

0580

IGCSE Maths.

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Mensuration

Revision

SP-20 | M-20 | S-20 | W-19 | W-20

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Mensuration

Change of units.

1. Chai says that 8cm^2 is same as 80mm^2 .
Explain why Chai is wrong.

[SP-20/02/Q4] ---[1]

Answer: $1\text{cm} = 10\text{mm}$

$$\Rightarrow 1\text{cm}^2 = 10 \times 10\text{mm}^2 = 100\text{mm}^2$$

$$\Rightarrow 8\text{cm}^2 = 8 \times 100 = 800\text{mm}^2 \text{ and not } 80\text{mm}^2$$

$\therefore$ Chai is wrong.

Note:

1. (i) $1\text{m}^2 = 100 \times 100 = 10000\text{cm}^2$

(ii) $1\text{km}^2 = 1000 \times 1000 = 1000000\text{m}^2$

2. (i) $1\text{cm}^3 = 10 \times 10 \times 10 = 1000\text{mm}^3$

(ii) $1\text{m}^3 = 100 \times 100 \times 100 = 1000000\text{cm}^3$

3. (i) $1\text{litre} = 10 \times 10 \times 10 = 1000\text{cm}^3 (\text{c.c}) = (1\text{dm}^3)$

(ii) $1\text{m}^3 = 100 \times 100 \times 100\text{cm}^3 = 1000 \times 1000\text{cm}^3$
 $= 1000\text{litres}$

2. Change 457000cm^2 into m^2

[W-20/23/Q7] ---[1]

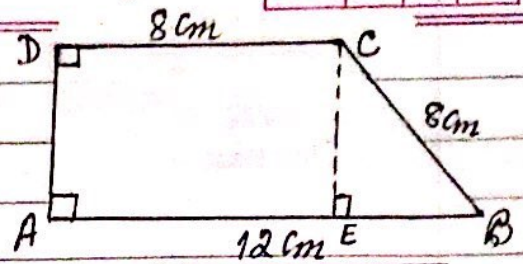
Solution: $475000 = \frac{457000}{100 \times 100}\text{m}^2$
 $= 45.7\text{m}^2 \checkmark$

$\left\{ \begin{array}{l} \because 1\text{m} = 100\text{cm} \\ 1\text{m}^2 = 100 \times 100\text{cm}^2 \end{array} \right.$

Triangles, Quadrilaterals and Polygons.

Perimeter and Area

1. Calculate the area of this trapezium.



Solution: Draw CE perpendicular to AB.

$$BE = AB - AE = AB - CD = 12 - 8 = 4$$

Now in $\triangle CEB$ using Pythagoras Theorem,

$$CE^2 + BE^2 = BC^2 \Rightarrow CE = \sqrt{BC^2 - BE^2} = \sqrt{8^2 - 4^2} = \sqrt{48}$$

$$\Rightarrow CE = 6.928 \text{ cm}$$

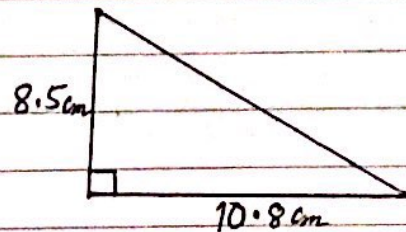
$$\begin{aligned} \text{Area of Trapezium} &= \frac{1}{2} (\text{Sum of Parallel Sides}) \times \text{distance between parallel sides} \\ &= \frac{1}{2} (12 + 8) \times 6.928 \\ &= \frac{1}{2} \times 20 \times 6.928 = 69.28 \\ &= \underline{69.3 \text{ cm}^2} \checkmark \end{aligned}$$

[SP-20/02/Q17]...[4]

2. The diagram shows a right-angled triangle.

(a) Calculate the area.

(b) Calculate the perimeter.



...[2]

...[3]

[M-20/22/Q7]

Solution (a) Area of right-angled triangle = $\frac{1}{2} \times$ the product of perp. sides.

$$= \frac{1}{2} \times 10.8 \times 8.5$$

$$= \underline{45.9 \text{ cm}^2}$$

(b) Hypotenuse = $\sqrt{10.8^2 + 8.5^2} = \sqrt{188.89} = 13.74 \text{ cm}$

$\therefore$ The perimeter of the triangle = $13.74 + 10.8 + 8.5$

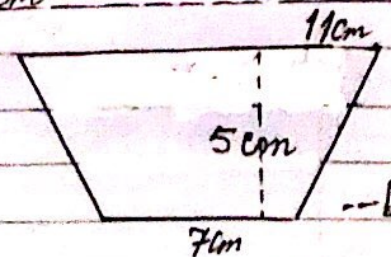
$$= \underline{33 \text{ cm}} \text{ (or } 33.04)$$

3. Calculate the area of the trapezium.

Solution: Area of trapezium = $\frac{1}{2} \times$ sum of parallel side $\times$ distance between the parallel sides.

$$= \frac{1}{2} (11 + 7) \times 5$$

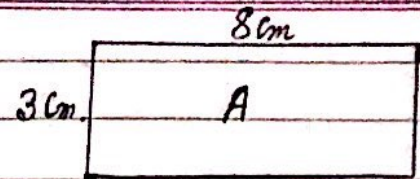
$$= \frac{1}{2} \times 18 \times 5 = \underline{45 \text{ cm}^2} \checkmark$$



