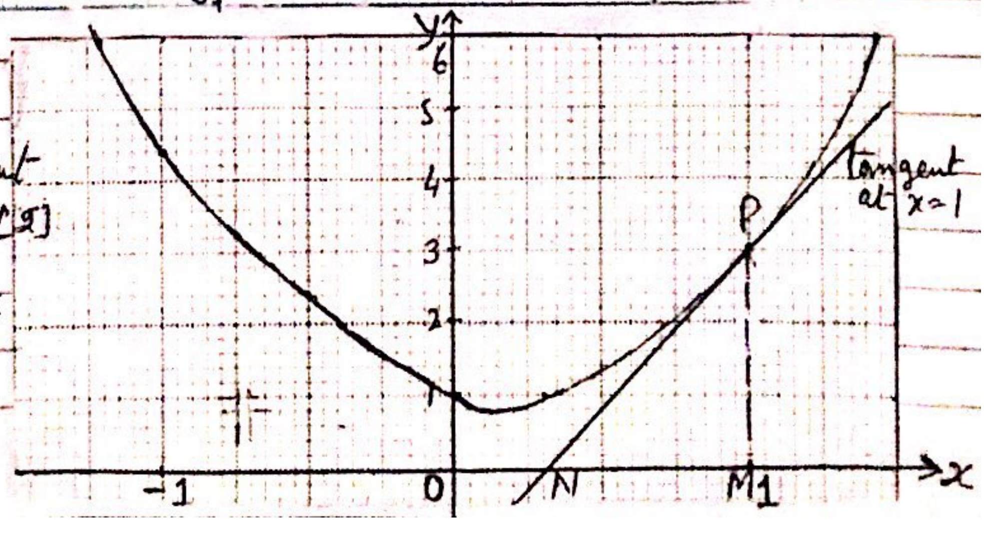
(ii) Use your tangent to estimate the gradient of the curve at $x=1$. -- [2]

(iii) Write down the equation of your tangent in the form $y = mx + c$ -- [2]

Solution: (ii) $m = \frac{PM}{NM} = \frac{3}{0.7}$

$m = 4.3 \checkmark$

(iii) $y = 4.3x - 2 \checkmark$



7 (i) Write $x^2+10x+14$ in the form $(x+a)^2+b$. ---[2]
(ii) On the axes, sketch the graph of $y=x^2+10x+14$, indicating the coordinates of the turning point. ---[3]

S-20/41/Q8(C)

Solution (i) $x^2+10x+14 = x^2+10x+(5)^2-25+14$ $\left\{\left(\frac{10}{2}\right)^2 \text{ add and subtract}\right.$

