

0580

IGCSE Maths

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Number

Revision. Part 2.

SP-20 | M-20 | S-20 | W-19 | W-20

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Fraction and Percentage; Population growth.
Simple interest and Compound interest.

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1. In a box of 80 glasses, 3 are broken.
Work out the percentage of broken glasses. --- [1]

SP-20/02/Q2

Solution: 3 glasses out of 80 are broken.

$$\therefore \text{Percentage of broken glasses} = \frac{3}{80} \times 100 = \frac{15}{4} = 3.75\% \checkmark$$

2. The population of a town decreases exponentially at a rate of 1.7% per year. The population now is 250 000.
Calculate the population at the end of 5 years.

Give your answer correct to the nearest hundred. --- [3]

M-20/22/Q12

Solution: decreases 1.7% per year, exponentially

$$\begin{aligned} \therefore \text{Population after 5 years} &= 250\,000 \times \left(1 - \frac{1.7}{100}\right)^5 \\ &= 250\,000 \times (0.983)^5 \\ &= 229\,460 \\ &= \underline{229\,500} \checkmark \text{ (To the nearest hundred)} \end{aligned}$$

3. Roberto buys a toy for \$5.00
He then sells it for \$4.60. Calculate his percentage loss. --- [2]

S-20/22/Q12

Solution: C.P = \$5

$$S.P = \$4.60$$

$$\text{loss} = 5 - 4.60 = \$0.40$$

$$\therefore \% \text{ loss} = \frac{0.40}{5} \times 100 = 8\%$$

$$\therefore \left. \begin{aligned} \text{loss} \% &= \frac{\text{loss}}{\text{C.P}} \times 100 \end{aligned} \right\}$$

4. Paddy and Anna each invest \$ 2000 for 5 years.
Paddy earns simple interest at a rate of 1.25 per year.
Anna earns compound interest at a rate of $r\%$ per year.
At the end of 5 years, Paddy's investment is worth the same as Anna's investment. Calculate the value of r . ---[5]
[S-20/23/Q16]

Solution: Principal = \$ 2000, for Paddy rate = 1.25%, simple interest, $t=5$ years
 \therefore Interest = $\frac{Prt}{100} = \frac{2000 \times 1.25 \times 5}{100} = \$ 125$
 \therefore Paddy's Amount = $2000 + 125 = 2125$ ✓ (1)

For Anna $A = P \cdot \left(1 + \frac{r}{100}\right)^n = 2000 \left(1 + \frac{r}{100}\right)^5$ (2)

from (1) & (2) $2000 \left(1 + \frac{r}{100}\right)^5 = 2125$

$\Rightarrow \left(1 + \frac{r}{100}\right)^5 = \frac{2125}{2000} = 1.0625$

$\Rightarrow 1 + \frac{r}{100} = (1.0625)^{1/5} = 1.01219$

$\Rightarrow \frac{r}{100} = 1.01219 - 1 = 0.01219$

$\Rightarrow r = 0.01219 \times 100 = 1.219\%$

or $r = 1.22\%$ ✓

5. Workout 5% of \$ 25 [W-19/21/Q1] ---[1]

Solution: $5\% \cdot 25 = \frac{5}{100} \times 25 = \$ 1.25$ ✓

6. Paula invests \$ 600 at a rate of $r\%$ per year simple interest.
At the end of 10 years, the total interest earned is \$ 90.
Find the value of r . ---[2]
[W-19/21/Q9]

Solution: $P = \$ 600$, $t = 10$ years, S.I = \$ 90, $r = ?$

S.I = $\frac{Prt}{100} \Rightarrow \frac{600 \times r \times 10}{100} = 90$

$\Rightarrow 60r = 90$

$r = \frac{90}{60} = 1.5$ ✓

7. (a) Kristian and Stephanie share some money in the ratio 3:2
 Kristian receives \$72, SP-20/04/Q1
- (i) Work out how much Stephanie receives. ---[2]
- (ii) Kristian spends 45% of his \$72 on a computer game.
 Calculate the price of computer game. ---[1]
- (iii) Kristian also buys a meal for \$8.40.
 Calculate the fraction of the \$72 Kristian has left after buying
 the computer game and the meal.
 Give your answer in its lowest terms. ---[2]
- (iv) Stephanie buys a book in a sale for \$19.20. This sale price is
 after a reduction of 20%. Calculate the original price of the book. ---[3]

Solution (a)(i) Stephanie receives = $\frac{72}{3} \times 2 = \48 ✓

(a)(ii) Computer game's price = $\frac{45}{100} \times 72 = \32.40 ✓

(iii) Kristian total expenditure = $\$32.40 + \$8.40 = \$40.80$

Kristian left with = $\$72 - \$40.8 = \$31.2$

∴ Required fraction = $\frac{31.2}{72} = \frac{312}{720} = \frac{13}{30}$ ✓

(iv) S.P of Book = \$19.20

reduction = 20%

S.P = $\left(\frac{100-20}{100}\right)$ original price = 19.20

∴ original price = $19.2 \times \frac{100}{80} = \24 ✓

- (b) Boris invests \$550 at a rate of 2% per year simple interest
 Calculate the value of the investment at the end of 10 years. --[3]

Solution: $P = \$550, r = 2\%, t = 10$ years

S.I = $\frac{Prt}{100} = \frac{550 \times 2 \times 10}{100} = \110

∴ A = value of the investment = $P + I = 550 + 110$
 $= \$660$ ✓

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7 (c) Marlene invests \$550 at a rate of 1.9% per year compound interest. Calculate the value of the investment at the end of 10 years.

Solution: $P = \$550$, $t = 10$ year, $r = 1.9\%$ per year C.I

$$\begin{aligned} \text{Amount } A &= P \left(1 + \frac{r}{100}\right)^t = 550 \left(1 + \frac{1.9}{100}\right)^{10} \\ &= 550 \left(\frac{101.9}{100}\right)^{10} = 550 / 1.019^{10} \\ &= \$ 663.90 \checkmark \end{aligned}$$

(d) Hans invests \$550 at a rate of $x\%$ per year compound interest. At the end of 10 years, the value of the investment \$638.30, correct to the nearest cent. Find the value of x . [SP-20/04/Q1] -- [3]

Solution: $P = \$550$, $\text{rate} = x\%$ per year C.I. Amount = 638.30

$$\begin{aligned} A &= P \left(1 + \frac{r}{100}\right)^t \Rightarrow 550 \left(1 + \frac{x}{100}\right)^{10} = 638.30 \\ \Rightarrow \left(1 + \frac{x}{100}\right) &= \left(\frac{638.30}{550}\right)^{\frac{1}{10}} = 1.015 \\ \Rightarrow \frac{x}{100} &= 1.015 - 1 = 0.015 \\ \Rightarrow x &= 0.015 \times 100 = \underline{1.5} \checkmark \end{aligned}$$

8. Dhannu has a model railway.

(a) He has a train that consists of a locomotive and 4 coaches. The mass of the locomotive is 87g and the mass of each coach is 52g.

