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0580

IGCSE_ Maths

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Geometry Notes_2

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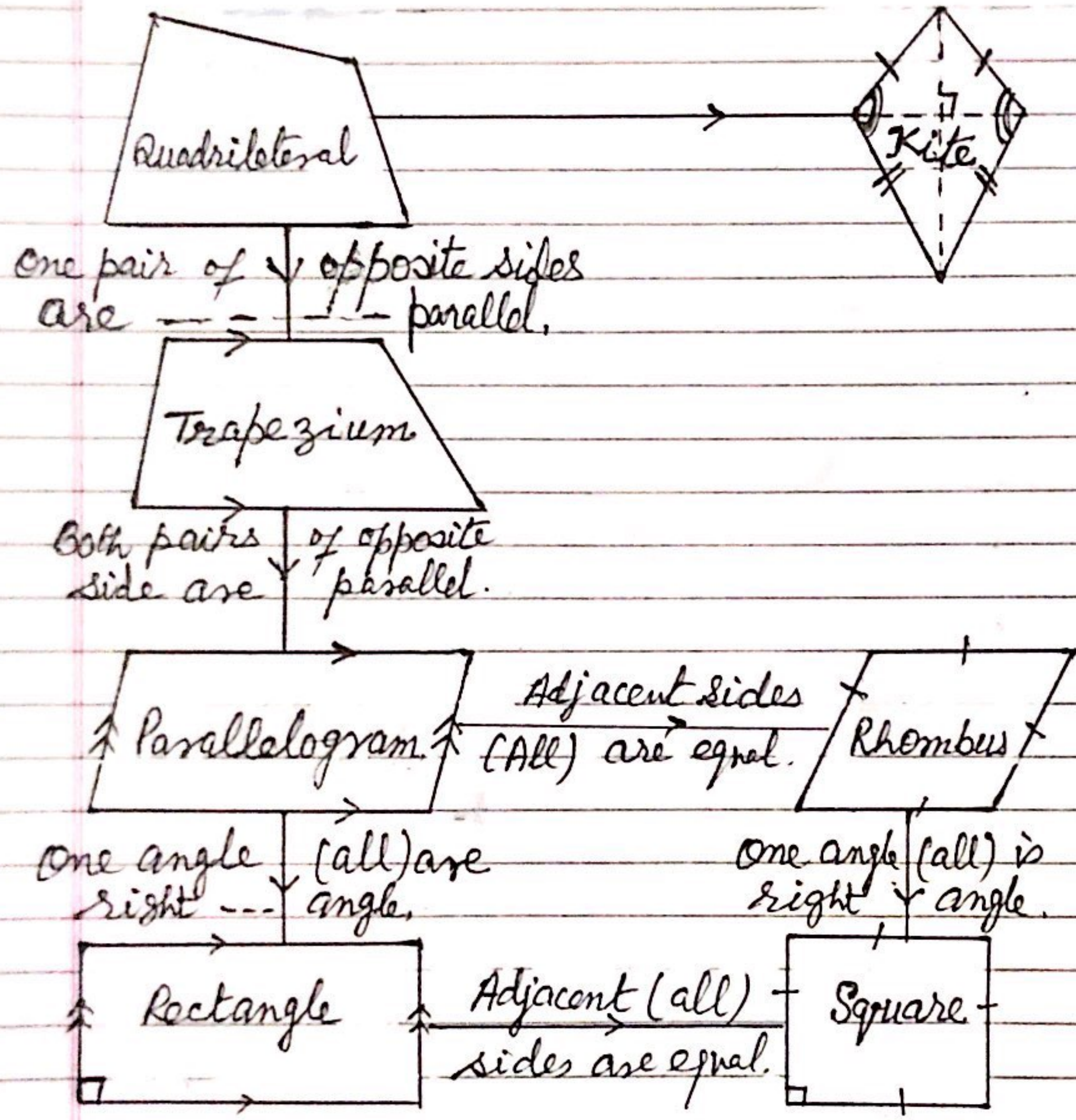
IG-Maths 0580

Quadrilaterals.

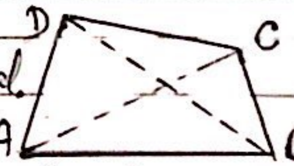
Geometry P-1

§

Types of Quadrilaterals
(Four sided closed geometric figures)



Quadrilateral: A plane figure ABCD, bounded by four line segment is called a quadrilateral.



