

0580

IGCSE Maths.

Probability

Revision

SP-20/M-20/S-20/W-19/W-20

Suresh Goel

(Former Director)

Alliance World School

Noida, Delhi, N.C.R.

INDIA,

(+91 9810444804)

1. A bag contains blue, red, yellow and green balls only.
A ball is taken from the bag at random.

The table shows some information about the probabilities.

Colour	Blue	Red	Yellow	Green
Probability	0.15	0.2		0.43

(a) Complete the table. ---[2]

- (b) Abdul takes a ball at random and replaces it in the bag.
He does this 200 times.

Find how many times he expects to take a red ball. --[1]

[S-20/21/Q4]

Solution: Let the prob. of getting yellow ball = p

$$(a) \Rightarrow 0.15 + 0.2 + p + 0.43 = 1 \quad \left[\because \sum p_i = 1 \right]$$

$$\Rightarrow p + 0.78 = 1 \Rightarrow p = 1 - 0.78 = 0.22 \quad \left(\begin{array}{l} \text{to be filled in the blank} \\ \text{space in the table} \end{array} \right)$$

(b) Prob. of getting a red ball $p = 0.2$

$n = \text{Number of times it is done} = 200$ [Expect

$$\text{Expectation of red ball} = 200 \times 0.2$$

$$= 40 \checkmark$$

2. Sofia has a bag containing 8 blue beads and 7 red beads only.
She takes one bead out of the bag at random and replaces it.
She does this 90 times.

Find the number of times she expects to take a red ball. ---[2]

[S-20/23/Q5]

Solution: Prob. of getting a red ball in one draw $p = \frac{\text{No. of red balls}}{\text{Total no. of balls}} = \frac{7}{8+7}$

$$\Rightarrow p = \frac{7}{15}$$

$n = \text{No. of draws with replacement} = 90$ [Prob. of getting red ball remains same every time]

$$\therefore \text{Number of times Sofia expects a red ball} = np \quad (\text{Expectation} = np)$$

$$= \frac{7}{15} \times 90 = 42 \checkmark$$

3.

Sulcika has six cards numbered 1 to 6.

1

2

3

4

5

6

(a) She takes one card at random, records the number and replaces the card.

(i) Write down the probability that the number is 5 or 6. --- [1]

(ii) Sulcika does this 300 times.

Find how many times she expects the number 5 or 6. --- [1]

(b) Sulcika ^{takes} two cards at random, without replacement.

(i) Find the prob. that the sum of the numbers on two cards is 5. --- [3]

(ii) Find the prob. that at least one of the numbers on the cards is a square number. [11-20/42] Q 6 [3]

Solution (a) (i) $P(5 \text{ or } 6) = \frac{2}{6} = \frac{1}{3} \checkmark$

(ii)

$n = 300$, Expectation of (5 or 6) $= np = 300 \times \frac{1}{3} = 100 \checkmark$

(b) When two cards are drawn without replacement,

Possible - outcomes $S = \{ (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) \}$

Number of outcomes $n(S) = 30 \checkmark$

(i) Sum is 5 $\rightarrow \{1, 2, 3, 4, 5, 6\}$
or $\{4, 3, 2, 1, \dots\}$

Sum 5 $\rightarrow (1, 4), (2, 3), (3, 2), (4, 1)$

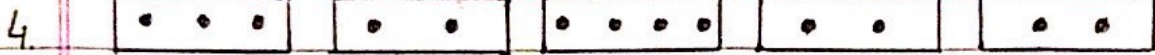
Number of favourable ways to get sum 5 = 4 $\checkmark$

$\therefore P(\text{Sum } 5) = \frac{4}{30} = \frac{2}{15} \checkmark$

(ii) Squared numbers = 1 and 4 (Non-sq numbers are four)

$P(\text{at least one}) = 1 - P(\text{None of 1 or 4}) \leftarrow \{2, 3, 5, 6\}$

$= 1 - \frac{4}{6} \times \frac{3}{5} = \frac{18}{30} = \frac{3}{5} \checkmark$



The diagram shows 5 cards.

(a) Donald chooses a card at random.