$= (x+5)^2-11$

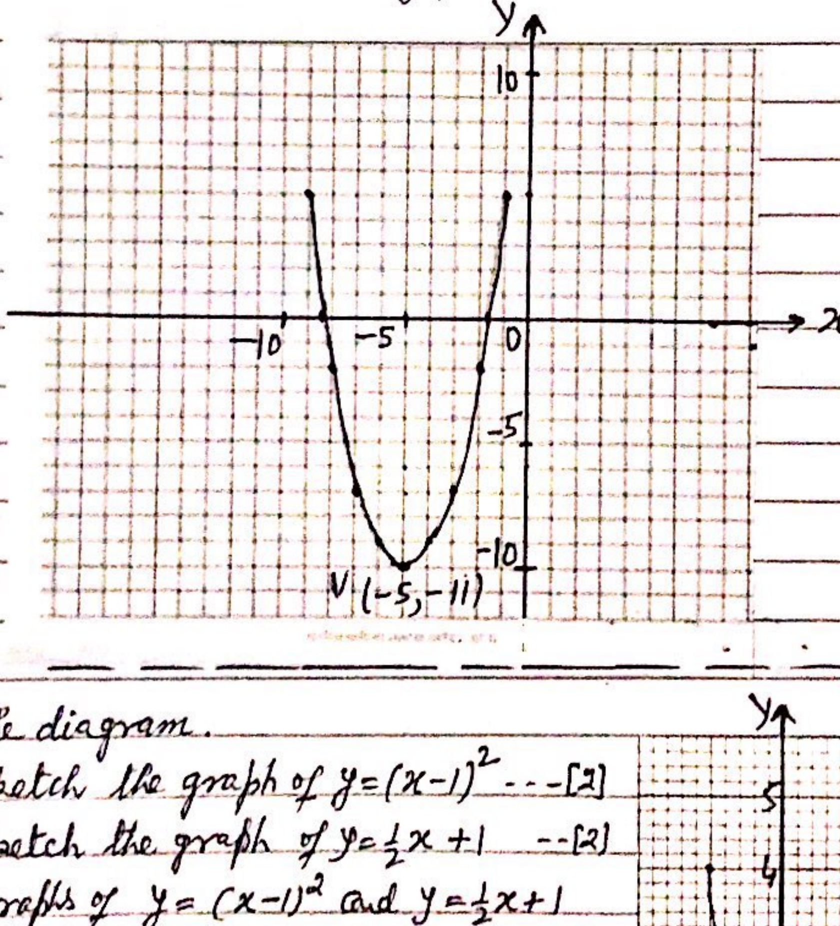
(ii) $y = x^2+10x+14$

$= (x+5)^2-11$ from Part (i)

or $= (x-(-5))^2+(-11)$

$V(-5,-11) \rightarrow$ Vertex of Parabola (or turning point).

[for a quadratic function: $y=(x-a)^2+b$ Vertex at (a,b).

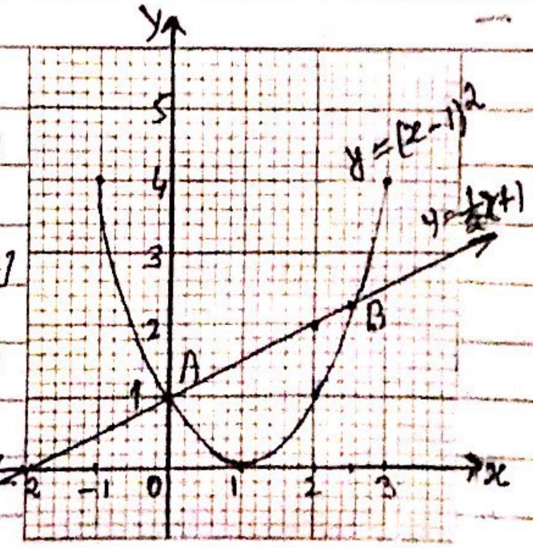


8 (i) On the diagram.
(a) Sketch the graph of $y=(x-1)^2$ ---[2]
(b) Sketch the graph of $y=\frac{1}{2}x+1$ --[2]
(ii) The graphs of $y=(x-1)^2$ and $y=\frac{1}{2}x+1$ intersect at A and B. Find the length AB. --[7]

Solution: $y=(x-1)^2$, $(1,0)$, $(2,1)$, $(3,4)$, $(0,1)$, $(-1,4)$
and $y=\frac{1}{2}x+1$, $(0,1)$, $(2,2)$, $(-2,0)$, $(4,3)$

S-20/42/Q9C

$AB = 2.8$ ✓



Graph

9. The table shows some values of: $y = \frac{x^2}{2} + \frac{1}{x^2} - \frac{2}{x}$, $x \neq 0$

x	-3	-2	-1	-0.5	-0.3	0.2	0.3	0.5	1	2	3
y	5.3	3.3		8.1	17.8		4.5	0.1	-0.5	1.3	

(a) Complete the table: 3.5 ✓ 15 ✓ 3.9 ✓ --[3]

(b) On the grid draw the graph of $y = \frac{x^2}{2} + \frac{1}{x^2} - \frac{2}{x}$ for $-3 \le x \le -0.3$ and $0.2 \le x \le 3$. --[5]

(c) Use your graph to solve $\frac{x^2}{2} + \frac{1}{x^2} - \frac{2}{x} \le 0$ --[2]

$0.6 \le x \le 1.5$ ✓

(d) Find the smallest integer value of k for which $\frac{x^2}{2} + \frac{1}{x^2} - \frac{2}{x} = k$ has two solutions for $-3 \le x \le -0.3$ and $0.2 \le x \le 3$

$k = 1$ ✓ ---[1]

