11.04.2021.

0580

IGCSE Maths

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Geometry

Notes_1

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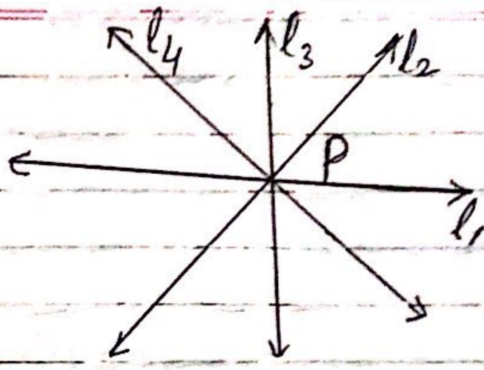
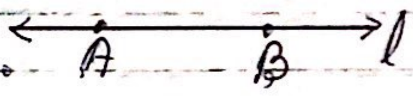
Geometry Notes

Lines and Angles:

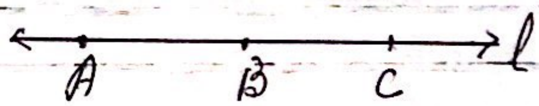
§1. (i) Through a given point, there pass infinitely many line. Three or more such lines are called **concurrent lines.**



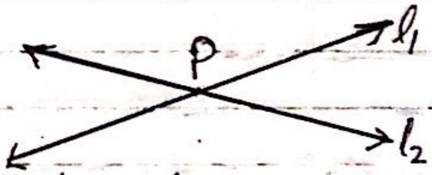
(ii) Given two distinct points, there is one and only one line containing them.



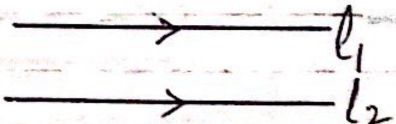
(iii) Three or more points lying on the same line are called **collinear** points.



(iv) Two lines are intersecting if they have one point in common. The common point is called the "point of intersection".



§2. (i) Parallel Lines: Two line in a plane which do not intersect when extendend.



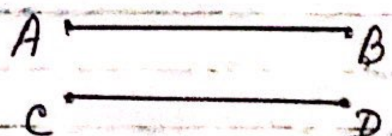
we write: $l_1 \parallel l_2$

(ii) Two lines which are parallel to the same line are parallel to each other.

$l_2 \parallel l_1$ and $l_3 \parallel l_1 \Rightarrow l_2 \parallel l_3$

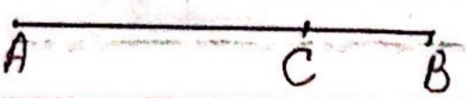
§3. Line Segments:

(i) Two congruent line segments are of equal length.



$\overline{AB} \cong \overline{CD} \Leftrightarrow$ length AB = length CD.

(ii) If a point 'C' lies in the interior of segment $\overline{AB}$. Then $l(AC) + l(CB) = l(AB)$.



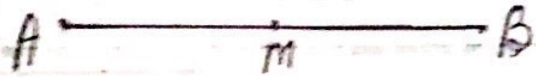
Geometry

Lines and Angles:

Line Segments:

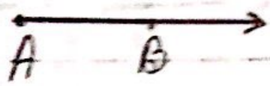
§3 (iii) Mid point 'M' of line segment $\overline{AB}$.

$AM = MB$.



(iv) Ray $\overrightarrow{AB}$.

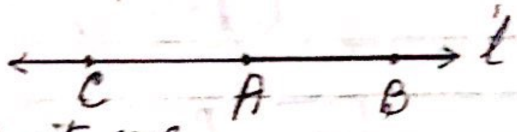
A is the initial point of ray $\overrightarrow{AB}$



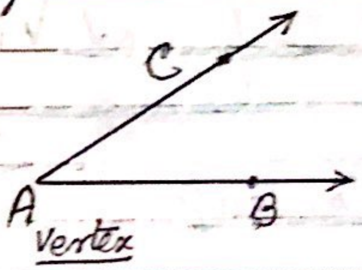
(v) Opposite Rays:

Point 'A' lies on line 'l'.

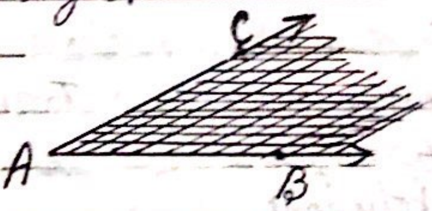
Then $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are opposite rays.



§4. (i) Angle: Angle is formed by two non-collinear rays with a common initial point, called Vertex of the angle, and rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are called the arms of the angle.



(ii) Interior of angle:



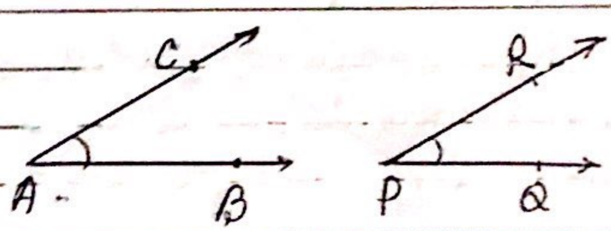
(iii) Measure of an angle:

Every angle has a measure.

Unit of angle measure is "Degree". 1 right angle $= 90^\circ$

(iv) Congruent angles:

Congruent angle have equal measure.

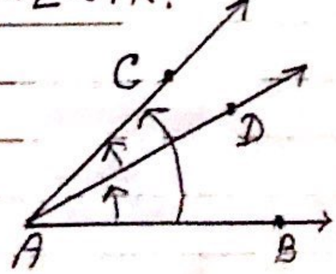


$\angle BAC \cong \angle QPR \Leftrightarrow m\angle BAC = m\angle QPR$.

(v) Angle addition axiom:

If D is point in the interior of $\angle BAC$. Then

$m\angle BAC = m\angle BAD + m\angle DAC$.



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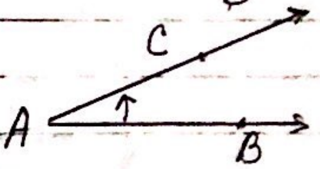
Lines and Angles:

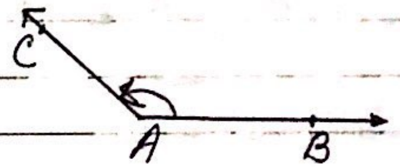
§ 5 Types of Angles:

(i) Right angle: whose measure is 90°.

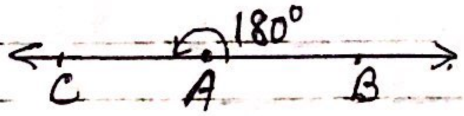
(ii) Acute angle: Whose measure is less than 90°.



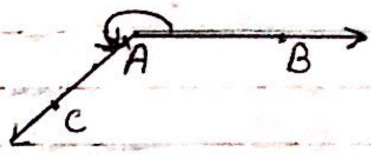
(iii) Obtuse angle: whose measure is more than 90° but less than 180°.



(iv) Straight angle: whose measure is 180°



(v) Reflex Angle: Whose measure is more than 180° but less than 360°.

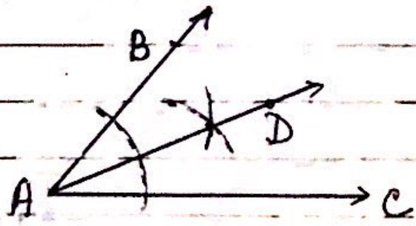


§ 6 Some Angles Relations:

(i) Angle Bisector: If D is point in the interior of $\angle BAC$, so that:

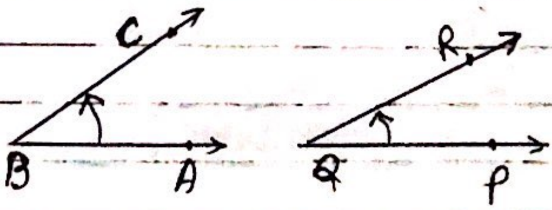
$m\angle CAD = m\angle BAD$

Ray $\overrightarrow{AD}$ is the bisector of angle BAC.



