04.03.21

0580

IGCSE_Maths

Contents:

Geometry

Revision

SP-20	M-20	S-20	W-19	W-20

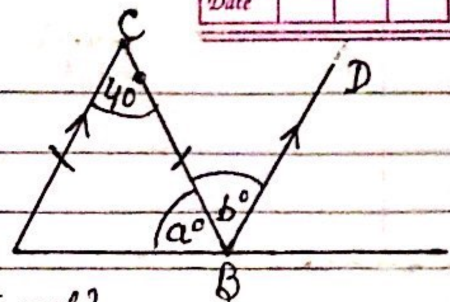
Suresh Goel
(Former Director)
Alliance World School,
Noida, Delhi. N.C.R.
INDIA.

(+91- 9810444804)

IG 0580

Geometry
Triangles

1. Triangle ABC is isosceles.
AC is parallel to BD.
Find the value of a and the value of b.



Solution: angle CBD = angle ACB ($\because$ Alternate angle)

$\Rightarrow \quad b = 40°$ ✓

--- [2]

SP-20/02/Q8

Now Isosceles Triangle ABC, AC = BD, the opposite angles,

$\therefore$ angle CAB = angle CBA = a°

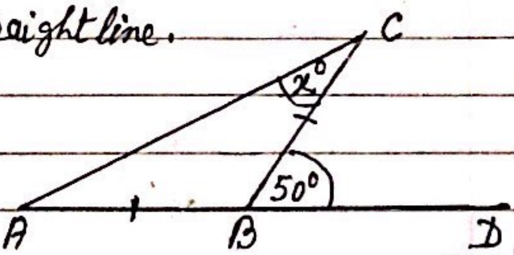
In $\triangle ABC$, $a° + a° + 40° = 180°$ (Sum of angles of a Triangle is 180°)

$\Rightarrow \quad 2a + 40 = 180$

$\Rightarrow \quad a = \frac{140}{2} = 70°$ ✓

$\therefore a = 70°$ and $b = 40°$

2. AB = BC and ABD is a straight line.
Find the value of x.



Solution: AB = BC in $\triangle ABC$

The opposite angle are equal

--- [2]

S-20/22/Q3

angle BAC = angle ACB = x° (given) — ①

Now angle BAC + angle ACB = 50° ($\because$ Exterior angle CBD is equal to the opp interior angle)

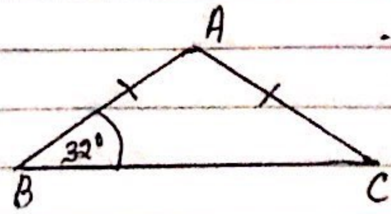
$\Rightarrow \quad x° + x° = 50°$ (from ①)

$2x = 50°$

$x = \frac{50}{2} = 25°$ $\quad \therefore \quad x = 50°$ ✓

3. Triangle ABC is isosceles.
Angle ABC = 32° and AB = AC
Find angle BAC.

--- [2]



Solution: In $\triangle ABC$, AB = AC, the opposite angles $\angle ACB = \angle ABC = 32°$ — ①

S-20/23/Q3

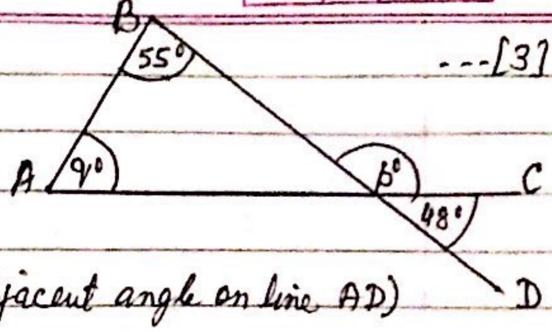
Now in $\triangle ABC$,

angle BAC + angle ACB + angle ABC = 180° (Angle sum property in a Triangle)

$\therefore$ angle BAC + 32° + 32° = 180°

$\therefore$ angle BAC = 180 − 32° − 32° = 116° $\quad \therefore$ angle BAC = 116°

4.(a) In the diagram, AC and BD are straight lines. ---[3]
Find the value of p and the value of q.



Solution: $p° + 48° = 180°$ (Adjacent angle on line AD)

$\therefore p = 180° - 48 = 132°$ —— ①

Now $q° + 55° = p$ (Exterior angle theorem in a triangle)

$q + 55° = 132°$ from ①

$\Rightarrow q = 132 - 55° = 77°$ $\therefore$ $p = 132°$ and $q = 77°$

(b) The angles of a quadrilateral are $x°$, $(x+5°)$, $(2x-25)°$ and $(x+10°)$. Find the value of x. ---[3]

W-19	41	Q 1(a)(b)

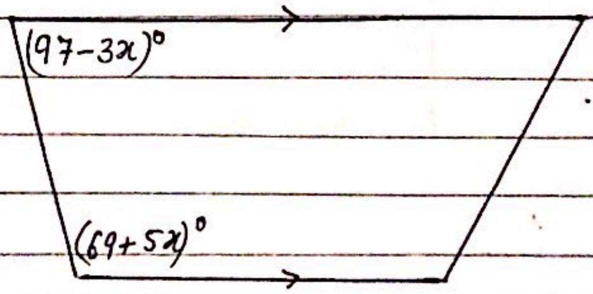
Solution: $x + (x+5) + (2x-25°) + (x+10) = 360°$ (Sum of interior angles of a quad. is 360°)

$\Rightarrow 5x - 10 = 360°$

$\Rightarrow 5x = 360 + 10 = 370 \Rightarrow x = \frac{370}{5} = 74°$

$\therefore x = 74°$

5. Work out the value of x.



S-20	21	Q6

---[3]

Solution:

$97 - 3x + 69 + 5x° = 180°$ (Sum of interior angle between two parallel lines made by by a transversal on of the same side of it is 180°)

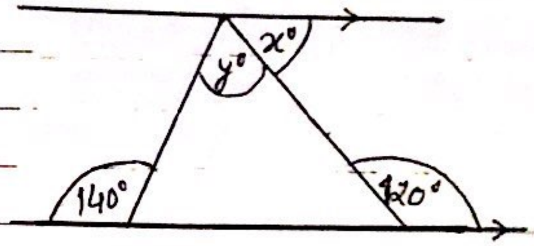
$166 + 2x = 180°$

$2x = 180 - 166 = 14$

$x = \frac{14}{2} = 7$

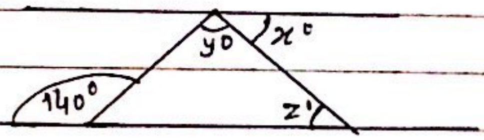
$\therefore x = 7°$

6. The diagram shows a triangle drawn between a pair of parallel lines.
Find the value of x and the value of y.



[W-20/22/Q4] ---[3]

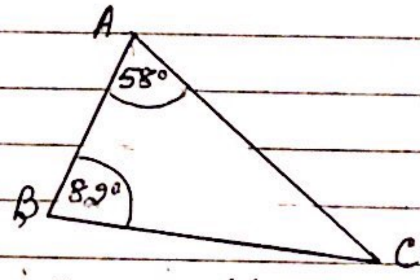
Solution: $x + 120° = 180°$ (Interior angles on the same side of tranversal between two parallel lines)
$\Rightarrow x = \underline{60°}$ ✓



Now $z = x = 60$ (Alt angle)
$y + z = 140$ (Exterior angle of Δ)
$y + 60 = 140° \Rightarrow y = 80°$ ✓

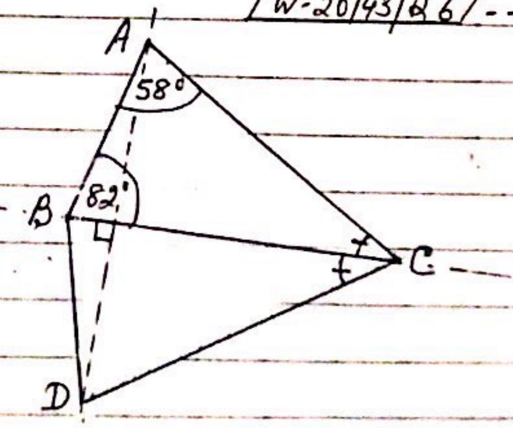
---

7. The diagram shows triangle ABC.
The triangle is reflected in the line BC to give a quadrilateral ABDC.



(a) Write down the mathematical name of the quadrilateral ABDC. --[1]
(b) Find angle ACD.