(e) (i) By drawing suitable straight line solve, $\frac{x^2}{2} + \frac{1}{x^2} - \frac{2}{x} = 3x+1$ for $-3 \le x \le -0.3$ and $0.2 \le x \le 3$ -- [3]

Graph $y = (3x+1)$ ✓

$(0,1)$, $(2,7)$, $(-1,-2)$

Solution is at P; $x = 0.35$ ✓

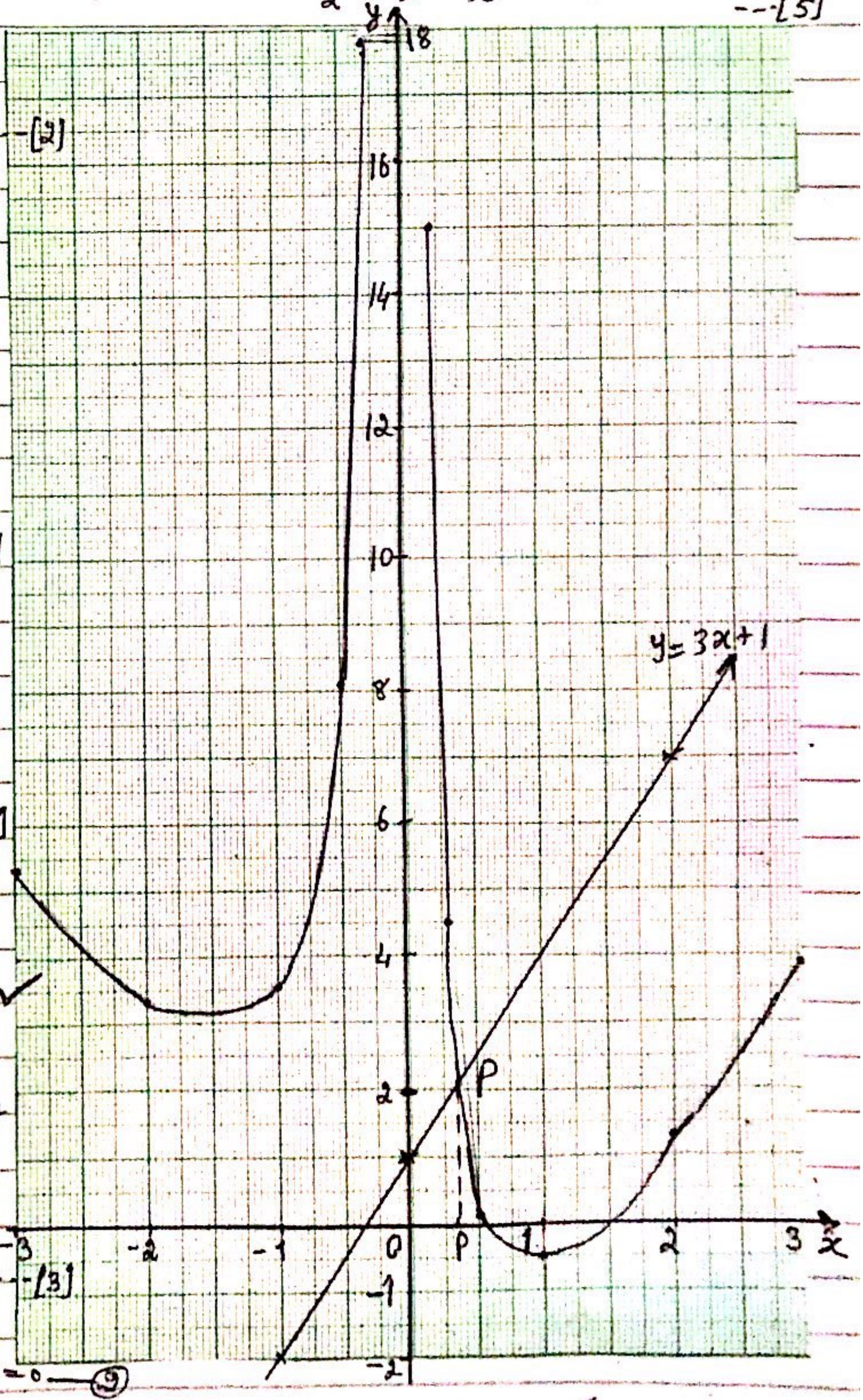
e(ii) The equation,

$\frac{x^2}{2} + \frac{1}{x^2} - \frac{2}{x} = 3x+1$ can be written as; —— ①

$x^4 + ax^3 + bx^2 + cx + 2 = 0$

Find the value of a, b, c. --[3]