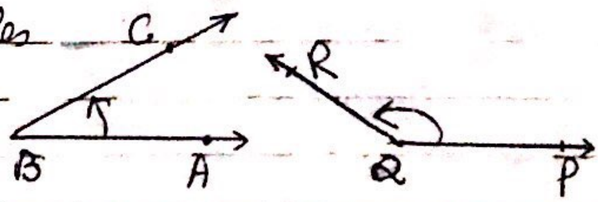
(ii) Complementary Angles:
Two angles ABC and PQR whose sum of measures is 90°

$m\angle ABC + m\angle PQR = 90°$

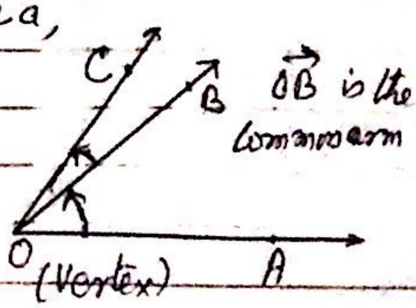


(iii) Supplementary Angles: Two angles sum of whose measures is 180°

$m\angle ABC + m\angle PQR = 180°$



(iv) Adjacent angles: Two angles which have a,
(i) common Vertex
(ii) common arm
(iii) the uncommon arms are on the either side of the common arm.
angle AOB and $\angle BOC$ are adjacent angles.



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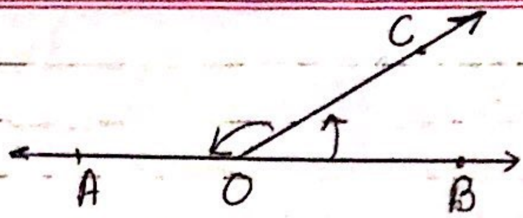
Geometry

§ 6. Some Angles Relations

(v) Linear Pair of Angles:



Two adjacent angles whose sum of measures is 180°.

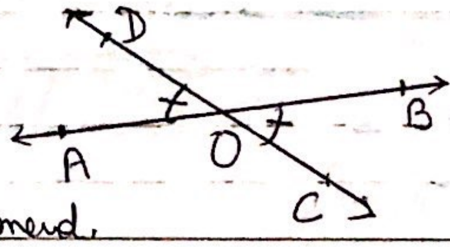
$m\angle BOC + m\angle COA = 180°$

Then $\angle BOC$ and $\angle COA$ form a linear pair, or The points A, O and B are collinear. ($\overrightarrow{OA}$ and $\overrightarrow{OB}$ are opp. rays)

Conversely: If on a line $\overleftrightarrow{AB}$ a ray $\overrightarrow{OC}$ stands

$m\angle BOC + m\angle AOC = 180°$

(vi) Vertically opposite angles:

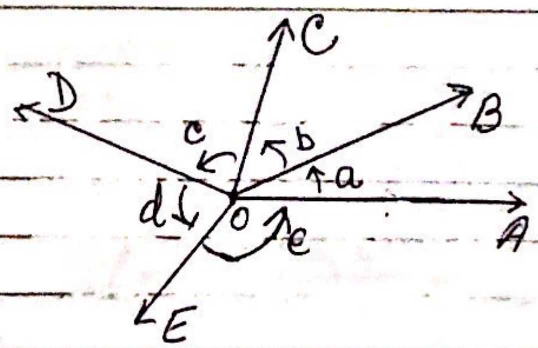


If two lines intersect at a point. Then two pairs of vertically opposite angles are formed.

(i) $\angle AOD$ and $\angle BOC$ (ii) $\angle BOD$ and $\angle COA$.

Vertically opposite angles are equal. $\begin{cases} \text{(i) } \angle AOD = \angle BOC \\ \text{(ii) } \angle BOD = \angle COA \end{cases}$

(vii) The sum of all angles around a point is 360°



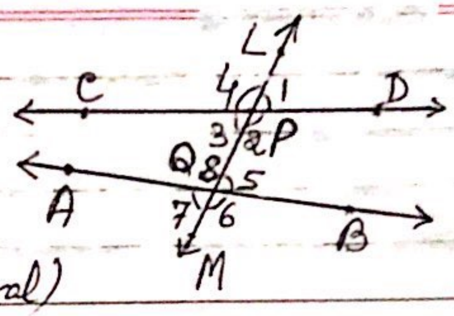
$m\angle AOB = a$, $m\angle BOC = b$

--- ----

$a + b + c + d + e = 360°$

Geometry

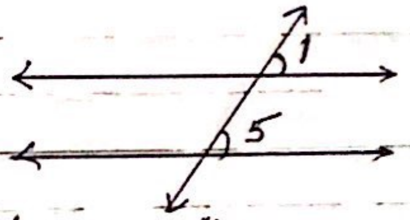
§7. Angles made by a Transversal with two lines:



Transversal:

Two lines AB and CD are intersected by a line LM (Transversal) at two distinct points P and Q.

(i) **Corresponding angles:**

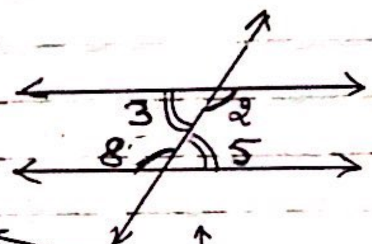


Two angles ($\angle 1$ and $\angle 5$) on the same side of the transversal and both of them are either above the two lines (or below the two lines)

There are four pairs of corresponding angles.

(i) $\angle 1$ and $\angle 5$ (ii) $\angle 2$ and $\angle 6$ (iii) $\angle 4$ and $\angle 8$ (iv) $\angle 3$ and $\angle 7$.

(ii) **Alternate Interior angles:**

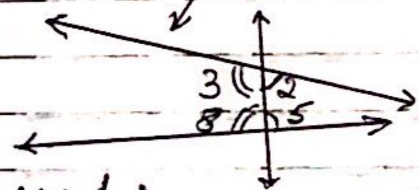


Two pairs (i) $\angle 2$ and $\angle 8$

(ii) $\angle 3$ and $\angle 5$

(iii) **Consecutive interior angles:**

(or Co-Interior angles)

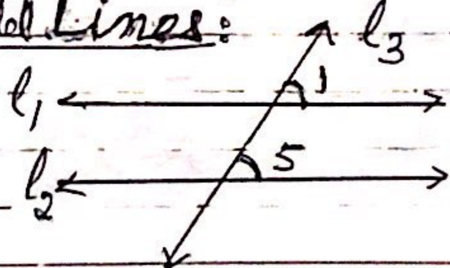


Two pair (i) $\angle 2$ and $\angle 5$

(ii) $\angle 3$ and $\angle 8$.

§8. Transversal intersecting Parallel Lines:

(i) **Corresponding angles axiom:**



If a transversal (l_3) intersects two parallel lines ($l_1 \parallel l_2$), then: the corresponding angles are equal. $\angle 1 = \angle 5$

so all the four pairs.

Conversely:

If two lines l_1 and l_2 are intersected by a transversal such that one pair of corresponding angles are equal then: lines are parallel, $l_1 \parallel l_2$

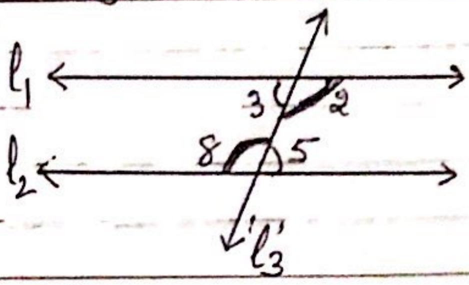
Geometry

Relation between the angles formed by two lines intersected by a transversal:

§ 8

(ii) **Alternate angles:**

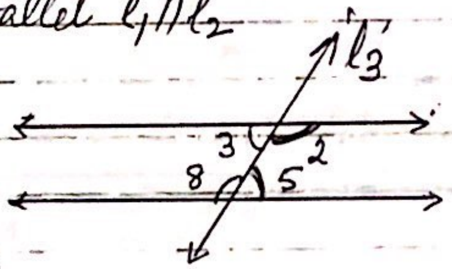
If two parallel lines ($l_1 \parallel l_2$) are intersected by a transversal 'l_3' then both the pairs of alternate angles are equal. (i) $\angle 3 = \angle 5$ and (ii) $\angle 2 = \angle 8$.