[W-20/43/Q6] --[2]



Solution: (a) Kite
(b) Angle $ACD = 2\angle ACB$
$= 2[180 - (58 + 82)]$
$= 2(180 - 140)$
$= 2 \times 40 = \underline{80°}$ ✓

Polygons.

1. A hexagon has five angles that each measure 115°.
Calculate the size of sixth angle. ---[3]
[SP-20/02/Q16]

Solution: Let the sixth angle $= x^\circ$

$\therefore \quad x + 5 \times 115^\circ = (6-2) \times 180^\circ$ — sum of interior angles of an n-sided polygon $= (n-2) \times 180^\circ$

$x + 575^\circ = 720$

$x = 720 - 575$

$\therefore \quad x = 145^\circ$

2. Find the interior angle of a regular polygon of 24 sides. --[2]
[M-20/22/Q4]

Solution: Interior angle of a n-sided regular polygon $= \dfrac{(n-2) \times 180^\circ}{n}$

$\therefore$ Interior angle of a 24-sided regular polygon:

$$= \frac{(24-2) \times 180^\circ}{24}$$

$$= \frac{22 \times 180^\circ}{24} = \underline{165^\circ}$$

3. A regular polygon has an interior angle 176°. ---[3].
Find the number of sides of this polygon. [W-19/22/Q12]

Solution: Let the number of sides of the polygon $= n$

Interior angle $= \dfrac{(n-2) \cdot 180^\circ}{n} = 176^\circ$ (given)

$$\Rightarrow (n-2) \cdot 180^\circ = 176n$$

$$\Rightarrow 180n - 360 = 176n$$

$$\Rightarrow 180n - 176n = 360$$

$$4n = 360^\circ$$

$$n = \frac{360}{4} = 90$$

$$\therefore \quad \underline{n = 90}$$

Polygons.

4. The interior angle of a regular polygon with n-sides is 150°. Calculate the value of n. [S-20/43/Q8(a)] --(2)

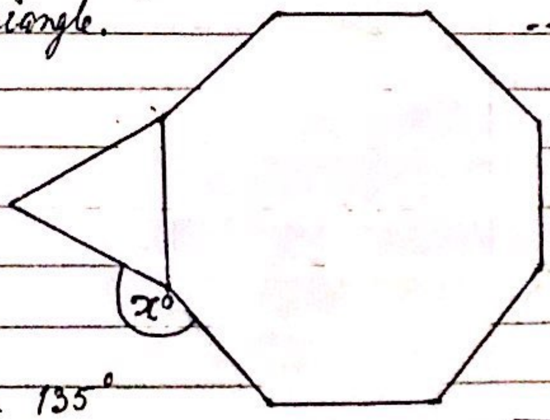
Solution: Interior angle $\frac{(n-2)\cdot 180^\circ}{n} = 150^\circ$ (given)

$$\Rightarrow (n-2)\cdot 180^\circ = 150 \cdot n$$
$$\Rightarrow 180n - 360^\circ = 150^\circ$$
$$\Rightarrow 180n - 150n = 360^\circ$$
$$\Rightarrow 30n = 360^\circ \Rightarrow n = \frac{360^\circ}{30} = 12$$
$$\therefore \underline{n = 12}$$

5. A regular polygon has 72 sides. --[3]
Find the size of an interior angle. [W-19/41/1(C)]

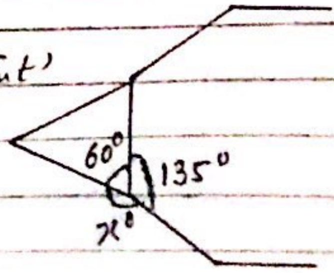
Solution: an interior angle of n-sides polygon $= \frac{(n-2)\cdot 180^\circ}{n}$
for $n = 72$,
$$= \frac{(72-2)\times 180^\circ}{72}$$
$$= \frac{70 \times 180^\circ}{72} = 175^\circ$$
$\therefore$ Required angle $= \underline{175^\circ}$

6. The diagram shows a regular octagon joined to an equilateral triangle. ---[3]
Workout the value of x.



Solution: Interior angle of a octagon ($n=8$)
$$= \frac{(n-8)\times 180^\circ}{n}$$
$$= \frac{(8-2)\times 180^\circ}{8} = \frac{6\times 180^\circ}{8} = 135^\circ$$

Now $x + 60^\circ + 135^\circ = 360^\circ$ (Angles at a point)
$$x + 195 = 360^\circ$$
$$x = 360 - 195 = 165^\circ$$
$$\therefore \underline{x = 165^\circ}$$



7. The interior angle of regular polygon with n-sides is 156°. Work out the value of n. [W-20|21|Q12] ---[2]

Solution: $\frac{(n-2)\cdot 180^\circ}{n} = 156^\circ \Rightarrow 180n - 360 = 156n$

$\Rightarrow 180n - 156n = 360$

$\Rightarrow 24n = 360 \Rightarrow n = 15$ ✓

8. Calculate the size of one interior angle of a regular polygon of 40 sides. --[2]

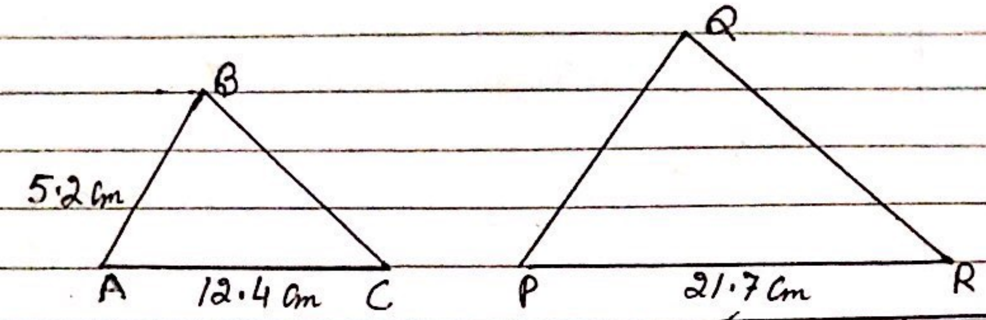
[W-20|22|Q8]

Solution: one angle $= \frac{(n-2)\times 180}{n} = \frac{(40-2)\times 180}{40}$

$= \frac{38\times 180}{40} = 171^\circ$

Similarity

1. Triangle ABC is similar to triangle PQR.
Find PQ. ...[2]



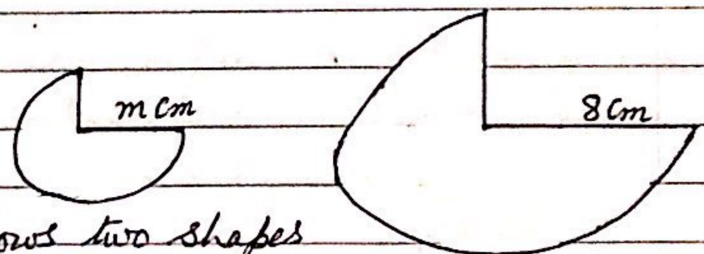
SP-20/02/Q12

Solution: $\triangle ABC \sim \triangle PQR$

$\dfrac{PQ}{AB} = \dfrac{PR}{AC}$ (Corresponding sides of similar triangles are proportional)

$\Rightarrow \dfrac{PQ}{5.2} = \dfrac{21.7}{12.4} \Rightarrow PQ = \dfrac{21.7}{12.4} \times 5.2 = 9.1$

$\therefore$ $PQ = 9.1$ Cm.

2. 

