

0580

IGCSE - Maths

ContentsPages.

- | | |
|---|---------|
| 1. Area of triangle. | A.1 - 2 |
| 2. Bearing / sine rule and cosine rule | B.1 - 7 |
| 3. Cosine rule and sine rule | C.1 - 5 |
| 4. Graph of trig. functions and solution of trig. equations | G.1 - 3 |
| 5. Three dimension problems. | T.1 - 4 |

Trigonometry
Revision.

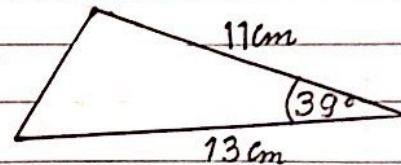
SP-20 | M-20 | S-20 | W-19 | W-20

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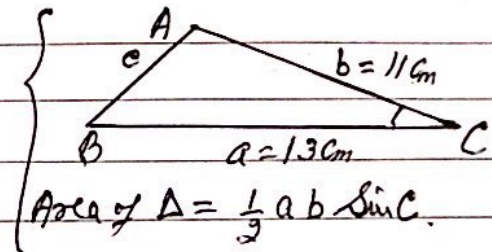
Area of Triangle.

1. Calculate the area of triangle.



Solution:

$$\begin{aligned} \text{Area of Triangle} &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} \times 13 \times 11 \times \sin 39^\circ \\ &= \underline{45 \text{ cm}^2} \checkmark \end{aligned}$$

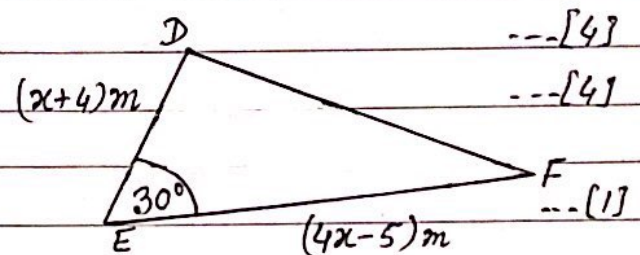


2. The area of triangle DEF is 70 m^2

(i) Show that $4x^2 + 11x - 300 = 0$

(ii) Use the quadratic formula to solve $4x^2 + 11x - 300 = 0$

(iii) Find the length of DE.



Solution:

$$\begin{aligned} \text{(i) Area of Triangle DEF} &= \frac{1}{2} (x+4)(4x-5) \sin 30 = 70 \text{ (Given)} \\ \frac{1}{2} \times (4x^2 + 11x - 20) \times \frac{1}{2} &= 70 \\ \Rightarrow 4x^2 + 11x - 20 &= 280 \\ \Rightarrow 4x^2 + 11x - 300 &= 0 \checkmark \end{aligned}$$

$$\begin{aligned} \text{(ii) } 4x^2 + 11x - 300 &= 0 & \left| \begin{array}{l} a=4, b=11, c=-300 \\ b^2 - 4ac = 11^2 - 4 \times 4 \times (-300) \\ = 4921 \end{array} \right. \\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} & \end{aligned}$$

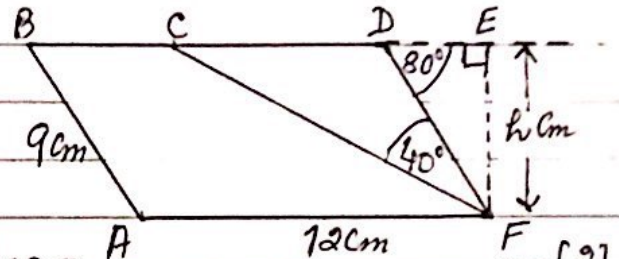
$$= \frac{-11 \pm \sqrt{4921}}{2 \times 4} = \frac{-11 \pm 70.15}{8} = \frac{-81.15}{8}, \frac{59.15}{8}$$

$$\therefore x = -10.14 \text{ or } x = 7.39 \checkmark$$

$$\begin{aligned} \text{(iii) } DE &= x + 4 \\ &= 7.4 + 4 & (x = 7.4 \text{ +ve value}) \\ &= \underline{11.4 \text{ m}} \checkmark \end{aligned}$$



3. $ABDF$ is a parallelogram,
and $BCDE$ is a straight line.
 $AF = 12 \text{ cm}$, $AB = 9 \text{ cm}$, $\angle CDF = 40^\circ$
and $\angle FDE = 80^\circ$



- (i) Calculate the height, h , of the parallelogram. [2]
(ii) Explain why triangle CDF is isosceles. [2]
(iii) Calculate the area of the trapezium $ABCF$. [3]

[W-20/41/Q4(b)]

Solution (i) In right triangle DEF , $\frac{EF}{DF} = \sin 80^\circ$ [DF = AB = 9 cm]
 $\Rightarrow \frac{h}{9} = \sin 80^\circ \Rightarrow h = 9 \times 0.9848 = 8.86 \text{ cm}$ ✓

(ii) $\angle CDF + \angle EDF = 180^\circ$
 $\Rightarrow \angle CDF + 80^\circ = 180^\circ \Rightarrow \angle CDF = 100^\circ$
Now In $\triangle CDF$, $\angle DCF + 100 + 40 = 180^\circ$
 $\Rightarrow \angle DCF = 40^\circ$ — ①
 $\angle DFC = 40^\circ$ — ② (Given)
From ① and ② $\angle DCF = \angle DFC$ — ③
 $\therefore$ Triangle CDF is isosceles (∵ from ③ opposite angles are equal)

(iii) Area Trapezium $ABCF = \frac{1}{2}(AF + BC) \times h$ — ④ (In $\triangle CDF$)
 $= \frac{1}{2}(12 + 3) \times 8.86$ $\left\{ \begin{array}{l} BC = BD - CD ; CD = DF \\ = 12 - 9 \quad (\because = AB) \\ = 3 \quad (= 9) \end{array} \right.$
 $= 7.5 \times 8.86$
 $= 66.47$ [h = 8.86 from part (i)]
 $\therefore \text{Area} = 66.5 \text{ cm}^2$

Three figures bearings and Sine rule/cosine rule

1. The bearing of B from A is 105°
Find the bearing of A from B.

Solution: Bearing of B from A is

angle $NAB = 105^\circ$ (from North clockwise)

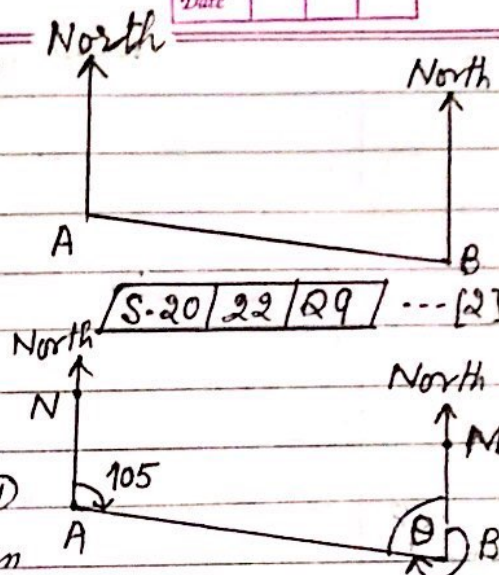
Now Bearing of A from B = angle MBA

$$= 360 - \theta \quad \text{--- (1)}$$

$105 + \theta = 180^\circ$ (Interior angle between two parallel lines on the same side of transversal are supplementary)

$$\Rightarrow \theta = 180 - 105 = 75^\circ$$

$\therefore$ from (1) The Bearing of the point A from B = $360 - 75 = 285^\circ$ ✓



2. The bearing of X from Y is 274°
Calculate the bearing of Y from X.

Solution: Given Bearing of X from Y = 274° .