Conversely: If any one pair of alternate angles are equal then: lines are parallel $l_1 \parallel l_2$

(iii) **Co-Interior angles:**

If two parallel lines ($l_1 \parallel l_2$) are cut by a transversal 'l_3' Then so formed interior angles on the same side are supplementary.
{ (i) $\angle 2 + \angle 5 = 180^\circ$
(ii) $\angle 3 + \angle 8 = 180$



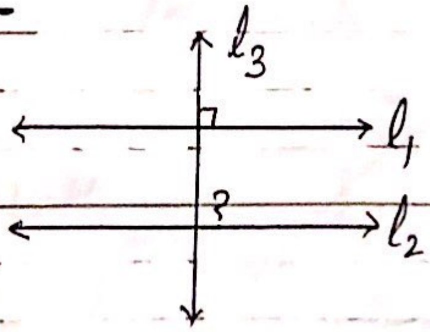
Conversely: If any one pair of interior angles on the same side are supplementary,

(1) $\angle 2 + \angle 5 = 180$ (or $\angle 3 + \angle 8 = 180$)

Then: $l_1 \parallel l_2$

(IV) If $l_1 \parallel l_2$ and $l_3 \perp l_1$

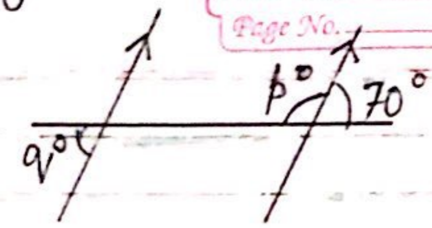
Then $l_3 \perp l_2$.



Geometry

Lines and Angles.

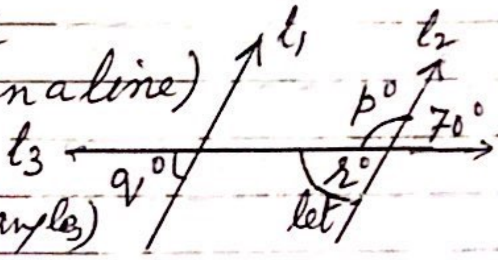
Example 1. The diagram shows a straight line intersecting two parallel lines. Find the value of p and the value of q. --- [2]



S-17/22/Q8

Solution: (i) $p + 70° = 180$ (Linear pair or angles on a line)

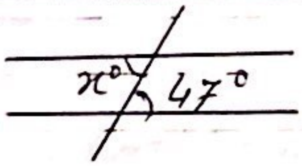
$\therefore p = 110°$ ✓



(ii) angle $r = 70°$ (Vertical angles)

$\therefore \angle q = \angle r = 70°$ ✓ ($l_1 \parallel l_2$ cut by trans. l_3 corresponding angles)

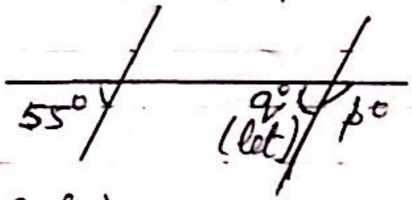
Example 2. Find the value of x. ---[1]



Ans: $x = 47°$ (Alternate Int angles)

M-16/22/Q4(a)

Example 3. Find the value of p. ---[2]



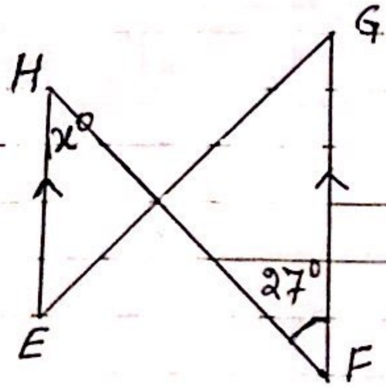
Solution: $p° + q° = 180°$ —(i) (linear pair)

$q = 55°$ (corresponding angles)

W-13/21/Q3

from (i) $p° + 55 = 180° \Rightarrow p = 125°$ ✓

Example 4. Given EH is parallel to FG. $\angle HFG = 27°$, find x ---[1]



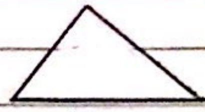
Ans: $EH \parallel FG$ cut by transversal HF.

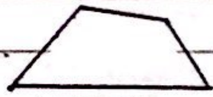
$\therefore x = 27°$ (Alternate angles)

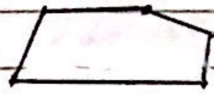
M-17/42/Q6(a)(i)

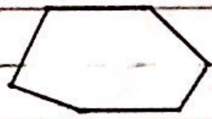
Polygons

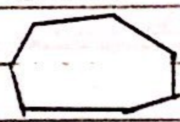
§9 (i) **Polygons:** A closed figure bounded by three or more line segments is called a polygon.

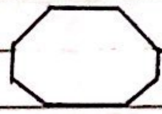
ⓐ Triangle - No. of Sides - 3. ------ 

ⓑ Quadrilateral. No of sides - 4. ------ 

ⓒ Pentagon - No of Sides 5. ---- 

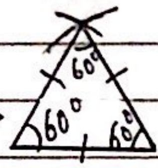
ⓓ Hexagon - No of Sides - 6. ----- 

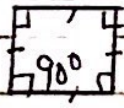
ⓔ Heptagon - No of sides - 7 ------ 

ⓕ Octagon - No of Sides - 8. --------- 

(ii) **Regular Polygon.**

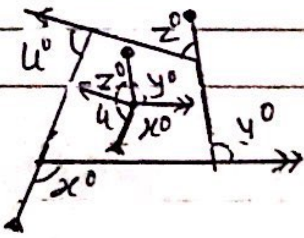
A polygon whose all sides and angles are equal.

ⓐ Equilateral triangle ------------→ 

ⓑ Square ----→ 

ⓒ Regular Pentagon ------------- 

(iii) Sum of Exterior angles of any polygon = $360°$



(iv) Sum of Interior angles of n-sided polygon = $(n-2)\cdot 180°$

(v) One (each) exterior angle of n-sided regular polygon = $\frac{360°}{n}$

(vi) One (Each) interior angle of n-sided regular polygon = $(n-2)\cdot 180° / n$

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Triangles

Geometry — Date P-9.

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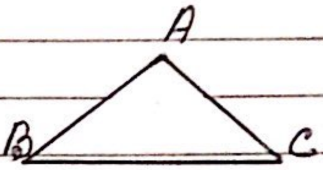
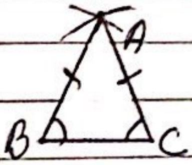
§ 10. Types of Triangles:

(i) On the basis of sides:

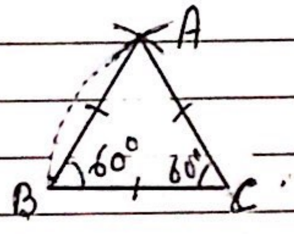
ⓐ Scalene triangle,
No two sides are equal.



ⓑ Isosceles triangle. Two of whose side are equal: AB = AC and the opposite angles are equal $\angle B = \angle C$.

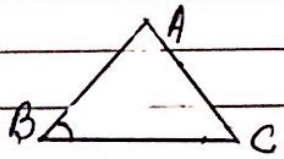


ⓒ Equilateral Triangle:
All three sides are equal. AB = BC = CA
and each angle = 60°

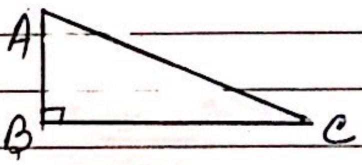


(ii) On the basis of angles:

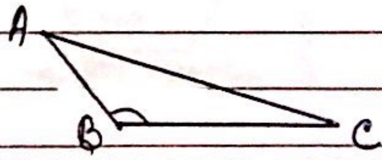
ⓐ Acute triangle: All the three angles are acute.