The diagram shows two shapes that are mathematically similar.
The smaller shape has area $52.5\ cm^2$ and the larger shape has area $134.4\ cm^2$.
Calculate the value of m. S-20/23/Q17 ...[3]

Solution: $\dfrac{\text{area of smaller shape}}{\text{area of larger shape}} = \left(\dfrac{m}{8}\right)^2$

$\Rightarrow \dfrac{52.5}{134.4} = \dfrac{m^2}{64}$

$\Rightarrow m^2 = \dfrac{52.5 \times 64}{134.4} = 25$

{ The areas of similar shapes are proportional to the square of the corresponding sides of the shapes.

$\therefore m = \sqrt{25} = 5$

$\therefore$ $m = 5$ cm.

3. The scale of a map is 1 : 10 000 000.
On the map, the area of Slovakia is $4.9 cm^2$.
Calculate the actual area of Slovakia.
Give your answer in square kilometres. [W-19|21|Q14] ...[3]

Solution: $\dfrac{\text{area of the map}}{\text{actual area}} = \left(\dfrac{1}{10\,000\,000}\right)^2$

$\Rightarrow \dfrac{4.9}{\text{actual area}} = \dfrac{1}{10^{14}}$

$\Rightarrow$ Actual area $= 4.9 \times 10^{14}$ sq cm

$= \dfrac{4.9 \times 10^{14}}{10^{10}}$

$= 4.9 \times 10^{4}$ sq km

$\left\{ \begin{array}{l} 1 \text{ Km} = 1000 \times 100 \\ \quad = 10^5 \text{ cm} \\ 1 \text{ sq km} \\ \quad = (10^5)^2 = 10^{10} \text{ sq cm} \end{array} \right.$

$\therefore$ Actual area of Slovakia $= \underline{49000 \text{ sq Km}}$.

4. Two mathematical similar containers have heights of 30 cm and 75 cm. The larger container has a capacity of 5.5 litres.
Calculate the capacity of the smaller container.
Give your answer in millilitres. [W-19|22|Q13] ...[3]

Solution: $\dfrac{V_1}{V_2} = \dfrac{\text{Capacity (volume) of smaller container}}{\text{Capacity (Volume) of larger container}} = \left(\dfrac{h_1}{h_2}\right)^3$

$\Rightarrow \dfrac{V_1}{5.5 \times 1000} = \left(\dfrac{30}{75}\right)^3$ [1 litre = 1000 c.c.

$V_1 = \left(\dfrac{2}{5}\right)^3 \times 5500 \text{ cm}^3$

$= \dfrac{8}{125} \times 5500 = 352 \text{ cm}^3$

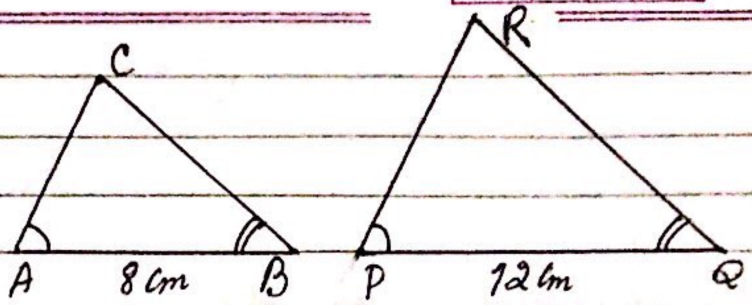
$= 352$ millilitres

$\left\{ \begin{array}{l} 1 \text{ litre} = 1000 \text{ cm}^3 \\ 1000 \text{ millilitres} \\ \quad = 1000 \text{ cm}^3 \\ \Rightarrow 1 \text{ millilitre} \\ \quad = 1 \text{ cm}^3 \end{array} \right.$

$\therefore$ Capacity of the smaller container

$= \underline{352 \text{ millilitres}}$.

5. Triangle ABC is mathematically similar to triangle PQR. The area of triangle ABC is $16\ cm^2$.



(i) Calculate the area of triangle PQR. ---[2]

(ii) The triangles are the cross-sections of prisms which are also mathematically similar. The volume of the smaller prism is $320\ cm^3$. Calculate the length of the larger prism. ---[3]

S-20 / 42 / Q8(a)

Solution (i) $\dfrac{\text{area of triangle PQR}}{\text{area of triangle ABC}} = \left(\dfrac{PQ}{AB}\right)^2$

$\Rightarrow \dfrac{\text{area of triangle PQR}}{16} = \left(\dfrac{12}{8}\right)^2$ [$\because$ Area $\Delta ABC = 16\ cm^2$]

$\Rightarrow$ area of triangle PQR $= \dfrac{144}{4} \times 16 = \underline{36\ cm^2}$

(ii) length of the smaller prism $= \dfrac{\text{Volume of small prism}}{\text{area of cross of cross-section}} = \dfrac{320}{16}$

$= 20\ cm.$ —— (1)

$\dfrac{\text{length of larger prism}}{\text{length of small prism}} = \dfrac{PQ}{AB}$

$\Rightarrow \dfrac{\text{length of larger prism}}{20\ cm} = \dfrac{12}{8}$ from (1)

$\therefore$ length of larger prism $= \dfrac{12}{8} \times 20 = 30\ cm$

$= \underline{30\ cm}$

6. Two cones are mathematically similar.
The total surface area of the smaller cone is $80\,cm^2$.
The total surface area of the larger cone is $180\,cm^2$.
The volume of the smaller cone is $168\,cm^3$. ---[3]
Calculate the volume of the larger cone. [W-19/41/Q4(c)]

Solution: $\dfrac{\text{area of smaller cone}}{\text{Area of larger cone}} = \left(\dfrac{r_1}{r_2}\right)^2$; r_1 and r_2 are the radii of the smaller and larger cones resp.

$$\Rightarrow \frac{80}{180} = \left(\frac{r_1}{r_2}\right)^2 = \frac{4}{9}$$

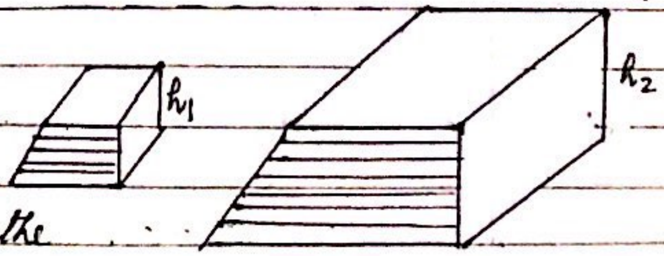
$$\Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \frac{4}{9} \Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{4}{9}} = \frac{2}{3} \quad \text{—— (1)}$$

Now $\dfrac{\text{Volume of of Smaller cone}}{\text{Volume of larger cone}} = \left(\dfrac{r_1}{r_2}\right)^3$

$$\Rightarrow \frac{168}{\text{Volume of larger cone}} = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

$$\Rightarrow \text{Volume of larger cone} = \frac{168}{8} \times 27 = \underline{567} ✓$$

7. The diagram shows two mathematically similar solid prisms. The volume of the smaller prism is $648\,cm^3$ and the Volume of the larger prism is $2187\,cm^3$. The area of cross-section of the smaller prism is $36\,cm^2$. [W-19/43/Q6(b)(i)]
Calculate the area of cross-section of the larger prism. ---[3]



Solution: $\dfrac{\text{Volume of larger prism}}{\text{Volume of smaller prism}} = \dfrac{2187}{648} = \left(\dfrac{h_1}{h_2}\right)^3 \rightarrow \dfrac{h_1}{h_2} = \left(\dfrac{2187}{648}\right)^{1/3} = \dfrac{3}{2}$ —— (1)

Now $\dfrac{\text{area of cross-section of larger cone}}{\text{area of cross-section of smaller cone}} = \left(\dfrac{h_1}{h_2}\right)^2 = \left(\dfrac{3}{2}\right)^2 = \dfrac{9}{4}$

$$\frac{\text{area of cross-section of larger cone}}{36} = \frac{9}{4}$$

$$\Rightarrow \text{Area of cross-section of larger cone} = \frac{9}{4} \times 36 = \underline{81\ cm^2} ✓$$