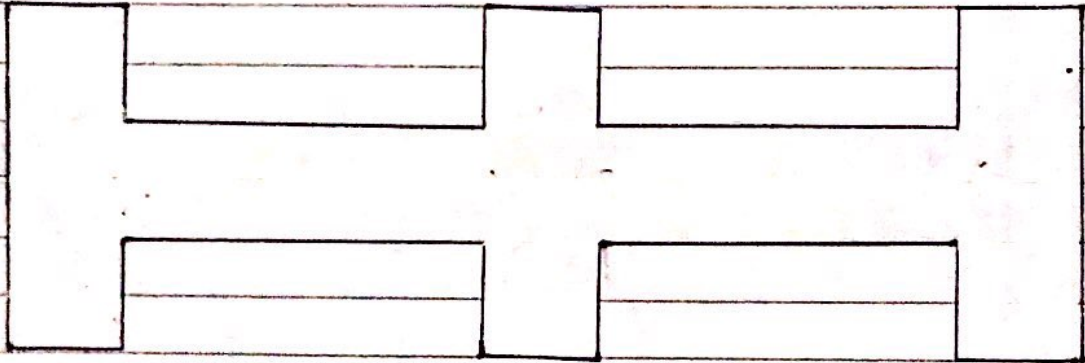
...[2]

[S-20/22/Q6]

4. Rectangle A measures 3cm by 8cm.



Five rectangles congruent to A are joined to make a shape.



Work out the perimeter of this shape. [S-20/21/Q1] ...[2]

Solution: Perimeter of a rectangle = $2(8+3) = 22 \text{ cm}$.

Perimeter of Shape = $5 \times 22 - 8 \times 3 = 86 \text{ cm}$ ✓

(or $8 \times 6 + 5 \times 4 + 3 \times 6 = 86$)

5. Find the area of a regular hexagon with side length 7.4 cm.

[S-20/21/Q22] ...[3].

Solution: Area of a regular hexagon

= $6 \times \text{area of equilateral } \triangle OAB$ — ①

Let the side = a .

Now Area of Equilateral Triangle OAB

$$= \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} \times a \times a \times \sin 60^\circ$$

$$= \frac{1}{2} a^2 \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} a^2$$

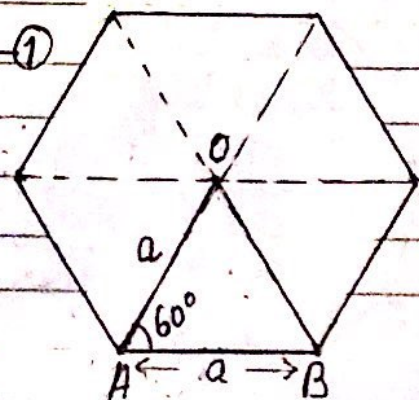
from ①

$$\text{Area of hexagon} = 6 \times \frac{\sqrt{3}}{4} a^2 = \frac{3\sqrt{3}}{2} a^2$$

$$\text{for } a = 7.4 \text{ cm} = \frac{3\sqrt{3}}{2} (7.4)^2$$

$$= 142.27$$

$$\therefore \text{Required Area} = 142 \text{ cm}^2 \checkmark$$

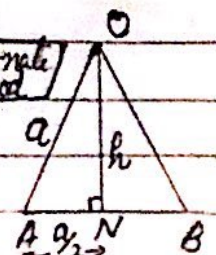


$$h = \sqrt{a^2 - \left(\frac{a}{2}\right)^2} \quad \text{[Alternate method]}$$

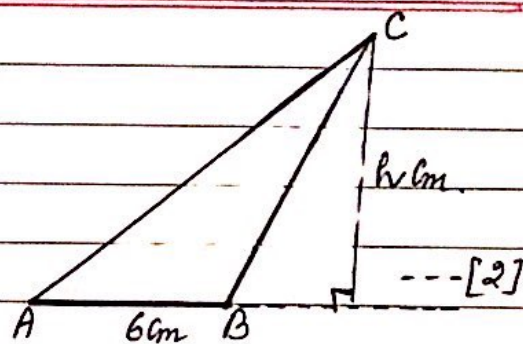
$$= \frac{\sqrt{3}}{2} a$$

$$\text{Area of } \triangle = \frac{1}{2} \times a \times h$$

$$= \frac{1}{2} \times a \times \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{4} a^2 \checkmark$$



6. The area of triangle ABC is 27 cm^2 ,
and $AB = 6 \text{ cm}$,
Find the value of h .



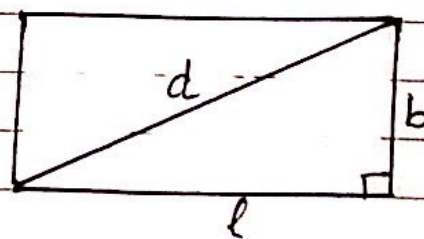
Solution: Area of triangle = $\frac{1}{2} \times \text{base} \times \text{Altitude}$
 $\frac{1}{2} \times 6 \times h = 27$ (Given)
 $3h = 27 \Rightarrow h = 9 \text{ cm}$ ✓

[W-20/22/Q7]

7. The length of one side of a rectangle is 12 cm .
The length of the diagonal of the rectangle is 13 cm .
Calculate the area of the rectangle.

[W-20/22/Q13] ---[3]

Solution: Length of one side $l = 12 \text{ cm}$.
Let the length of other side = $b \text{ cm}$.
and diagonal $d = 13 \text{ cm}$.