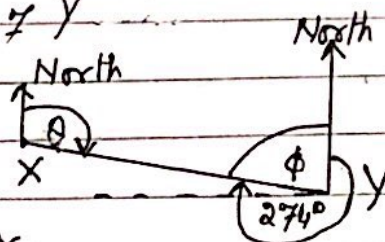
The bearing of Y from X.

$$\theta = 180 - \phi \quad \text{--- (1) } (\because \theta \text{ and } \phi \text{ are Co-Interior angles})$$

$$\text{Now } \phi = 360 - 274 = 86^\circ$$

$$\text{from (1) } \theta = 180 - 86 = 94^\circ$$

$\therefore$ The bearing of Y from X = 094° ✓



3. The bearing of P from B is 102°
Find the bearing of B from P.

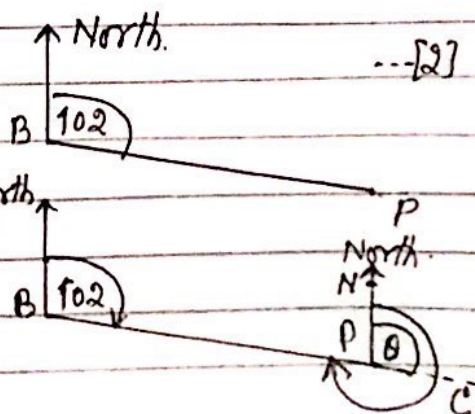
Solution: Let angle $NPC = \theta$ (BP is extended)

$\theta = 102^\circ$ (Corresponding angles) --- (1)

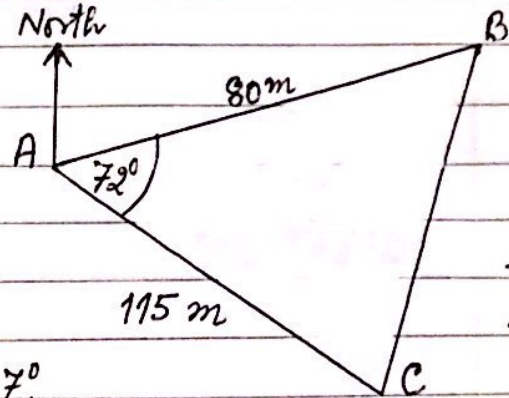
$$\therefore \text{Bearing of B from P} = \theta + 180^\circ$$

$$= 102 + 180^\circ$$

$$= 282^\circ \quad \checkmark$$



4. The diagram shows the positions of three points A, B, and C in a field.



- (a) Shows that BC is 118.1m correct to 1 d.p.

---[3]

- (b) Calculate angle ABC.

---[3]

- (c) The bearing of C from A is 147° .

Find the bearing of (i) A from B

---[3]

(ii) B from C

[S-20/41/Q7] --[2]

- (d) Mitchell takes 35 seconds to run from A to C.

Calculate the average running speed in kilometres per hour, ---[3]

- (e) Calculate the shortest distance from B to AC.

---[3]

Solution: (a) $BC^2 = AB^2 + AC^2 - 2AB \cdot AC \cos BAC$ (Using cosine rule)
 $= 80^2 + 115^2 - 2 \times 80 \times 115 \times \cos 72^\circ$ ($a^2 = b^2 + c^2 - 2bc \cos A$)
 $= 6400 + 13225 - 18400 \times 0.309 = 19625 - 5685.6 = 13939.4$

$BC = \sqrt{13939.4} = 118.06 \checkmark$

- (b) In $\triangle ABC$ using Sine Rule ($\frac{a}{\sin A} = \frac{b}{\sin B}$)

$\frac{BC}{\sin 72} = \frac{AC}{\sin ABC} \Rightarrow \frac{118.06}{\sin 72} = \frac{115}{\sin ABC} \Rightarrow \sin ABC = \frac{115 \times \sin 72}{118.06}$
 $= 0.9264$

$\therefore \text{Angle } ABC = \sin^{-1}(0.9264) = 67.88^\circ \checkmark$

- (c) Bearing of C from A = 147°
 $\angle NAC = 147^\circ$

- (i) Bearing of A from B.

$= 180^\circ + \text{angle } MBD$

$= 180 + 75^\circ$ ($\because \angle MBD = \angle NAB$)

$= 225^\circ \checkmark$

$= 147 - 72 = 75^\circ$
 (1)

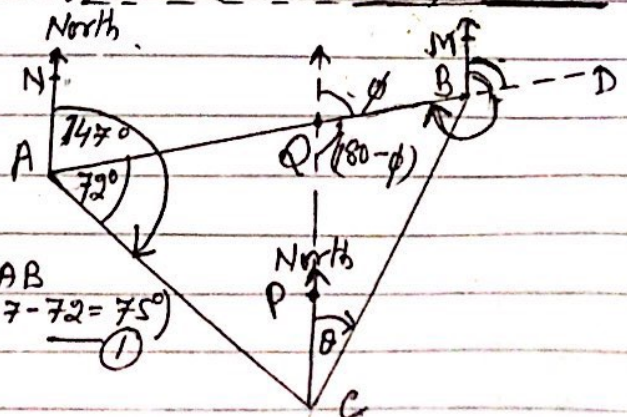
- (ii) Bearing of B from C = θ let

In $\triangle QCB$, $\theta + (180 - \phi) + \angle ABC = 180^\circ$

$\theta + 180 - 75 + 67.88 = 180^\circ$

$\theta = 75 - 67.88 = 7.12^\circ \checkmark$

($\because \phi = \angle MBD = 75^\circ$ from (1))



(Continued $\rightarrow$)

(continued →)

4(d) average speed to run from A to C (115m) in 35 second

$$= \frac{115}{35} = 3.2857 \text{ m/s}$$

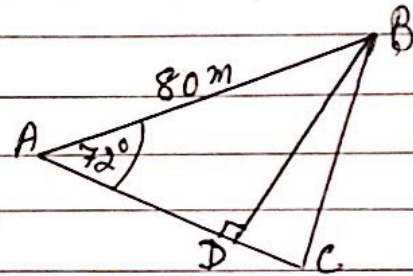
$$= 3.2857 \times 3.6 \text{ km/h}$$

$$= 11.82 \text{ km/h} \checkmark \quad (\because 1 \text{ m/s} = 3.6 \text{ km/h})$$

e. draw BD perpendicular to AC.

The shortest distance of B from AC = BD.

In $\triangle ABD$, $\frac{BD}{AB} = \sin 72^\circ$
 $\Rightarrow BD = 80 \times \sin 72^\circ$
 $= 76.08 \text{ m} \checkmark$

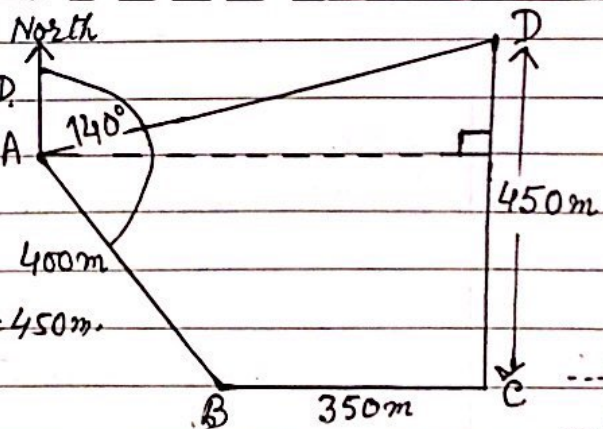


5. The diagram shows a field ABCD.

The bearing of B from A is 140°

C is due east of B and D is due north of C.

$AB = 400 \text{ m}$, $BC = 350 \text{ m}$ and $CD = 450 \text{ m}$.



(a) Find the bearing of D from B. ---[2]

(b) Calculate the distance from D to A. ---[6]

(c) Jono runs around the field from A to B, B to C, C to D and D to A. He runs at a speed of 3 m/s .

Calculate the total time Jono takes to run around the field. ---[4]

Give your answer in minutes and seconds, correct to nearest second.

