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IGCSE Maths

Coordinate Geometry

Revision

SP-20	M-20	S-20	W-19	W-20

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Coordinate Geometry

1. A is the point (3,5) and B is the point (1,-7)
Find the equation of the line perpendicular to AB that passes through the point A. Give your answer in the form $y = mx + c$. | M-20 | 22 | Q17 | ---[4]

Solution: Gradient of line $AB = \frac{-7-5}{1-3} = \frac{-12}{-2} = 6$ $\left[\because \text{Gradient } m = \frac{y_2-y_1}{x_2-x_1}\right]$

$\therefore$ Gradient line perp. to $AB = -\frac{1}{m}$
$= -\frac{1}{6}$ $\left\{\because \text{Grad. of perp. line} = -\frac{1}{m}\right.$

$\therefore$ Equation of line through A(3,5), and gradient $-\frac{1}{6}$,

$$y-5 = -\tfrac{1}{6}(x-3) \qquad [\because y-y_1 = m(x-x_1)]$$
$$6y-30 = -x+3$$
$$\Rightarrow 6y = -x+33 \Rightarrow y = -\tfrac{1}{6}x + \tfrac{11}{2} \checkmark$$

2. A Rhombus ABCD has a diagonal AC where A is the point (-3,10) and C is the point (4,-4).
(i) Calculate the length AC. ---[3]
(ii) Show that the equation of line AC is $y = -2x+4$ ---[2]
(iii) Find the equation of the line BD. | S-20 | 41 | Q 10(a) | --[4]

Solution (i) A(-3,10), C(4,-4) $\left[\because A = (x_1,y_1),\ B(x_2,y_2).\right.$ $\left. AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\right.$

$$AC = \sqrt{(-4-10)^2+(4+3)^2}$$
$$= \sqrt{(-14)^2+7^2} = \sqrt{245} = 15.65 \checkmark$$

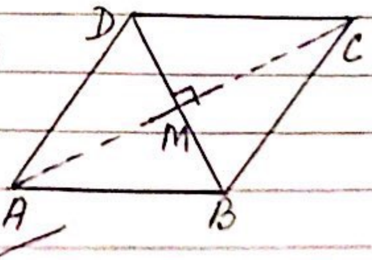
(ii) Gradient of AC = $m = \frac{(-4-10)}{(4+3)} = \frac{-14}{7} = -2$

$\therefore$ Equation of line through A(-3,10); $y-10 = -2(x+3)$
$$\Rightarrow y-10 = -2x-6$$
$$\Rightarrow y = -2x+4 \checkmark$$

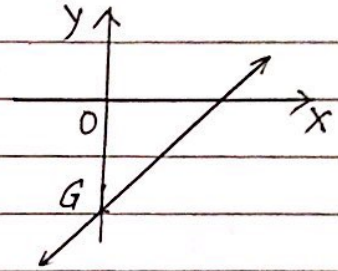
(iii) Diagonal BD and AC of a rhombus bisect each other at right angle. $M\left(\frac{-3+4}{2}, \frac{10-4}{2}\right)$

Gradient of BD $= -\frac{1}{m} = -\frac{1}{-2} = \frac{1}{2}$ | $M\left(\frac{1}{2}, 3\right)$

$\therefore$ Equation BD, passing through $M(\frac{1}{2},3)$, and gradient $\frac{1}{2}$ is $y-3 = \frac{1}{2}(x-\frac{1}{2}) \Rightarrow y = \frac{1}{2}x + \frac{11}{4} \checkmark$



3. The line $y = 3x - 2$ crosses the y-axis at G.
Write down the coordinates of G. [S-20|42|Q10(C)] ---[1]



Solution: line $y = 3x - 2$, crosses y axis at
$x = 0 \Rightarrow y = 0 - 2 \Rightarrow y = -2$
$\therefore G(0, -2)$ ✓

4.(a) The equation of line L is $3x - 8y + 20 = 0$
(i) Find the gradient of L. ---[2]
(ii) Find the coordinates of the point where line L cuts the y-axis. ---[1]
(b) The coordinates of P are $(-3, 8)$ and coordinates of Q are $(9, -2)$.
(i) Calculate the length PQ. ---[3]
(ii) Find the equation of line parallel to PQ that passes through the point $(6, -1)$ ---[3]
(iii) Find the perpendicular bisector of PQ. [S-20|43|Q9] --[4]

Solution (a)(i) line L: $3x - 8y = 20 \Rightarrow -8y = -3x + 20$
$\Rightarrow y = \frac{+3}{+8}x + \frac{20}{-8} \Rightarrow y = \frac{3}{8}x + \frac{5}{2}$ —— ①
$\therefore$ Grdient of line L $= 3/8$ from ① $[y = mx + c)$.
(ii) Line L cuts y-axis for $x = 0$ in ① $\Rightarrow y = 0 + 5/2 = 2.5$
$\therefore$ Point $(0, 2.5)$ ✓

(b)(i) $P(-3, 8), Q(9, -2) \Rightarrow PQ = \sqrt{(9+3)^2 + (-2-8)^2} = \sqrt{144 + 100} = \sqrt{244} = 15.62$ ✓
(ii) Gradient a line parallel to line = grad of PQ $= \frac{-2-8}{9+3} = -\frac{10}{12} = -\frac{5}{6}$
$\therefore$ Eqn of line through $(6, -1)$, $m = -\frac{5}{6}$ —— ②
$y + 1 = -\frac{5}{6}(x - 6) \Rightarrow y = -\frac{5}{6}x + 4$ ✓