(i) Write down the probability that the number of dots on this card is an even number. ---[1]

(ii) Write down the prob. that the number of dots on this card is a prime number. --[1]

(b) Donald chooses two of the five cards at random, without replacement. He works out the total number of dots on these two cards.

(i) Find the prob. that the total number of dots is 5. ---[3]

(ii) Find the prob. that the total number of dots is an odd number. ---[3]

[W-19/41/Q8]

Solution (a)(i) Total number of cards = 5

Cards with even number of dots = 4 (2 dots & 4 dots)

$$\therefore P(\text{even no. of dots}) = \frac{4}{5} \checkmark$$

(ii) cards with prime no. of dots (2 or 3) = 4

$$\therefore P(\text{prime no. of dots}) = \frac{4}{5} \checkmark$$

b(i) Number of dots: $\begin{matrix} D_1 & D_2 & D_3 & D_4 & D_5 \\ 3 & 2 & 4 & 2 & 2 \end{matrix}$

Total no. of outcomes = $5 \times 4 = 20$ (without replacement)

Sum of dots 5, $\left. \begin{matrix} (3, 2), (2, 3), (3, 2) \\ \text{or } (2, 3), (2, 3), (2, 3) \end{matrix} \right\}$ Total 6 ways

$$\therefore P(\text{Total 5}) = \frac{6}{20} \checkmark$$

(ii) Total no. of dots is odd $\rightarrow (4, 3) \text{ or } (3, 4) \text{ and } (3, 2), (2, 3), (3, 2), (2, 3), (2, 3), (2, 3)$

$$\therefore \text{Total} = 8$$

$$\therefore P(\text{odd no.}) = \frac{8}{20} \checkmark$$

5. The prob. that the school bus is late is $\frac{9}{10}$.
If the school bus is late, the prob. that Seb travels on school bus is $\frac{15}{16}$.
If the school bus is on time, the prob. that Seb travels on bus is $\frac{3}{4}$.
Find the prob that Seb travels on the bus [3]
[W-19/21/Q20]

Solution: $P(\text{Seb travels by bus/bus is late}) = \frac{9}{10} \times \frac{15}{16} = \frac{135}{160}$ — ①

$P(\text{Seb travels by bus/bus is on time}) = \frac{1}{10} \times \frac{3}{4} = \frac{3}{40}$ (' $P(\text{bus is on time}) = 1 - \frac{9}{10} = \frac{1}{10}$)
— ②

$\therefore P(\text{Seb travels on bus}) = \frac{135}{160} + \frac{3}{40}$ (from ① & ②)
 $= \frac{135 + 12}{160} = \frac{147}{160}$ ✓

6. Tanya plants some seeds.

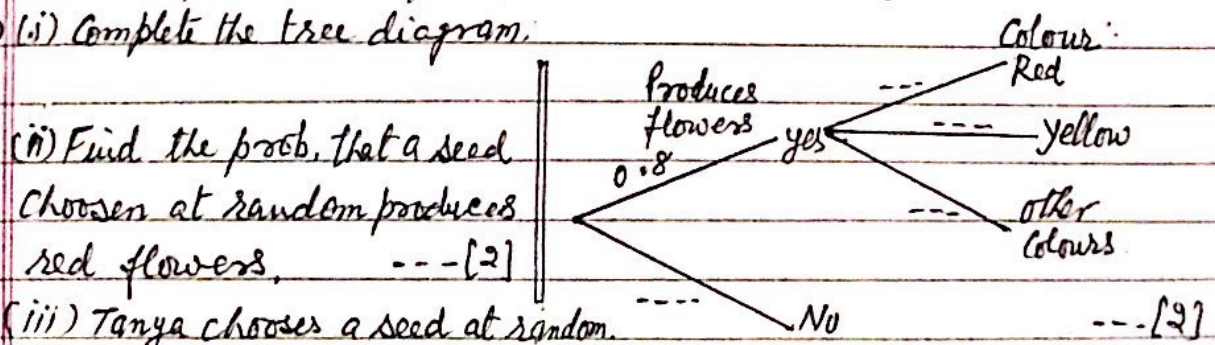
The prob. that a seed will produce flowers is 0.8.

When a seed produces flowers, the prob. that the flowers are red is 0.6 and the prob. that the flowers are yellow is 0.3.

- (a) Tanya has a seed that produces flowers.