ⓑ Right angled Triangle: One angle is 90°, $\angle B = 90°$

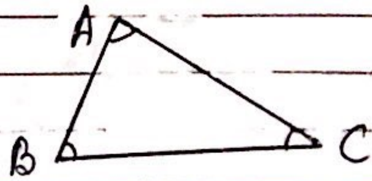


ⓒ Obtuse Triangle: One angle is obtuse angle. $90° < \angle B < 180°$

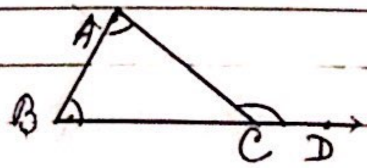


(iii) Properties of Triangle:

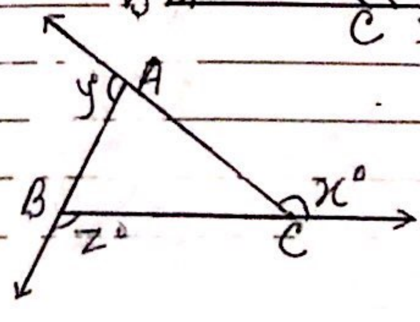
ⓐ Angle sum property:
Sum of the three angles is 180°.
$\angle A + \angle B + \angle C = 180°$



ⓑ Exterior angle Theorem:
If one side of a triangle is produced, the ext. angle $\angle ACD = \angle A + \angle B$



ⓒ Sum of three exterior angles of a triangle is 360°.
$x + y + z = 360°$

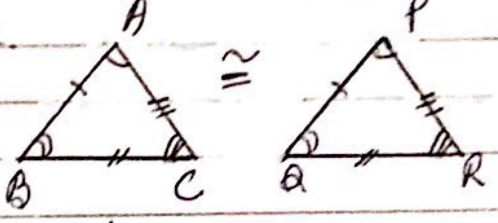


Triangles.

Geometry.

§ 11 Congruent triangles:

(i) Given two congruent triangles,
$\triangle ABC \cong \triangle PQR$ Then.



(a) The corresponding sides are congruent:

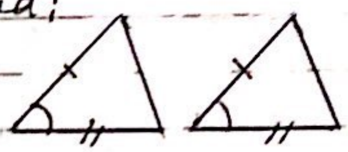
$AB = PQ$; $BC = QR$ and $AC = PR$

and, (b) The corresponding angles are equal:

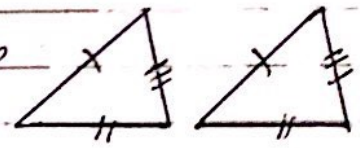
$\angle A = \angle P$; $\angle B = \angle Q$ and $\angle C = \angle R$.

(ii) Conditions required for two triangles to be congruent:
One of the following four is required:

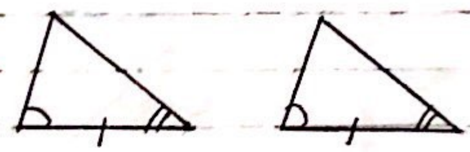
(a) SAS Two sides and included angle.



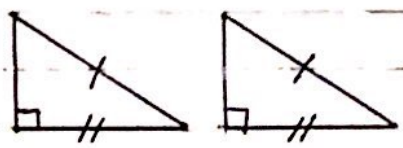
(b) SSS All three corresponding sides be equal.



(c) ASA (or SAA) One side and the two corresponding angles.

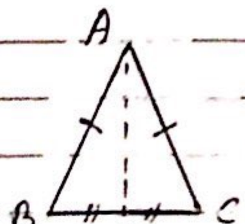


(d) RHS, In two right angled triangles, Hypentenuse and one of the corresponding sides.



§ 12 Theorems:

(i) In an isosceles triangle the line joining the vertex to the mid point of the base is perp to the base.

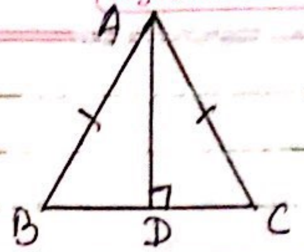


Given: Isosceles $\triangle ABC$, $AB = AC$ and D is the mid point of BC ($BD = DC$).

T.P Then: $AD \perp BC$.
(or $\angle ADB = 90^\circ$)

Triangles

§ 12(ii) In an isosceles triangle, perp from vertex, bisects the base.



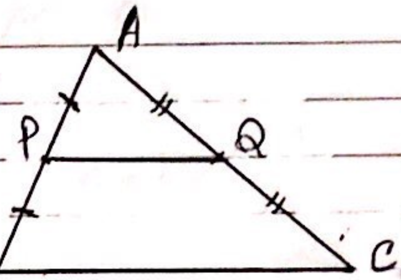
Given: $AD \perp BC$; $AB = AC$

To Prove: $BD = DC$

(Converse of 12(i))

(iii) Mid point theorem:

The line joining the mid points of two sides of a triangle is parallel to the third and half of it.



Given: P and Q are the mid points of sides AB and AC resp.

To Prove: $PQ \parallel BC$ and $PQ = \frac{1}{2} BC$.

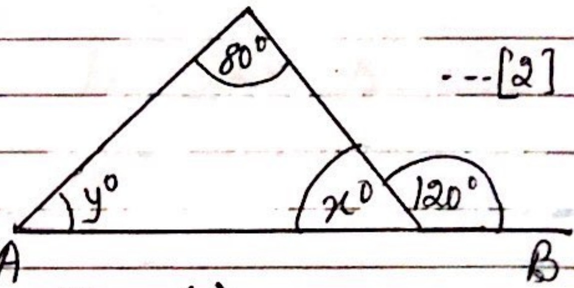
(iv) Converse of (iii)

Given P is the mid point of AB, and $PQ \parallel BC$,

Then, To Prove: Q is the mid point of AC.

← × — × →

Example 5: Find the value of x and the value of y. ---[2]



Solution

(i) $x + 120° = 180°$ (Straight angle)

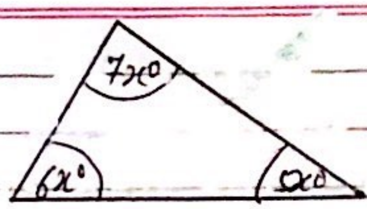
$\therefore x = 60°$ ✓

(ii) $y + 80 = 120$ (Ext angle theo. in a triangle)

or $y = 40°$ ✓

W-17/22/Q3

Example 6: The three angles in a triangle are $5x°$, $6x°$ and $7x°$



(a) Find the value of x ---[2]

(b) Work out the size of the largest angle in the triangle. ---[1]

S-17/23/10

Solution (a) $5x+6x+7x = 180°$ (Sum of Interior angle of a triangle is 180°)

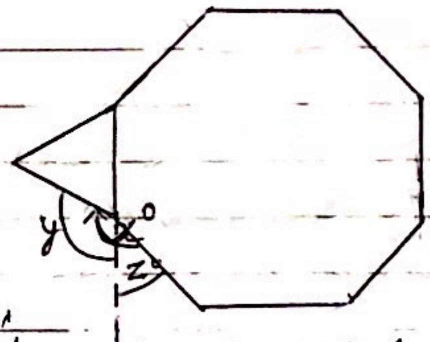
or $18x = 180$

$x = 10$ ✓

(b) largest angle $= 7x° = 7 \times 10 = 70°$ ✓

Example 7. The diagram shows a regular octagon joined to an equilateral triangle.

Work out the Value of x.



Solution:

$x = y + z$

$= 120° + 45°$

$= 165°$ ✓

Ext angle of n-sided regular polygon $= \frac{360°}{n}$

y = Ext angle of Equilateral triangle $= \frac{360}{3} = 120°$

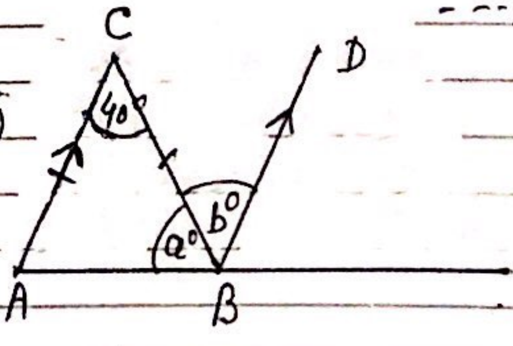
z = Ext. angle of Octagon

$= \frac{360}{8} = 45°$

S-17/21/Q14

Example 8: Triangle ABC is isosceles and AC is parallel to BD. ---[2]

Find the value of a and value of b.



Solution: $b = 40°$ ✓ (Alt interior angle) AC // BD.