8. A model of a statue has a height of 4 cm.
The volume of the model is $12\,cm^3$.
The Volume of the statue is $40500\,cm^3$. Calculate the height of the statue.
[W-20/22/Q20] --- [3]

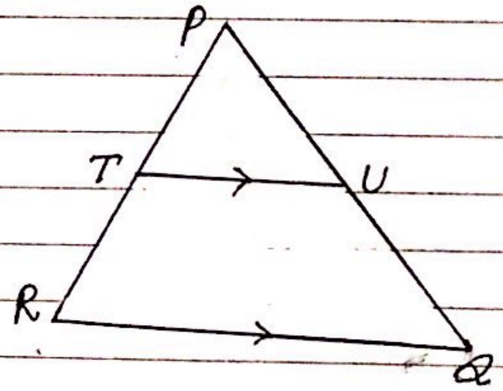
Solution: $\dfrac{\text{Volume of the statue}}{\text{Volume of the model}} = \left(\dfrac{\text{Height of the statue 'h'}}{\text{Height of the model}}\right)^3$

$$\Rightarrow \frac{40500}{12} = \left(\frac{h}{4}\right)^3 \Rightarrow \left(\frac{h}{4}\right)^3 = 3375 = (15)^3$$

$$\Rightarrow \frac{h}{4} = 15 \Rightarrow h = 60\,cm \checkmark$$

---

9. PQR is a triangle. T is a point on PR and U is a point on PQ.
RQ is parallel to TU.



(i) Explain white triangle PQR is similer to triangle PUT. Give reason for each statement. --- [3]
(ii) PT : TR = 4:3
(a) Find the ratio PU : PQ. --- [1]
(b) The area of ΔPUT is $20\,cm^2$
Find the area of quadrilateral QRTU -- [3]
[W-20/43/Q5(b)]

Solution (i) Angle PTU = angle PRQ (Corresponding angles)
angle PUT = angle PQR (Corresponding angles)
angle RPQ is common in both Δ's
$\therefore$ Triangle PQR is similar ΔPUT (AAA)

(ii)(a) $\dfrac{PU}{PQ} = \dfrac{PT}{PR} = \dfrac{4}{4+3} = \dfrac{4}{7} \checkmark$ $\left[\dfrac{PT}{TR} = \dfrac{4}{3}\right]$

(b) $\dfrac{\text{area } \Delta PUT}{\text{area } \Delta PQR} = \left(\dfrac{PU}{PR}\right)^2 = \left(\dfrac{4}{7}\right)^2$ from (ii)(a)

$\Rightarrow \dfrac{20}{\text{area } \Delta PQR} = \dfrac{16}{49} \Rightarrow$

$\Rightarrow$ ar $PQR = \dfrac{20 \times 49}{16} = 61.25$ $\Rightarrow$

area of quad RRTU
$= ar\,\Delta PQR - ar\,\Delta PUT$
$= 61.25 - 20 = 41.25\,cm^2$

Scanned with CamScanner

10. A paperweight has height 4 cm and volume 38·4 cm^3.
A mathematically similar paperweight has height 7 cm.
Calculate the volume of this paperweight. --[3]

[W-20/23/Q16]

Solution:

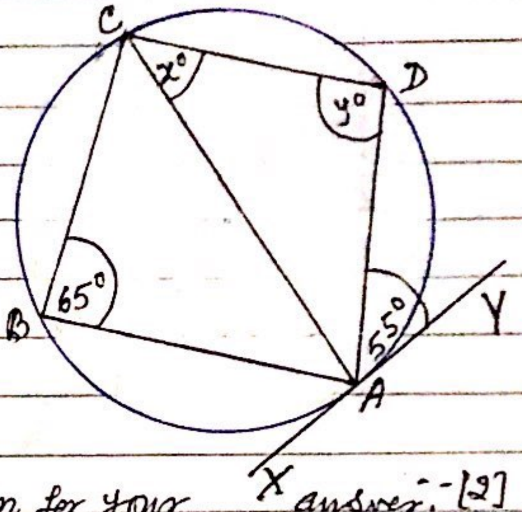
$$\frac{\text{Volume of paperweight 1}}{\text{Volume of paperweight 2}} = \left(\frac{h_1}{h_2}\right)^3$$

$$\Rightarrow \quad \frac{38\cdot4}{V} = \left(\frac{4}{7}\right)^3 = \frac{64}{343}$$

$$\Rightarrow \quad V = \frac{38\cdot4 \times 343}{64} = 205.8 \text{ cm}^3 \checkmark$$

Circles

1\. A, B, C and D are points on the circumference of the circle.
The line XY is a tangent to circle at A.



(a) Find the value of x, giving the reason for your answer. ---[2]

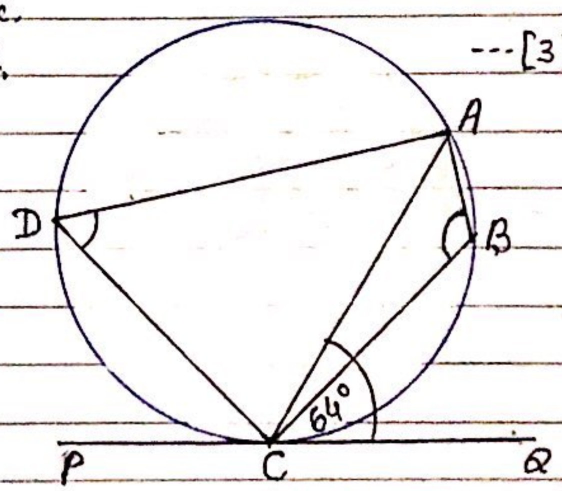
Solution: $x = 55°$ ✓ because angles in the alternate segment of circle are equal.

(b) Find the value of y, giving a reason for your answer. ---[2]

Solution: $y = 115°$ because the opposite angles in the cyclic quadrilateral ABCD are supplementary. $y + 65 = 180° \Rightarrow y = 115°$ ✓

SP-20 | 02 | Q26

2\. A, B, C and D lie on the circle.
PCQ is a tangent to the circle at C. ---[3]
Angle $ACQ = 64°$
Work out angle ABC, giving reasons for your answer.



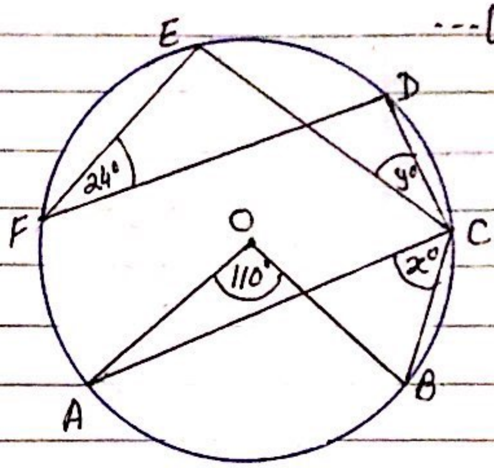
M-20 | 22 | Q15

Solution: angle $ABC = 116°$ ✓
because angle $\angle ADC = 64°$
angles in alternate circle.
and $\angle ADC + \angle ABC = 180$, opp. angle in the cyclic quad ABCD. $\therefore 64° + \angle ABC = 180°$
or angle $ABC = 180 - 64 = 116°$

$\therefore$ Angle $ABC = 116°$ ✓

3. A, B, C, D, E and F lie on the circle, centre O. ---[2]
Find the value of x and the value of y.

[S-20/21 | Q10]



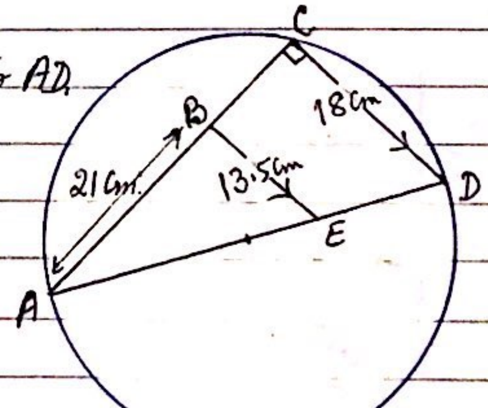
<u>Solution</u>: Angle subtended by the arc AB in the alternate segment of the circle 'x' is half the angle subtended by it at the centre.