Equation ① can be simplified to $x^4 - 6x^3 - 2x^2 - 4x + 2 = 0$ —— ②



Comparing ① & ② $\Rightarrow$ $a = -6$, $b = -2$ and $c = -4$ ✓

W-19/42/Q5

Graph

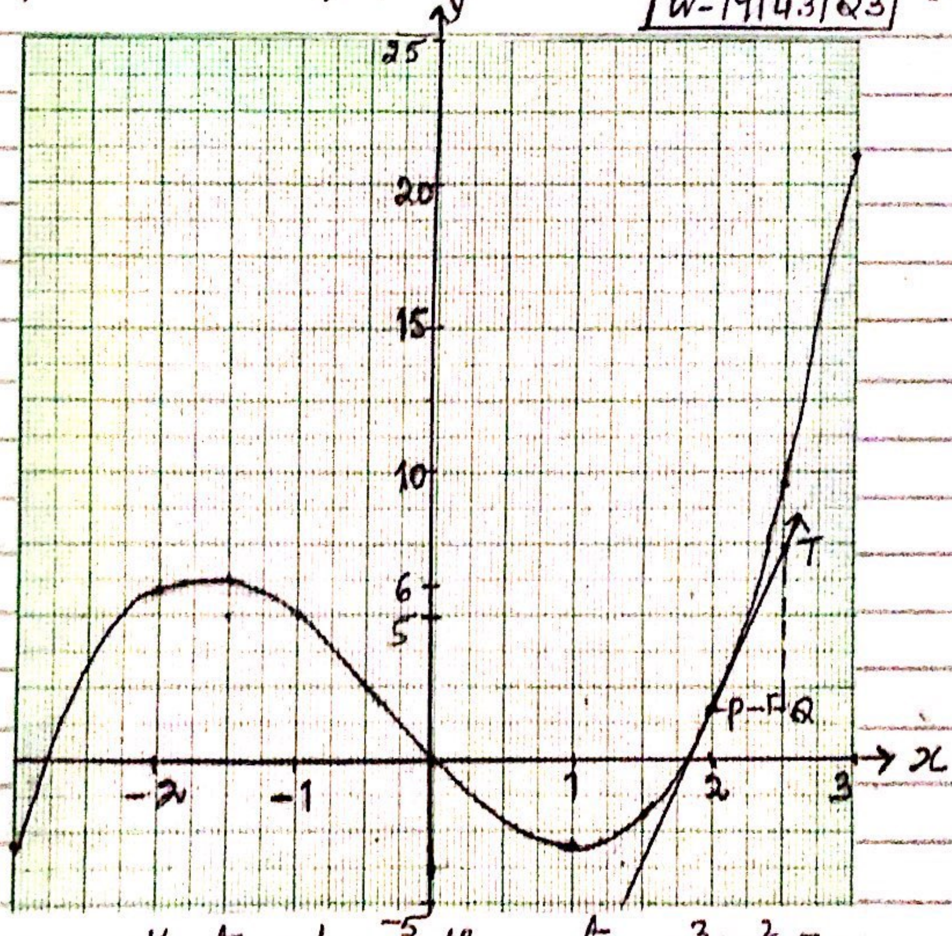
10. The table shows some values for $y = x^3 + x^2 - 5x$

x	-3	-2	-1.5	-1	0	1	1.5	2	2.5	3
y	-3	6	6.4	....	0	....	-1.9	2	9.4	....