(iii) Mid point of PQ $= M\left(\frac{-3+9}{2}, \frac{8-2}{2}\right) \equiv M(3, 3)$
Gradient of line perp to PQ $= -\frac{1}{m} = -\frac{1}{-\frac{5}{6}} = \frac{6}{5}$ (from ② $m = -\frac{5}{6}$)

$\therefore$ The eqn of Perp. bisector PQ, $y - 3 = \frac{6}{5}(x - 3)$
Passes through $M(3,3)$ and grad. $\frac{6}{5}$ $\Rightarrow$ or $y = \frac{6}{5}x - \frac{18}{5} + 3 \Rightarrow y = \frac{6}{5}x - \frac{3}{5}$ ✓

5. A line joins $A(1,3)$ to $B(5,8)$

(i) Find the mid point of AB. --[2]

(ii) Find the equation of AB. --[3]

Give your answer in the form $y = mx + c$. W-19/42/Q3(a)

Solution: Given $A(1,3)$, $B(5,8)$ { Given (x_1, y_1), (x_2, y_2)

(i) Mid point of $AB \left(\frac{1+5}{2}, \frac{3+8}{2}\right) = (3, 5.5)$ { Mid point $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

(ii) Gradient of line $AB = \frac{8-3}{5-1} = \frac{5}{4}$

$\therefore$ Equation of line AB, passing through $A(1,3)$ is,

$$y - 3 = \frac{5}{4}(x-1) \Rightarrow y - 3 = \frac{5}{4}x - \frac{5}{4} \Rightarrow \underline{y = \frac{5}{4}x + \frac{7}{4}} \checkmark$$

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6. The curve $y = x^2 - 2x + 1$ is drawn on a grid.

A line is drawn on the same grid.

The points of intersection of the line and the curve are used to solve the equation $x^2 - 7x + 5 = 0$

Find the equation of the line in the form $y = mx + c$. --[1]

S-20/21/Q20

Solution: Curve : $y = x^2 - 2x + 1$ —— ①

and line : $y = mx + c$ —— ②

To find the points of intersection of ① & ②

$$x^2 - 2x + 1 = mx + c$$

$$\Rightarrow x^2 - 2x - mx + 1 - c = 0$$

$$\Rightarrow x^2 - (m+2)x + (1-c) = 0 \quad \text{—— ③}$$

But given the points of intersection of ① and ② are given by

$$x^2 - 7x + 5 = 0 \quad \text{—— ④}$$

Comparing the coefficients of x and the constant term of ③ & ④

$$-(m+2) = -7 \Rightarrow m = 5 \checkmark$$

and $\quad 1 - c = 5 \Rightarrow c = -4$

$\therefore$ from ② the equation of line: $\underline{y = 5x - 4} \checkmark$

7. Find the gradient of a line that is perpendicular to $8y+4x=5$ ---[2]

W-20/21/Q14

Solution: Given line: $8y+4x=5 \Rightarrow 8y=-4x+5 \Rightarrow y=-\frac{1}{2}x+\frac{5}{8}$ —①

Gradient of the line①, $m_1=-\frac{1}{2}$

hence Gradient of the perpendicular line $m_2=-\frac{1}{-\frac{1}{2}}=\underline{2}$ ✓

$(\because m_1m_2=-1)$

8. A line from the point $(2,3)$ is perpendicular to the line $y=\frac{1}{3}x+1$. The two lines meet at the point P. Find the coordinates of P.

W-20/22/Q24 ---[5]

Solution: Given line l_1: $y=\frac{1}{3}x+1$ —①

Gradient of line① $m_1=\frac{1}{3}$

Gradient of the line l_2 perp. to l_1,

$m_2=-\frac{1}{m_1}=\frac{-1}{\frac{1}{3}}=-3$

$\therefore$ Eqn of line l_2, passing through $(2,3)$, $y-3=-3(x-2)$

$\Rightarrow y=-3x+9$ —②

l_1 and l_2 intersect at P

solving ① & ②

$\frac{1}{3}x+1=-3x+9$

$\Rightarrow 3x+\frac{1}{3}x=8$

$\frac{10}{3}x=8$

$x=\frac{8\times 3}{10}=2.4$ ✓

for① $y=\frac{1}{3}\times 2.4+1=1.8$ ✓

$\therefore$ $\underline{P(2.4, 1.8)}$ ✓

9. A straight line, l, has equation $y=5x+12$

(a) Write down the gradient of line l. ---[1]

(b) Find the coordinates of the point where line l crosses the x-axis. --[2]

(c) A line perpendicular to line l has gradient k. Find the value of k.

W-20/23/Q12 --[1]

Solution (a) Given line l: $y=5x+12$ —①

Gradient of line① $m_1=\underline{5}$ ✓

(b) The line l: $y=5x+12$ crosses x-axis, $y=0 \Rightarrow 0=5x+12$

$x=-\frac{12}{5}$

$\therefore \underline{\left(-\frac{12}{5},0\right)}$ ✓

(c) Gradient of line perp to line① $=k$

$k\times 5=-1$

$\Rightarrow k=\underline{-\frac{1}{5}}$ ✓

$(m_1\times m_2=-1$ for $l_1\perp l_2)$