We know In Δ $l^2 + b^2 = d^2$

$$12^2 + b^2 = 13^2 \Rightarrow b^2 = 169 - 144 = 25$$

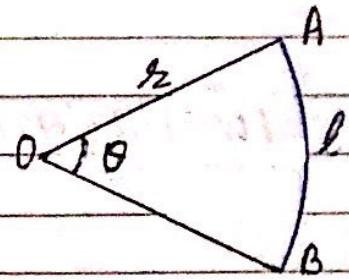
$$\Rightarrow b = 5 \text{ ✓}$$

$\therefore$ Area of the rectangle = $l \times b = 12 \times 5 = 60 \text{ cm}^2$ ✓

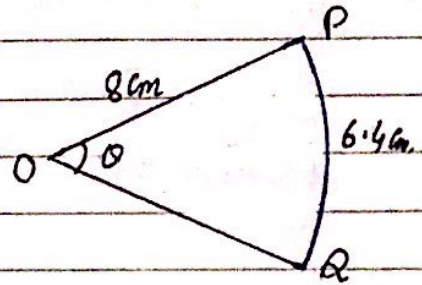
Length of arc and Area of Sector of a circle

1. length arc AB, $l = \frac{\theta}{360^\circ} \times 2\pi r$

2. Area of Sector OAB = $\frac{\theta}{360} \times \pi r^2$



1. The diagram shows a sector of a circle of radius 8 cm. The length of arc PQ is 6.4 cm. Find the area of the sector.



Solution: Let the sector angle POQ = θ

[S-20/22/Q24] --- [4]

$$\text{length of arc PQ} = \frac{\theta}{360} \times 2\pi r = 6.4$$

$$\Rightarrow \frac{\theta}{360} \times \pi r = \frac{6.4}{2} = 3.2 \quad \text{--- (1)}$$

$$\text{Now area of Sector} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{\theta}{360} \times \pi r \times r = 3.2 \times 8 \quad \text{for (1)}$$

$$= 25.6 \text{ cm}^2 \checkmark$$

[Alternatively: Area of Sector = $\frac{1}{2} l r$ (l is the length of arc)]

$$= \frac{1}{2} \times 6.4 \times 8$$
$$= 25.6 \text{ cm}^2$$

2. Calculate the area of the sector of a circle with radius 65 mm, and sector angle 42° . Give your answer in cm^2 . --- [3]

[S-20/23/Q9]

Solution: Area of Sector = $\frac{\theta}{360} \times \pi r^2$

$$[r = 65 \text{ mm} = 6.5 \text{ cm}]$$

$$= \frac{42}{360} \times \pi \times (6.5)^2$$

$$= 15.485$$

$$\therefore \text{Area of Sector} = 15.5 \text{ cm}^2 \checkmark$$

Area of sector of a circle

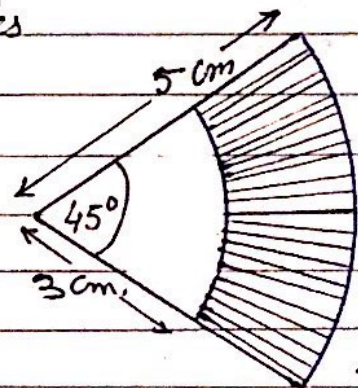
3. The total perimeter of a semicircle is 19.02 cm. --- [3]
Calculate the radius of the semicircle. [S-20/23/Q12]

Solution: Perimeter of semicircle = $2r + \pi r = 19.02$

$$\Rightarrow r(2 + \pi) = 19.02$$

$$\Rightarrow r = \frac{19.02}{(\pi + 2)} = \underline{3.7 \text{ cm}} \checkmark$$

4. The diagram shows two sectors of circles with the same centre.
Calculate the shaded area.



Solution: Area of shaded region

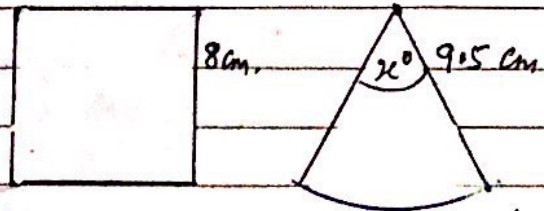
$$= \frac{\theta}{360} \times \pi (r_1^2 - r_2^2)$$

$$= \frac{45}{360} \times \pi (5^2 - 3^2)$$

$$= \frac{45}{360} \times \pi \times 16 = 2\pi = \underline{6.28 \text{ cm}^2} \checkmark$$

[W-19/21/Q17]

5. The diagram shows a square with side 8 cm and a sector of a circle with radius 9.5 cm and sector angle x° .



The perimeter of the square is equal to the perimeter of the sector. Calculate the value of x . --- [3]

[W-20/41/Q4(d)]

Solution: Perimeter of square = $4l = 4 \times 8 = 32 \text{ cm}$ --- (1)

$$\text{Perimeter of sector} = 2r + \frac{x}{360} \times 2\pi r = 2 \times 9.5 + \frac{x}{360} \times 2\pi \times 9.5$$

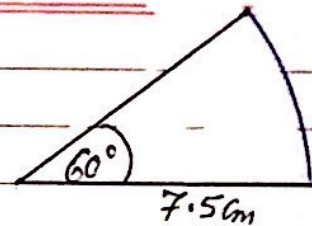
$$= (19 + \frac{19\pi x}{360}) \text{ --- (2)}$$

Given (1) & (2) equal.

$$19 + \frac{19\pi x}{360} = 32 \Rightarrow \frac{19\pi x}{360} = 32 - 19 = 13$$

$$\Rightarrow x = \frac{13 \times 360}{19\pi} = \frac{4680}{59.69} = \underline{78.4^\circ} \checkmark$$