$\therefore \; x = \frac{1}{2} \times 110° = 55°$ $\quad \therefore \underline{x = 55°}$

and $\underline{y = 24°}$ ✓ as the angles in the same segment of the circle subtended by arc ED.

---

4. C lies on a circle with diameter AD.
B lies on AC and E lies on AD,
such that BE is parallel to CD.
AB = 21 cm, CD = 18 cm and BE = 13.5 cm.
Work out the radius of the circle.



<u>Solution</u>: In ACD, BE ∥ CD

$\therefore \; \triangle ABC$ and $\triangle ACD$ are similar. ---[5]

[S-20/2 | Q13]

$\frac{AB}{AC} = \frac{BE}{CD}$ ($\because$ Corresponding sides of the similar triangles are proportional)

$\Rightarrow \frac{21}{AC} = \frac{13.5}{18} \Rightarrow AC = \frac{21 \times 18}{13.5} = 28.$

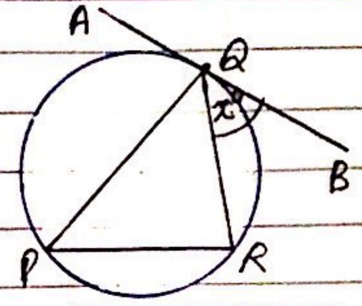
Now using Pythagoras theorem in rt $\triangle$ ACD ($\because$ AD is diameter)

$$AD^2 = AC^2 + CD^2$$
$$= 28^2 + 18^2 = 1108$$

diameter $AD = \sqrt{1108} = 33.286 \Rightarrow 2r = 33.28$
$r = 16.64$

$\therefore$ radius $= \underline{16.64 \text{ cm}}$

5. P, R and Q are points on the circle.
AB is a tangent to the circle at Q,
QR bisects angle PQB.
angle BQR = x° and $x < 60$
Use this information to show that
triangle PQR is an isosceles triangle.



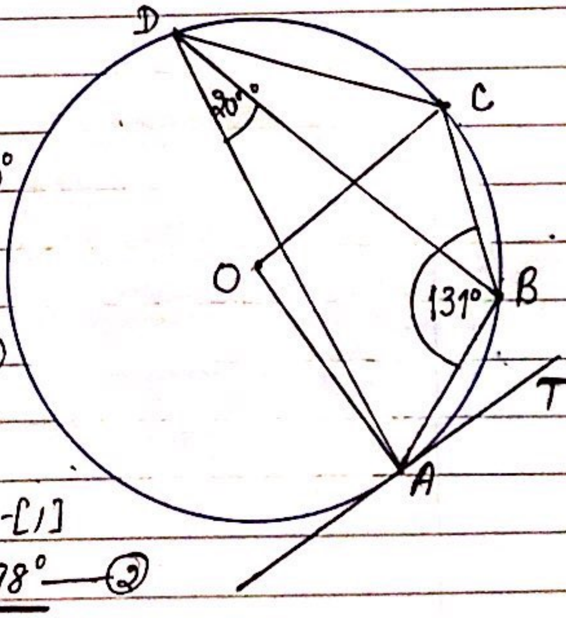
[S-20/21/Q15]

Give a geometrical reason for each step of your work. ---[3]

Solution: angle PQR = ∠BQR = x° —① (∵ QR bisect angle PQB)
angle QP = angle BQR = x —② (∵ Angle in the Alternate segment of a circle
from ① & ② ∠QPR = ∠PQR = x
∴ In Triangle PQR opposite sides PR = QR
∴ Triangle PQR is an isosceles Triangle.

---

6. A, B, C, D lie on circle, Centre O.
TA is tangent to the circle at A.
Angle ABC = 131° and angle ADB = 20°
Find.



(a) angle ADC. ---[1]
Answer: angle ADC = 180° − 131° = 49° —①
(∵ ABCD is a cyclic quad.
angle ADC + 131° = 180°)

(b) angle AOC ---[1]
Answer: angle AOC = 2 × ∠ADC = 2 × 49° = 98° —②

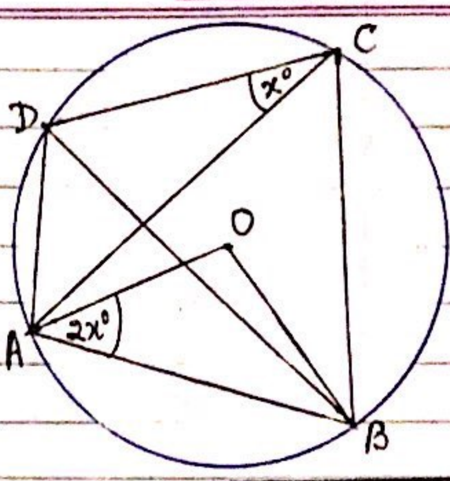
(c) angle BAT ---[1]
Answer: angle BAT = angle ADB = 20° (angles in the alternate segment) —③

(d) angle OAB
Answer: angle OAB + angle BAT = 90° (∵ OA is perp AT (rad ⊥ tangent) at A.
angle OAB + 20° = 90°
angle OAB = 90 − 20 = 70° ✓

[S-20/22/Q20]

7. In the diagram, A, B, C and D lie on the circumference of a circle, centre O. Angle ACD = x° and angle OAB = $2x^\circ$

Find an expression, in terms of x, in its simplest form for



(a) angle AOB ---[1]

Answer: angle AOB = $\underline{180 - 4x}$ ✓ —— ①

(In ΔOAB, $OA = OB = r \Rightarrow \angle OAB = \angle OBA = 2x$ and sum of angles of a triangle is 180°)

(angle AOB $+ 2x + 2x = 180^\circ$.)

W-19 | 22 | Q19

(b) angle ACB. ---[1]

Answer: angle ACB = $\frac{1}{2}$ angle AOB $= \frac{1}{2}(180 - 4x) = \underline{90 - 2x^\circ}$

($\because$ Angle AOB at the centre is double the angle in alternate seg. of arc $\overset{\frown}{AB}$)

(c) angle DAB ---[2]

Answer: In triangle ABD, $\angle DAB + \angle ADB + \angle ABD = 180^\circ$

$\Rightarrow \angle DAB = 180 - (\angle ADB + \angle ABD)$

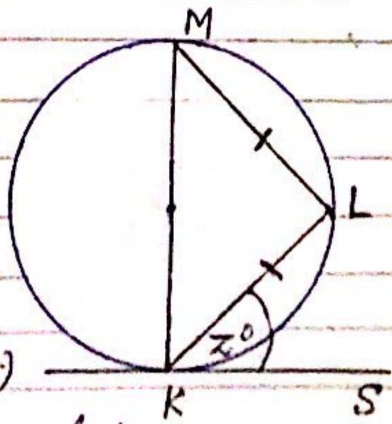
$= 180 - (\angle ACB + \angle ACD)$

$= 180 - (90 - 2x + x) = 180 - (90 - x) = \underline{(90 + x)^\circ}$ ✓

---

8 (a) (i) K, L and M are points on the circle. KS is a tangent to the circle at K. KM is a diameter and triangle KLM is isosceles.