Find the prob. that the flowers are not red and not yellow. --- [1]

- (b) (i) Complete the tree diagram.



- (c) Two of the seeds are chosen at random.

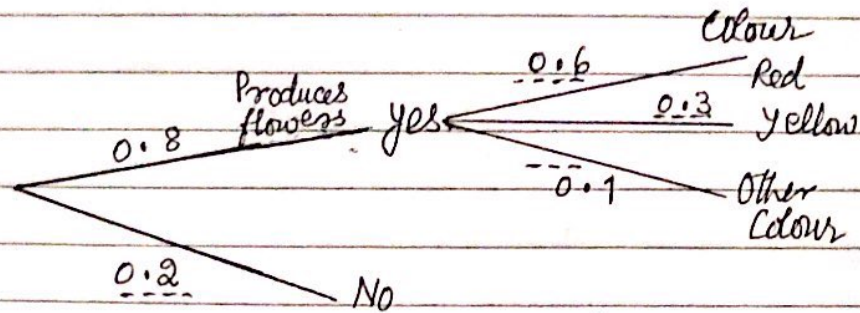
Find the prob. that one produces flowers and one does not produce flowers. [S-20/42/Q7] --- [3]

(solution on the next page) →

(continued)

6. Solution (a). $P(\text{not red or yellow}) = 1 - (\text{either red or yellow})$
 $= 1 - (0.6 + 0.3) = \underline{0.1} \checkmark$

(b) (i)



(ii) $P(\text{seed chosen at random produce red f.}) = 0.8 \times 0.6 = \underline{0.48} \checkmark$

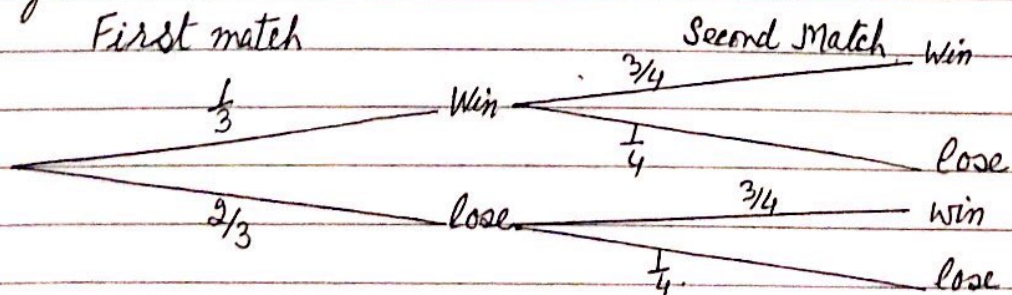
(iii) $P(\text{chosen at random and produces not red, not yellow}) =$
 $= P(\text{seed does not produce flowers}) + (P(\text{produce not red and not yellow}))$
 $= (1 - 0.8) + (0.8 \times 0.1)$
 $= 0.2 + 0.08 = \underline{0.28} \checkmark$

(c) $P(\text{Two seed are chosen at random and one produces flowers and the other does not}) = 0.8 \times 0.2 + 0.2 \times 0.8$
 $= 0.16 + 0.16 = \underline{0.32} \checkmark$

7.

A soccer team plays two matches.

The tree diagram shows the prob. of the team winning or losing the matches.



Find the prob. that the soccer team wins at least one of the two matches.

[SP-20/02/Q22] --- [3]

Solution: $P(\text{at least one match win}) = 1 - P(\text{both matches lost})$
 $= 1 - \frac{2}{3} \times \frac{1}{4} = 1 - \frac{2}{12} = \underline{\frac{10}{12}} \checkmark$

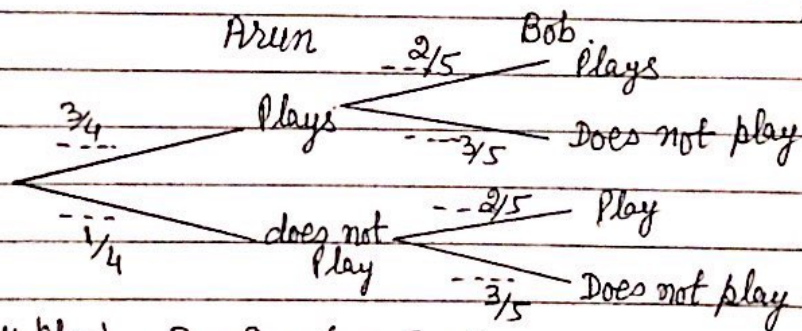
8. On any Saturday, the probability that Arun plays football is $\frac{3}{4}$. On any Saturday, the prob. that Bob plays football is $\frac{2}{5}$.