6. Calculate the area of this sector of circle. --- [2]



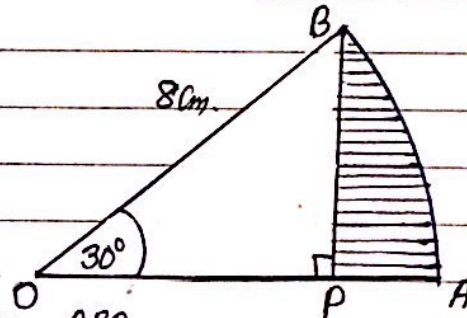
Solution: Area of sector = $\frac{\theta}{360} \times \pi r^2$

$$= \frac{60^\circ}{360} \times \pi \times 7.5^2 = 29.5 \text{ cm}^2 \checkmark \quad [W-20/22/Q15] \quad (\text{or } 29.45)$$

7. OAB is a sector of a circle, centre O.
OB = 8 cm and angle AOB = 30°
BP is perpendicular to OA.

(a) Calculate AP. --- [3]

(b) Work out the area of the shaded region APB. --- [3]



[W-20/23/Q1]

Solution(a) In right $\triangle OBP$, $\frac{OP}{OB} = \cos 30^\circ \Rightarrow \frac{OP}{8} = \cos 30^\circ \Rightarrow OP = 8 \cos 30^\circ = 6.93 \text{ cm}$
 $\therefore AP = OA - OP = 8 - 6.93 = 1.07 \text{ cm} \checkmark$

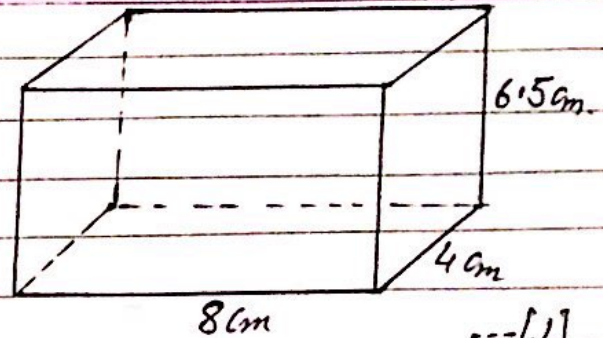
(b) Area of Sector OAB = $\frac{\theta}{360} \times \pi r^2 = \frac{30}{360} \times \pi \times 8^2 = 16.755 \text{ cm}^2$

Area of $\triangle OPB = \frac{1}{2} ab \sin C = \frac{1}{2} \times 8 \times 6.93 \times \sin 30^\circ = 13.86$

$\therefore$ Area of the shaded region = $16.755 - 13.86 = 2.9 \text{ cm}^2 \checkmark \quad (\text{or } 2.895)$

Cuboids / Cylinder

1. The diagram shows a cuboid.
Calculate the volume of cuboid.



Solution: $V = lbh$
 $= 8 \times 4 \times 6.5$
 $= \underline{208 \text{ cm}^3} \checkmark$

8 cm
4 cm
6.5 cm
---[1]
[S-20/23/Q2]

2. A cuboid measures 5 cm by 7 cm by 9.5 cm.
Work out the surface area of the cuboid.

---[3]
[W-19/23/Q9]

Solution: Surface area of cuboid $= 2[lb + bh + lh]$
 $= 2[5 \times 7 + 7 \times 9.5 + 5 \times 9.5]$
 $= \underline{298 \text{ cm}^2}$

3. Water from Manjeet's shower flows at a rate of 12 litres per minute. The water from the shower flows into a tank that is a cuboid of length 90 cm and width 75 cm.
Calculate the increase in the level of water in the tank when the shower is used for 7 minutes.

[M-20/42/Q3(C)] [3]

Solution: Water flow through the shower in 7 minutes $= 12 \times 7$ litres
 $= 84$ litres --- (1)

Let the increase in water level in the tank $= h$ cm.

$\therefore$ Increase in Volume in cuboid tank $= l \times b \times h$
 $= 90 \times 75 \times h \text{ cm}^3$
 $= 6750h \text{ cm}^3$ --- (2)

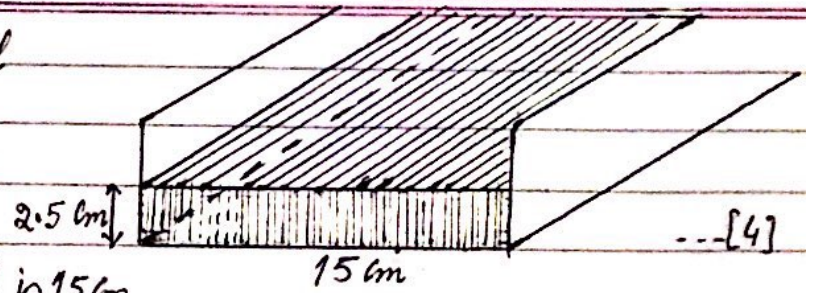
from (1) & (2) $6750h = 84 \times 1000$ [$\because 1 \text{ litre} = 1000 \text{ cm}^3$]

$$h = \frac{84000}{6750} = 12.44 \text{ cm}$$

$\therefore h = \underline{12.4 \text{ cm}} \checkmark$

Cuboid / Cylinder.

4. Water flows at a speed of 20 cm/s along a rectangular channel into a lake.



The width of the channel is 15 cm.

The depth of the water is 2.5 cm. Calculate the amount of water from the channel into the lake in 1 hour.

Give your answer in litres.