[5-20/42/Q5]

Solution (a) Let $\angle BDC = \theta \Rightarrow \tan \theta = \frac{350}{450} \Rightarrow \theta = 37.87^\circ$

$\therefore$ Bearing of D from B = $\theta = 37.87^\circ \checkmark$

(b) $BE = 400 \sin 50^\circ = 306.4 \text{ m}$ --- (1) ($BE \perp AF$)
 $\Rightarrow DF = 450 - FC = 450 - BE = 450 - 306.4$
 $= 143.6$ --- (2)

Now $AE = 400 \cos 50^\circ = 257$ --- (3)

$\Rightarrow AF = AE + EF = AE + BC = 257 + 350 = 607$ --- (4)

$\therefore$ In $\triangle AFD$, $AD = \sqrt{AF^2 + DF^2} = \sqrt{(607)^2 + (143.6)^2}$
 $= \sqrt{389069.96} = 623.8 \text{ m}$

$\therefore AD = 624 \text{ m} \checkmark$

(continued →)

(continued →)

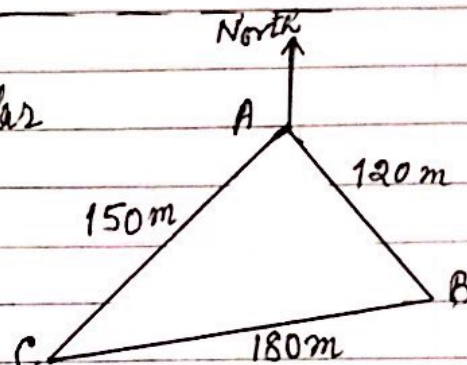
5 (C) $AB + BC + CD + DA = 400 + 350 + 450 + 624 = 1824 \text{ m}$

Speed = 3 m/s

$\therefore \text{Time} = \frac{\text{distance}}{\text{Speed}} = \frac{1824}{3} = 608 \text{ s} = 10 \text{ m. } 8 \text{ s} \checkmark$

6. The diagram shows a triangular field, ABC, on horizontal ground.

(a) Olav runs from A to B at a constant speed of 4 m/s and then from B to C at a constant speed of 3 m/s . He then runs at a constant speed from C to A. His average speed for the whole journey is 3.6 m/s . Calculate his speed when he runs from C to A. -- [3]



(b) Use Cosine rule to find angle BAC. -- [4]

(c) The bearing of C from A is 210°

(i) Find the bearing of B from A. -- [1]

(ii) Find the bearing of A from B. -- [2]

(d) D is the point on AC that is nearest to B. -- [2]

Calculate the distance from D to A.

[W-19/41/Q5]

Solution (a) let the speed from C to A = $x \text{ m/s}$

Total Time from A to B, B to C and C to A = $\left(\frac{120}{4} + \frac{180}{3} + \frac{150}{x}\right)$ second -- (1)

also the average speed is 3.6 m/s

Total time $\Rightarrow \frac{(120 + 180 + 150)}{3.6} = 125 \text{ second}$ -- (2)

from (1) & (2) $\left(30 + 60 + \frac{150}{x}\right) = 125 \Rightarrow \frac{150}{x} = 35 \Rightarrow x = \frac{150}{35} = 4.286 \text{ m/s} \checkmark$

$\therefore$ Speed from C to A = $4.29 \text{ m/s} \checkmark$

(b) $\cos BAC = \frac{150^2 + 120^2 - 180^2}{2 \times 150 \times 120} = \frac{4500}{36000}$ $\left(\cos A = \frac{b^2 + c^2 - a^2}{2bc}\right)$
 $= 0.125$

$\therefore$ Angle BAC = $\cos^{-1} 0.125$
 $= 82.82^\circ \checkmark$

(continued →)

(Continued →)

6 (c) (i) Bearing of B from A = 210° - angle BAC

$$\theta = 210 - 82.82 = 127.18^\circ \approx 127.2^\circ$$

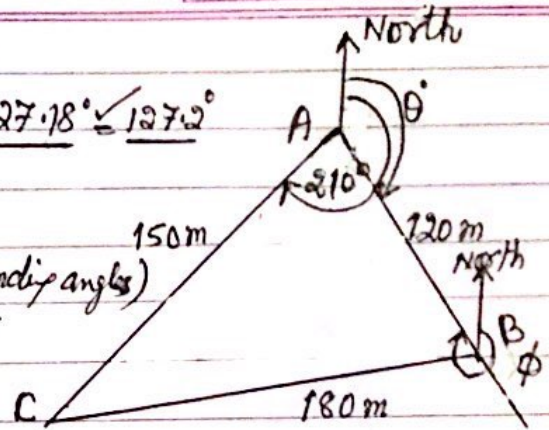
(ii) Bearing of A from B

$$= 180 + \phi$$

$$= 180 + \theta$$

$$= 180 + 127.2 = 307.2^\circ \checkmark$$

($\phi = \theta$
Corresponding angles)



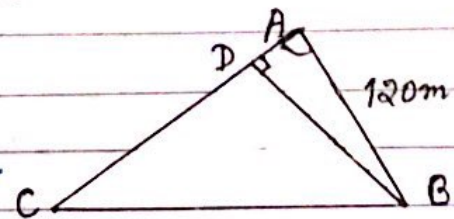
(d) Draw $BD \perp AC$

D is the nearest point on AC from B.

In $\triangle ADC$, $\frac{AD}{120} = \cos A$

$$AD = 120 \times \cos 82.82^\circ \quad (\text{from part (b) angle } BAC = 82.82^\circ)$$

$$= 15m \checkmark$$



7. The diagram shows a field, ABCD, on a horizontal ground.

(a) There is a vertical post at C.

From B, the angle of elevation of the top of the post is 19° .

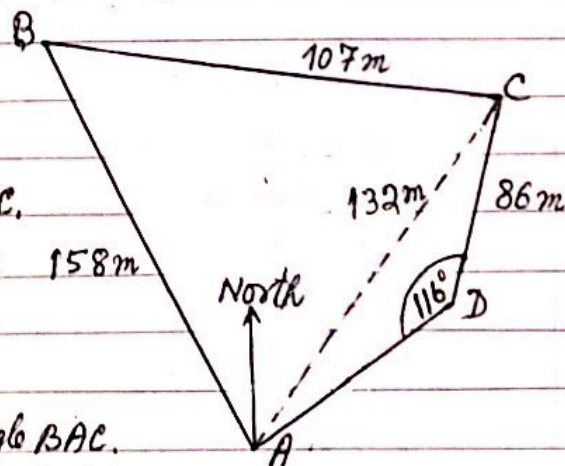
Find the height of the post.

(b) Use the cosine rule to find angle BAC.

(c) Use sine rule to find angle CAD.

(d) Calculate the area of the field.

(e) The bearing of D from A is 070° . Find the bearing of A from C.



---[2]

---[4]

---[3]

---[3]

---[2]

[W-19/43/Q4]

Solution (a) In $\triangle PCB$, $\frac{h}{107} = \tan 19^\circ$

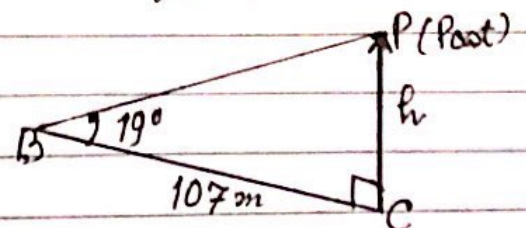
$$\Rightarrow h = 107 \times \tan 19^\circ = 36.84m$$

$$\therefore h = 36.84m \checkmark$$

(b) $\cos BAC = \frac{158^2 + 132^2 - 107^2}{2 \times 158 \times 132} = 0.7417$ ($\because \cos A = \frac{b^2 + c^2 - a^2}{2bc}$)

$$\text{angle } BAC = \cos^{-1} 0.7417 = 42.12^\circ \checkmark$$

(Continued →)



Bearings / Sine rule and Cosine rule.

(Continued →)

7(c) $\frac{86}{\sin CAD} = \frac{132}{\sin 116^\circ}$ (Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B}$)

$\sin CAD = \frac{86 \cdot \sin 116}{132} = 0.5856$
 $\Rightarrow \text{Angle CAD} = \sin^{-1} 0.5856 = 35.84^\circ$

(d) Area ABCD = area BAC + area ACD

area BAC = $\frac{1}{2} bc \sin A$ — ①

$= \frac{1}{2} \times 132 \times 158 \cdot \sin 42.1^\circ$

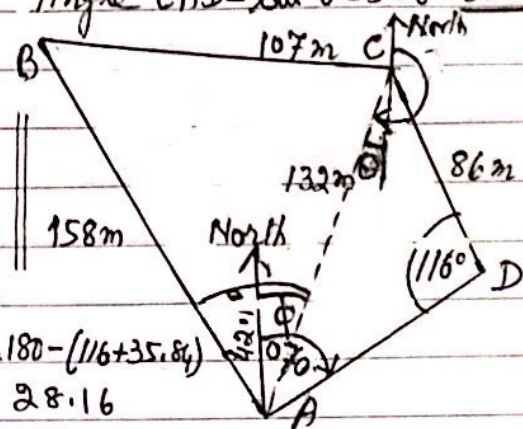
$= 6991.2$ — ②

area ACD = $\frac{1}{2} \times 132 \times 86 \cdot \sin ACD$

$= \frac{1}{2} \times 132 \times 86 \cdot \sin 28.16^\circ$ [angle ACD = $180 - (116 + 35.84)$]