(i) Work out the total mass of the train. -- [2]

(ii) Work out the mass of the locomotive as a percentage of the total mass of the train. -- [1]

Solution (a)(i) Total mass = $87 + 4 \times 52 = \underline{295 \text{ g}}$. ✓ [M-20/42/Q1(a)(d)]

(a)(ii) $\text{Req } \% = \frac{87}{295} \times 100 = \underline{29.5 \text{ g}}$. ✓

(d) The cost of a railway magazine increases by 12.5% to \$2.70. Calculate the cost of the magazine before this increase. -- [2]

Solution: Let the original cost of the magazine = \$ x

$$\begin{aligned} \text{After increase of } 12.5\% &= \left(1 + \frac{12.5}{100}\right)x = 2.70 \Rightarrow \frac{112.5}{100}x = 2.70 \\ \Rightarrow x &= \frac{2.70 \times 100}{112.5} = \underline{\$ 2.4} \checkmark \end{aligned}$$

9. Manjeet uses 220 litres of water each day. She reduces the amount of water she use by 15%. Calculate the number of litres of water she now uses, [M-20/42/Q3(a)] ---[2]

Solution: Reduction = 15%

$$\begin{aligned} \therefore \text{After reduction the amt of water} &= \left(1 - \frac{15}{100}\right) \times 220 \\ &= \frac{85}{100} \times 220 = \underline{187 \text{ L}} \end{aligned}$$

10(a) In 2018, Gretel earned \$32000.

(i) She paid tax of 24% on these earnings. [S-20/41/Q1]
Workout the amount she paid in tax in 2018. ---[2]

(ii) In 2019, Gretel earnings increased by 7%.
Workout her earnings in 2019.

(b) Gretel invests \$5000 at a rate of 2% per year compound interest.
Calculate the value of her investment at the end of 3 years. ---[2]

Solution(a) (i) Tax paid = $\frac{24}{100} \times 32000 = \underline{\$7680}$ ✓

(ii) Earnings in 2019 = $\left(1 + \frac{7}{100}\right) \times 32000 = \frac{107}{100} \times 32000 = \underline{\$34240}$ ✓

(b) P = \$5000, rate of C.I per year r = 2%, t = 3 years

$$\begin{aligned} \therefore \text{Value of investment } A &= P \left(1 + \frac{r}{100}\right)^t = 5000 \left(1 + \frac{2}{100}\right)^3 \\ &= 5000 \cdot (1.02)^3 = \underline{\$5306.04} \end{aligned}$$

(c) One month, Gretel spent a total of \$360 on presents. She spent $\frac{1}{5}$ of this total on presents for her parents. She spent $\frac{2}{3}$ of the remaining money on presents for her friends, She spent rest of the money on presents for her sisters. Calculate the percentage of \$360 that she spent on presents for her sisters

Solution: Total money for presents = \$360.

To parents = $\frac{1}{5} \times 360 = \72 ,

To friends = $\frac{2}{3} (360 - 72) = \frac{2}{3} \times 288 = \192

For presents to sisters = $360 - (72 + 192) = 360 - 264 = \96

\therefore Presents to sisters as % of total = $\frac{96}{360} \times 100 = 26.666 = \underline{26.67\%}$ ✓

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10 (d). Arjun earned \$ 36515 in 2019. This was an increase of 9% on his earnings in 2018. Work out his earnings in 2018. ---[2]

[S-20/41/Q1]

Solution: Let earnings in 2018 = \$ x

In 2019 earnings increased by 9%

$$\therefore \text{earnings in 2019} = x \cdot \left(1 + \frac{9}{100}\right) = \$ 36515$$

$$\Rightarrow \frac{109}{100} x = 36515$$

$$\Rightarrow x = \frac{36515 \times 100}{109}$$

$$\therefore \text{Earnings in 2018} \rightarrow = \$ 33500 \checkmark$$

11. (a) (i) Divide \$24 in the ratio 7:5 (done on page-11) ---[2]

(ii) Write \$24.60 as a fraction of \$ 2870. ---[2]

Give your answer in its lowest terms.

(iii) Write \$ 1.92 as a percentage of \$ 1.60. ---[1]

(b) In a sale the original prices are reduced by 15%

(i) Calculate the sale price of a book that has an original price of \$12. ---[2]

(ii) Calculate the original price of a jacket that has a sale price of \$ 38.25. ---[2]

[S-20/42/Q1]

Solution (a)(ii) Fraction = $\frac{24.60}{2870} = \frac{2460}{287000} = \frac{3}{350}$ ✓ (Dividing N^o and D^o by 82.)

(iii) % = $\frac{1.92}{1.60} \times 100 = 120\%$ ✓

(b) (i) Book's original price = \$12, reduction = 15%

$$\therefore \text{Sale price} = 12 \cdot \left(1 - \frac{15}{100}\right) = 12 \cdot \frac{85}{100} = \$ 10.20 \checkmark$$

(ii) let original price = \$ x

Reduction = 15%, and sale price = \$ 38.25

$$\text{Now S.P} = \left(1 - \frac{15}{100}\right)x = 38.25$$

$$\Rightarrow \frac{85}{100} x = 38.25 \Rightarrow x = \frac{38.25}{85} \times 100 = \$ 45$$

$$\therefore \text{Original price of jacket} = \$ 45 \checkmark$$

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11. (C) (i) Dean invests \$500 for 10 years at a rate of 1.7% per year simple interest. Calculate the total interest earned during 10 years. --- [2]
 (ii) Ollie invests \$200 at a rate of 0.0035% per day compound interest. Calculate the value of Ollie's investment at the end of 1 year. -- [2]
 (iii) Edna invests \$500 at a rate of $r\%$ per year compound interest. At the end of 6 years, the value of Edna's investment is \$559.78. Find the value of r . [5-20/42/Q1] -- [3]

Solution (i) $P = \$500$, $t = 10$ years, $r = 1.7\%$ per year S.I.

$$\text{Interest} = \frac{P \times r \times t}{100} = \frac{500 \times 1.7 \times 10}{100} = \$85.$$

(ii) $P = \$200$, $r = 0.0035$ per day C.I., $t = 1$ year = 365 days

$$\text{Value (or amount)} \quad A = P \left(1 + \frac{r}{100}\right)^t = 200 \left(1 + \frac{0.0035}{100}\right)^{365}$$

$$= 200 \left(\frac{100.0035}{100}\right)^{365} = 200 \times (1.000035)^{365}$$

$$= 200 \times 1.0128$$

$$= \$202.57 \checkmark$$

(iii) $P = \$500$, rate = $r\%$ per year C.I., $t = 6$ years, Amount = 559.78

$$A = P \left(1 + \frac{r}{100}\right)^t$$

$$\Rightarrow 500 \left(1 + \frac{r}{100}\right)^6 = 559.78$$

$$\Rightarrow \left(1 + \frac{r}{100}\right) = \left(\frac{559.78}{500}\right)^{1/6} = 1.019$$

$$\Rightarrow \frac{r}{100} = 1.019 - 1 = 0.019$$

$$r = 0.019 \times 100 = 1.9$$

$$\therefore \underline{r = 1.9\%}$$

12. (i) In September the total income at the campsite was \$ 37054. This was a decrease of 4.5% on the total income in August. Calculate the total income in August. ---[2]
- (ii) The value of the campsite has increased exponentially by 1.5% every year since it opened 30 years ago. Calculate the value of the campsite now as a percentage of its value 30 years ago. [2]
- S-20/43/Q1(b)(c)