Find the value of z. ---[2]



Solution: z = angle KML (Alternate segment) ①

Now angle KLM = 90° (KM is diameter, angle in the semicircle is right angle)

$\angle KML = \angle MKL$ (Isosceles Δ) —②

$\angle KML + \angle MKL = 90$ —③ from ① (angle sum prop. of Δ)

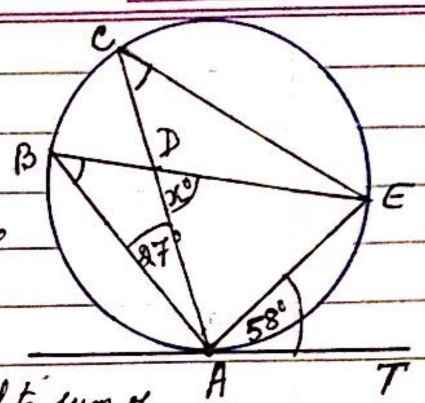
fr ② & ③ $\angle KML + \angle KML = 90 \Rightarrow 2\angle KML = 90 \Rightarrow \angle KML = 45^\circ$ ④

from ① & ② $\underline{z = 45^\circ}$ ✓

(Continued →) S.20 | 43 | Q8(b)(c)

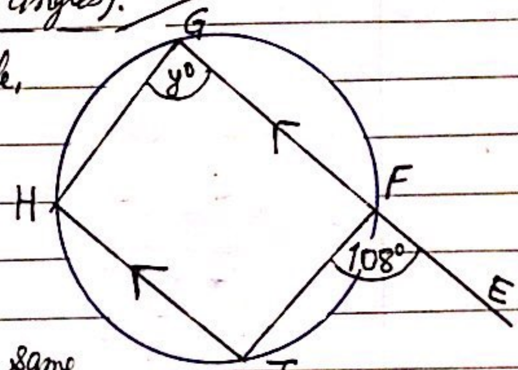
(continued →)

8 (a)(ii) AT is a tangent to the circle at A.
Find the value of x. ---[2]



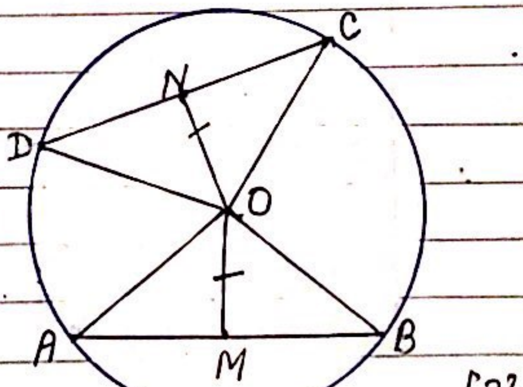
Solution: angle ACE = 58 (Alternate segment theorem)
$\angle ABE = ACE$ (angles in the same segment of a circle)
$\therefore \angle ABD = 58^\circ$ —— (1)
Now $x = 58 + 27$ (In ΔDAB, x is the exterior angle, equal to sum of opp. interior angles).
$\therefore x = 85^\circ$ ✓

(iii) F, G, H and J are points on the circle.
EFG is a straight line parallel to JH.
Find the value of y. ---[2]



Solution: angle $H = 108^\circ$
$y + \text{angle}H = 180^\circ$ (Interior Angles on the same
$\Rightarrow y + 108 = 180^\circ$ (side of transversal between parallel lines)
$\Rightarrow y = 72^\circ$ ✓

(b) A, B, C and D are points on the circle, centre O.
M is the mid point of AB and N is the mid point of CD; OM = ON.
Explain, giving reasons, why triangle OAB is congruent to triangle OCD.



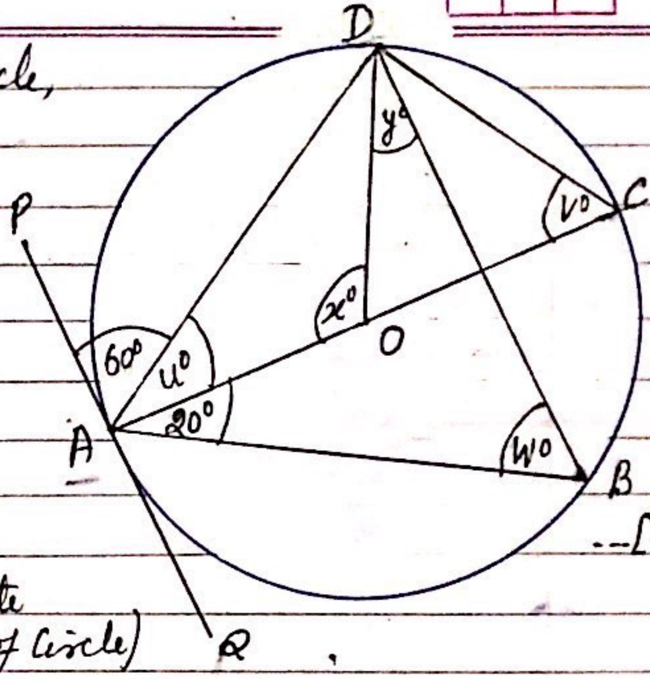
---[3]

Solution: $OM = ON \Rightarrow AB = CD$ (Chords equidistant from centre are equal)

Now $AB = CD$ —— (1)
$OA = OC = r$
$OB = OD = r$
$\therefore$ Triangle $OAB \cong$ Triangle OCD (SSS theorem of congruency)

S-20 | 43 | Q8(b)(c)

9(a) A, B, C and D lie on the circle, centre O, with diameter AC.
PQ is a tangent to the circle at A.
angle PAD = 60° and
angle BAC = 20°
Find the value of u, v, w, x and y.



---[6]

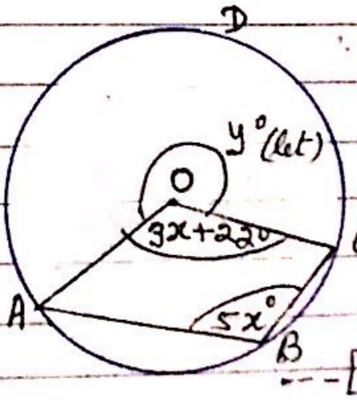
Solution: $v = w = 60°$ (Alternate segment of circle)

$\angle ADC = 90°$ (angle in semicircle)
$\Rightarrow u + v = 90 \Rightarrow u + 60 = 90 \Rightarrow u = 30°$

$x = 2w = 2 \times 60 = 120°$ (angle at the centre is double the angle in the remaining circle)
$\therefore x = 120°$

$\angle OAD = \angle ODA$ $(OA = OD)$
$30 = \angle ODA$ and $\angle BDC = 20°$
$\therefore y + 30° + 20° = 90° \Rightarrow y = 40°$

9 (b) A, B and C lie on the circle, centre O.
Angle $AOC = (3x + 22)°$ and
angle $ABC = 5x°$. Find the value of x.



---[4]

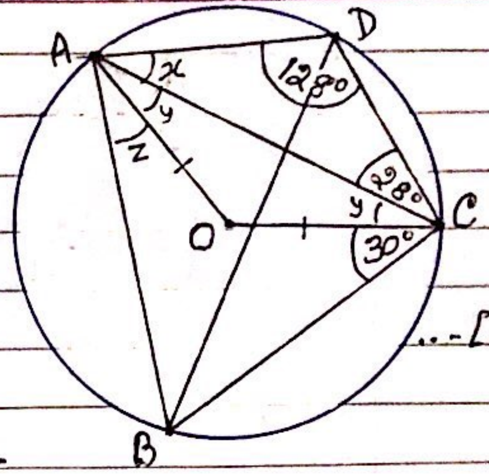
Solution: Angle subtended by arc ADC at the centre angle $y° = 360 - (3x+22)$
$= (338 - 3x)°$

Now $338 - 3 = 2 \times 5x°$ (angle subtended by arc ADC is double the angle in the opp. segment of circle)

$\boxed{W-19/41/Q1(d)(e)}$

$338 - 3x = 10x$
$\Rightarrow 13x = 338$
$x = \frac{338}{13} = 26°$
$\therefore x = 26°$

10. In the diagram A, B, C and D lie on the circle, centre O.
Angle $ADC = 128^\circ$, angle $ACD = 28^\circ$ and angle $BCO = 30^\circ$



(i) Show that obtuse angle $AOC = 104^\circ$
Give a reason for each step. ...[3]

Answer: angle $ABC = 180 - 128 = 52^\circ$ — ①
(∵ the opp. angles of cyclic Quad ABCD are supplementary)
∴ obtuse angle $AOC = 2 \times 52 = 104^\circ$ ✓ (∵ angle at the centre is double the angle at circumference by an arc $\widehat{ADC}$

(ii) Find angle BAO. ---[2]