(i) Adjacent side: AD and AB with a Common vertex A.

(ii) Opposite sides: Two pairs — AD & BC, AB & CD

(iii) Adjacent Angles: $\angle A$ and $\angle B$ with a common side (or $\angle B$ & $\angle C$) ...

(iv) Opposite angle — $\angle A$ and $\angle C$, $\angle B$ and $\angle D$.

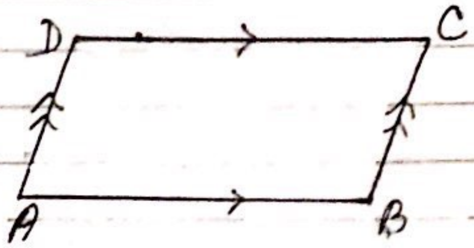
(v) Diagonals: AC and BD

(vi) Angle Sum Property:
$\angle A + \angle B + \angle C + \angle D = 360^\circ$

Quadrilaterals

§ Various Types of Quadrilaterals:

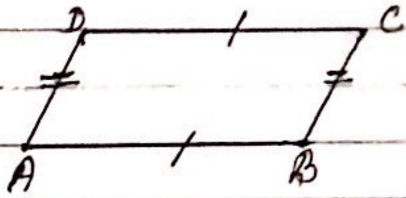
(a) Parallelogram:



Def: A quad. in which both pairs of opposite sides are parallel.

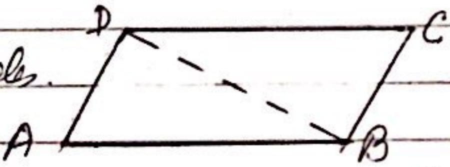
AB ∥ DC and AD ∥ BC.

Properties of a parallelogram:

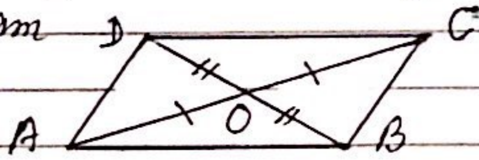


(i) Both pairs of opposite sides are equal. AB = CD and AD = BC

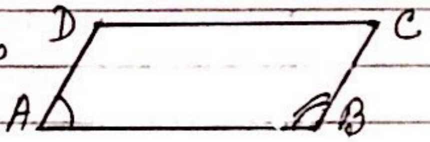
(ii) Both pairs of opp. angles are equal.
$\angle A = \angle C$ and $\angle B = \angle D$.



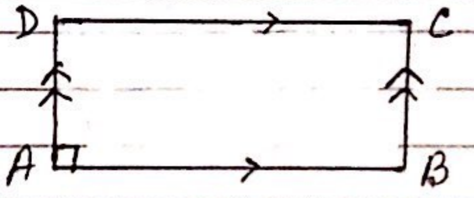
(iii) Each diagonal of a parallelogram divides it into two congruent triangles.
Triangle DAB ≅ Triangle BCA



(iv) The diagonal of a parallelogram bisect each other.
OA = OC and OB = OD.



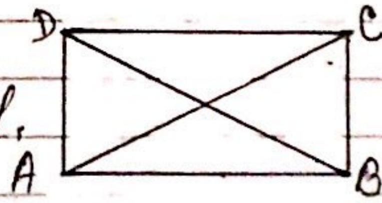
(V) Sum of two consecutive angles is 180°.
$\angle A + \angle B = 180°$



(b) Rectangle:

Rectangle is a parallelogram whose one angle (hence all) is right angle 90°.

(i) All the properties of parallelogram are satisfied by a rectangle. Extra properties

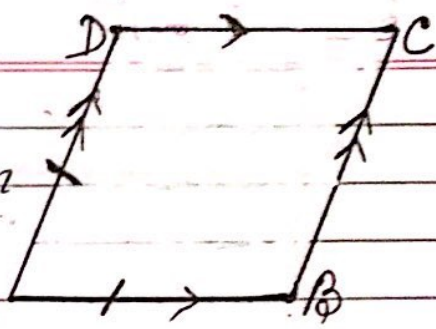


(ii) One angle (all) is 90°

(iii) Diagonals of a rectangle are equal.
AC = BD

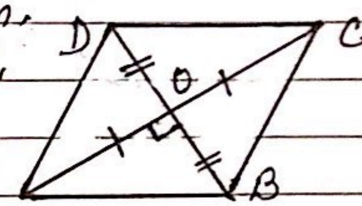
Quadrilaterals.