- (a) (i) Complete the tree diagram, ---[2]
(ii) Calculate the prob. that, one Saturday, Arun and Bob both play football. ---[2]
(iii) Calculate the prob. that, one Saturday, either Arun plays football or Bob plays football, but not both. ---[3]
(b) Calculate the prob. that Bob plays football for 2 of the next 3 Saturdays. ---[3]
(c) When Arun plays football, the prob. that he scores the winning goal is $\frac{1}{7}$. Calculate the prob. that Arun scores the winning goal one Saturday. ---[2]

[S-20/43/Q7]

Solution

a(i)



(ii) $P(\text{both play}) = \frac{3}{4} \times \frac{2}{5} = \frac{6}{20} = \frac{3}{10} \checkmark$

(iii) $P(\text{Either Arun play or Bob plays but not both}) = \frac{3}{4} \times \frac{3}{5} + \frac{1}{4} \times \frac{2}{5} = \frac{11}{20} \checkmark$

(b) $P(\text{Bob plays for 2 out of 3 Saturdays}) = \left(\frac{2}{5}\right)^2 \times \frac{3}{5} \times 3 = \frac{36}{125} \checkmark$

(c) $P(\text{Arun plays and scores the winning goal}) = \frac{3}{4} \times \frac{1}{7} = \frac{3}{28} \checkmark$

- (a) A bag contains 4 red marbles and 2 yellow marbles.
Behnaz picks two marbles at random without replacement.
Find the probability that:
- (i) the marbles are both red. ---[2]
(ii) the marbles are not both red. ---[1]
- (b) Another bag contains 5 blue marbles and 2 green marbles.
Bryn picks one marble at random without replacement.
If this marble is not green, he picks another marble at random without replacement. He continues until he picks a green marble.
Find the prob. that he picks a green marble on his first, second or third attempt. [W-19/43/Q8] --[4]

Solution: (a)(i) 4 Red & 2 yellow

$$P(\text{both red}) = \frac{4}{6} \times \frac{3}{5} = \frac{12}{30} = \frac{2}{5} \checkmark$$

$$(ii) P(\text{not both red}) = P(1R \& 1G \text{ or } 1G \& 1R \text{ or both green})$$

$$= \frac{4}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{4}{5} + \frac{2}{6} \times \frac{1}{5} = \frac{18}{30} = \frac{3}{5} \checkmark$$

Alternate method:

$$= 1 - P(\text{both red})$$

$$= 1 - \frac{4}{6} \times \frac{3}{5} = 1 - \frac{12}{30} = \frac{18}{30} = \frac{3}{5} \checkmark$$

(b) 5 Blue & 2 Green

$P(\text{getting Green in I}^{\text{st}} \text{ or II}^{\text{nd}} \text{ or III}^{\text{rd}} \text{ attempt})$

$$= P(G) + P(B) \times P(G) + P(B)P(B) \times P(G)$$

$$= \frac{2}{7} + \frac{5}{7} \times \frac{2}{6} + \frac{5}{7} \times \frac{4}{6} \times \frac{2}{5}$$

$$= \frac{2}{7} + \frac{10}{42} + \frac{40}{210}$$

$$= \frac{150}{210}$$

$$= \frac{5}{7} \checkmark$$

- 10 Harris is taking a driving test.
The probability that he passes the driving test at the first attempt is 0.6.
If he fails, the prob. that he passes at any further attempt is 0.75.
Calculate the probability that Harris.
- (a) passes the driving test at the second event. ---[2]
(b) takes no more than three attempts to pass the driving test. ---[2]

W-19/22/Q18

Solution:

(a) $P(\text{passes the test in the second event})$
 $= P(\text{Fails 1st time}) \times P(\text{passes second time})$
 $= (1 - 0.6) \times 0.75$
 $= 0.4 \times 0.75$
 $= \underline{0.3} \checkmark$