In iso $\triangle ABC$ $\angle A = \angle B = a$

$\therefore \angle A + \angle B + C = 180°$

$a + a + 40 = 180°$

$\therefore 2a = 140$ or $a = 70$ ✓

S-16/21/Q9

Example 9. Five angles of hexagon are each $115°$. Calculate the size of the sixth angle. [S-16/21/Q17] --[3]

Solution: Let sixth angle $= x°$

$\therefore x + 115° \times 5 = 720°$

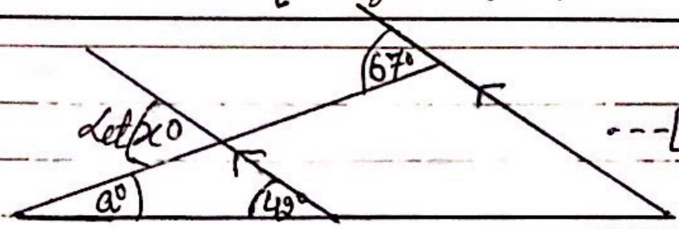
$\therefore x = 720° - 575°$

$x = 145°$ ✓

$\because$ For n-sided polygon sum of int. angle $= (n-2)180°$
$\because$ Sum of six angle of hexagon $= (6-2) \times 180° = 720°$

Example 10.

Find the value of a. ---[2]



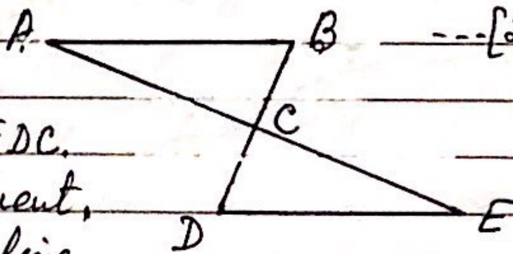
Solution: $x° = 67°$ (corresponding angles) [W-16/22/Q2]

also $x° = a + 42°$ (Ext angle is equal to the sum of opp. int. angles of a triangle)

$\therefore a + 42 = 67°$

$\therefore a = 25°$ ✓

Example 11: The diagram shows two straight lines, AE and BD, intersecting at C. Angle ABC = angle EDC. Triangles ABC and EDC are congruent. Write down two properties of the line segments AB and DE. ---[2]



[S-15/22/Q7]

Solution: Given $\Delta ABC \cong \Delta EDC$

$\therefore AB = ED$ ✓ (corresponding sides)

and angle ABC = angle EDC

$\therefore AB \parallel DE$

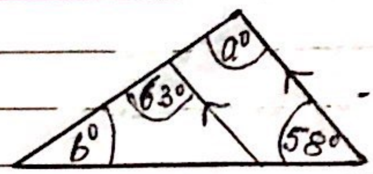
($\because$ angle formed by lines AB and DE cut by transversal BD are Alternate angles equal.)

$\therefore$ Segments AB and DE are equal and parallel.

Example 12: Complete the statements:

$a =$ .... because ----

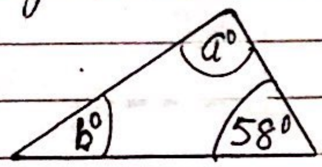
$b =$ .... because -----



--- [4]

M-18/22/Q15

Solution: $a = \underline{63^\circ}$ ✓ because --- corresponding angles.

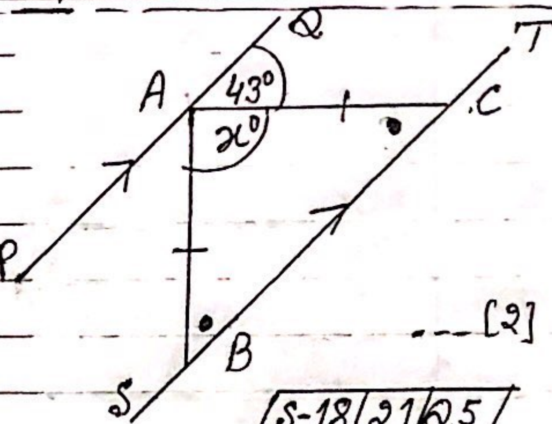


$a + b + 58^\circ = 180^\circ$ [Angle sum prop. in a triangle)

$63 + b + 58 = 180$

$b = 180 - (63 + 58) = 180 - 121 = \underline{59^\circ}$ ✓

Example 13: The diagram shows two parallel lines PAQ and SBCT, AB = AC and angle QAC = 43°. Find the value of x.



--- [2]

S-18/21/Q5

Solution: In ΔABC, AB = AC

$\Rightarrow$ Angle ABC = angle ACB = y (let)

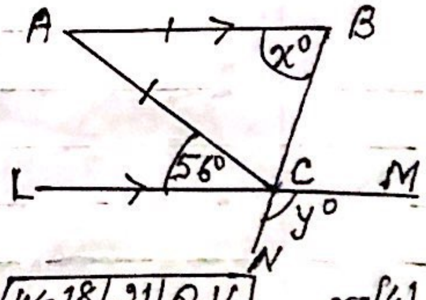
$x^\circ + y + y = 180^\circ$

$+ 2y = 180^\circ$ --- (1)

angle $y = 43$ --- (2) (AQ ∥ BCT, cut by transeversal AC alt int. angle)

from (1) & (2) $\quad x + 2\times 43 = 180 \Rightarrow x = 180 - 86 = \underline{94^\circ}$ ✓

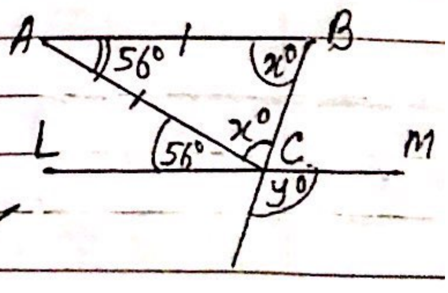
Example 14: The diagram shows an isosceles triangle ABC, with AB = AC. LCM and BCN are straight lines and LCM is parallel to AB. Angle ACM = 56°. Find the value of x and the value of y.



W-18/21/Q16

--- [4]

Solution: Angle BAC = angle ACL = 56° (Alt angles)



$\angle ACB = \angle ABC = x^\circ$ ($\because$ AB = AC, isosc. Δ)

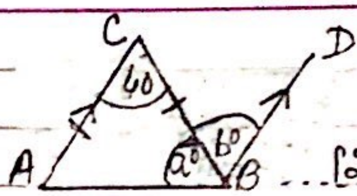
In ΔABC, $x + x + 56 = 180^\circ \Rightarrow 2x + 56^\circ = 180^\circ$

$\Rightarrow 2x = 124 \Rightarrow x = 62^\circ$ ✓

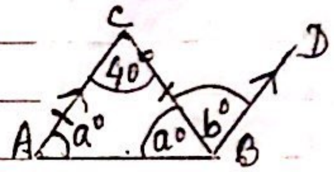
$y =$ angle BCL $= x^\circ + 56^\circ = 62 + 56 = \underline{118}$ ✓

(Vertically opposite angles).

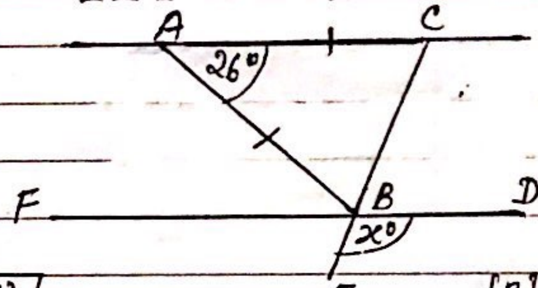
Example 15: Triangle ABC is isosceles,
AC is parallel to BD
Find the value of a and b.
[SP-20/02/Q8] ...[2]



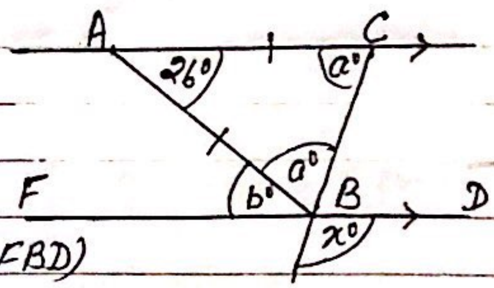
Solution: In isosceles ΔACB, $CA = CB$
$\therefore \angle CAB = \angle CBA = a^\circ$
In ΔABC, $a + a + 40 = 180^\circ$ (angle sum property)
$\Rightarrow a = 70^\circ$ ✓
$b = 40^\circ$ ✓ (Alt interior angle as $AC \parallel BD$ cut by BC)