(a) Complete the table: 5 ✓ -3 ✓   21 ✓ ---[3]

(b) On the grid, draw the graph of $y = x^3 + x^2 - 5x$ for $-3 \leq x \leq 3$

[W-19/43/Q3] ---[4]



(c) Use your graph to solve the equation $x^3 + x^2 - 5x = 0$ ---[2]

ans: $-2.8, 0, 1.8$ ✓

(d) By drawing suitable tangent, find an estimate of the gradient to the curve at $x = 2$. ---[3]

ans: PT is the tangent at $x=2$, gradient $= \frac{TQ}{PQ} = \frac{6}{.5} = 12$ ✓

(e) Write down the largest value of the integer k, [1] so that the equation $x^3 + x^2 - 5x = k$ has three solutions for $-3 \leq x \leq 3$.

ans: $k = 6$ ✓

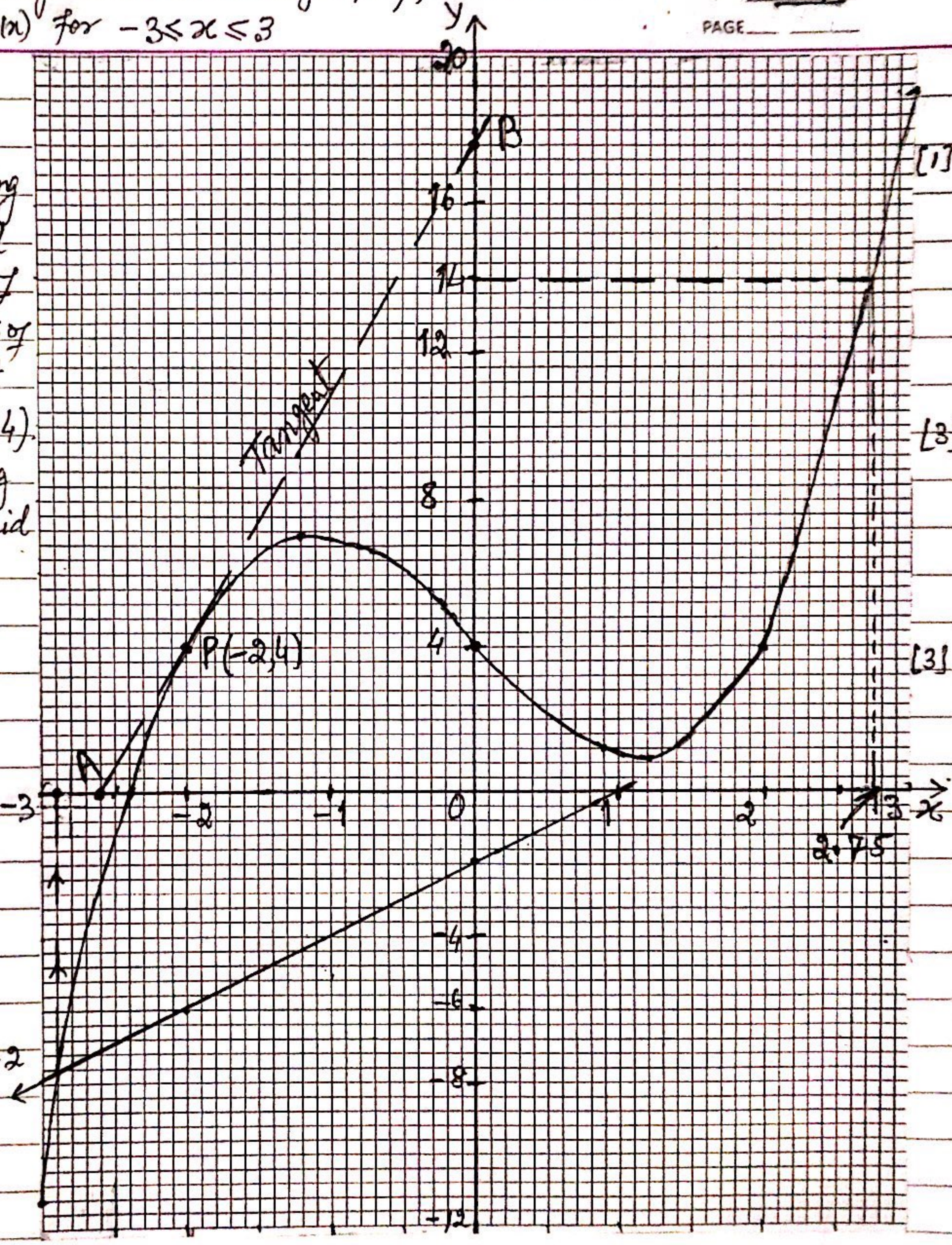
The diagram shows the graph of, C-9

11. $y = f(x)$ for $-3 \leq x \leq 3$

(i). Solve $f(x) = 14$ [1]