Solution (i) Let the income in August = \$ x
In Sept, it is a decrease of 4.5% in August.

$$\begin{aligned} \therefore \text{Income in September} &= x \left(1 - \frac{4.5}{100}\right) = 37054 \\ &= x \left(\frac{95.5}{100}\right) = 37054 \\ \Rightarrow x &= \frac{37054 \times 100}{95.5} = \$ 38800 \end{aligned}$$

\therefore Income in August = \$ 38800 ✓

(ii) Let the value of the campsite 30 year ago = \$ x
r = 1.5% increase exponentially.

$$\begin{aligned} \therefore \text{Value now} &= x \left(1 + \frac{1.5}{100}\right)^{30} \\ &= x \left(\frac{101.5}{100}\right)^{30} \\ &= (1.015)^{30} \cdot x \\ &= 1.563 \cdot x \end{aligned}$$

$$\therefore \frac{\text{Value now}}{x} = 1.563 = 156.3\%$$

\therefore Value of Campsite now is 156.3% of the value 30 years ago. ✓

- 13 (i) In a sale, Ali buys a television for \$ 195.80
The original price was \$ 220.
Calculate the percentage reduction on the original price. -- [3]
- (ii) In a sale, Mo buys a jacket for \$63.
The original price was reduced by 25%. -- [3]
Calculate the original price of the jacket.

W-19/41 Q2(b)(c)

Solution (i) Let the % reduction = x ; original price = \$220
Sale price = $220\left(1 - \frac{x}{100}\right) = 195.80$

$$\Rightarrow \frac{1-x}{100} = \frac{195.80}{220} = 0.89$$

$$\Rightarrow \frac{x}{100} = 1 - 0.89 = 0.11$$

$$\Rightarrow x = 0.11 \times 100 = 11$$

% reduction in price = 11% ✓

(ii) Let the original price = \$ x ; reduction = 25%; S.P = \$63

$$\text{Sale price} = x\left(1 - \frac{25}{100}\right) = 63 \text{ (Given)}$$

$$\Rightarrow \frac{75}{100} x = 63$$

$$\Rightarrow x = \frac{63 \times 100}{75} = \underline{\$84} \checkmark$$

- 14 (a) Dina invests \$600 for 5 years at a rate of 2% per year compound interest. Calculate the value of this investment at the end of the 5 years, ... [2]

Solution: $P = \$600$, $t = 5$ years, $r = 2\%$ per year C.I

$$\text{Value after 5 years } A = P\left(1 + \frac{r}{100}\right)^t = 600\left(1 + \frac{2}{100}\right)^5$$

$$= 600(1.02)^5 = \$662.448 = \underline{\$662.45} \checkmark$$

(continued →)

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14 (b) The value of a gold ring increases exponentially at a rate 5% per year. The value is now \$882.

(i) Calculate the value of the ring 2 years ago.

(ii) Find the number of complete years it takes the ring's value of \$882 to increase to value greater than \$1100. --[2]

W-19/41 Q3

Solution: Let the value 2 year ago = \$x, Present value A = \$882

(b)(i)

$$A = P \left(1 + \frac{r}{100}\right)^t$$

$$= x \left(1 + \frac{5}{100}\right)^2 = 882$$

$$\Rightarrow x(1.05)^2 = 882 \Rightarrow 1.1025x = 882$$

$$\Rightarrow x = \frac{882}{1.1025} = \$800 \checkmark$$

(b)(ii) time $t = ?$, $P = \$882$, $A = \$1100$
 $r = 5\%$

$$\text{Value (or amount) } A = P \left(1 + \frac{r}{100}\right)^t = 882 \left(1 + \frac{5}{100}\right)^t > 1100$$

$$\Rightarrow (1.05)^t > \frac{1100}{882} \Rightarrow (1.05)^t > 1.2471$$

$$[(1.05)^4 = 1.2155 \& (1.05)^5 = 1.2762] \Rightarrow t = 5 \text{ year } \checkmark$$

15 (i) The apples cost Mohsin \$0.85 per kilogram to produce. He sells them at a profit of 60%.

Work out the selling price per kilogram of the apples. --[2]

W-19/42/1(b)(ii)

Solution: C.P = \$0.85, Profit = 60%.

$$SP = C.P \left(1 + \frac{r}{100}\right) = 0.85 \left(1 + \frac{60}{100}\right)$$

$$= 0.85 \times 1.6$$

$$= \$1.36 \checkmark$$

(Continued →)

- 15 (ii) Mohsin exports some of his pears to a shop in Belgium. The shop buys the pears at \$ 1.50 per kilogram. The shop sells the pears for 2.30 euros per kilogram. The exchange rate is \$1 = 0.92 euros. --- [5]
Calculate the percentage profit per kilogram made by the shop.
- (iii) Mohsin's earnings increase exponentially at a rate of 8.7% each year. During 2018 he earned \$195600. -- [3]
During 2027, how much more does he earn than during 2018?
N-19/42 Q1(c)(d)

Solution (ii) Exchange rate \$1 = 0.92 euros
for the shop C.P = \$1.50 = 1.50×0.92 euros = 1.38 euros ✓
S.P = 2.30 euros
Profit per kg = $2.30 - 1.38 = 0.92$ euros
 \therefore Profit % = $\frac{\text{Profit} \times 100}{\text{C.P}} = \frac{0.92 \times 100}{1.38} = \underline{66.7\%}$ ✓

(iii) $P = \$195600$ (In 2018), Increase rate $r = 8.7\%$, $t = 2027 - 2018 = 9$ years.

$$\begin{aligned} \therefore \text{Value in 2027, } A &= P \left(1 + \frac{r}{100}\right)^t = 195600 \left(1 + \frac{8.7}{100}\right)^9 \\ &= 195600 (1.087)^9 \\ &= 195600 \times 2.11868 \\ &= 414413.8 \text{ euros} \end{aligned}$$

$$\begin{aligned} \therefore \text{Increase in earnings in 2027 to 2018} \\ &= 414413.8 - 195600 \\ &= 218814 \text{ euros } \checkmark \end{aligned}$$

16 (a) In a cycling club, the number of members in the ratio male:female = 8:3. The club has 342 females.