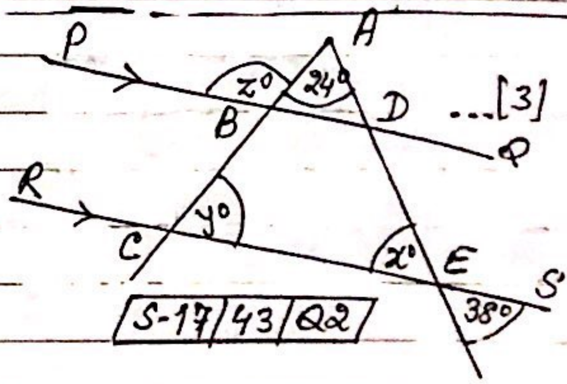
Example 16: AC is parallel to FBD,
ABC is an isosceles triangle,
CBE is a straight line.
Find the value of x.
[S-19/42/Q2(a)] ---[3]



Solution: ABC is isosceles, $AB = AC$
opposite angles $\angle ABC = \angle ACB = a^\circ$ (let)
In ΔABC, $a + a + 26 = 180^\circ$
$\Rightarrow 2a = 154^\circ \Rightarrow a = 77^\circ$ --- ①
$b^\circ = 26^\circ$ --- ② (Alt interior angles $AC \parallel FBD$)
from ① & ② $x = \angle FBC$ (Vertical angles)
$= a + b = 77 + 26 = 103^\circ$ ✓



Example 17: PQ is parallel to RS,
ABC and ADE are straight lines.
Find the values of x, y and z.
...[3]



[S-17/43/Q2]

Solution: $x = 38^\circ$ ✓ (Vertical angles)
In ΔACE, $24^\circ + y + x = 180^\circ$
$\Rightarrow 24 + y + 38 = 180$ ($\because x = 38^\circ$)
$\Rightarrow y = 118^\circ$ ✓

Now $z = \angle ACR$ (Corresponding angles) $= x + 24^\circ = 38 + 24 = 62^\circ$ ✓
(Ext angle)

IG-Maths 0580

Geometry

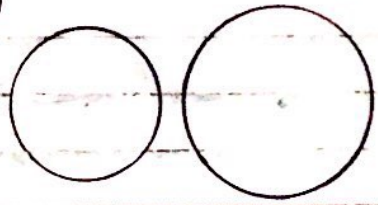
Similar Figures

Date P-16
Page No.

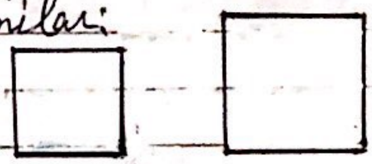
§13. **Similar figures:**

Geometric figures having the same, size may vary.

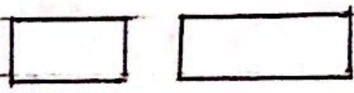
(i) (a) any two circles are similar.



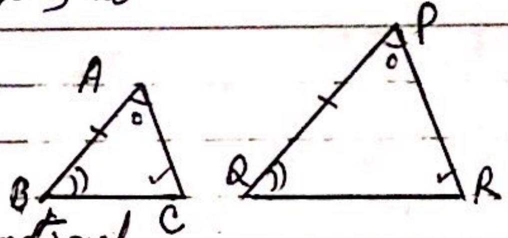
(b) any two squares are Similar.



(c) Are any two rectangles similar? why/why not?



(ii) **Similar triangles:**



(a) $\Delta ABC \sim \Delta PQR$ (Given)

Then:

Corresponding sides are proportional.

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = k$$

and the corresponding angles are equal.

$$\angle A = \angle P, \quad \angle B = \angle Q \text{ and } \angle C = \angle R$$

Conversely:

(b) Conditions required for similar triangles:

One of the following is required for two triangles be similar;

"AAA" (i) Corresponding angles are equal.

$$\angle A = \angle P, \angle B = \angle Q \text{ and } \angle C = \angle R$$

'SSS' or (ii) Corresponding sides are proportional.

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = k.$$

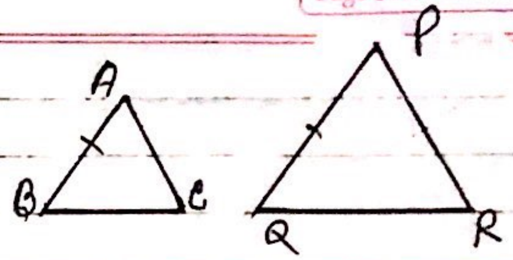
SAS

or (iii) Two pairs of sides are proportional and the respective included angle are equal.

$$\frac{AB}{PQ} = \frac{BC}{QR} \text{ and } \angle B = \angle Q$$

Properties of Similar figs — Geometry

§14 Given $\Delta ABC \sim \Delta PQR$



(a) Corresponding sides are in the same ratio

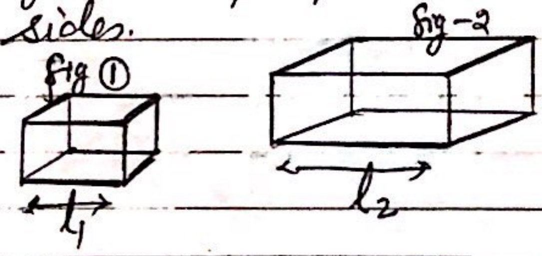
$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = k$$

(b) Areas of similar triangles are proportial to the square of the corresponding sides.

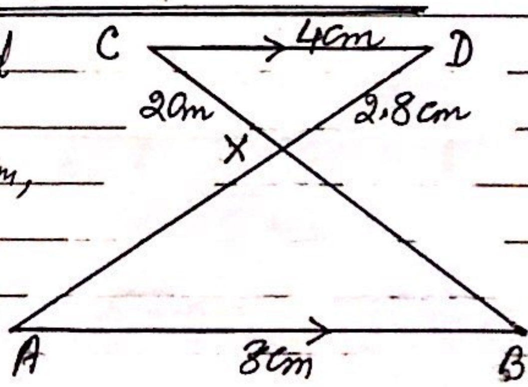
$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta PQR} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2 = k^2$$

(c) Volumes of two solid figures are proportial to the cubes of the corresponding sides.

$$\frac{\text{Volume of fig 1}}{\text{Volume of fig 2}} = \left(\frac{l_1}{l_2}\right)^3$$



Example 18: In the diagram, AB and CD are parallel. AD and BC intersect at X. AB = 8 cm, CD = 4 cm, CX = 2 cm and DX = 2.8 cm.



(a) Complete the statement: ---[1]
Triangle ABX is --- to Triangle DCX.

(b) Calculate AX ----[2]

(c) The area of triangle ABX is $y\ \text{cm}^2$.
Find the area of triangle DCX in terms of y. --[1]

S-17/22/Q24

Solution (a) similar ✓ (AAA similarity theo.) $\because$ $\angle A = \angle D$ Alt angle; $\angle B = \angle C$ Alt angle; $\angle X = \angle X$ Vertical

(b) $\frac{AX}{DX} = \frac{AB}{CD}$ (corresponding sides of similar Δs)

$\frac{AX}{2.8} = \frac{8}{4}$

$\therefore AX = 5.6\text{cm}$ ✓

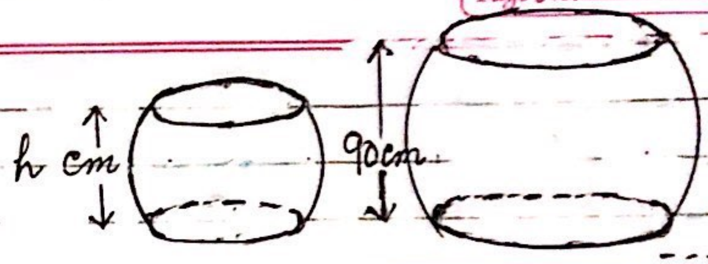
(c) $\frac{\text{Area of } \Delta ABX}{\text{Area of } \Delta DCX} = \left(\frac{AB}{DC}\right)^2$

or $\frac{y}{\text{area } DCX} = \left(\frac{8}{4}\right)^2 = 4$

$\therefore$ Area $DCX = \frac{y}{4}\ \text{cm}^2$

Example 19:

The two barrels in the diagram are mathematically similar.