Varioues types of Quadrilateral:

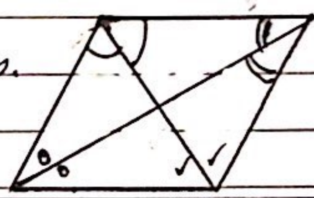


(C) **Rhombus**: Rhombus is a parallegram whos adjacent (all) sides are equal.

(i) All the properterties of parallel gram.
(ii) Adjacent (all) sides are equal.
(iii) Diagonals bisect at right angle.
$OA = OC$ and $OB = OD$.
angle $AOB = 90°$.



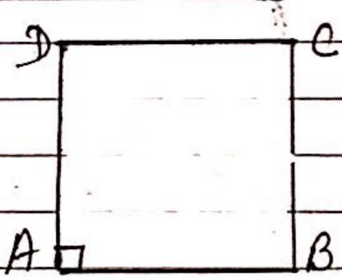
(iv) Diagonals bisect vertial angles.



(d) **Square**: It a rhombus with one (all) angle right angle.

Or

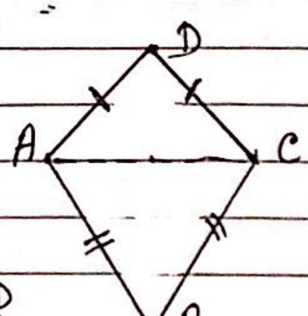
It is a rectangle with adjacent (all) sides equal.



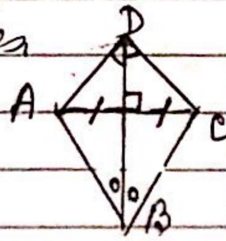
(i) It has all the ~~prop.~~ of parallelogram, rectangle and rhombus.

$\therefore$
- all angles are righ angles.
- all sides are equal / diagonal are equal.
- diagonals bisect eachother at right angle.

(e) **Kite**: Kite has two isosceles triangles of the opposite sides of a diagonal.
$AD = DC$ and $BA = BC$.



(i) Diagonal DB is the perp. bisector of the diagonal AC.
(ii) BD bisects the vertical angles $\angle B$ and $\angle D$.
(iii) $\Delta ADB \cong \Delta CDB$
congruent.



(iv) One pair of opposite angles are equal. $\angle DAC = \angle DCB$.

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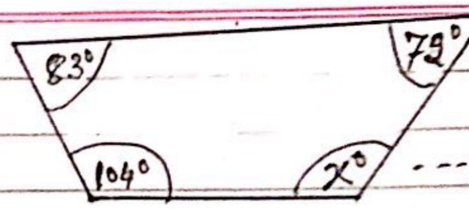
Quadrilaterals and Polygons

Geometry

Example 1 : The diagram shows a quadrilateral.
Find the value of x.



---[1]

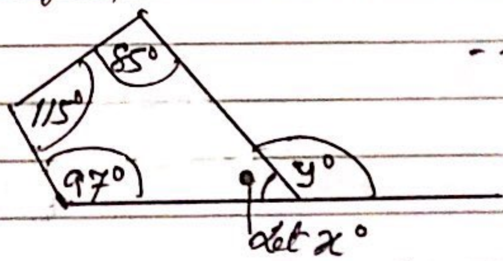
Solution: $x° + 104 + 83 + 72 = 360°$

$\therefore x = 101°$ ✓

[W-17/21/Q1]

(Sum of Interior angles of a quad. is 360°)

Example 2: Find the value of y. ---[2]



Solution $x + 97° + 115° + 85° = 360°$

$\therefore x = 63°$

Now $x + y° = 180°$

or $63 + y° = 180$

$\therefore y = 117°$ ✓

[M-16/22/Q18(b)]

Example 3 : ABCD is a kite, the diagonals AC and BD intersect at X. AC = 12 cm, BD = 20 cm, and DX : XB = 3 : 2

(a) Calculate angle ABC ---[3]

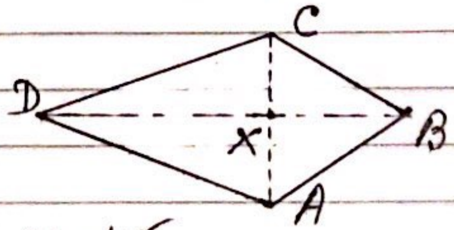
(b) Calculate the area of the kite. ---[2]