$= 2678.7$ — ③ = 28.16

from ② & ③ in ① $\Rightarrow \text{Area ABCD} = 6991.2 + 2678.7 = 9669.9 \checkmark$

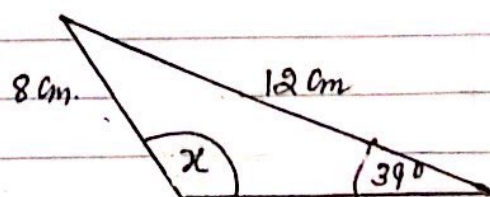


(e) Bearing of D from A = 070°

Bearing of A from C = $180 + \theta = 180 + \phi$ (Alternate angles)
 $\theta = \phi$
 $= 180 + (70 - \text{angle CAD})$
 $= 180 + (70 - 35.84)$
 $= 214.16$
 $= 214.2^\circ$

8. Calculate the obtuse angle x in the triangle.

--- [3]



Solution:

using sine rule:

$\frac{12}{\sin x} = \frac{8}{\sin 39}$ ($\frac{a}{\sin A} = \frac{b}{\sin B}$)

$\Rightarrow \sin x = \frac{12 \sin 39}{8} = 0.944$

$x = \sin^{-1} 0.944$

$= 70.7^\circ$ or $(180 - 70.7)$

$= 70.3^\circ$ $109.3^\circ \checkmark$ (obtuse angle)

$\therefore x = 109.3^\circ$

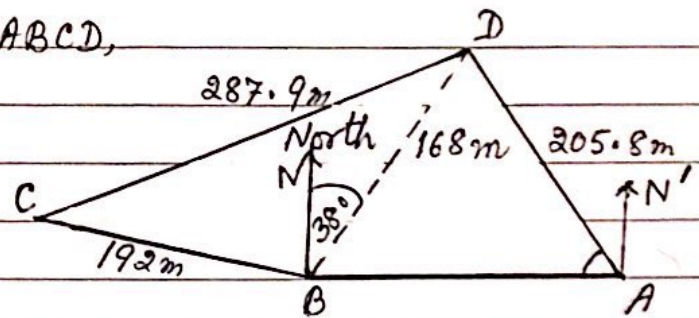
[W-19/21/Q16]



9. The diagram shows a field, ABCD, on horizontal ground.

$BC = 192\text{ m}$, $CD = 287.9\text{ m}$,

$BD = 168\text{ m}$ and $AD = 205.8\text{ m}$.



- (a) (i) Calculate angle CBD and show that it rounds to 106.0°

Correct to 1 decimal place.

-- [4]

- (ii) The bearing of D from B is 038° . Find the bearing of C from B. -- [1]

- (iii) A is due east of B. Calculate the bearing of D from A. -- [5]

- (b) (i) Calculate the area of triangle BCD.

-- [2]

- (ii) Thomas buys the triangular part of the field BCD. The cost is \$35 750 per hectare. Calculate the amount he pays. Give your answer correct to the nearest \$100. [1 hectare = $10\,000\text{ m}^2$] -- [2]

[W-20/41/Q6]

In $\triangle BCD$,

Solution (i)

$$\cos CBD = \frac{192^2 + 168^2 - 287.9^2}{2 \times 192 \times 168} = -0.276 = -\cos 73.98^\circ$$

$$= \cos(180 - 73.98^\circ)$$

$$= 106.2^\circ$$

$$\Rightarrow \text{angle } CBD = 106.2^\circ \checkmark \quad \text{--- (1)}$$

- (ii) Bearing C from B = $360 - \text{angle } CBN = 360 - (\text{angle } CBD - 38^\circ)$
 $= 292^\circ$ $\{ = 360 - (106.2 - 38^\circ) = 291.8^\circ$

- (iii) In $\triangle DBA$, $\frac{\sin A}{168} = \frac{\sin(90 - 38)}{205.8} \Rightarrow \sin A = \frac{\sin 52 \times 168}{205.8} = 0.6432$
 $\Rightarrow A = 40^\circ \checkmark$

$$\text{Hence Bearing of D from A} = 270^\circ + 40^\circ = 310^\circ$$

(b)

(i) Area of $\triangle BCD = \frac{1}{2} \times BC \times BD \times \sin 106.2^\circ = \frac{1}{2} \times 192 \times 168 \times 0.96$
 $= 15487\text{ m}^2$
 $= 15500\text{ m}^2 \checkmark$

- (ii) Cost of triangular plot BCD.

$$= \$15500 \times \frac{35750}{10,000} = \$55412$$

$$= \$55400 \text{ (correct to the nearest \$100)}$$

$$\therefore \text{Cost} = \$55400 \checkmark$$

1 The diagram shows a trapezium ABCD.

AB is parallel to DC

AB = 55m, BD = 70m,

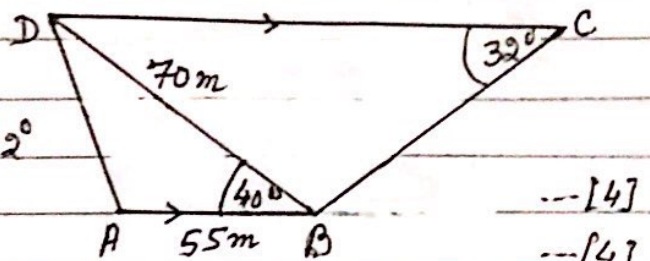
angle ABD = 40° and angle BCD = 32°

(a) Calculate AD

(b) Calculate BC

(c) Calculate area of ABCD.

(d) Calculate the shortest distance from A to BD.



---[4]

---[4]

--[3]

--[2]

[SP-20/04/Q9]

Solution: In ΔABD

(a) $AD^2 = 70^2 + 55^2 - 2 \times 70 \times 55 \cos 40^\circ$ (Cosine rule)
 $= 2026.5$

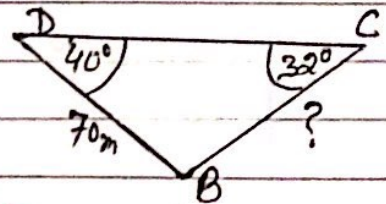
$\Rightarrow AD = \sqrt{2026.5} = 45.02 \text{ m}$

(b) Angle BDC = $\angle ABD = 40^\circ$ (Alternate angle)

$\frac{BC}{\sin 40^\circ} = \frac{70}{\sin 32^\circ}$ (Sine Rule)
 $\frac{a}{\sin A} = \frac{b}{\sin B}$

$BC = \frac{70 \times \sin 40^\circ}{\sin 32^\circ} = 84.9$

$\Rightarrow BC = 84.9 \text{ m} \checkmark$



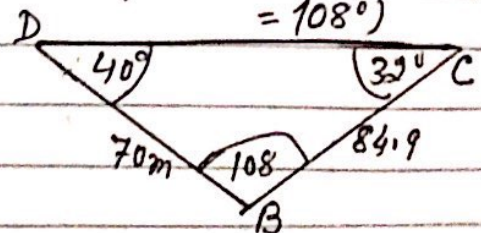
(c) area ABCD = ar ΔABD + ar ΔBCD — (1)

area $\Delta ABD = \frac{1}{2} \times 70 \times 55 \sin 40^\circ = 1237.36$ (ar $\Delta = \frac{1}{2} ab \sin C$)

area $\Delta BCD = \frac{1}{2} \times 70 \times 84.9 \sin 108^\circ$ $\because \angle DBC = 180 - (40 + 32) = 108^\circ$
 $= 2826.06$

$\therefore$ area ABCD = $1237.36 + 2826.06$

$= 4063.42 \text{ sqm.} \checkmark$

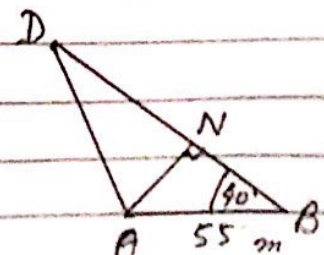


(d) Draw AN perp. to BD, AN is the required shortest dis.