- (i) Find the total number of members. -- [2]
(ii) Find the percentage of the total number of members that are female. -- [1]

Solution (i) Total number of members = $x \Rightarrow \frac{3}{11} \times x = 342 \Rightarrow x = \frac{342 \times 11}{3} = 1254$ ✓

(ii) female % = $\frac{\text{female} \times 100}{\text{Total m}} = \frac{342 \times 100}{1254} = 27.3\%$ ✓ (Continued →)

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- 16 (b) The price of a bicycle is \$1020
Club members receive a 15% discount on the price.
Find how much a club member pays for this bicycle. ---[2]
- (c) In 2019, the membership fee of the cycling club is \$79.50
This is 6% more than last year.
Find the increase in the cost of membership. ---[3]
- (e) Bryan invested \$480 in an account 4 years ago.
The account pays compound interest at a rate of 2.1% per year.
Today, he uses some of the money in this account to buy a bicycle costing \$430. Calculate how much money remains in his account. --[3]

[W-19/43/21]

Solution (b) Price = \$1020, discount = 15%, SP = ?

$$S.P = P \left(1 - \frac{r}{100}\right) = 1020 \left(1 - \frac{15}{100}\right) = 1020 \times \frac{85}{100} = \$867 \checkmark$$

(c) Let cost of membership last year = \$x ; increase = 6%
 Cost now = $x \left(1 + \frac{6}{100}\right) = 79.50$ (Given)

$$\Rightarrow 1.06 \cdot x = 79.50$$

$$\Rightarrow x = \frac{79.50}{1.06} = \$75$$

$$\therefore \text{Increase in cost} = 79.50 - 75 = \$4.50 \checkmark$$

(e) P = \$480, r = 2.1 per year C.I, t = 4 years.
 Present Value $A = P \left(1 + \frac{r}{100}\right)^t = 480 \left(1 + \frac{2.1}{100}\right)^4$

$$= 480 \times (1.021)^4$$

$$= \$521.6$$

 Spent on Bicycle = \$430

$$\therefore \text{Remaining money} = 521.6 - 430$$

$$= \$91.6 \checkmark$$

17. Ahmed increases 40 by 300%, from this list put a ring around the correct calculation: 40×1.300 , 40×3 , 40×400 , 40×4 , 40×300 . ---[1]

Solution: 40 increased by 300% = $40 \times \frac{300}{100} = 40 \times 3 \checkmark$ [W-20/21/29(a)]

18. A town has a population of 45 000.
This population increases exponentially at a rate of 1.6% per year.
Find the population of the town at the end of 5 years, --- [3]
Give your answer correct to the nearest hundred, [W-20/21/Q10]

Solution: $P = 45\ 000$, $r = 1.6\%$, time = 5 years

$$A = P \left(1 + \frac{r}{100}\right)^n = 45\ 000 \left(1 + \frac{1.6}{100}\right)^5 = 45\ 000 (1.016)^5$$

$$= 45\ 000 \times 1.0826$$

$$= 48\ 717 = \underline{48\ 700} \text{ [To the nearest hundred]}$$

19. The cost of a premium ticket is reduced from \$114 to \$96.90.
Calculate the percentage reduction in the cost of a ticket, --- [2]
[W-20/41/Q26(ii)]

Solution: Reduction in the cost = $114 - 96.90 = \$17.10$

$$\therefore \text{Percentage reduction} = \frac{17.10}{114} \times 100 = 15\% \checkmark$$

20. Increase 42 by 16% [W-20/22/Q5] --- [2]

Solution: $42 \cdot \left(1 + \frac{16}{100}\right) = 42 \times \frac{116}{100} = \underline{48.72} \checkmark$

21. The selling price of a shirt is \$26.50. This includes a tax of 6%. Calculate the price of the shirt before the tax was added, --- [2]

Solution: Let the price of shirt before tax = \$ x

$$\text{price after tax} = x \left(1 + \frac{6}{100}\right) = 26.50$$

$$\Rightarrow \frac{106}{100} x = 26.50$$

$$\Rightarrow x = \frac{26.50 \times 100}{106} = \underline{\$25} \checkmark$$

22. (a) Beth invests \$2000 at a rate of 2% per year compound interest.
- Calculate the value of this investment at the end of 5 years. --- [2]
 - Calculate the overall percentage increase in the value of Beth's investment at the end of 5 years. --- [2]
 - Calculate the minimum number of complete years it takes for the value of Beth's investment to increase from \$2000 to more than \$2500. --- [3]
- (b) The population of a village decreases exponentially at a rate of 4% each year. The population is now 255. Calculate the population 16 years ago. --- [3]

W-20/42/Q3

Solution (a)(i) $A = P(1 + \frac{2}{100})^n = 2000(1 + \frac{2}{100})^5 = 2000(1.02)^5 = \$ 2208.16$ ✓

(ii) Percentage increase = $\frac{(2208.16 - 2000) \times 100}{2000} = \frac{208.16}{20} = 10.4\%$

(iii) $2000(1.02)^n > 2500 \Rightarrow (1.02)^n > 1.25$
 $\Rightarrow n = 12$ ✓

∴ $(1.02)^{11} = 1.243$
 $(1.02)^{12} = 1.268$

(b) Let the population 16 years ago = P

Now $A = P(1 - \frac{4}{100})^{16} = 255$

$\Rightarrow P \times (.96)^{16} = 255 \Rightarrow P = \frac{255}{(.96)^{16}} = \frac{255}{0.52}$
 $\Rightarrow P = 490$ ✓

- 23 (a)(ii) The earth has a surface area of approximately 510 100 000 km². Water covers 70.8% of the Earth's surface. Work out the area of the Earth's surface covered by water. --- [2]

W-20/43/Q1

Solution: Area covered by water = $\frac{70.8}{100} \times 510100000 = 361150800$ km² ✓

(continued →)

23(b) The table shows the surface area of some countries and their estimated population in 2017.

Country	Surface Area (km^2)	Estimated population in 2017
Brunel	5.77×10^3	433100
China	9.60×10^6	1388000000
France	6.41×10^5	67000000
Maldives	3.00×10^2	374600

(ii) The ratio surface area of the Maldives : surface area of China can be written in the form $1:n$, Find the value of n . --[2]

(iii) Find the surface area of France as a percentage of the surface area of China. --[2]

(iv) Find the population density of the Maldives.
(Population density = population \div surface area) --[2]

(c) The population of the Earth in 2017 was estimated to be 7.53×10^9
The population of the Earth in 2000 was estimated to be 6.02×10^9

(i) Work out the percentage increase in the Earth's estimated population from 2000 to 2017. --[2]

(ii) Assume that the population of the Earth increased exponentially by $y\%$ each year for these 17 years. Find the value of y . --[3]

[W-20/43/21]

Solution (b)(ii) Maldives : China (S. Area)

$$= 3.00 \times 10^2 : 9.60 \times 10^6$$

$$= 1 : \frac{9.60 \times 10^6}{3 \times 10^2}$$

$$= 1 : 32000 \quad (1:n)$$

$$\therefore n = 32000 \checkmark$$

$$(iii) \frac{\text{France S. area} \times 100}{\text{China S. area}} = \frac{6.41 \times 10^5 \times 100}{9.60 \times 10^6}$$

$$= 6.68\% \checkmark$$

(iv) Population density of Maldives

$$= \frac{374600}{3 \times 10^2} = 1248.66$$

$$\text{or } = 1250 \checkmark$$

(c)(i) Percentage increase of the population = $\frac{(7.53 \times 10^9 - 6.02 \times 10^9)}{6.02 \times 10^9} \times 100$

$$= \frac{1.51 \times 10^9}{6.02 \times 10^9} \times 100$$

$$= 25.1\% \text{ (or } 25.08)$$

(ii)

$$7.53 \times 10^9 = 6.02 \times 10^9 \left(1 + \frac{y}{100}\right)^{17}$$

$$\Rightarrow 1 + \frac{y}{100} = \sqrt[17]{\frac{7.53}{6.02}} = \sqrt[17]{1.2508}$$

$$\Rightarrow 1 + \frac{y}{100} = 1.01325$$

$$\Rightarrow \frac{y}{100} = 0.01325$$

$$y = 1.325\% \text{ (or } 1.33)$$

24. Adil and Brian are paid the same wages,
 Adil is given a 7% pay decrease and his new wage is \$ 427.80,
 Brian is given a 7% pay increase, Workout Brian's new wage. ---[3]

W-20/23 | Q17

Solution: Let the initial wage of Adil and Brian each = \$ x .