[W-13/21/Q21]

Solution: $DX : XB = 3 : 2$

Let $DX = 3x$

and $XB = 2x$



$\therefore DB = DX + XB = 20$

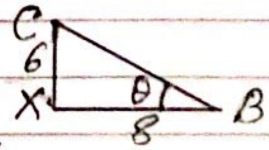
$= 3x + 2x = 20$

or $5x = 20 \rightarrow \therefore x = 4$ ✓

$\therefore XB = 2x = 2 \times 4 = 8$ ✓ cm

$\therefore CX = \frac{AC}{2} = \frac{12}{2} = 6$ cm ($\because$ BD is perp. bisector of AC)

(a) $\tan\theta = 6/8 = 3/4$ (Let angle CBX = θ)



$\therefore \theta = 36.87°$

$\therefore \angle ABC = 2\theta = 2 \times 36.87 = \underline{73.74°}$ ✓

(b) Area of kite = 2 × area of triangle DCB

$= \cancel{2} \times \frac{1}{\cancel{2}} \times DB \times CX$

$= 20 \times 6 = 120$ sq. cm. ✓

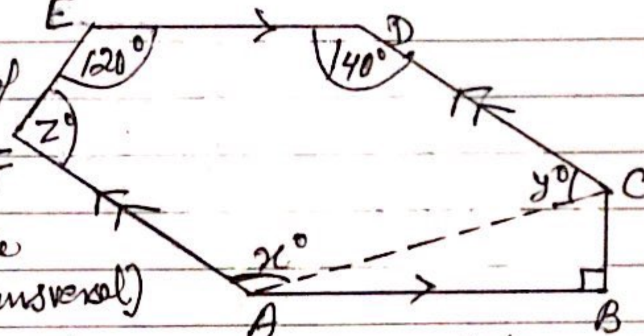
Quadrilaterals and Polygons.

Example 4: In the hexagon ABCDEF, AB parallel to ED, and AF is parallel to CD. angle $ABC = 90°$, Angle $CDE = 140°$ and angle $DEF = 120°$, Calculate angle EFA. ---[4]

[S-15/43/Q6(a)]

Solution: To find $z°$.

Join AC.



AF // CD cut by transverse AC.

$\angle x + \angle y = 180$ —(i)

(Int. angles on the same side of transversal)

Now $x° + y° + 140° + 120° + z° = 540$

$180° + 140 + 120° + z° = 540$

$\therefore z° = 100°$

$\therefore$ Required angle $EFA = 100°$ ✓

Sum of Interior angles of n-sided poly $= (n-2) \times 180°$
In Pentagon ($n=5$)
Sum $= (5-2) \times 180° = 540°$

Example 5 ABCDEF is a hexagon. AB is parallel to ED and BC is parallel to FE. YFE and YABX are straight lines. Angle $CBX = 32°$ and angle $EFA = 90°$.

[S-14/43/Q8(a)]

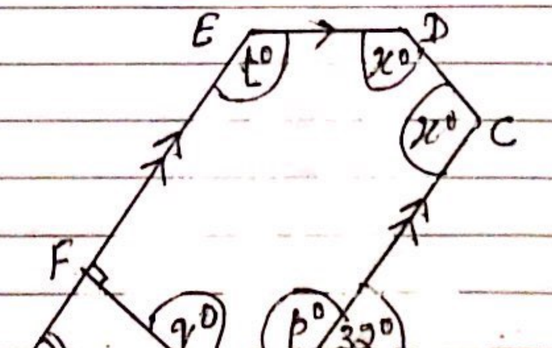
Calculate the value of:

(i) p, ---[1]

(ii) q, [2]

(iii) t, [1]

(iv) x [3]



Solution: (i) $p + 32 = 180°$ (Linear Pairs)

$\therefore p = 148°$ ✓

(ii) $\angle y = 32°$ [YE // BC, corresponding angles]

In ΔFAY

$q = Y + F = 32° + 90 = 122°$ ✓ [$\because \angle F = 90°$]

(iii) $t + \angle Y = 180$ [YAB // ED, Int. angle on the same side of YE]