[S-20/43/Q6(b)]--[4]

Solution: Water flown in 1 hour through the channel (length of water column)
 $= 20 \times 60 \times 60 \text{ cm}$
 $l = 72000 \text{ cm}.$

$\therefore$ Volume of water through the channel in 1 hr.

$$= l \cdot b \cdot h$$

$$= 72000 \times 15 \times 2.5$$

$$= 2700000 \text{ cm}^3$$

$$= 2700 \text{ litres} \checkmark (\because 1 \text{ litre} = 1000 \text{ cm}^3)$$

5. A solid cylinder has radius 3 cm and height 4.5 cm. Calculate the total surface area of the cylinder. --[4]

[S-20/23/Q10]

Solution: Total surface area of cylinder $= 2\pi rh + 2\pi r^2$
 $= 2\pi r(h+r)$

$$= 2\pi \times 3(4.5+3) \quad \begin{matrix} \because r=3 \text{ cm} \\ h=4.5 \text{ cm} \end{matrix}$$

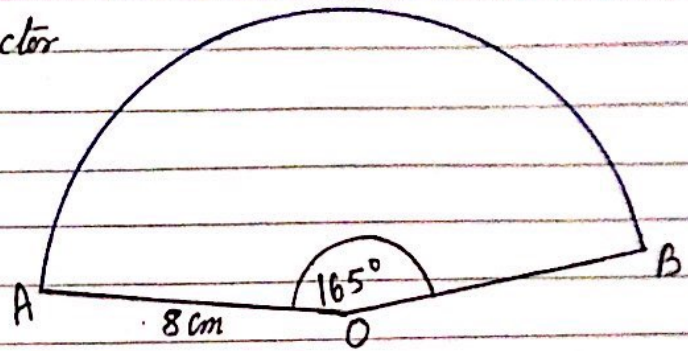
$$= 2\pi \times 3 \times 7.5$$

$$= 141.37 \text{ cm}^2$$

$$= 141 \text{ cm}^2 \checkmark$$

Spheres

1. The diagram shows a sector of a circle with centre O, radius 8 cm and sector angle 165° .



The surface area of a sphere is same as the area of the sector.

Calculate the radius of the sphere.

Solution: Area of sector = $\frac{\theta}{360} \times \pi r^2 = \frac{165}{360} \times \pi \times 8^2$ ——— (1)

Surface area of sphere = $4\pi R^2$ { Radius of sphere = R (let) }

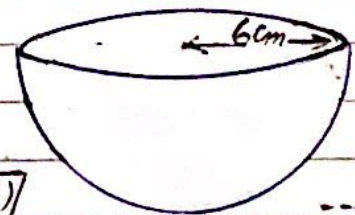
$\therefore 4\pi R^2 = \frac{165}{360} \times \pi \times 64$ from (1)

$R = \frac{165 \times 64 \times 1}{360 \times 4 \times 1440} = 7.333$

$R = \sqrt{7.333} = 2.7079$

$\therefore R = 2.71 \text{ cm} \checkmark$

2. The diagram shows a hemisphere with radius 6 cm. Calculate the volume. Give units of your answer.



[W-19/42/Q4(a)]

---[3]

Solution: Volume of hemisphere = $\frac{2}{3} \pi r^3$
 $= \frac{2}{3} \pi \times 6^3$

$= 452.389 \text{ cm}^3$

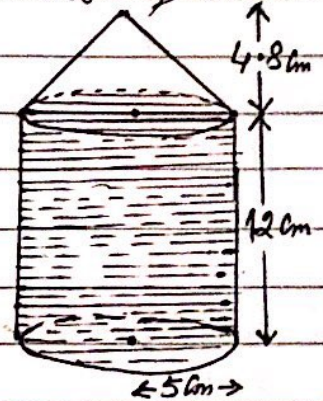
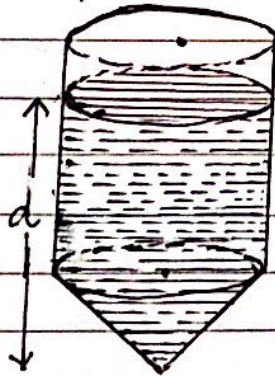
$= \underline{452.4 \text{ cm}^3} \checkmark$

Compound Shapes (Area and Volume)

1. A container is made from a cylinder and a cone, each of radius 5 cm. The height of the cylinder is 12 cm and the height of the cone is 4.8 cm.

The cylinder is filled completely with water. The container is turned upside down.

Calculate the depth, d , of the water.



---[5]

[W-19/23/Q22]

Solution: Case I: Cylinder is full of water Volume $V = \pi r^2 h$

$$= \pi \times 5^2 \times 12$$

$$= 300\pi \text{ cm}^3 \text{ --- (1)}$$

Case II Cone is full of water $V_1 = \frac{1}{3} \pi r^2 h_1$

$$V_1 = \frac{1}{3} \pi \times 5^2 \times 4.8 = 40\pi \text{ --- (2)}$$

Now let the height of water level in the cylinder = h_1

Now Volume of water in cylinder = $\pi r^2 h_1$

$$V_2 = \pi \times 5^2 \times h_1 = 25\pi h_1 \text{ --- (3)}$$

$$V_1 + V_2 = V$$

$$40\pi + 25\pi \cdot h_1 = 300\pi$$

$$\Rightarrow 25\pi h_1 = 300\pi - 40\pi = 260\pi$$

$$h_1 = \frac{260\pi}{25\pi} = 10.4$$

$$\therefore d = h_1 + 4.8$$

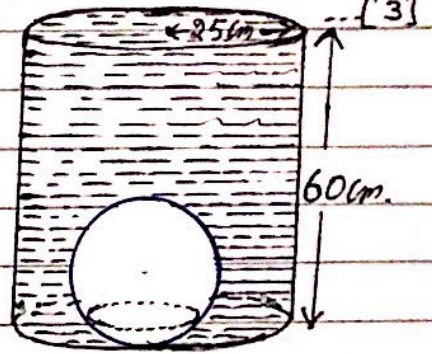
$$= 10.4 + 4.8 = \underline{15.2 \text{ cm}} \checkmark$$

2. (a) Show that the volume of a metal sphere of radius 15cm is 14140 cm^3 , correct to 4 significant figures. ---[2]