(ii) By drawing tangent, find an estimate of the gradient of the graph at the point $(-2,4)$. [3]

(iii) By drawing line on the grid Solve $f(x) = 2x - 2$ for $-3 \leq x \leq 3$ [3]



(i) $f(x) = 14 \Rightarrow x = 2.75$ (approx)

(ii) Gradient of the tangent to the curve at $(-2,4) = \frac{OB}{OA} = \frac{17.5}{2.6}$
$= \underline{6.7}$ ✓

(iii) Consider $y = 2x - 2$ and $y = f(x)$, $-3 \leq x \leq 3$
$(0,-2), (1,0), (-2,-6)$ $\Rightarrow \underline{x = -2.9}$ ✓

12 $y = x^2 + \frac{1}{x}$, $x \neq 0$

(a) Complete the table:

x	0·2	0·3	0·5	1	1·5	2	2·5
y	5	3·4	2·3		2·9		6·7

[2]

(b) On the grid, draw the graph of $y = x^2 + \frac{1}{x}$ for $0.2 \le x \le 2.5$

The graph of $y = x^2 + \frac{1}{x}$ for $-2.5 \le x \le -0.2$ has been drawn.

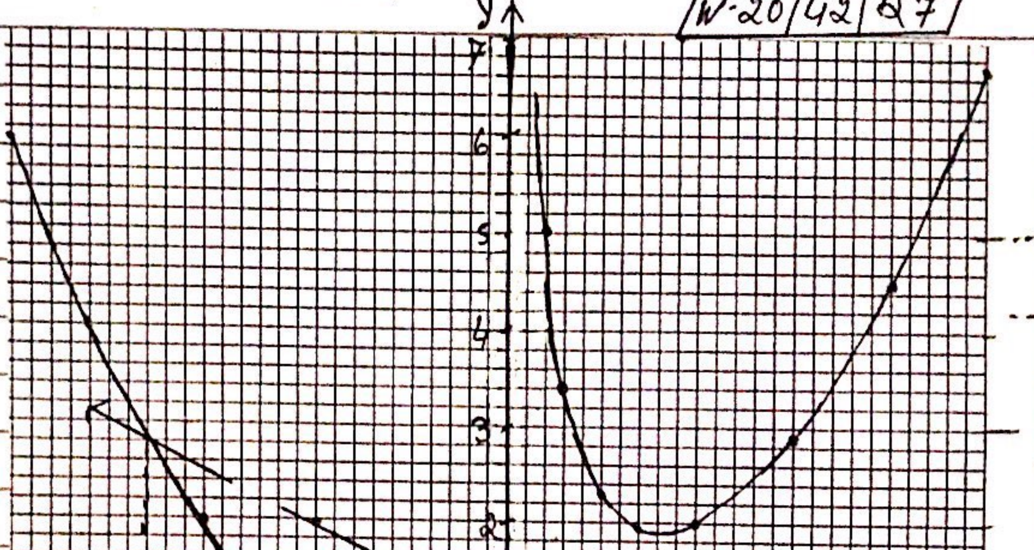
W-20/42/Q7

(c) By drawing line on the grid, Solve the following equation:

(i) $x^2 + \frac{1}{x} = -2$ ---[1]

(ii) $x^2 + \frac{1}{x} + x - 1 = 0$ --[2]

(d) K is an integer and the equation $x^2 + \frac{1}{x} = k$ has three solutions. write down a possible value of K. --[1]



[4]

Solution:

(a) 2, 4.5 ✓

(b) Graph in the Ist Quadrant

C (i) Consider line $y = -2$, intersects curve $y = f(x)$ at $x = -0.5$ ✓

(ii) consider line $y = 1 - x$ intersects the curve $y = f(x)$ at $x = -1.9$ ✓

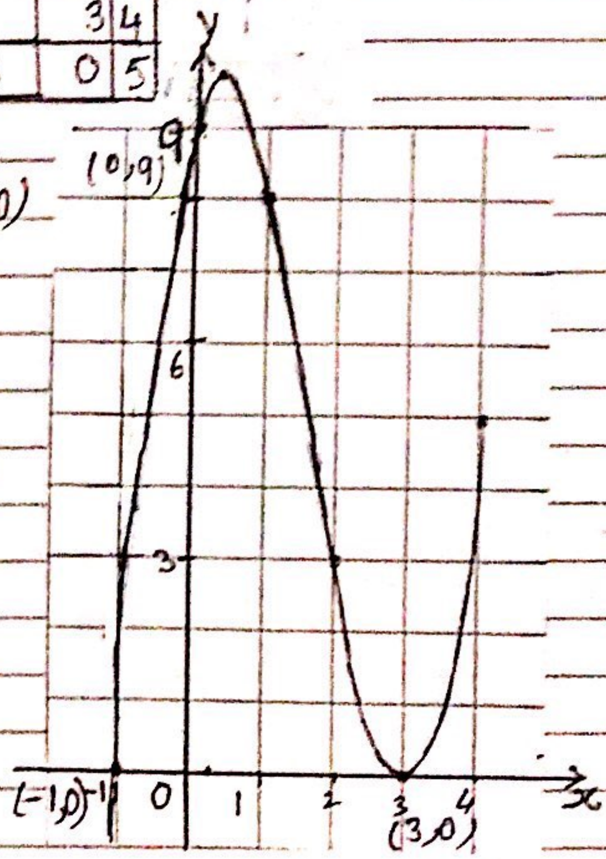
(d) Any integer ≥ 2.

13(i) Sketch the graph of $y=(x+1)(x-3)^2$. Label the values where the graph meets the x-axis and the y-axis. --[4]
(ii) Write $(x+1)(x-3)^2$ in the form ax^3+bx^2+cx+d --[3]

W-20|43|Q7

Solution (i)

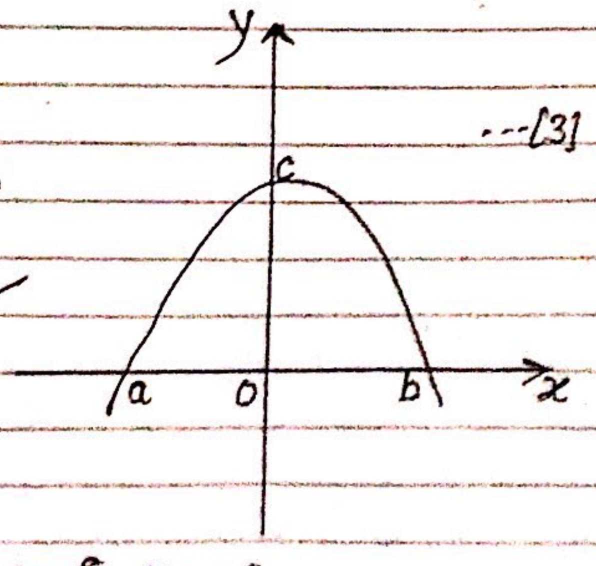
x		-1	0	1	2	3	4
y		0	9	8	3	0	5



Graph intersects -
X-axis at $(-1,0)$ and $(3,0)$
and Y-axis at $(0,9)$ ✓

(ii) $(x+1)(x-3)^2$
$= (x+1)(x^2-6x+9)$
$= x^3-6x^2+9x+x^2-6x+9$
$= \underline{x^3-5x^2+3x+9}$ ✓

Q(ii) The diagram shows a sketch of; $y=24+5x-x^2$
Work out the values of a, b, c. ---[3]



Solution: $y=24+5x-x^2$ —— ①
intersects y-axis at $x=0$
$y=24 \Rightarrow c=24$ ✓
Curve intersects the x-axis at $y=0$
from ① $24+5x-x^2=0$
$24+8x-3x-x^2=0$
$8(3+x)-x(3+x)=0$
$(8-x)(3+x)=0 \rightarrow x=8,\ x=-3$
$\therefore \underline{a=-3}$ ✓, $\underline{b=8}$ ✓ and $\underline{c=24}$ ✓