$t + 32 = 180°$

$t = 148°$ ✓

(iv) Sum of Int. angles $= (n-2) \cdot 180°$

for hexagon $n = 6$

Sum $= (6-2) \cdot 180 = 720°$

$q + p + x + x + t + 90 = 720$

$122° + 148 + 2x + 148 + 90 = 720°$

$\therefore x = 106°$ ✓

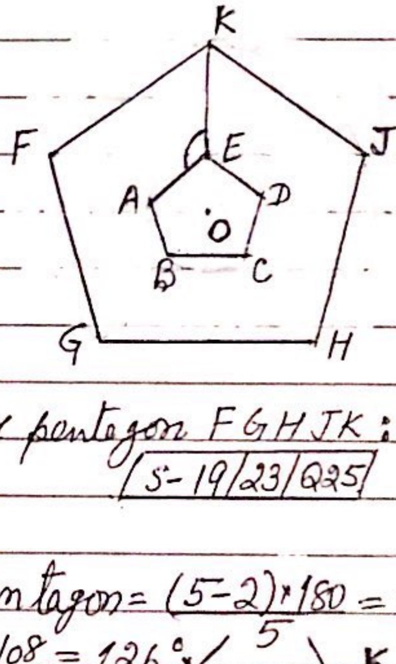
Example 6. Complete the statement:
A quadrilateral with only one pair of parallel sides is called a ...
[S-19/22/Q4] [1]

Solution: Trapezium.

Example 7: A hexagon has five angles that each measures 115°. Calculate the size of the sixth angle. [SP-20/02/Q16] --[3]

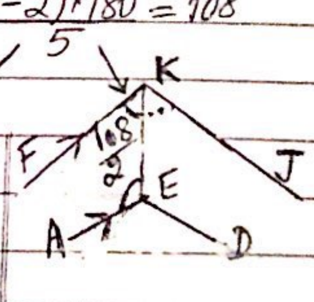
Solution: Sum of interior angles of a hexagon = $(6-2)\times 180 = 720°$
Sum of the measures of 5 angles = $5\times 115° = 575°$
$\therefore$ Measure of sixth angle = $720-575 = \underline{145°}$ ✓

Example 8: The diagram shows two regular pentagons.
Pentagon FGHJK is an enlargement of pentagon ABCDE, Centre O.



(a) Find angle AEK. ---[4]
(b) The area of pentagon FGHJK is $73.5\ cm^2$. The area of pentagon ABCDE is $6\ cm^2$. Find the ratio, perimeter of pentagon FGHJK : perimeter of pentagon ABCDE. [S-19/23/Q25] --[2]

Solution: Measure of one angle of regular pentagon = $\frac{(5-2)\times 180}{5} = 108°$
(a) angle AEK = $180 - \frac{\text{angle } FKJ}{2} = 180 - \frac{108}{2} = \underline{126°}$ ✓



(b) $\frac{\text{Area of pentagon } FGHJK}{\text{Area of pentagon } ABCDE} = \left(\frac{\text{Perimeter of } FGHJK}{\text{Perimeter of } ABCDE}\right)^2$

$\therefore \frac{\text{Perimeter of } FGHJK}{\text{Perimeter of } ABCDE} = \sqrt{\frac{73.5}{6}} = \sqrt{12.25}$
$= 3.5 = \frac{7}{2}$

$\therefore$ Required ratio = $\underline{7:2}$ ✓

Example 9: The exterior angle of a reguler polygon is $x°$ and the interior angle is $8x°$. Calculate the number of sides of the polygon. ---[3]

S-18/41/Q 8

Solution: Interior angle of regular polygon $= 8x°$

Exterior angle $= x°$

$\therefore 8x° + x° = 9x° = 180° \Rightarrow x° = \frac{180}{9} = 20°$ --- (1)

Exterior angle $= \frac{360}{n} = 20$ from (1)