In ΔANB

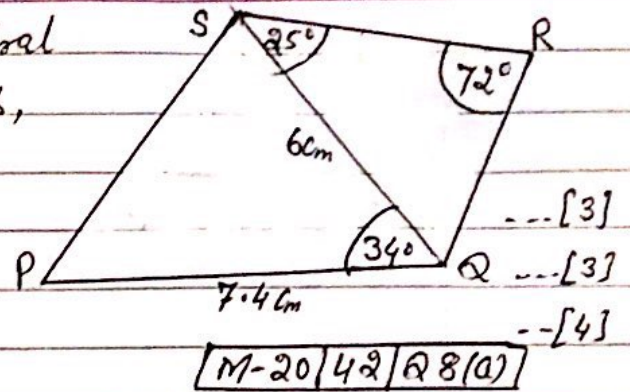
$\frac{AN}{55} = \sin 40^\circ \Rightarrow AN = 55 \sin 40^\circ$
 $= 35.35 \text{ m}$

$\therefore$ Shortest dis = 35.35 m $\checkmark$



2. The diagram shows a quadrilateral PQRS formed from two triangles, PQS and QRS. Calculate:

- (i) QR
(ii) PS
(iii) Area of quadrilateral PQRS.



Solution (i) In ΔQRS

$$\frac{QR}{\sin 25^\circ} = \frac{6}{\sin 72^\circ} \quad (\text{Using Sine Rule. } \frac{a}{\sin A} = \frac{b}{\sin B})$$

$$\Rightarrow QR = \frac{6 \sin 25^\circ}{\sin 72^\circ} = 2.67 \text{ cm} \checkmark$$

(ii) In ΔPQS .

$$\left\{ \begin{array}{l} \text{Cosine Rule} \\ a^2 = b^2 + c^2 - 2bc \cos A \end{array} \right.$$

$$PS^2 = 6^2 + 7.4^2 - 2 \times 6 \times 7.4 \cos 34^\circ$$

$$= 90.76 - 73.61 = 17.14$$

$$\therefore PS = \sqrt{17.14} = 4.14 \text{ cm} \checkmark$$

(iii) area PQRS = ar ΔPQS + ar ΔQRS — (1)

$$\text{area } \Delta PQS = \frac{1}{2} \times 7.4 \times 6 \times \sin 34^\circ \quad (\text{ar } \Delta = \frac{1}{2} ab \sin C)$$

$$= 12.414 \text{ — (2)}$$

$$\text{ar } \Delta QRS = \frac{1}{2} \times QR \times SQ \times \sin \angle SQR$$

$$= \frac{1}{2} \times 2.67 \times 6 \times \sin 83^\circ$$

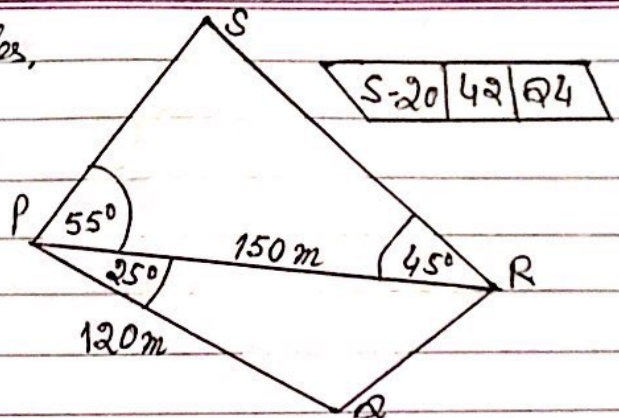
$$= 7.95 \text{ — (3)}$$

$\left\{ \begin{array}{l} \text{angle } SQR = 180^\circ - (72^\circ + 25^\circ) \\ = 83^\circ \\ QR = 2.67 \text{ Part (i)} \end{array} \right.$

from (2) & (3) in (1)

$$\text{Area of Quad. PQRS} = 12.41 + 7.95 = 20.36 \text{ cm}^2 \checkmark$$

3. The diagram shows two triangles,
 (a) Calculate QR. ---[3]
 (b) Calculate RS. ---[4]
 (c) Calculate the total area of the two triangles. ---[3]



Solution: In ΔPQR (cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$)

$$\begin{aligned} (a) \quad QR^2 &= 120^2 + 150^2 - 2 \times 120 \times 150 \times \cos 25^\circ \\ &= 36900 - 32627 = 4273 \\ QR &= \sqrt{4273} = \underline{65.37} \checkmark \end{aligned}$$

$$(b) \quad \text{angle } PSQ = 180 - (55 + 45) = 80^\circ$$

Using sine rule in ΔPSQ ,

$$\frac{RS}{\sin 55^\circ} = \frac{150}{\sin 80^\circ}$$

$$\left(\frac{a}{\sin A} = \frac{b}{\sin B} \right)$$

$$\Rightarrow RS = \frac{150 \times \sin 55^\circ}{\sin 80^\circ} = \frac{122.87}{0.9848} = \underline{124.80m} \checkmark$$

$$(c) \quad \text{Total area} = \text{ar } \Delta PQR + \text{ar } \Delta PRS \quad \text{--- (1)}$$

$$\begin{aligned} \text{ar } \Delta PQR &= \frac{1}{2} \times 120 \times 150 \times \sin 25^\circ \quad (\text{ar } A = \frac{1}{2} ab \sin C) \\ &= 3803.56 \quad \text{--- (2)} \end{aligned}$$

$$\begin{aligned} \text{ar } \Delta PRS &= \frac{1}{2} \times PR \times RS \times \sin 45^\circ \quad \text{Part (ii)} \\ &= \frac{1}{2} \times 150 \times 124.8 \times \sin 45^\circ \quad [RS = 124.8] \\ &= 6618.52 \quad \text{--- (3)} \end{aligned}$$

from (2) & (3) in (1)

$$\begin{aligned} \text{Total area} &= 3803.56 + 6618.52 \\ &= \underline{10422} \checkmark m^2 \end{aligned}$$

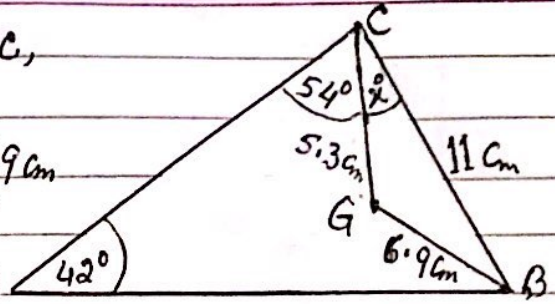
Sine Rule and Cosine Rule

4. The diagram shows triangle ABC, with point G inside.

$CB = 11\text{cm}$, $CG = 5.3\text{cm}$ and $BG = 6.9\text{cm}$

Angle $CAB = 42^\circ$ and angle $ACG = 54^\circ$

- (i) Calculate the value of x . --- [4]
 (ii) Calculate AC.



--- [4]
S-20/43 Q6(a)

Solution: (i) $\cos x^\circ = \frac{11^2 + 5.3^2 - 6.9^2}{2 \times 11 \times 5.3}$ [Cosine Rule, $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$]

$$= \frac{101.48}{116.6} = 0.87$$

$$\Rightarrow x = \cos^{-1}(0.87) = 29.5^\circ \checkmark$$

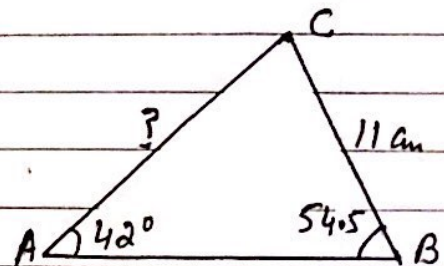
(ii) angle $ABC = 180 - \{42 + (54 + 29.5)\} = 54.5$

Using sine rule in $\triangle ABC$.

$$\frac{AC}{\sin 54.5} = \frac{11}{\sin 42}$$

$$\Rightarrow AC = \frac{11 \times \sin 54.5}{\sin 42}$$

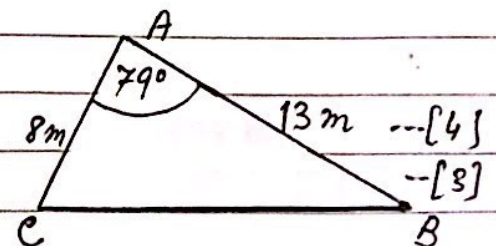
$$= 13.38 \text{ or } AC = 13.4\text{cm} \checkmark$$



5. The diagram shows triangle ABC.

- (i) Use cosine rule to calculate BC.