Solution: Volume of sphere $= \frac{4}{3} \pi r^3 = \frac{4}{3} \pi \times 15^3 = 14137.16 \text{ cm}^3$
 $= 14140 \text{ cm}^3 \text{ (4s.f.)} \text{ --- (1)}$

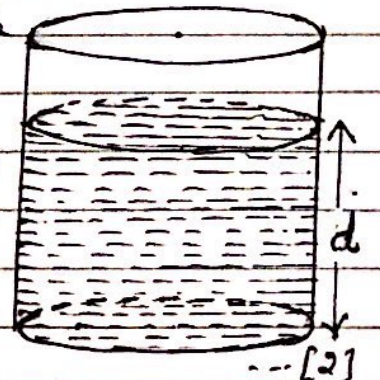
- (b) (i) The sphere is placed inside an empty cylindrical tank of radius 25cm and height 60cm.

The tank is filled with water. Calculate the volume of water needed to fill the tank.



Solution: Volume of water = Volume of cylinder - Volume of sphere
 $= \pi R^2 H - 14137 \text{ (from (1))}$
 $= \pi \times 25^2 \times 60 - 14137$
 $= 117810 - 14137 = 103673 \text{ cm}^3 \checkmark \text{ --- (2)}$

- (b) (ii) The sphere is removed from the tank. Calculate the depth, d , of water in the tank.

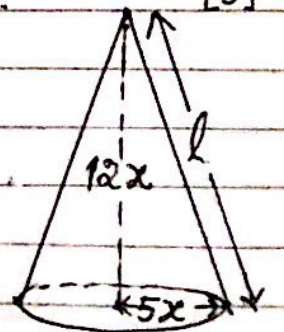


Solution: Volume of water $= \pi r^2 d = 103673 \text{ (from (2))}$
 $\pi \times 25^2 d = 103673$
 $d = \frac{103673}{625\pi} = 52.8 \text{ cm} \checkmark$

- (c) A cone has radius $5x \text{ cm}$ and height $12x \text{ cm}$.

The sphere has radius 8 cm .

The cone has the same total surface area as the sphere. Show that $x^2 = \frac{45}{2}$



Solution: Total surface area of sphere $= 4\pi r^2 \text{ --- (1)}$

Let the slant height of cone is $l = \sqrt{(5x)^2 + (12x)^2} = 13x$

Total surface area of cone $= \pi R^2 + \pi R l$
 $= \pi \cdot (5x)^2 + \pi \times 5x \times 13x = 90x^2 \pi \text{ --- (2)}$

from (1) & (2)

$4\pi r^2 = 90x^2 \pi$

$\Rightarrow r^2 = \frac{90}{4} x^2 = \frac{45}{2} x^2 \checkmark$

$\therefore r^2 = \frac{45}{2} x^2 \checkmark$

SP-20/04/2010

3. (a) A cylinder with radius 6 cm. and height h cm has the same volume as a sphere with radius 4.5 cm. Find the value of h . ---[3]

(b) A solid metal cube of side 20 cm is melted down and made into 40 solid spheres r cm. Find the value of r . ---[3]

Solution (a) Volume of cylinder = $\pi r^2 h = \pi \times 6^2 \times h = 36\pi h$ --- (1)

Volume of sphere = $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi \cdot (4.5)^3 = 121.5\pi$ --- (2)

from (1) and (2) $121.5\pi = 36\pi h$

$$\therefore h = \frac{121.5}{36} = 3.375 \text{ cm} \checkmark$$

(b) Volume of cube = $20^3 = 8000 \text{ cm}^3$ --- (3)

Volume of one sphere = $\frac{4}{3}\pi r^3$ --- (4)

Given $40 \times \frac{4}{3}\pi r^3 = 8000$ (from (3) & (4))

$$\Rightarrow r^3 = \frac{8000 \times 3}{160 \times \pi} = \frac{24000}{502.65} = 47.74$$

$$r = \sqrt[3]{47.74} = 3.627 \checkmark$$

(c) A solid cylinder has radius x cm. and height $\frac{7x}{2}$.

The surface area of a sphere with radius R cm is equal to the total surface area of the cylinder.

Find an expression for R in terms of x . ---[3]

[5-20/42/Q8(b,c,d)]

Solution: Surface area of sphere = $4\pi R^2$ --- (5)

Total surface area of cylinder = $2\pi r h + 2\pi r^2$

$$\begin{aligned} &= 2\pi r (h + r) \\ &= 2\pi \cdot x \left(\frac{7x}{2} + x \right) \quad \left(\begin{array}{l} r = x \text{ cm} \\ h = \frac{7x}{2} \text{ cm} \end{array} \right) \\ &= 9\pi x^2 \text{ --- (6)} \end{aligned}$$

$$\text{from (5) \& (6)} \quad \Rightarrow 4\pi R^2 = 9\pi x^2$$

$$\Rightarrow R^2 = 9\pi x^2 \times \frac{1}{4\pi}$$

$$R^2 = \frac{9}{4} x^2$$

$$\Rightarrow R = \frac{3}{2} x \checkmark$$

4. A solid metal cone has radius 1.65 cm and slant height 4.70 cm.

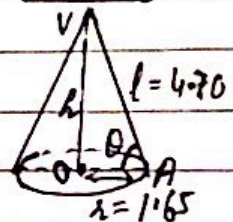
(a) Calculate the total surface area of the cone. ---[2]

(b) Find the angle the slant height makes with the base of the cone. ---[2]

Solution: (a) Total surface area of cone = $\pi r^2 + \pi r l$
 $= \pi r (r + l)$ [M-20/42/Q4]
 $= \pi \times 1.65 \times (1.65 + 4.70)$
 $= \pi \times 1.65 \times 6.35 = 32.91 \text{ cm}^2 \checkmark$