$\Rightarrow 20n = 360 \Rightarrow n = \frac{360}{20} = \underline{18}$ ✓

Example 10: In the pentagon ABCDE,

angle EAB = angle ABC = 110°

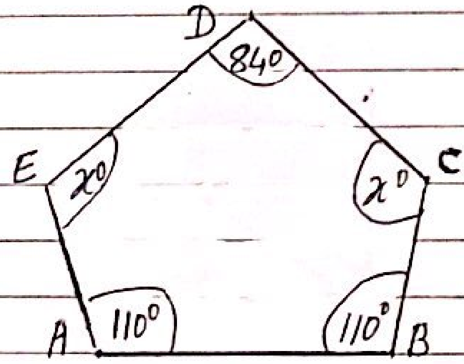
and angle CDE = 84°,

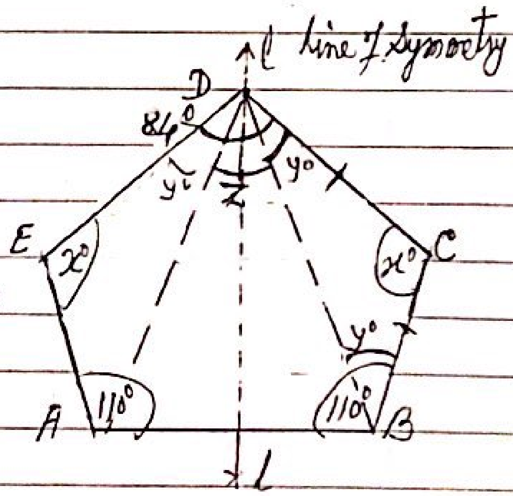
angle BCD = angle DEA = $x°$

(i) Calculate the value of x. ---[2]

(ii) BC = CD, Calculate angle CBD ---[1]

(iii) The pentagon also has one line of symmetry. Calculate angle ADB, ---[1]





Solution:

(i) Sum of Interior angles of the

$= 110 + 110 + x + x + 84 = (5-2) \times 180°$

$\Rightarrow 2x + 304 = 540 \Rightarrow x = \underline{118°}$ ✓ --- (1)

(ii) $BC = CD \Rightarrow$ In $\triangle BCD$,

angle CBD = angle CDB = y (let)

$\therefore 2y + x = 180 \Rightarrow 2y + 118 = 180$

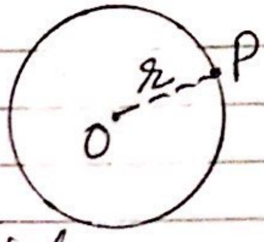
$\Rightarrow y = \underline{31°}$ ✓

(iii) l is the line of Symmetry;

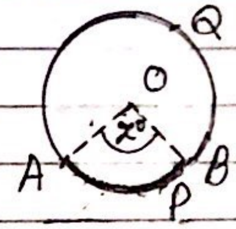
$\therefore$ angle ADB $= 84 - 2y = 84 - 2 \times 31 = \underline{22°}$ ✓

Geometry – Circles

§ **Circle**: Circle is the locus of point 'P' which is at a constant distance 'r' from a fixed point 'O'.
Fixed point 'O' is the centre and the constant distance 'r' is the radius of the circle.

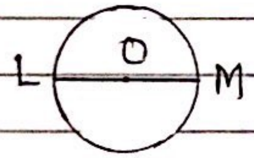


1 (i) **Arc of a circle**: If A and B are any two point of circle, then part of circle.
$\overset{\frown}{APB}$ is Minor arc.
and $\overset{\frown}{AQB}$ is the Major arc.
measure of minor arc $m\overset{\frown}{APB} = x^\circ < 180^\circ$ and $m\overset{\frown}{AQB} = (360 - x)^\circ$

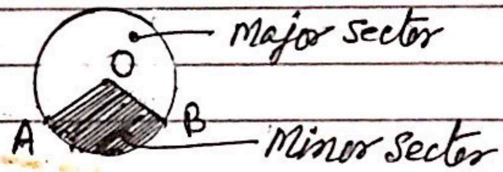


(ii) **Chord** $\overline{AB}$: Segment AB.

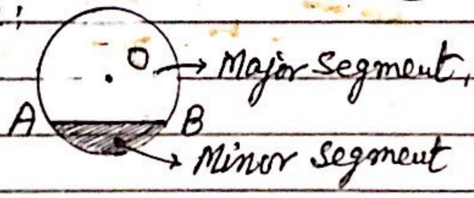
(iii) **Diameter**:
Chord LM passing through the centre.



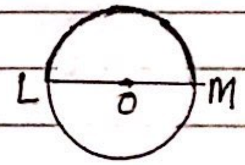
(iv) **Sector**:



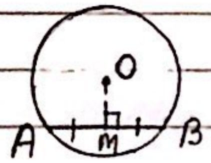
(V) **Segment of a circle**:



(VI) **Semi circle**:
Arc $\overset{\frown}{LQM}$
LM is diameter.

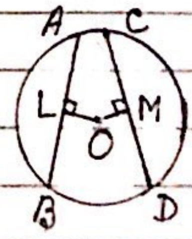


2. **Properties of a circle**:
(i) Perpendicular from the centre to a chord bisects the chord. "OM perp AB"
Then AM = MB.