--- [3]

The small barrel has a height 'h' cm and a capacity of 100 litres. The larger barrel has a height of 90 cm, and a capacity of 160 litres, work out the value of h. [S-17/21/Q11]

Solution:

$$\frac{\text{Volume of small barrel}}{\text{Volume of larger barrel}} = \left(\frac{h_1}{h_2}\right)^3$$

$$\text{or } \frac{100000}{160,000} = \left(\frac{h}{90}\right)^3 \quad [\text{1 Litre} = 1000 \text{ c.c.}$$

$$\Rightarrow \frac{5}{8} = \frac{h^3}{729000}$$

$$\Rightarrow h^3 = \frac{5}{8} \times 729000 = 455,625$$

$$\therefore h = \sqrt[3]{455,625} = 76.95 \text{ cm}$$

Example 20. A model of a house is made using a scale of 1:30. The model has a volume of 2400 cm³. Calculate the volume of the actual house. Give your answer in cubic metres. ----- [3] [W-17/23/Q10]

Solution:

$$\frac{\text{Volume of model house}}{\text{Volume of actual house}} = \left(\frac{l_1}{l_2}\right)^3 = \left(\frac{1}{30}\right)^3$$

$$\text{or } \frac{2400}{V} = \frac{1}{27000}$$

$$\Rightarrow V = 2400 \times 27000 \text{ c.c}$$

$$= \frac{2400 \times 27000}{1000000} \text{ cu. m} \quad [\text{1 cu. m} = 1000000 \text{ cc}]$$

$$V = 64.8 \text{ cubic metres}$$

Similarity

Example 21: A map is drawn on a scale of 1 : 1000000. A forest on the map has an area of $4.6\ cm^2$. Calculate the actual area of the forest in square kilometres. [S-16/22/Q7] ---[2]

Solution:

$$\frac{\text{Area of forest on the map}}{\text{Actual Area of the forest}} = \left(\frac{l_1}{l_2}\right)^2$$

$$\therefore \frac{4.6}{\text{Actual area}} = \left(\frac{1}{1000000}\right)^2$$

[1 Km = 100000 cm

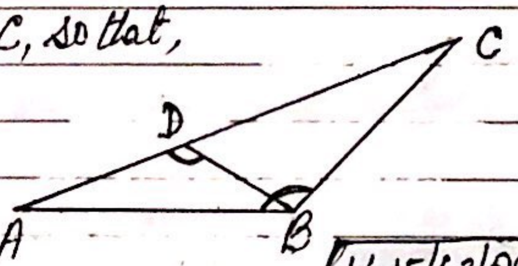
$1\ Km^2 = (100000)^2$]

$$\therefore \text{Actual area} = 4.6 \times 1000000 \times 1000000\ cm^2$$

$$= \frac{4.6 \times 1000000 \times 1000000}{100000 \times 100000}$$

$$= 4.6 \times 10 \times 10 = 460 \text{ sq. kilometres}$$

Example 22. In the diagram, D is on AC, so that, angle ADB = angle ABC.



(i) Show that angle ABD = angle ACB ---[2]

(ii) Complete the statement,

triangle ABD and ACB are ---- [1] [W-15/43/Q8]

(iii) AB = 12 cm, BC = 11 cm, and AC = 16 cm. Calculate the length of BD -- [2]

Solution (i) (a) $\angle ADB = \angle ABC$ (Given)

(b) $\angle CAB$ is common.

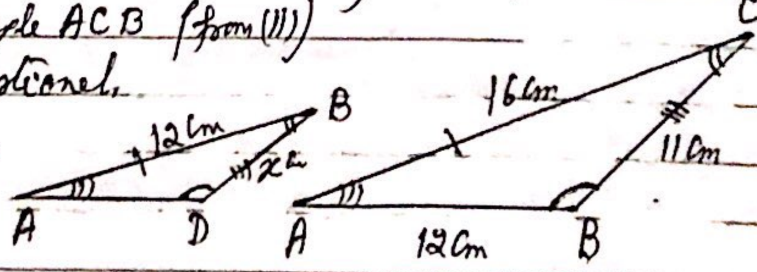
$\therefore$ The third angle ABD = angle ACB ($\because$ Sum of three angles of a triangle is 180°) [and if two angles are equal, then third angles will also be equal.]

(ii) Similar ($\because$ AAA Similarity Theorem)

(iii) Given triangle ADB ~ triangle ACB (from (ii))

Corresponding sides are proportional.

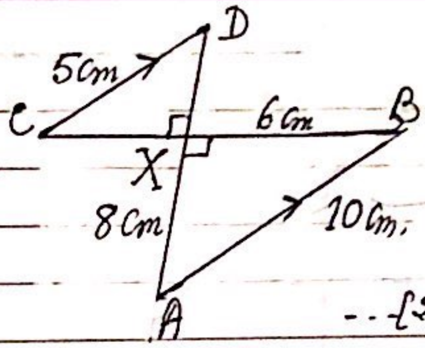
Let BD = x cm.



$$\frac{BD}{BC} = \frac{AB}{AC}$$

$$\Rightarrow \frac{x}{11} = \frac{12}{16} \Rightarrow x = \frac{12 \times 11}{16} = 8.25\ cm$$

Example 23: In the diagram AB and CD are parallel. AD and BC intersect at right angle at the point X.
$AB = 10$ cm, $CD = 5$ cm, $AX = 8$ cm and $BX = 6$ cm.



(i) Use similar triangles to calculate DX. --[2]
(ii) Calculate angle XAB. --[2]

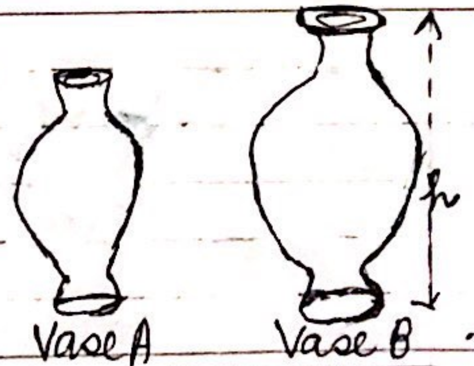
[W-18/43/Q8(a)]

Solution:
(i) angle CDX = angle XAB (Alt int. angles CD∥AB)
angle DCX = angle ABX (----same)
$\angle DXC = \angle AXB$ (both right angle/vertical angles)
$\therefore \Delta DXC \sim AXB$ (AAA similarity)
$\therefore \frac{DX}{AX} = \frac{DC}{AB}$ (corresponding sides)

or $\frac{DX}{8} = \frac{5}{10} \Rightarrow DX = \frac{5}{10} \times 8 = \underline{4 \text{ cm}}$ ✓

(ii) In rt ΔXAB, $\tan XAB = \frac{6}{8} \Rightarrow$ angle $XAB = \tan^{-1}\frac{6}{8} = \underline{36.9°}$ ✓

Example 24: The diagram shows two mathematically similar vases.
Vase A has height 20 cm and volume 1500 cm^3.
Vase B has volume 2592 cm^3.
Calculate h, the height vase B. ---[3]



[W-16/22/Q16]

Solution: $\frac{\text{Volume of Vase B}}{\text{Volume of Vase A}} = \left(\frac{h}{20}\right)^3$

$\Rightarrow \frac{h^3}{8000} = \frac{2592}{1500} \Rightarrow h^3 = \frac{2592}{1500} \times 8000 = 13824$

$h = \sqrt[3]{13824} = 24$

$\therefore h = \underline{24 \text{ cm}}$ ✓

Example 25: Two bottles and their labels are mathematically similar. The smaller bottle contains 0.512 litres of water and has a label with area 96 cm^2. The larger bottle contains 1 litre of water. Calculate the area of the larger label. --[3]

Solution: Let the height of the larger bottle = h_1 cm.
And the height of the smaller bottle = h_2 cm.