Adil is given 7% decrease; his new wage = $x \left[1 - \frac{7}{100} \right] = 427.80$
 (given)

$$\Rightarrow x \times \frac{93}{100} = 427.80 \Rightarrow x = \frac{427.80 \times 100}{93}$$

$$x = \$460 \checkmark$$

Now Brian is given 7% increase,

hence his new wage = $x \left[1 + \frac{7}{100} \right] = 460 \times \frac{107}{100} = \$492.2 \checkmark$

Time, Speed and time and Average speed.

DATE: / /
PAGE: I - 1

1. A train leaves Zurich at 2240 and arrives in Vienna at 0732 the next day. Work out the time the train takes. ---[1]

[SP-20/22/Q1]

Solution: 2240 to 2400 = 1hr 20m

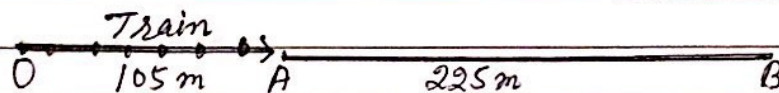
and 0000 to 0732 = 7h 32m

$$\therefore \text{Total time taken by train} = 1\text{hr } 20\text{m} + 7\text{h } 32\text{m} \\ = \underline{8\text{hr } 52\text{m}} \checkmark$$

2. A train of length 105m takes 11 seconds to pass completely through a station of length 225m. Calculate the speed of train in km/h ---[3]

[S-20/22/Q17]

Solution:



Total distance travelled to pass through = $105 + 225 = 330\text{m}$

Time = 11s.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{330}{11} = 30\text{ m/s}$$

$$\text{or Speed} = 30 \times 3.6 \text{ km/h} \\ = \underline{108 \text{ km/h}}$$

[$\because 1\text{m/s} = 3.6\text{km/h}$]

3. A train journey takes 5 hours 54 minutes.

(a) The journey starts at 0915. Find the time that the journey ends. ---[1]

(b) The average speed of the train for this journey is 80 km/h.

Calculate the distance travelled. ---[2]

[S-20/23/Q4]

Solution (a) Journey ends at $0915 + 5\text{hr } 54\text{m} = \underline{1509} \checkmark$

(b) distance = Speed \times time

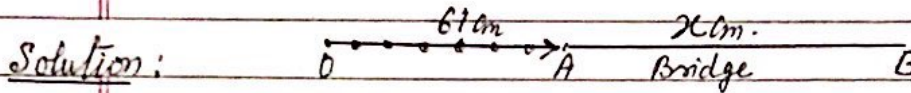
$$= 80 \times 5.9$$

(5hr 54m = 5.9hrs)

$$= \underline{472 \text{ km}}$$

4. A train is 61m long and travels at a speed of 18 cm/s. It takes 4 second for the whole of the train to cross the bridge. Calculate the length of the bridge. -- [2]

[M-20/42/Q1(b)]



Total distance to cross = $(x + 61)$ m

Time = 4 seconds, speed = 18 cm/s

Now speed = $\frac{\text{Distance}}{\text{Time}} \Rightarrow \frac{x + 61}{4} = 18$

$\Rightarrow x + 61 = 18 \times 4 = 72$

length of the bridge. $x = 72 - 61 = 11$ m. ✓

5. Car A and Car B take part in a race around a circular track. One lap of the track measures 7.6 km. Car A takes 2 minutes and 40 seconds to complete each lap of the track. Car B takes 2 minutes and 25 seconds to complete each lap of the track.

(a) Calculate the speed of Car A. Give your answer in km/h. -- [3]

(b) both car start the race from the same position, S, at the same time.

(i) Find the time taken when both car A and Car B are next at position S at the same time. Give your answer in minutes and seconds. --- [4]

(ii) Find distance that car A has travelled at this time. [W-19/42/Q9] -- [2]

Solution (a) speed of Car A = $\frac{\text{Distance}}{\text{Time}} = \frac{7.6}{4/90} = 7.6 \times \frac{90}{4} = 171 \text{ km/h}$ [2 m 40 s = $\frac{4}{90}$ hrs]

(b)(i) for one lap of track time taken by Car A = 2 m 40 s = 160 s = $2^5 \times 5$

by Car B = 2 m 25 s = 145 s = 5×29

Both the cars will reach at S, L.C.M = $2^5 \times 5 \times 29 = 4640$ s

= 77 m, 20 s ✓

(ii) Distance for Car A = Speed \times time = $171 \times \frac{4640}{3600} = 220.4 \text{ km}$ [77 m 20 s = $\frac{4640}{3600}$ h ✓]

6. Asif cycles a distance of 105 km.

on the first part of his journey he cycles 60 km in 2 hr 24 min. --- [4]

on the second part of his journey he cycles 45 km. at 20 km/h

Find his average speed for the whole journey.

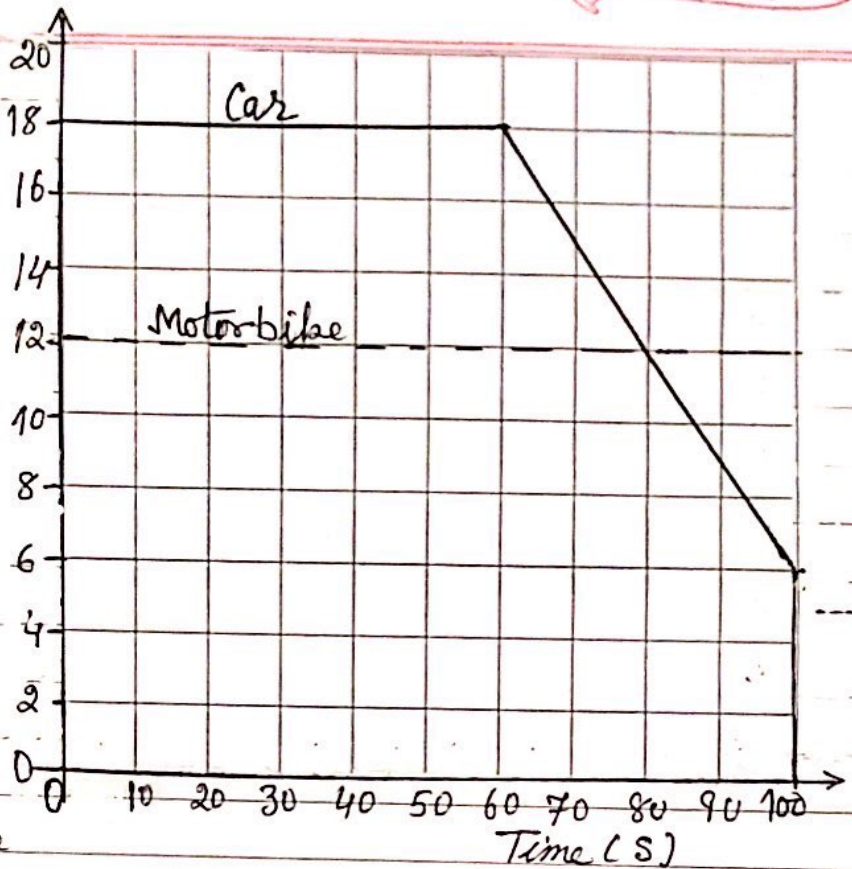
[W-19/43/Q1(d)]