Conversely: segment joining centre to mid point 'M' of chord, is perp. to chord. $OM \perp AB$

(ii) OL perp AB and $OM \perp CD$.
Equal chords are equidistant from the centre
ie AB = CD, Then OL = OM



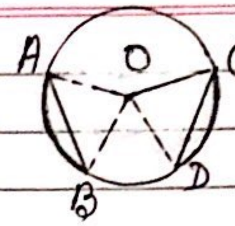
Conversely: If OL = OM then AB = CD

Circles

Properties of circles:

2(iii) If two arc are equal, then the corresponding chords are equal.

$\widehat{AB} = \widehat{CD} \Rightarrow \overline{AB} = \overline{CD}$

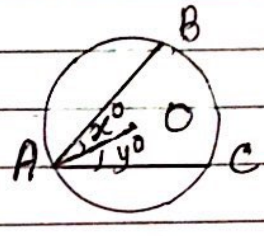


and Conversely $\overline{AB} = \overline{CD} \Rightarrow$ arc $\widehat{AB} = \widehat{CD}$

(iv) Given AB and AC are equal, then AO is the bisector of angle BAC,

$AB = AC \Rightarrow x^\circ = y^\circ$

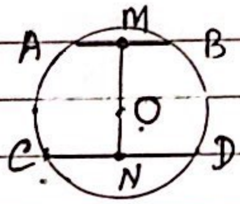
and conversely.



(v) $\overline{AB} \parallel \overline{CD}$ line joining their mid-points passes through centre.

M is the mid point of AB and N is the mid point of CD, Then

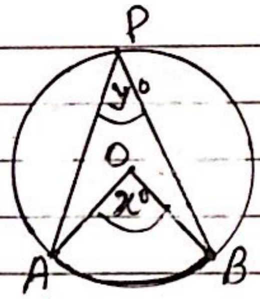
MN passes through the centre O.



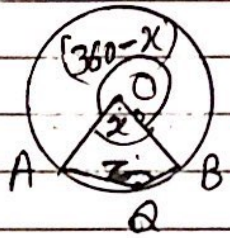
(vi) The angle subtended by an arc $\widehat{AB}$

(a) at the centre x° is double the angle subtended by it any 'P' point on the remaining part of the circle y°.

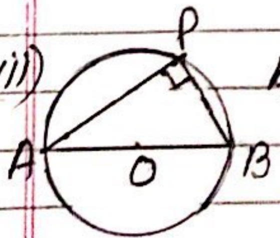
$\therefore x^\circ = 2y^\circ$ or $y = \frac{1}{2}x$



(b) $z^\circ = \frac{1}{2}(360 - x^\circ)$



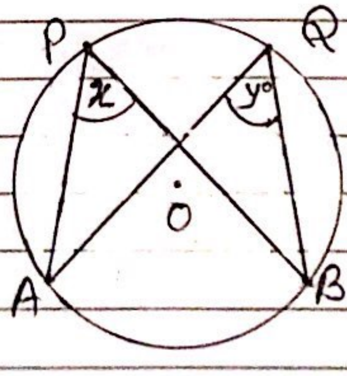
(vii)



Angle in semicircle is a right angle

$\angle APB = 90^\circ$ and conversely.

(viii) Angles in the same segment of a circle are equal $\angle x = \angle y$.



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Geometry

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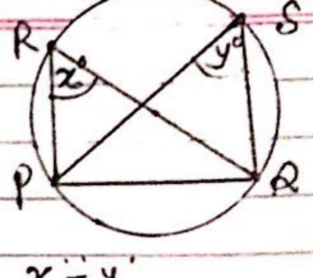
Circles

Properties of Circles:

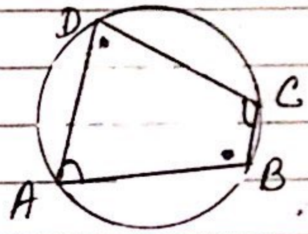
2(IX) (Converse of (viii))

If a line segment $\overline{PQ}$ subtends equal angles at two other points 'R' and 'S' lying on the same side of it, then the four points are concyclic.



(Given $x = y$ Then four points PQRS lie on the same circle.)

(X) The opposite angles of a cyclic quadrilateral are supplementary.