- (ii) Use sine rule to calculate angle ACB.



Solution: (i) In $\triangle ABC$.

$$BC^2 = 8^2 + 13^2 - 2 \times 8 \times 13 \times \cos 79^\circ$$
[Cosine Rule, $a^2 = b^2 + c^2 - 2bc \cos A$]

$$= 64 + 169 - 39.69 = 193.31 \Rightarrow BC = \sqrt{193.31} = 13.9\text{ m} \checkmark$$

(ii) $\frac{13}{\sin C} = \frac{13.9}{\sin 79}$ ($\therefore$ Sine Rule, $\frac{a}{\sin A} = \frac{b}{\sin B}$)

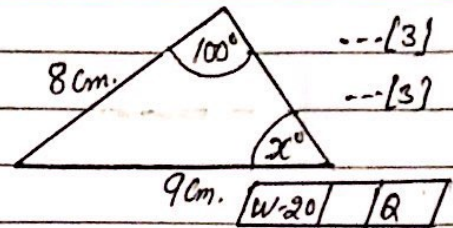
$$\sin C = \frac{13 \times \sin 79}{13.9} = 0.918$$

$$\therefore \angle ACB = \sin^{-1}(0.918) = 66.64^\circ \checkmark$$

W-19/42 Q6(a)



6. (a) Calculate the value of x .
(b) Calculate the area of the triangle.



Solution (a) using sine rule, $\frac{\sin x^\circ}{8} = \frac{\sin 100^\circ}{9} \Rightarrow \sin x^\circ = \frac{8 \times \sin 100^\circ}{9} = 0.8753$
 $\Rightarrow x = 61.1^\circ \checkmark (60.09)^\circ$

(b) Area of triangle = $\frac{1}{2} ab \sin C = \frac{1}{2} \times 8 \times 9 \times \sin [180 - (100 + 61.1)]$
 $= 36 \sin 18.9^\circ =$
 $= 11.7 \text{ cm} \checkmark (11.66)$

Graph of Trig. functions / Solution of Trig. eqnⁿ

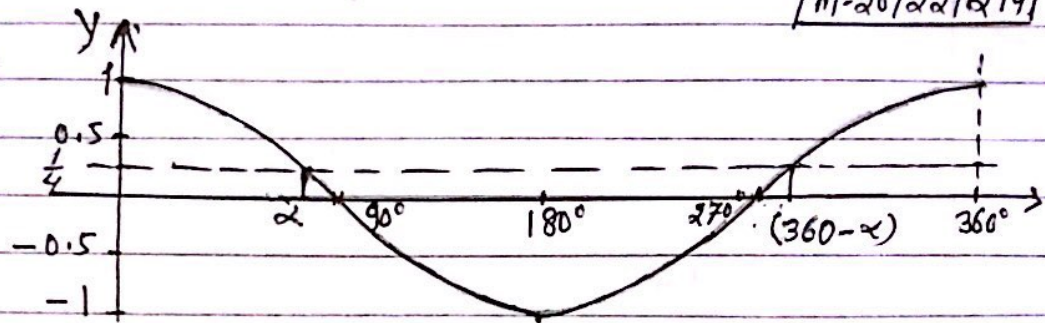
$$0 \leq \theta \leq 360^\circ$$

Page No.	G.1.
Date	

1. (a) On the diagram, sketch the graph of $y = \cos x$ for $0 \leq x \leq 360^\circ$ --- [2]
 (b) Solve the equation $4\cos x + 2 = 3$ for $0 \leq x \leq 360^\circ$ --- [3]

[M-20/22/Q19]

Solution:



$$\begin{aligned} (b) \quad 4\cos x + 2 &= 3 \Rightarrow \cos x = \frac{1}{4} = \cos \alpha \text{ (let)} \\ \alpha &= \cos^{-1} \frac{1}{4} = 75.5^\circ \\ \therefore x &= 75.5^\circ \text{ and } 360 - 75.5^\circ \\ x &= 75.5^\circ \text{ and } 284.5^\circ \checkmark \end{aligned}$$

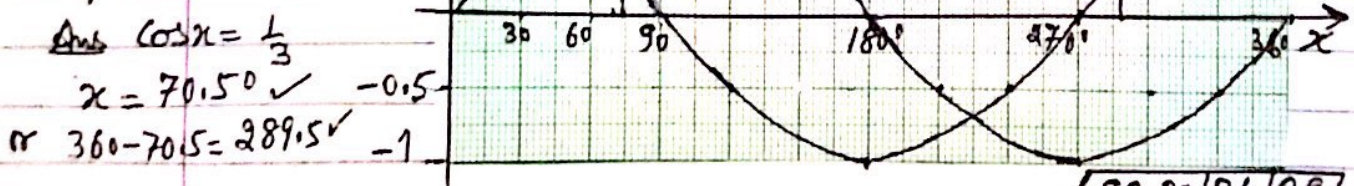
2. When $\sin x^\circ = 0.36$, find

- (a) the acute angle x° , --- [1]
 (b) the obtuse angle x° --- [1]

Solution (a) $\sin x^\circ = 0.36$
 $\Rightarrow x^\circ = \sin^{-1} 0.36 = 21.1^\circ \checkmark$
 (b) Obtuse angle $= (180 - 21.1)$
 $= 158.9^\circ \checkmark$

3. The grid shows the graph of $y = \cos x$ for $0^\circ \leq x \leq 360^\circ$

- (a) Solve the equation $3\cos x = 1$ for $0 \leq x \leq 360^\circ$ [4]



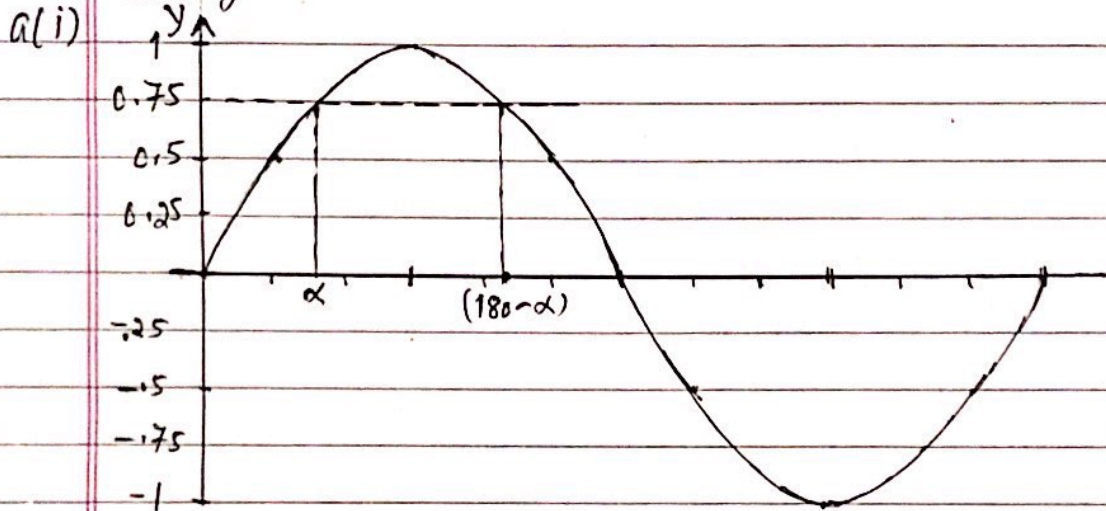
[SP.20/04/Q8]

- (b) On the same grid, sketch the graph of $y = \sin x$ for $0 \leq x \leq 360^\circ$ --- [2]

4. (a) (i) On the axis, sketch the graph of $y = \sin x$ for $0 \leq x \leq 360^\circ$ --- [2]
 (ii) Describe fully the symmetry of the graph of $y = \sin x$; $0 \leq x \leq 360^\circ$ --- [2]
 (b) Solve $4 \sin x - 1 = 2$ for $0 \leq x \leq 360^\circ$ --- [3]

[S-20/41/Q8(a)(b)]