Solution: Time for the first part = 2 hr 24 min = 2.4 hr \Rightarrow Total Time = 2.4 + 2.25 = 4.65 hr
Time for the second part = $\frac{45}{20} = 2.25$ hr \therefore Average speed = $\frac{105}{4.65} = 22.6 \text{ km/h}$ ✓

7.

The diagram shows the speed-time graph for 100 seconds of the journey of a car and of a motorbike.

- (a) Find the deceleration of the car between 60 and 100 seconds.
- (b) Calculate how much further the car travelled than the motorbike during the 100 seconds.



[W-20/21/Q15] --[1]

Solution (a) Deceleration of the car between 60 and 100s = $\frac{18-6}{60-100} = \frac{12}{-40} = -0.3$
 deceleration = 0.3 m.s^{-2} ✓

(b) Distance travelled by car = area under the curve
 in 100s
 $= 18 \times 60 + \frac{1}{2} \times 40 \times (18+6)$
 $= 1080 + 480 = 1560 \text{ m}$

Distance travelled by Motorbike = $12 \times 100 = 1200 \text{ m}$

∴ Car is ahead of Motorbike by $= 1560 - 1200 = 360 \text{ m}$ ✓

8. When the local time in Athens is 0900, the local time in Berlin is 0800. A plane leaves Athens at 1315. It arrives Berlin at 1505 local time.

(i) Find the flight time from Athens to Berlin. --[1]

(ii) The distance the plane flies from Athens to Berlin is 1802 km.

Calculate the average speed of the plane.

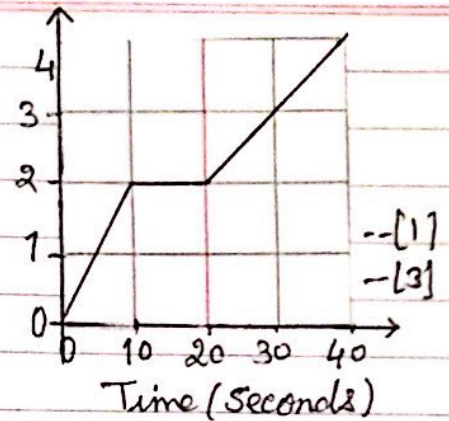
[W-20/41/Q2(C)] --[2]

Solution (i) Time of flight = $(1505 + 1\text{hr}) - (1315) = 1605 - 1315 = 2\text{hr } 50\text{m}$.

(ii) Average speed = $\frac{\text{Dis}}{\text{Time}} = \frac{1802}{2.833} = 636 \text{ km/h}$ [2hr 50m = 2.833hr]

9. The diagram shows the speed-time graph for the first 40 seconds of a cycle ride.

Speed (m/s)



- (a) Find the acceleration between 20 and 40 Sec.
(b) Find the total distance travelled.

[W-20/22/Q17]

Solution(a) between 20 and 40 seconds.

$$\text{acc} = \frac{\text{Speed Change}}{\text{Time}} = \frac{4-2}{40-20} = \frac{2}{20} = \underline{0.1 \text{ m/s}^2} \checkmark$$

(b) Total distance = area under the graph

$$= \frac{1}{2} (20+10) \times 2 + \frac{1}{2} (2+4) \times 20 = 30 + 60 = \underline{90 \text{ m}} \checkmark$$

10. Karel travelled from London to Johannesburg and then from Johannesburg to Windhoek.

- (a) The flight from London to Johannesburg took 11 hours 10 minutes. The average speed was 813 km/h.

Calculate the distance travelled from London to Johannesburg.

Give your answer correct to the nearest 10 km. ---[3]

- (b) The total time for Karel's journey from London to Windhoek was 15 hours 42 minutes. The total distance travelled from London to Windhoek was 10 260 km.

[W-20/42/Q17]

(i) Calculate the average speed for this journey. ---[2]

(ii) The cost of Karel's journey from London to Windhoek was \$470.

(a) Calculate the distance travelled per dollar. ---[1]

(b) Calculate the cost per 100 km of this journey. Give ans. to nearest cent. ---[2]

Solution:

$$\begin{aligned} \text{(a) Distance} &= \text{Speed} \times \text{Time} = 813 \times (11 \text{ h } 10 \text{ m}) \\ &= 813 \times \frac{67}{10} = 9078.5 \text{ km} \\ &= \underline{9080} \text{ (correct to nearest 10 km)} \end{aligned}$$

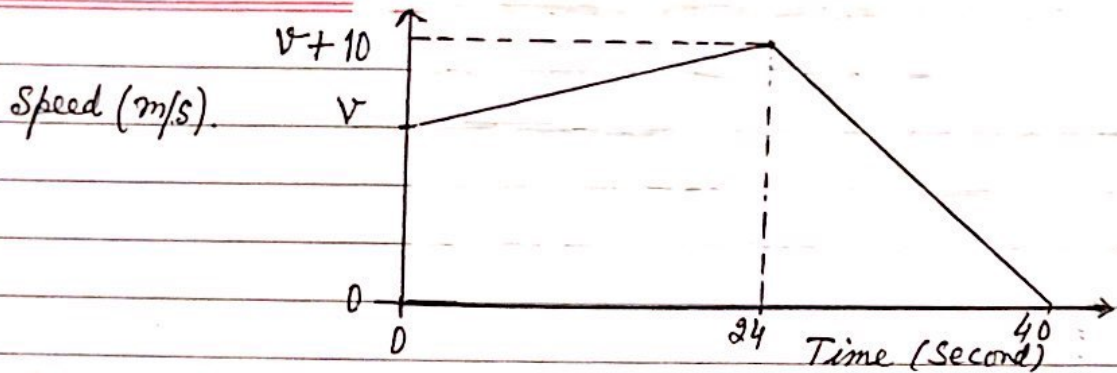
$$\begin{aligned} \text{(b)(i) Average Speed from London to Windhoek} \\ &= \frac{\text{Distance}}{\text{Time}} = \frac{10260}{15.7} = 653.5 \quad (15 \text{ h } 42 \text{ m} = 15.7 \text{ h}) \\ &= \underline{654 \text{ km/h}} \checkmark \end{aligned}$$

$$\begin{aligned} \text{(b)(ii) (a) distance per dollar} \\ &= \frac{10260}{470} = \underline{21.82 \text{ km/\$}} \checkmark \end{aligned}$$

(b) Cost for 100 km

$$\frac{470}{10260} \times 100 = \underline{\$4.58} \checkmark$$

11.