(b) $\cos \theta = \frac{r}{l} = \frac{1.65}{4.70} = 0.351$

$\therefore \theta = \cos^{-1}(0.351) = 69.44^\circ \checkmark$



(c) (i) Calculate the volume of the cone. ---[4]

Solution: The perpendicular height h of the cone: $h = \sqrt{l^2 - r^2}$
 $= \sqrt{(4.7)^2 - (1.65)^2} = 4.4 \text{ cm}$

$\therefore$ Volume of cone = $\frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \times (1.65)^2 \times 4.4 = 12.54 \text{ cm}^3 \checkmark$ ---①

(ii) A metal sphere with radius 5 cm is melted down to make cones identical to this one. Calculate the number of complete identical cones that are made. ----[4]

Solution: Volume of sphere = $\frac{4}{3} \pi R^3 = \frac{4}{3} \pi \times 5^3 = 523.6 \text{ cm}^3$ ---②

Number of cones = $\frac{\text{Volume of sphere}}{\text{Volume of one cone}}$

$= \frac{523.6}{12.54}$ (from ① & ②)

$= 41.75$

$\therefore$ Complete number of cones = 41

5. (a) (i) Calculate the external curved surface area of a cylinder with radius 8m and height 19m. --[2]
- (ii) The surface is painted at a cost of \$0.85 per square metre. Calculate the cost of painting this surface. --[2]

Solution: Curved surface area of a cylinder = $2\pi r h$

(a) (i) $= 2\pi \times 8 \times 19 = 955 \text{ m}^2 \checkmark$

(ii) Cost of painting = $955 \times 0.85 = \$811.75$ (or \$812) $\checkmark$

- (b) A solid metal sphere with radius 6cm is melted down and all the metal is used to make a solid cone, with radius 8cm and height h cm.

(i) Show that $h = 13.5$ --[2]

(ii) Calculate the slant height of the cone. --[2]

(iii) Calculate the curved surface area of the cone. --[1]

[W-19/41/Q4]

Solution (i) Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 6^3 = 288\pi \text{ cm}^3$ --- (1)

Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 8^2 \times h = \frac{64}{3}\pi h$ --- (2)

from (1) & (2) $\frac{64}{3}\pi h = 288\pi$

$\Rightarrow h = \frac{288\pi \times 3}{64\pi} = 13.5 \text{ cm} \checkmark$

(ii) Slant height 'l' of cone $l = \sqrt{h^2 + r^2}$
 $= \sqrt{(13.5)^2 + 8^2} = 15.69 \text{ cm}$

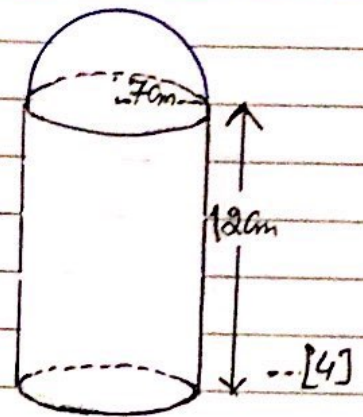
(iii) Curved surface area of cone = $\pi r l$
 $= \pi \times 8 \times 15.69$
 $= 394.39$
 $= 394.4 \text{ cm}^2 \checkmark$

6. The diagram shows a solid made from a cylinder and a hemisphere, both of radius 7 cm.

The cylinder has length 12 cm.

Work out the total surface area of the solid.

[W-20/21/218]



Solution: Total surface area of the solid:

= curved surface area of cylinder + curved surface area of the hemisphere + circular base

$$= 2\pi rh + 2\pi r^2 + \pi r^2$$

$$= 2\pi rh + 3\pi r^2$$

$$= \pi r [2h + 3r]$$

$$= \pi \times 7 [2 \times 12 + 3 \times 7] = 7\pi \times 45 = 7 \times 45 \times \pi = 989.6 \text{ cm}^2$$

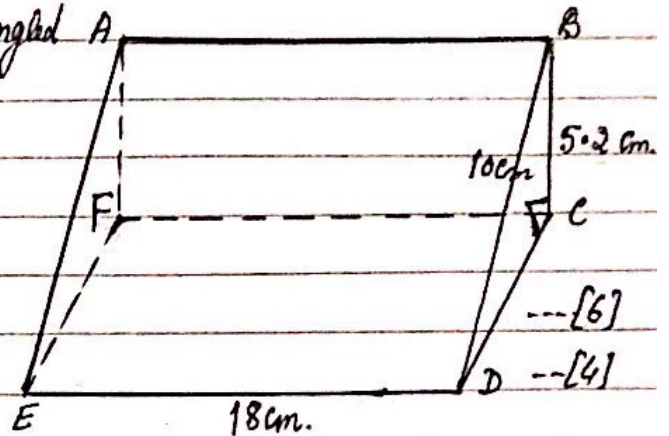
$$\therefore \text{Total surface area} = \underline{990 \text{ cm}^2} \checkmark$$

$$\left[\text{C.S. Area of hemisphere} = \frac{1}{2} \times 4\pi r^2 \right]$$

1. The diagram shows a prism ABCDEF.

The cross-section is a right-angled triangle BCD

BD = 10cm, BC = 5.2cm and ED = 18cm.



(i) (a) Work out the volume of the prism.

(b) Calculate angle BEC.

(ii) The point G lies on the line ED and GD = 7cm. Work out angle BGE.