$$\frac{V_1}{V_2} = \frac{\text{Volume of larger bottle}}{\text{Volume of the smaller bottle}} = \frac{1000\ cm^3}{512\ cm^3} = \left(\frac{h_1}{h_2}\right)^3 \quad \left[\begin{array}{l} 1\text{ Litre} = 1000\ cm^3 \\ 0.512\text{ lt} = 512\ cm^3 \end{array}\right.$$

$$\Rightarrow \left(\frac{h_1}{h_2}\right)^3 = \frac{1000}{512} = \left(\frac{10}{8}\right)^3 \Rightarrow \frac{h_1}{h_2} = \frac{10}{8} \text{ ---(1)}$$

Now $\dfrac{\text{Area of larger label}}{\text{Area of smaller label}} = \left(\dfrac{h_1}{h_2}\right)^2$

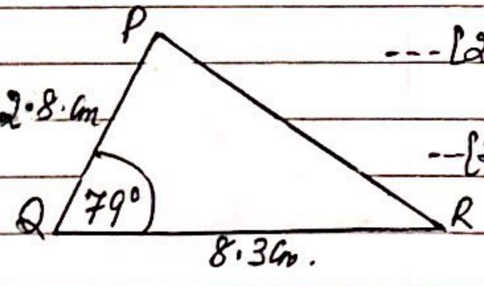
$$\Rightarrow \frac{\text{Area of larger label}}{96} = \left(\frac{10}{8}\right)^2$$

$$\Rightarrow \text{Area of larger label} = \frac{100}{64} \times 96 = \underline{150\ cm^2}$$

Example 26(a) Calculate the area of triangle PQR. ---[2]

(b) Triangle PQR is enlarged by scale factor 4.5, Calculate the area of the enlarged triangle. --[2]

[M-17/22/Q19]

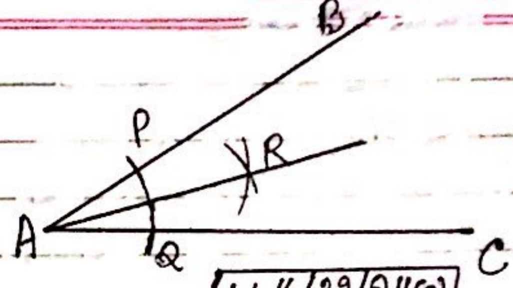


Solution(a) Area of triangle PQR $= \frac{1}{2} \cdot PQ \cdot QR \cdot \sin 79 = \frac{1}{2} \times 2.8 \times 8.3 \times \sin 79°$
$= 11.4\ cm^2$ --(1)

(b) $\dfrac{\text{Area of enlarged triangle}}{\text{Area of triangle PQR}} = \left(\dfrac{l_1}{l_2}\right)^2 = (4.5)^2$ $\left[\text{Given scale factor } \dfrac{l_1}{l_2} = 4.5\right]$

$\Rightarrow$ Area of enlarge triangle $= (4.5)^2 \times$ Area ΔPQR
$= (4.5)^2 \times 11.4$ (from (1))
$= \underline{231\ cm^2}$

§15(i) To draw bisector of an angle:



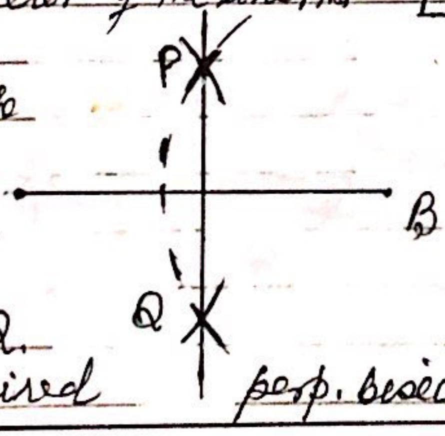
Example 17. Using compasses and a straight edge only, construct the bisector of angle BAC. ---- [2]

[W-16/22/Q11(a)]

With the vertex A as centre draw an arc, intersecting the sides AB and AC at points 'P' and 'Q' respectively.
Now taking radius more than half the distance PQ, draw the arcs with centres 'P' and 'Q', intersecting at the point 'R'.
Join AR and produce, AR is the required angle bisector.

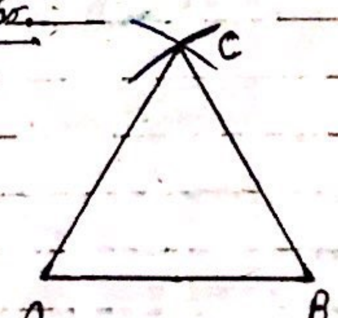
§15(ii) To draw the Perpendicular bisector of a line segment:

Example 18. Using a straight edge and compasses only, construct the perpendicular bisector of the line AB [S-16/23/Q6] --[2]



Taking more than half the length AB, as radius draw arcs with centres 'A' and B. to intersect at P and Q.
Join PQ, is the required perp. bisector.

§15(iii) To construct an Equilateral Triangle:



Example 19. The line AB is one side of an equilateral triangle ABC. Using a straight edge and compasses only, construct triangle ABC. --- [2]

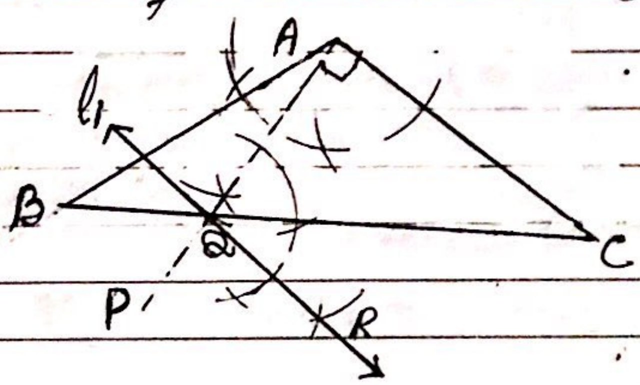
Taking length AB as radius draw the arcs with centre A and as centre B; The two arcs intersect at C.
Join AC and BC. Triangle ABC is the required equilateral triangle.

[M-17/22/Q2]

Constructions

§ 15(iv) Given a line 'l_1', To draw another line 'l_2' at a distance of 'p' units from l_1. ($l_2 \parallel l_1$ and p is the perp. distance between l_1 and l_2)

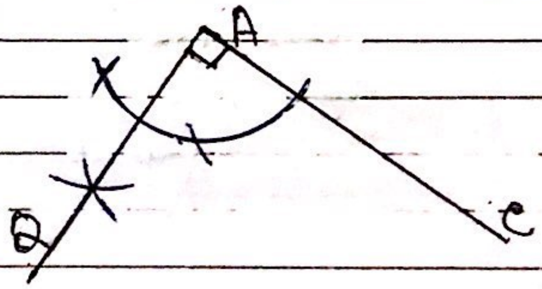
Example 20. Draw a line on B- side of line AC that is 3 cm from AC. --[1]



Draw a line AP through A and perp. to AC. Take a point 'Q' on AP so that AQ = 3 cm, again draw QR perp. to line AQ at Q.

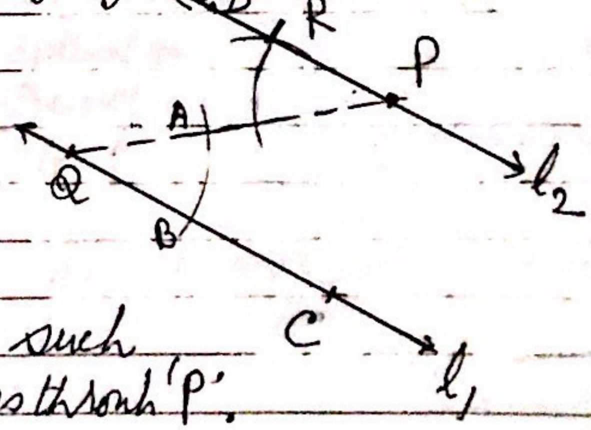
Then 'QR' or l_1 is the required line at a distance 3 cm from line AC on the B-side.

To draw a line AQ perp. to AC at A



AQ is perp. to AC.

§15(v) To draw a line 'l_2' parallel to a given line 'l_1', passing through a point 'P' not lying on l_1.



Take point Q on line l_1.
Join PQ.
Draw angle DPQ equal to angle PQC.
Then line PD or 'l_2' is such that $l_2 \parallel l_1$ and passes through 'P'.