Answer: $x = 180 - (128 + 28)$ (angle sum prop in ΔADC)
$x = 24^\circ$ — ②
In isosceles ΔOAC, $2y + 104 = 180 \Rightarrow y = 38^\circ$ — ③
Now $\angle BAD + \angle BCD = 180$ (opp angle of cyclic quad ABCD)
$\Rightarrow (z + 38 + 24) + (28 + 38 + 30) = 180^\circ$
$z + 168 = 180 \Rightarrow z = \angle BAO = 180 - 168 = \underline{22^\circ}$ ✓

(iii) Find angle ABD ---[1]

Answer: angle ABD = angle $ACD = \underline{28^\circ}$ ✓ (Angles in the same segment of circle)

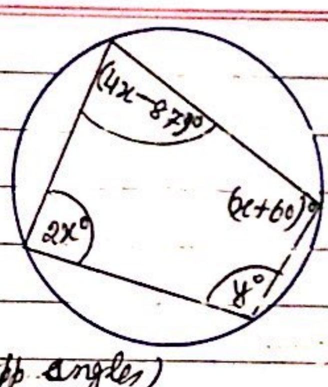
(iv) The radius, OC, of the circle is 9.6 cm.
Calculate the total perimeter of sector OADC. ---[3]

Answer: Perimeter of sector OADC $= OA + OC + \text{arc } \widehat{ADC}$
$= 9.6 + 9.6 + \frac{104}{360} \times 2\pi \times 9.6$
$= 19.2 + 17.425$
$= \underline{36.62 \text{ cm}}$ ✓

W-19/43/Q6(a)

11. The diagram shows a cyclic quadrilateral.
Find the value of y. ---[4]

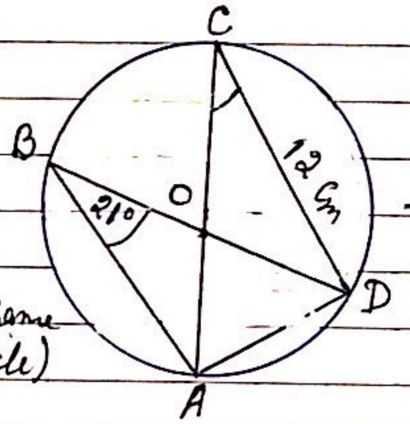
[W-20/21/Q20]



Solution: $2x + (x+60)^\circ = 180^\circ$ [opposite angles of a cyclic quad]
$3x + 60 = 180$
$\Rightarrow 3x = 120 \Rightarrow x = 40^\circ$ ---①
Also $4x - 87^\circ + y = 180^\circ$ (Another pair of opp angles are supplmentary)
$4\times40^\circ - 87 + y = 180^\circ$ [from ① $x = 40^\circ$]
$\Rightarrow y = 180 + 87 - 160 = 107^\circ \Rightarrow$ $\underline{y = 107^\circ}$ ✓

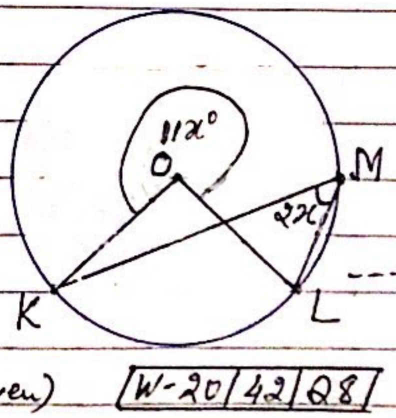
12. A, B, C and D are the points on the circle, centre O. Angle $ABD = 21^\circ$ and $CD = 12$ cm.
Calculate the area of the circle. ---[5]

[W-20/41/Q4(c)]



Solution: $\angle ACD = \angle ABD = 21^\circ$ (angles in the same segment of circle)
AC is diameter $\Rightarrow \angle ADC = 90^\circ$
Now In rt ΔACD, $\frac{12}{AC} = \cos 21^\circ \Rightarrow AC = \frac{12}{\cos 21^\circ} = 12.853$
radius $r = \frac{AC}{2} = \frac{12.853}{2} = 6.4267$
$\therefore$ Area of the circle $= \pi r^2 = \pi.(6.4267)^2 = 130\text{ cm}^2$ (or 129.8)

13.(b) In the diagram K, L and M lie on a circle, centre O. Angle $KLM = 2x^\circ$ and reflex angle $KOL = 11x^\circ$
Find the value of x. ---[3]

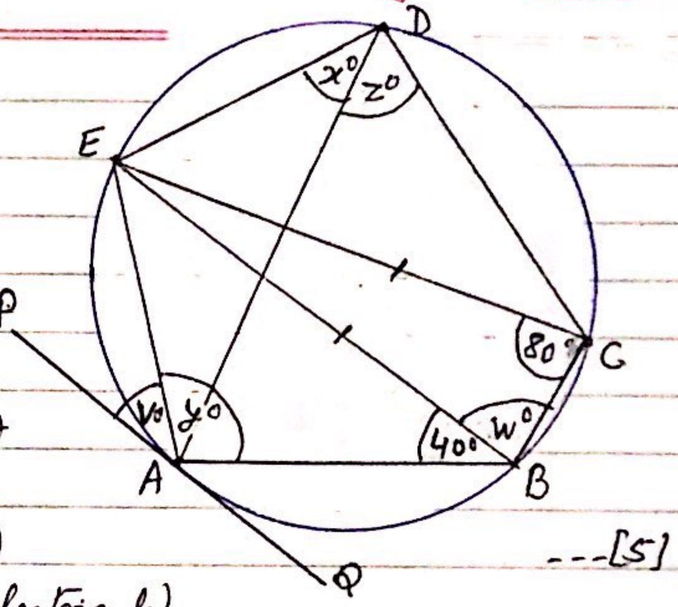


Solution: angle $KOL = 2\times2x^\circ = 4x^\circ$ (angle at the centre is double---)
$\therefore$ reflex angle $KOL = 360 - 4x = 11x$ (given) [W-20/42/Q8]
$\Rightarrow 15x = 360^\circ \Rightarrow x = \frac{360}{15}$
$\Rightarrow \underline{x = 24^\circ}$ ✓

(continued →)

(Continued)

13 (a) The points A, B, C, D and E lie on the circle. PAQ is a tangent to the circle at A and EC = EB. Angle ECB = 80° and angle AEB = 40°. Find the values of v, w, x, y, and z.



---[5]

Solution: $v = 40^\circ$ (Angle between the tangent and chord is equal to the angle in the alt circle)

$w = 80^\circ$ (ΔBEC is isosceles triangle)

$x = \angle ABE = 40^\circ$ (angles in the same segment of circle)

$y = 100^\circ$ ($\because y + 80 = 180^\circ$ opposite angle of the cyclic quad ABCE)

$z = 180 - (40 + w)$ [opposite angles in the cyclic quad ABCD)

or $z = 180 - (40 + 80) = 60^\circ$ ✓

(C) The diagonals of the cyclic quadrilateral ABCD intersect at X.