(b) $P(\text{takes no more than three attempts to pass the test})$
 $= 1 - P(\text{does not pass in three attempts})$
 $= 1 - 0.4 \times 0.25 \times 0.25$
 $= 1 - 0.25$
 $= \underline{0.75} \checkmark$

Alternate method:

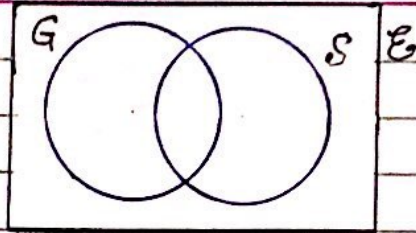
$$P = p + q p + q q p$$

$$= 0.6 + 0.4 \times 0.75 + 0.4 \times 0.25 \times 0.75$$

$$= \underline{0.75} \checkmark$$

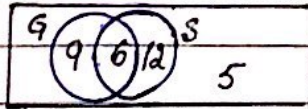


11. (a) There are 32 students in a class.
5 do not study any languages.
15 study German (G)
18 study Spanish (S)



- (i) Complete the Venn diagram to show this information. --[2]
(ii) A student is chosen at random. Find the probability that the student studies Spanish but not German. --[1]
(iii) A student who study German is chosen at random. Find the prob. that this student also studies Spanish. --[1]
(b) A bag contains 54 red marbles and some blue marbles. 36% of the marbles in the bag are red. Find the number of blue marbles in the bag. ---[2]
(c) Another bag contains 15 red beads and 10 yellow beads. Ariana picks a bead at random, record its colour and replaces it in the bag. She then picks another bead at random.
(i) Find the probability that she picks two red beads. --[2]
(ii) Find the prob. that she does not pick two red beads. --[1]
(d) A box contains 15 red pencils, 8 yellow pencils and 2 green pencils. Two pencils are picked at random without replacement. Find the prob. that at least one pencil is red. [W-20/41/Q9] ---[3]

Solution (a) (i)



(ii) $P(S \text{ and not } G) = \frac{12}{32} = \frac{3}{8} \checkmark$

(iii) $\frac{P(G \cap S)}{P(G)} = \frac{6/32}{15/32} = \frac{6}{15} = \frac{2}{5} \checkmark$

(b) Let Total no. of marbles = n
 $36\% \text{ of } n = 54 \Rightarrow \frac{36}{100} \times n = 54 \Rightarrow n = 150$
blue = $64\% \text{ of } 150 = \frac{64}{100} \times 150 = 96 \checkmark$

(c) (i) $P(R, R) = \frac{15}{25} \times \frac{14}{24} = \frac{7}{10} \checkmark$

(ii) $P(\text{not two red}) = 1 - \frac{7}{10}$
 $= \frac{3}{10} \checkmark$

(d) Red 15, Yellow 8, Green 2, Total 25

$P(\text{at least one red}) = 1 - P(2)$

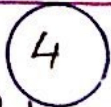
$= 1 - \left[\frac{10}{25} \times \frac{9}{24} \right]$

$= 1 - \frac{90}{600} = 1 - \frac{3}{20}$

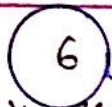
$= \frac{17}{20} \checkmark$



12



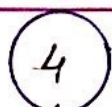
Red



Yellow



Blue



Blue



Yellow



Blue

The diagram shows six discs.

Each has a colour and a number.

(a) One disc is picked at random. Write down the probability that,

(i) the disc has number 4. --- [1]

(ii) the disc is red and has the number 3. --- [1]

(iii) the disc is blue and has the number 4. --- [1]

(b) Two of the six discs are picked at random without replacement.

Find the prob. that (i) both discs have the number 3. --- [2]

(ii) both discs have the same colour. -- [3]

(c) Two of the six discs are picked at random with replacement.