The diagram shows the speed-time graph for the final 40 seconds for a car journey. At the start of the 40 seconds the speed is v m/s.

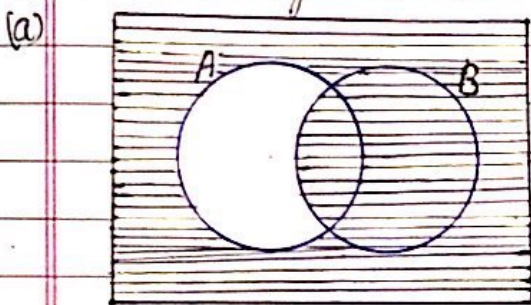
- (a) Find the acceleration of the car during the first 24 seconds. --- [1]
 (b) The total distance travelled during the 40s is 1.24 kilometres. Find the value of v . [W-20/23/219] --- [4]

Solution (a) Acceleration = $\frac{\text{Change in speed}}{\text{Time}} = \frac{(v+10)-v}{24} = \frac{10}{24} = 0.417 \text{ m/s}^2$ (or 0.4166) ✓

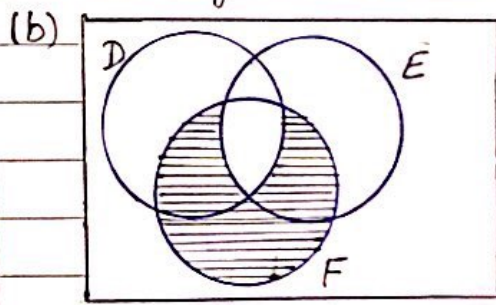
(b) In 40 seconds, the total distance travelled = area under the graph = 1.24 km
 $= \frac{1}{2} (v + (v+10)) \times 24 + \frac{1}{2} (v+10)(40-24) = 1240 \text{ m}$
 $\Rightarrow 12(2v+10) + 8(v+10) = 1240$
 $\Rightarrow 24v + 120 + 8v + 80 = 1240$
 $\Rightarrow 32v = 1040 \Rightarrow v = \frac{1040}{32} = 32.5 \text{ m/s}$ ✓

Sets and Venn diagrams.

1. Shade the region in each of the Venn diagrams below.



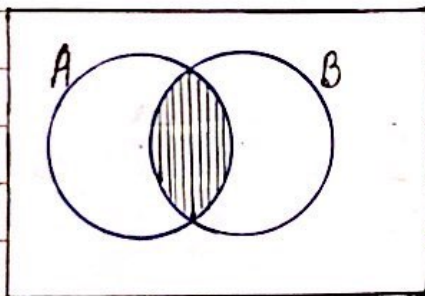
$A' \cup B'$ --- [1]



$(D \cap E \cap F)'$

[SP-20/02/Q18] --- [1]

2.

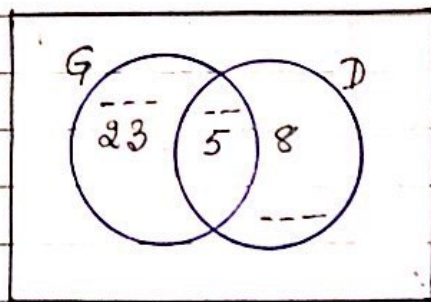


on the Venn diagram,
shade the region $A \cap B$. --- [1]

[S-20/22/Q7]

3. (a) In a class of 40 students:

- 28 wears glasses (G)
- 13 has driving lessons (D)
- 4 do not wear glasses and do not have driving lessons.



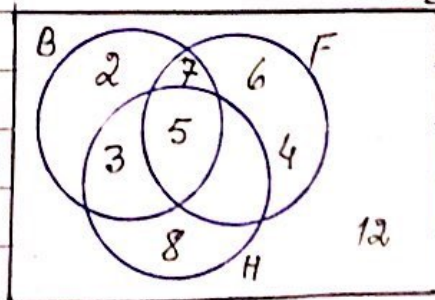
(i) Complete the Venn diagram.

Solution: either glasses or driving lesson $G \cup D = 40 - 4 = 36 = n(G) + n(D) - n(G \cap D)$
 $\therefore 36 = 28 + 13 - n(G \cap D)$
 $\Rightarrow n(G \cap D) = 5 \checkmark$
 only G = $28 - 5 = 23$
 only D = $13 - 5 = 8$

(ii) use set notation to describe region that contain a total of 33 student

Answer: $G \cup D'$ --- [1]

(b) This Venn diagram shows information about the number of students who play basketball (B), football (F) and hockey (H).
Find $n(B \cup F \cap H')$

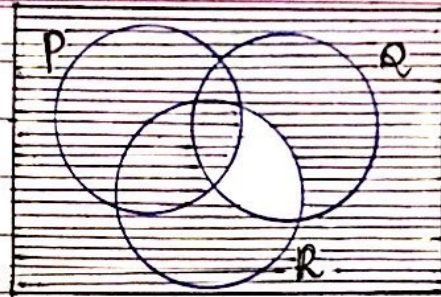


Solution: $n(B \cup F \cap H') = 2 + 7 + 6 = 15 \checkmark$ --- [1]

(continued →)

(Continued →)

3. (c) Shade the region $P \cup (Q \cap R)'$



S-20/23/Q19

---[13]

4. $E = \{0, 1, 2, 3, 4, 5, 6\}$ $A = \{0, 2, 4, 5, 6\}$ $B = \{1, 2, 5\}$

Complete each of the following statements.