[W-19/42/4(b)]

Solution (i) (a) In $\triangle BCD$, $CD = \sqrt{BD^2 - BC^2}$

$$= \sqrt{10^2 - (5.2)^2}$$

$$= 8.54$$

Area of cross-section of the prism = area of triangle BCD

$$= \frac{1}{2} \times CD \times BC$$

$$= \frac{1}{2} \times 8.54 \times 5.2 = 22.2 \text{ cm}^2 \quad \text{--- (1)}$$

$\therefore$ Volume of the prism = area of cross-section $\times$ length

$$= 22.2 \times 18 = 399.749$$

$$= 400 \text{ cm}^3 \checkmark$$

(i) (b) $EC = \sqrt{ED^2 + DC^2} = \sqrt{18^2 + 8.54^2} = \sqrt{396.9} = 19.92 \text{ cm}$

$$\tan BEC = \frac{BC}{EC} = \frac{5.2}{19.92} = 0.261$$

$$\therefore \text{angle } BEC = \tan^{-1}(0.261) = 14.6^\circ \checkmark$$

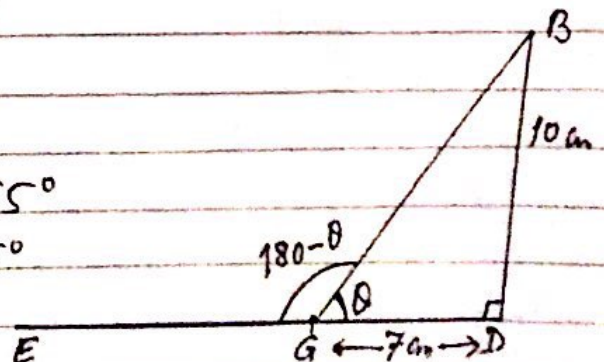
(ii) angle BGE = $180 - \theta$ --- (2)

$$\tan \theta = \frac{BD}{GD} = \frac{10}{7}$$

$$\theta = \tan^{-1} \frac{10}{7} = 55^\circ$$

$$\therefore \text{from (2) angle } BGE = 180 - 55^\circ$$

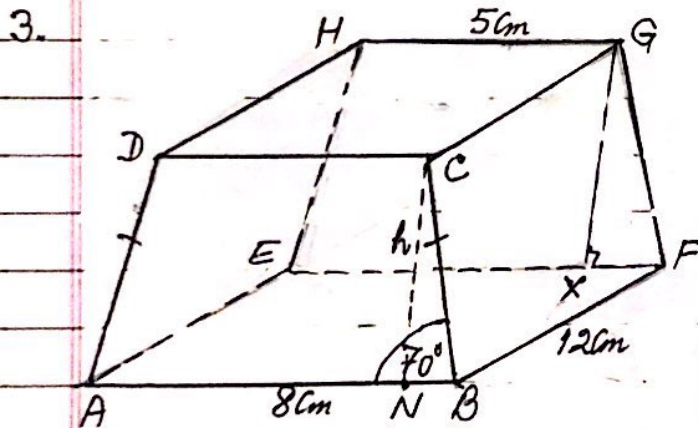
$$= 125^\circ \checkmark$$



2. A pipe is completely full of water. Water flows through the pipe at a speed of 1.2 m/s into a tank. The cross-section of the pipe has an area of 6 cm^2 . Calculate the number of litres of water flowing into the tank in 1 hour, [W-19/21/Q22/-..[4]

Solution: Volume = Area of cross-section $\times$ length. — (1)
 Area of cross-section of the pipe = 6 cm^2 — (2)
 length of water column flown in 1 second = $1.2 \text{ m} = 120 \text{ cm}$.
 hence length of water column flown in 1 hour = $120 \times 60 \times 60 = 432000 \text{ cm}$ — (3)
 $\therefore$ Volume of water flown in 1 hr = 6×432000 [from (1)]
 $= 2592000 \text{ c.c}$
 $= 2592 \text{ litres} \checkmark$

[1 litre = 1000 c.c.]



The diagram shows a prism with a rectangular base ABFE. The cross-section, ABCD, is a trapezium with $AD = BC$. $AB = 8 \text{ cm}$, $GH = 5 \text{ cm}$, $BF = 12 \text{ cm}$, and angle $ABC = 70^\circ$.

(a) Calculate the total surface area of the prism. — [6]

(b) The perpendicular from G on EF meets EF at X [W-20/42/Q 9]

(i) Show that $EX = 6.5 \text{ cm}$. — [1] (b)(i) $EX = 8 - \frac{1}{2}(8-5) = 6.5 \checkmark$

(ii) Calculate AX. — [2] (ii) $AX = \sqrt{12^2 + 6.5^2} = 13.6 \checkmark$

(iii) Calculate the angle between the diagonal AG and the base ABFE. — [2] (iii) In $\triangle GAX$,
 $\tan GAX = \frac{GX}{AX} = \frac{5}{13.6} = 0.303$

angle $GAX = \tan^{-1} 0.303 = 16.8^\circ$
 $\therefore$ Angle between the diagonal AG and base ABFE = $16.8^\circ \checkmark$

Solution (a) h is height of the trap $\Rightarrow \tan 70^\circ = \frac{h}{1.5}$

$$\Rightarrow h = \tan 70^\circ \times 1.5 = 4.12 \quad \frac{1}{2}(8-5)$$

$$BC = \frac{1}{2}(8-5) = 4.3857$$

$$\text{Total surface area} = 2 \times \text{Area of Trap ABCD} + 2 \times \text{Side BC GF} + \text{base ABFE} + \text{Top CDHG}$$

$$= 2 \times \frac{1}{2}(8+5) \times 4.12 + 2 \times (12 \times 4.3857) + 8 \times 12 + 12 \times 5 = 315 \text{ cm}^2 \checkmark$$