Find the prob. that both discs have the same colour. -- [3]

W-20/42/Q6

Solution (a) (i) $P(\text{number } 4) = \frac{2}{6} = \frac{1}{3} \checkmark$

(ii) $P(\text{red and number } 3) = \frac{0}{6} = 0 \checkmark$

(iii) $P(\text{blue, number } 4) = \frac{1}{6} \checkmark$

(b) (i) $P(\text{without replacement blue and } 3) = \frac{2}{6} \times \frac{1}{5} = \frac{1}{15} \checkmark$

(ii) $P(\text{without rep; both yellow or both blue}) = \frac{2}{6} \times \frac{1}{5} + \frac{3}{6} \times \frac{2}{5} = \frac{4}{15} \checkmark$

(c) $P(\text{with rep; both red or both yellow or both blue})$

$$= \frac{1}{6} \times \frac{1}{6} + \frac{2}{6} \times \frac{2}{6} + \frac{3}{6} \times \frac{3}{6} = \frac{1+4+9}{36}$$

$$= \frac{14}{36}$$

$$= \frac{7}{18} \checkmark$$



13.

P O S S I B I L I T Y

Morgan picks two of these letters, at random, without replacement.

(a) Find the probability that he picks

(i) the letter Y first.

---[1]

(ii) the letter B then the letter Y.

---[2]

(iii) two letters that are the same.

--[3]

(b) Morgan now picks a third letter at random.

Find the probability that,

(i) all three letters are the same.

--[2]

(ii) exactly two of the three letters are the same.

--[5]

(iii) all three letters are different.

--[2]

W-20/43/Q4

Solution (i) $P(Y - \text{one more without rep}) = \frac{1}{11} \times \frac{10}{10} = \frac{1}{11} \checkmark$

(a)

(ii) $P(B, Y) = \frac{1}{11} \times \frac{1}{10} = \frac{1}{110} \checkmark$

(iii) $P(\text{both S or both I}) = \frac{2}{11} \times \frac{1}{10} + \frac{3}{11} \times \frac{2}{10} = \frac{8}{110} = \frac{4}{55} \checkmark$

↓ (3 letters without replacement)

(b) (i) $P(3I) = \frac{3}{11} \times \frac{2}{10} \times \frac{1}{9} = \frac{6}{990} = \frac{1}{165} \checkmark$

(ii) $P(2S \text{ and one more or } 2I \text{ and one other than } I)$

$= 3 \left(\frac{2}{11} \times \frac{1}{10} \times \frac{9}{9} \right) + 3 \left(\frac{3}{11} \times \frac{2}{10} \times \frac{8}{9} \right)$

$= \frac{54 + 144}{11 \times 10 \times 9} = \frac{198}{11 \times 10 \times 9} = \frac{1}{5} \checkmark$

(iii) $P(\text{all three are different})$

$= 1 - [P(\text{all three same}) + P(\text{exactly two are same})]$

$= 1 - \left[\frac{1}{165} + \frac{1}{5} \right]$

$= 1 - \frac{34}{165}$

$= \frac{131}{165}$

from
(b)(i) and b(ii)]



14. A bag contains 7 red discs, 5 green discs and 2 pink disc.
- (a) Helan takes one disc at random, record the colour and replaces it in the bag. She does that 140 times.
Find how many times she expects to take a green disc. --- [2]
- (b) Helan adds 9 green discs and some pink discs to the discs already in the bag. The prob. of taking a green disc is now $\frac{2}{7}$.
Find the number of pink discs that Helan added to the bag. --- [2]

[W-20/23/Q 11]

Solution: 7 red, 5 Green and 2 pink $\rightarrow$ Total = 140

(a) $P(1 \text{ Green}) = \frac{5}{14}$

Number of trials with replacement = 140,

Prob. will remain same every time.

$\therefore$ Expectation of getting number green discs = $n \times p$
 $= 140 \times \frac{5}{14} = 50 \checkmark$

(b) Let the number pink discs added = x

$$\text{Now} \rightarrow \begin{cases} \text{Pink} = x + 2 \\ \text{Green} = 5 + 9 = 14 \\ \text{Red} = 7 \end{cases}$$

$$\text{Total} = x + 2 + 14 + 7 = x + 23$$

$$\text{Now, } P(1 \text{ Green}) = \frac{14}{(x+23)} = \frac{2}{7} \quad (\text{Given})$$

$$\rightarrow 2x + 46 = 98 \Rightarrow 2x = 52$$

$$\Rightarrow \underline{x = 26 \checkmark}$$