W-19/21/Q23

Q/Solution: $A \cap B = \{ \quad \quad \quad \} = \{2, 5\} \checkmark$

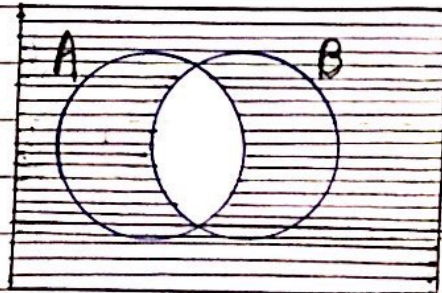
$n(B) = \quad \quad \quad = 3 \checkmark$

$\{0, 4, 6\} = \quad \quad \quad = A \cap B' \checkmark$

$\{2, 4\} = \quad \quad \quad = CA \checkmark$

---[4]

5. On the Venn diagram, shade the region $(A \cap B)'$



W-19/22/Q4

---[1]

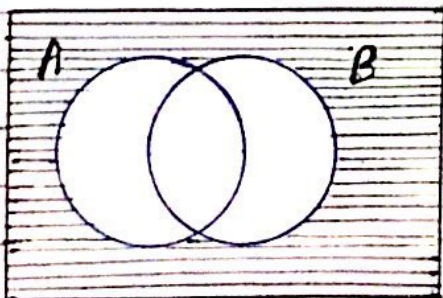
6 (a) $M = \{x; x \text{ is an integer and } 2 \leq x < 6\}$

Q/Ans. (i) Find $n(M) = 4 \checkmark$

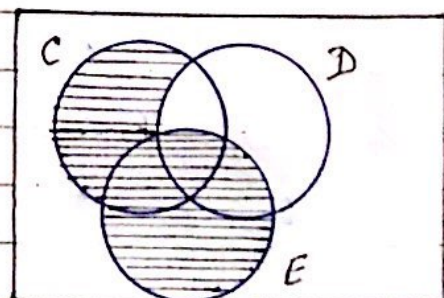
---[1]

(ii) Write down a set N where $N \subset M$ and $N \neq \emptyset$ and at least one. \checkmark

(b) In each Venn diagram, shade the required region.



$(A \cup B)'$



$(C \cap D)' \cup E$

W-19/23/Q18

---[2]

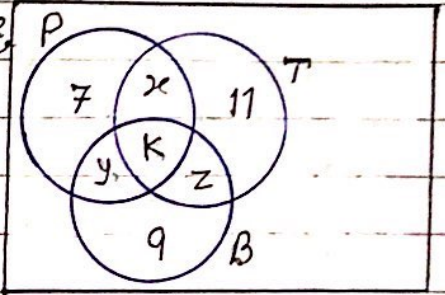
7. This year, 40 students have each travelled by one or more of Plane (P) ; boat (B) or train (T).

7 have travelled only by plane. \mathcal{E}

11 have travelled only by train.

9 have travelled only by boat.

$n(P \cap T) = 8$, $n(B \cap T) = 3$; $n(B \cap P) = 6$



(a) Complete the Venn diagram, --- [3]

(b) Find $n((P \cup B)')$ --- [1]

(c) Use set notation to complete the statement $(P \cup T \cup B)' = \dots$ --- [1]

(d) Two students are chosen at random.

Calculate the probability that they both have travelled only by plane. --- [2]

(e) Two students are chosen at random from those who have travelled by train. Calculate the prob. that they both have also travelled by plane. [M-20/42/R9] --- [2]

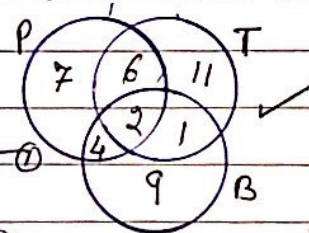
Solution (a) ^{let} $n(P \cap T \cap B) = k$

Now $x + y + z + k = 40 - (7 + 11 + 9) = 40 - 27 = 13$ --- (1)

and $(x+k) + (y+k) + (z+k) = 8 + 3 + 6 = 17$

$\Rightarrow (x+y+z) + 3k = 17$ --- (2)

(2) - (1) $\Rightarrow 2k = 4 \Rightarrow k = 2 \Rightarrow$ $\begin{cases} x+2=8 \Rightarrow x=6 \\ y+2=6 \Rightarrow y=4 \\ z+2=3 \Rightarrow z=1 \end{cases}$



(b) $n((P \cup B)') = 11$ ✓

(c) ϕ or $\{\}$ ✓

(d) $\frac{7}{40} \times \frac{6}{39} = \frac{7}{260}$ ✓

(e) $n(T) = 20$ $n(T \cap P) = 8$

$P(\text{Two students travelled by Plane} / \text{Travelled by Train})$

$= \frac{8}{20} \times \frac{7}{19} = \frac{14}{95}$ ✓

8. x is an integer, $E = \{x; 41 \leq x \leq 50\}$

$A = \{x; x \text{ is an odd number}\}$

$B = \{x; x \text{ is a multiple of } 3\}$

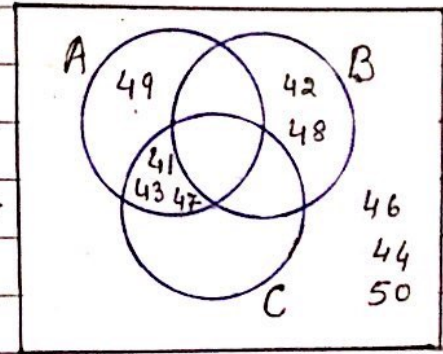
$C = \{x; x \text{ is a prime number}\}$

(a) Complete the Venn diagram to show this information. --- [3] →

(b) List the elements of (i) $A \cap C$ --- [1]

(ii) $(B \cup C)'$ --- [1]

(c) Find $n(A \cap B \cap C)$ --- [1]



[S-20/41/25]

Solution (b) (i) $A \cap C = 41, 43, 47$ ✓

(ii) $(B \cup C)' = 44, 46, 49, 50$ ✓

(c) 0 ✓

9. There are 32 students in a class.

5 do not any language,

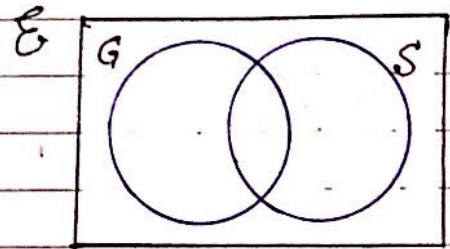
15 study German (G)

18 study Spanish (S)

(i) Complete the Venn diagram to show this information. --- [2]

(ii) A student is chosen at random. Find the probability that the student studies Spanish but not German. --- [1]

(iii) A student who studies German is chosen at random. Find the probability that this student also studies Spanish. --- [1]



Solution:

(i) No. of students studying G or S or both = $32 - 5 = 27$.

Let number of student studying both = x .

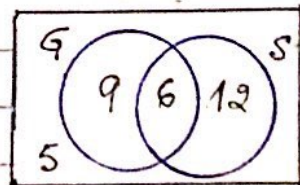
$\therefore n(G) + n(S) - x = 27 \Rightarrow 15 + 18 - x = 27$

$\therefore n(G \text{ only}) = 15 - 6 = 9$ and $n(S \text{ only}) = 18 - 6 = 12$

(ii) $P(\text{Spanish but not German}) = \frac{12}{32} = \frac{3}{8}$ ✓

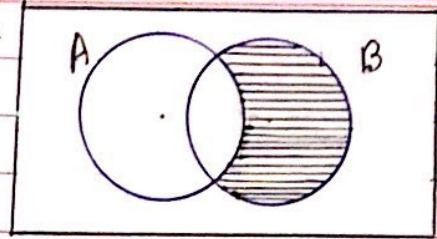
(iii) $P(\text{Studies Spanish, given that he studies German}) = \frac{6}{15} = \frac{2}{5}$ $\left[P(S/G) = \frac{P(S \cap G)}{P(G)} \right]$

[W-20/41/29(a)]



10. Use set notation to describe the shaded region.

[W-20/23/Q13]



---[1]

Solution: $A' \cap B$