Solution: $y = \sin x$ $0 \leq x \leq 360^\circ$



(ii) Rotational symmetry of order 2, centre $(180^\circ; 0)$

(iii) Solve: $4 \sin x - 1 = 2$ $0^\circ \leq x \leq 360^\circ$

$$\sin x = \frac{3}{4} = 0.75$$

$$x = \sin^{-1} 0.75 = 48.6^\circ$$

$$\therefore x = 48.6^\circ ; 180 - 48.6^\circ$$

$$x = 48.6^\circ ; x = 131.4^\circ \checkmark$$

5 (i) Sketch the graph of
 $y = \tan x$ for $0 \leq x \leq 360^\circ$

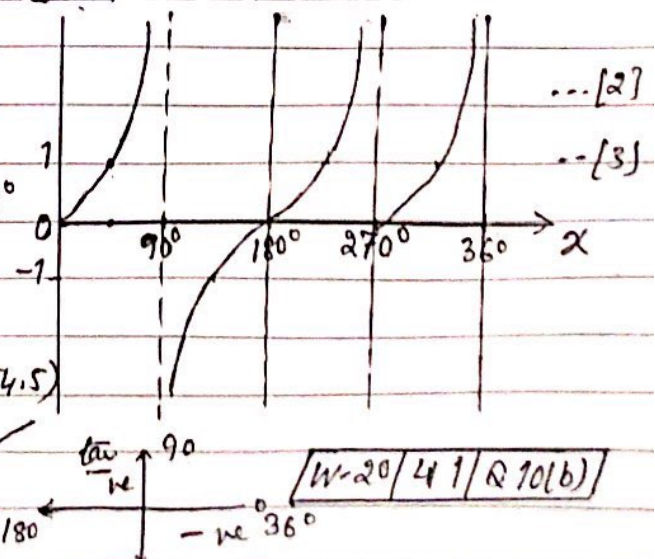
(ii) Solve the equation, $5 \tan x = -7$,
 for $0 \leq x \leq 360^\circ$

Solution: (ii) $5 \tan x = -7$

$$\tan x = -\frac{7}{5} = -\tan 54.5^\circ$$

$$\therefore x = (180 - 54.5), (360 - 54.5)$$

$$= 125.5^\circ ; 305.5^\circ \checkmark$$



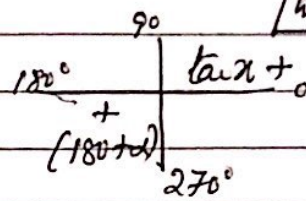
[W-20/41/Q10(b)]

Solution: $\tan x = 2 = \tan 63.43^\circ$

$$\therefore x = 63.4^\circ$$

$$\alpha = 63.4^\circ; 180 + 63.4^\circ$$

$$= 63.4^\circ; 243.4^\circ$$

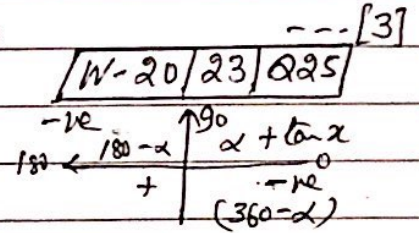


Solution: $3 \tan x = -4 \Rightarrow \tan x = -\frac{4}{3}$

$$= -\tan 53.18^\circ$$

$$x = (180 - 53.1), (360 - 53.1)$$

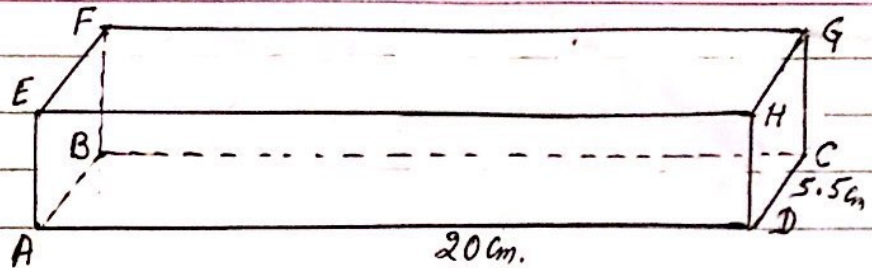
$$= 126.1 \text{ or } 306.1^\circ \checkmark$$



Three dimensions problems.

PAGE: T-1

1.



The diagram shows cuboid ABCDEFGH of length 20 cm and width 5.5 cm. The volume of the cuboid is 495 cm^3 .

Find the angle between the line AG and the base of the cuboid ABCD.

[S-20/21/Q19]

Solution: Volume of cuboid = $l \times b \times h$ [let $GC = h$ cm]

$$= 20 \times 5.5 \times h = 495 \text{ (Given)}$$

$$\Rightarrow h = \frac{495}{20 \times 5.5} = 4.5 \text{ cm.} \quad \text{--- (1)}$$

Now In rectangular base ABCD, $AC^2 = 20^2 + 5.5^2 = 430.25$

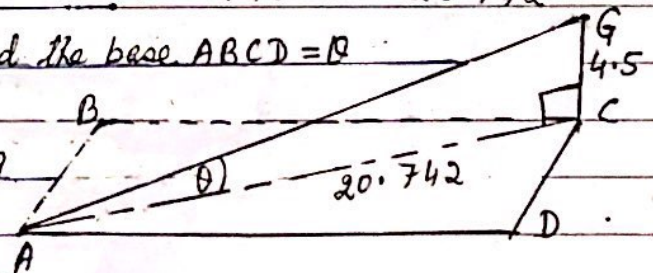
$$AC = \sqrt{430.25} = 20.742$$

Let the angle between AG and the base ABCD = θ

$$\tan \theta = \frac{GC}{AC} = \frac{4.5}{20.742} = 0.2169$$

$$\therefore \theta = \tan^{-1} 0.2169$$

$$\text{Angle GAC} = 12.24^\circ \checkmark$$



2. The diagram shows a cuboid, $AB = 8 \text{ cm}$.

$AD = 6 \text{ cm}$, $BE = 6 \text{ cm}$

Calculate angle HAF.

Solution: Using Cosine Rule:

$$AF = \sqrt{8^2 + 6^2} = 10$$

$$FH = \sqrt{8^2 + 6^2} = 10$$

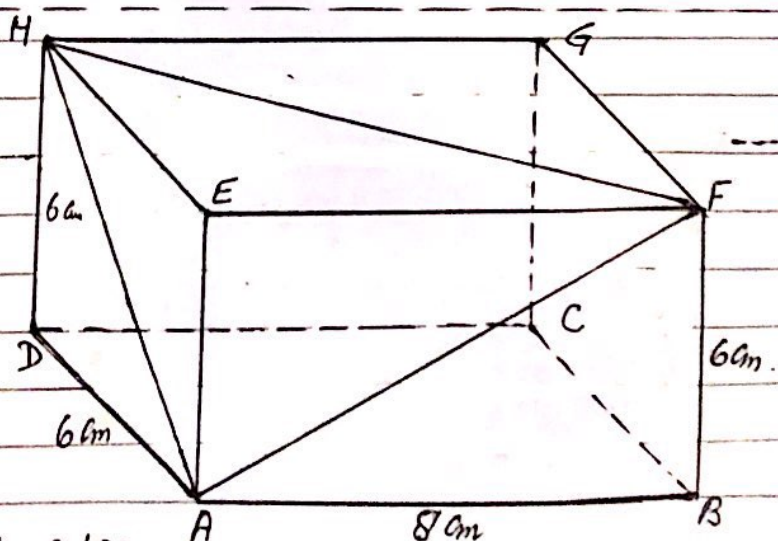
$$AH = \sqrt{6^2 + 6^2} = 6\sqrt{2}$$

$$\cos HAF = \frac{100 + 72 - 100}{2 \times 10 \times 6\sqrt{2}} = \frac{72}{170} = 0.4235$$

$$\Rightarrow \text{Angle HAF} = \cos^{-1}(0.4235) = 64.9^\circ \checkmark$$

$$\therefore \cos HAF = \frac{AF^2 + AH^2 - FH^2}{2 \times AF \times AH}$$

[S-20/22/Q27]



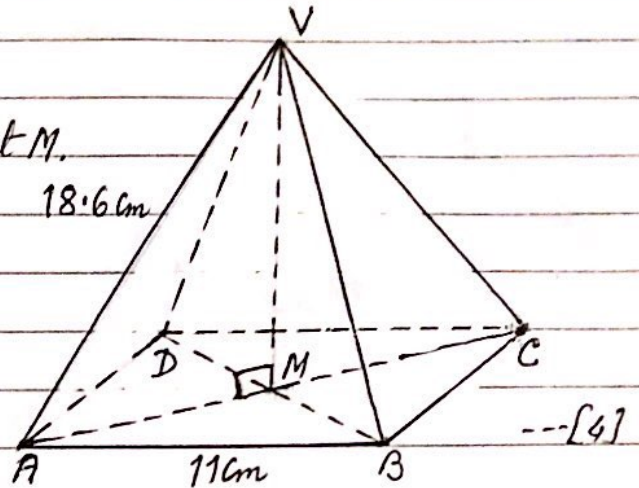
3. The diagram shows a pyramid with a square base ABCD.