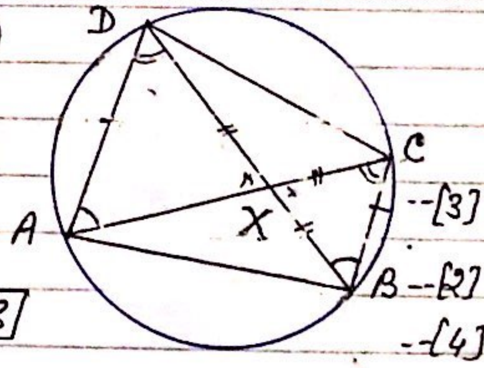
(i) Explain why ΔADX is similar to ΔBCX. --[3]

(ii) AD = 10 cm, BC = 8 cm, BX = 5 cm and CX = 7 cm

(a) Calculate DX. --[2]

(b) Calculate BXC. --[4]

W-20/42/Q8



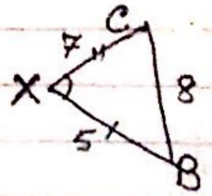
Solution (i) $\Delta ADX \sim \Delta BCX$ [AAA $\because$ (i) $\angle AXD = \angle BXC$ vertical opp. angles
(ii) $\angle DAX = \angle CBX$ angles in the same
(iii) $\angle ADX = \angle BCX$ ← same segment)

(ii) (a) $\Delta ADX \sim \Delta BCX$ from Part (i)

Corresponding sides are proportional $\Rightarrow \dfrac{DX}{CX} = \dfrac{AD}{BC}$

$\Rightarrow \dfrac{DX}{7} = \dfrac{10}{8} \Rightarrow DX = \dfrac{10 \times 7}{8} = \underline{8.75}$ ✓

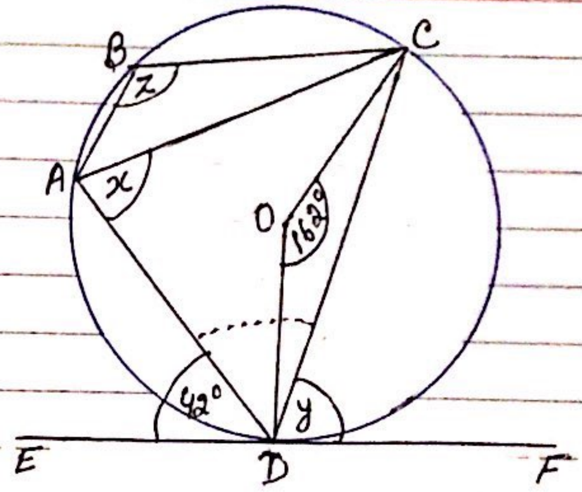
(b) $\cos BXC = \dfrac{5^2 + 7^2 - 8^2}{2 \times 5 \times 7} = \dfrac{1}{7}$



$\therefore$ angle $BXC = \cos^{-1}\left(\frac{1}{7}\right) = 81.8^\circ$ (81.78)

$\therefore$ angle $BXC = \underline{81.8^\circ}$

14.(a) A, B, C and D are points on the circle, centre O. EF is a tangent to the circle at D.
Angle ADE = 42° and angle COD = 162°.
find the following angles, giving reasons for each of your answers.
(i) Angle x ---[2]
(ii) Angle y --[2]
(iii) Angle z --[3]



W-20	43	Q5(a)

Solution(i) $x = \frac{162}{2} = \underline{81°}$ because the angle at the centre is twice the angle at the circumference.

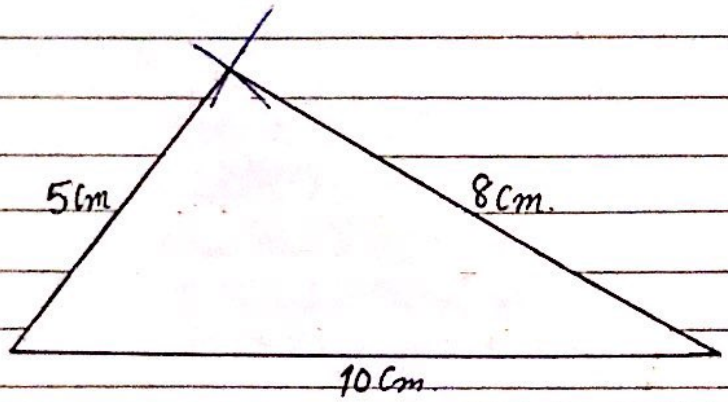
(ii) $y = x = \underline{81°}$ angle between the tangent and chord is equal to the angle in the alternate segment of circle.

(iii) $\angle ADC = 180° - (42+y) = 180-(42+81) = 57°$ [Straight angle]

$\therefore z = 180 - \angle ADC = 180 - 57 = \underline{123°}$ [opposite angles in the cyclic quad. ABCD]

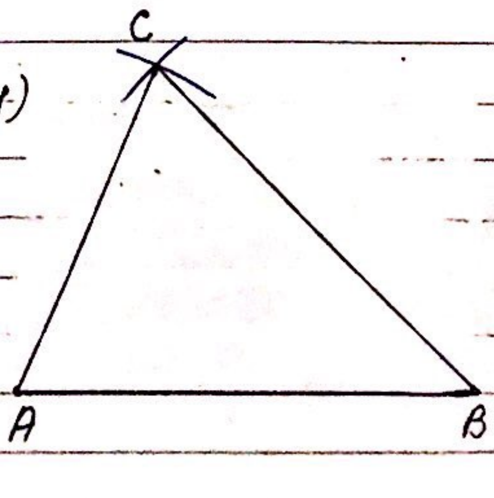
Constructions

1. Using a ruler and pair of compasses only, construct a triangle with sides 5cm, 8cm and 10cm.
Leave in your construction arcs. ---[2]
[SP-20/02/Q7]

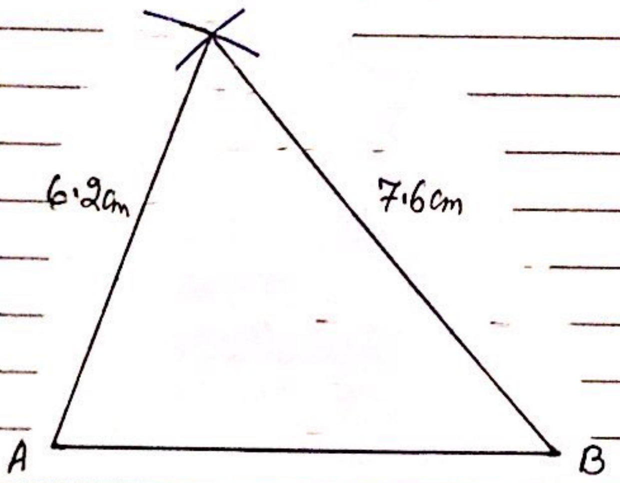


2. A field, ABC, is in the shape of a triangle.
$AC = 500m$ and $BC = 650m$.
Using a ruler and compasses only, complete the scale drawing of the field ABC. Leave in your construction arcs.
Use a scale of 1cm to represent 100m, The side AB has been drawn for you. [W-20/21/Q2] --[3]

Solution: $100m = 1cm$ (on drawing)
$AC = 500m = 5cm$ (on drawing)
$BC = 650m = 6.5cm$ (on drawing)

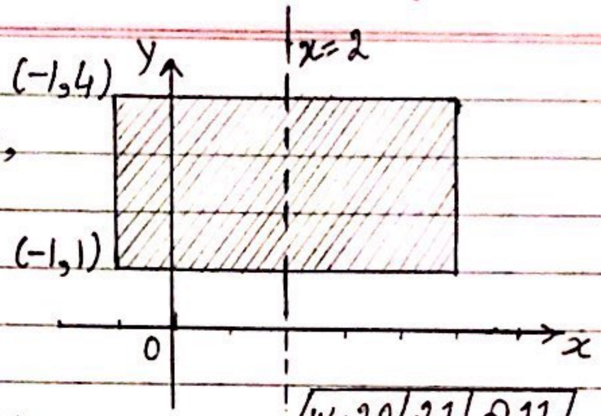


3 In triangle ABC, BC = 7.6 cm and AC = 6.2 cm.
Using ruler and compass only, construct triangle ABC.
Leave in your construction arcs. --[2]
The side AB has been drawn for you. [W-20/23/Q 3]



Rotational and line Symmetry

1. The diagram shows a rectangle with a line of symmetry at $x=2$. Two vertices of the rectangle are at $(-1,1)$ and $(-1,4)$. The shaded region is defined by the inequalities $a \le x \le b$ and $c \le y \le d$. Find the values of a, b, c and d.



W-20/21/Q11 ...[2]

Solution: $a = -1$, ✓
$b = ? = 5$ ✓
$c = 1$ ✓
$d = 4$ ✓

$\left\{ \begin{array}{l} \frac{b+(-1)}{2} = 2 \\ \Rightarrow b - 1 = 4 \\ \Rightarrow b = 5 \checkmark \end{array} \right.$

($x=2$ is the line of symmetry)