The diagonals AC and BD intersect at M.

The vertex V is vertically above M. 18.6cm

AB = 11cm and AV = 18.6cm.

Calculate the angle that AV makes with the base.



Solution: $AC = \sqrt{11^2 + 11^2} = 15.556$

$$AM = \frac{1}{2} AC = \frac{1}{2} \cdot 15.556 = 7.778 \text{ cm}$$

Angle between AV and base = angle VAM = θ (let)

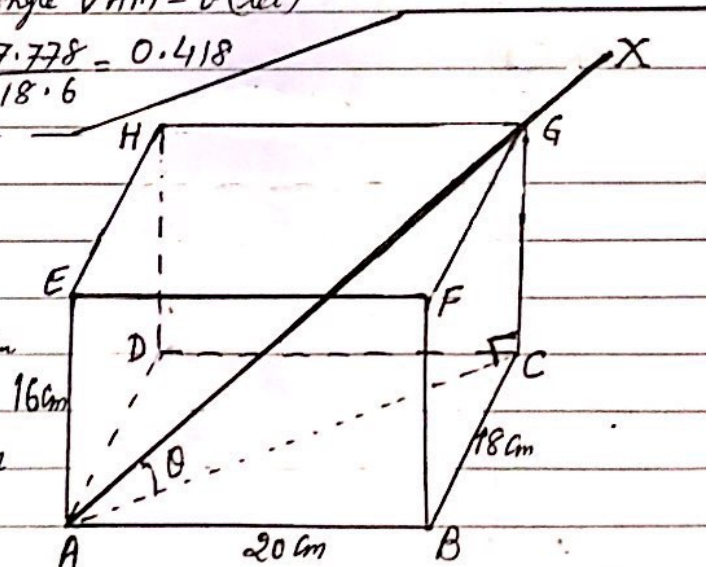
$$\cos \theta = \frac{AM}{VA} = \frac{7.778}{18.6} = 0.418$$

$$\Rightarrow \theta = \cos^{-1} 0.418 = 65.28^\circ$$

4. The diagram shows an open box ABCDEFGH in the shape of a cuboid. AB = 20cm, BC = 18cm and AE = 16cm.

A thin rod AGX rests partly in the box as shown.

The rod is 40cm long.



- (i) Calculate GX, the length of the rod which is outside the box. ---[4]

- (ii) Calculate the angle the rod makes with the base of the box. ---[3]

Solution: $AG = \sqrt{AC^2 + GC^2} = \sqrt{(AB^2 + BC^2) + GC^2} = \sqrt{20^2 + 18^2 + 16^2} = 31.3$

(i) $\therefore GX = AX - AG = 40 - 31.3 = 8.7$

- (ii) Let the angle between the rod AG and base = $\angle GAC = \theta$ (let)

In $\triangle ACG$: $\sin \theta = \frac{GC}{AG} = \frac{16}{31.3} = 0.5112$

$$\therefore \theta = \sin^{-1} 0.5112 = 30.74^\circ$$

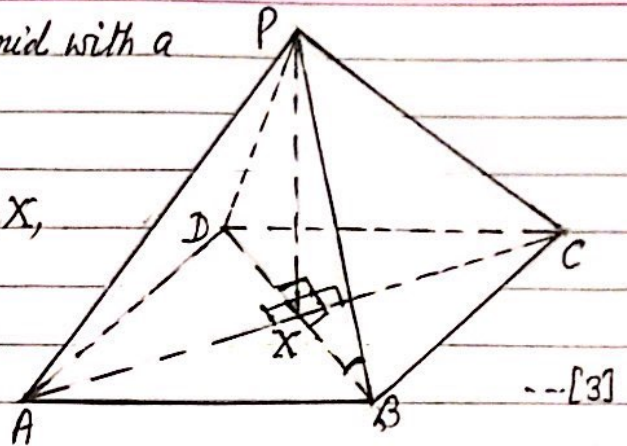
$$\therefore \theta = 30.7^\circ$$

5. The diagram shows a pyramid with a square base ABCD.

$$DB = 8\text{ cm.}$$

P is vertically above the centre X, of the base and $PX = 5\text{ cm.}$

Calculate the angle between PB and the base ABCD.



---[3]

Solution: $XB = \frac{1}{2} BD = \frac{1}{2} \times 8 = 4\text{ cm}$

$$PX = 5\text{ cm (Given)}$$

In $\triangle PXB$,

$$(\text{angle } PXB = 90^\circ)$$

$$\tan \theta = \frac{PX}{BX} = \frac{5}{4} = 1.25 \quad (\text{let angle } PBX = \theta)$$

$$\therefore \theta = \tan^{-1} 1.25 = 51.34^\circ$$

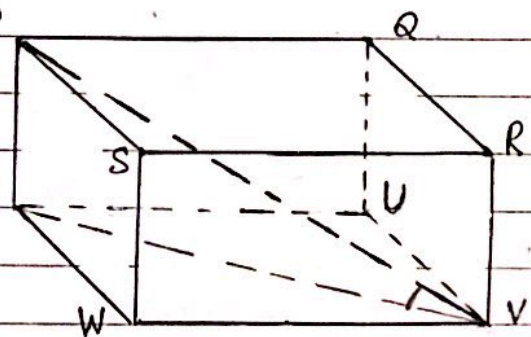
$$\therefore \text{angle between PB and the base ABCD} = 51.34^\circ \checkmark$$

6. The diagram shows a cuboid PQRS TUVW.

$$PV = 17.2\text{ cm.}$$

The angle between the line PV and the base TUVW of the cuboid is 43° .

Calculate PT.



[3]

Solution: In right triangle PTV,

$$\frac{PT}{PV} = \sin 43$$

$$\Rightarrow PT = PV \sin 43^\circ$$

$$= 17.2 \times 0.6819$$

$$= 11.73\text{ cm.}$$

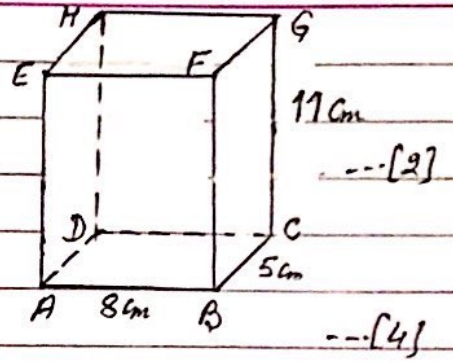
$$PT = 11.7\text{ cm} \checkmark$$

7. $ABCDEFGH$ is a cuboid.

$AB = 8 \text{ cm}$, $BC = 5 \text{ cm}$ and $CG = 11 \text{ cm}$.

- (a) Work out the volume of the cuboid,
(b) Ivana has a pencil of length 13 cm.
Does this pencil fit completely inside
the cuboid? show how you decide.

- (c) (i) Calculate angle CAB,
(ii) Calculate angle GAC.



W-20/43/Q6

Solution (a) Volume of the cuboid = $l \times b \times h = 8 \times 5 \times 11 = 440 \text{ c.c.}$ ✓

(b) length of diagonal AG = $\sqrt{8^2 + 5^2 + 11^2} = \sqrt{210} = 14.5 \text{ in } (14.49)$

$\therefore$ A pencil of length 13cm. can be completely fit in the cuboid. $\checkmark$

(c) (i) $\tan \text{angle CAB} = \frac{5}{8} = 0.625 \Rightarrow \text{angle CAB} = \tan^{-1} 0.625$
 $= 32.0^\circ$

(ii) $AG = 14.49$ from Part (b)

$$\sin GAC = \frac{GC}{AG} = \frac{11}{14.49} = 0.7591$$

hence

$$\text{angle GAC} = \sin^{-1} 0.7591$$
$$= 49.4^\circ (49.38) \checkmark$$