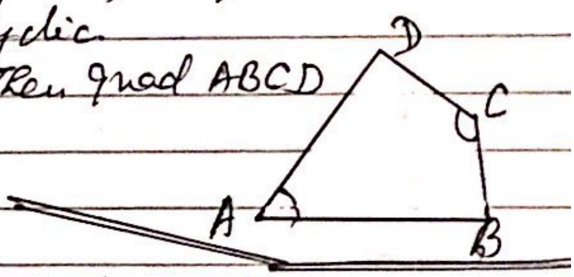


Given a cyclic quad ABCD (vertices of quad. lie on a circle) Then.

$\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$

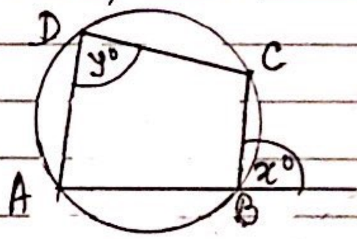
[XI]. Conversely: If one pair of opp. angles of a quad. ABCD are supplementary, the quad is cyclic.

if $\angle A + \angle C = 180$ (or $\angle B + \angle D = 180$) Then quad ABCD is cyclic.



(xii) If one side of a cyclic quad. is produced then the exterior angle so formed is equal to the opposite interior angle.

$\angle x = \angle y$

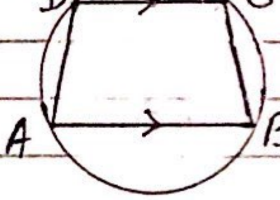


(xiii) If two sides of a cyclic quad (ABCD) are parallel (AB ∥ DC)

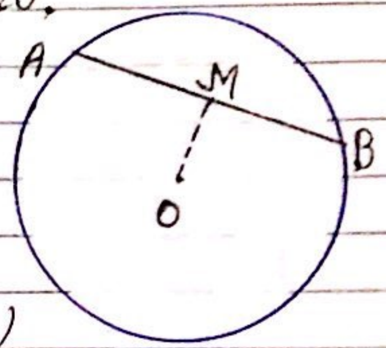
Then:

Remaining two sides are equal AD = BC

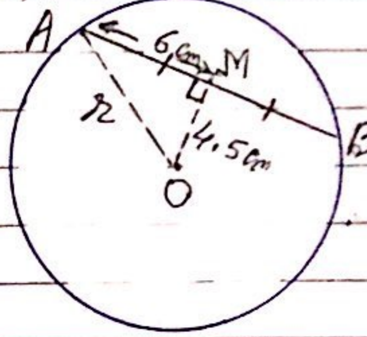
and diagonals are equal AC = BD



Example 11: The diagram shows a circle, centre O.
AB is a chord of length 12cm. M is the mid-point of AB and OM = 4.5cm.
Calculate the radius of the circle. ---[3]
[S-18/22/Q16]



Solution: OM ⊥ to AB (line joining the centre to the mid point of a chord is perp to it)



$AM = \frac{AB}{2} = \frac{12}{2} = 6cm$

$OM = 4.5cm$ (Given)

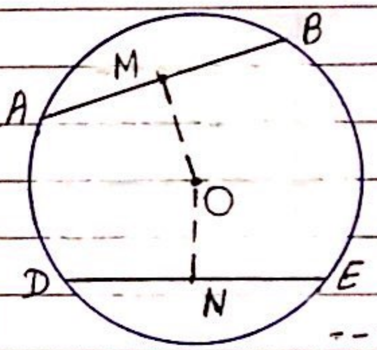
In right $\triangle OAM$, let radius $= r$

$r^2 = AM^2 + OM^2$

$= 6^2 + 4.5^2 = 56.25$

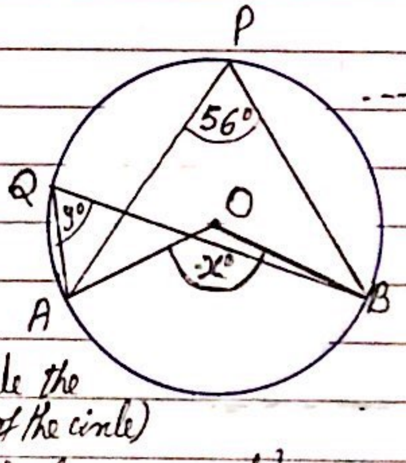
$\therefore r = \sqrt{56.25} = \underline{7.5 cm}$ ✓

Example 12: The diagram shows a circle, centre O,
AB and DE are chords of the circle.
M is the mid point of AB and N is the mid point of DE.
AB = DE = 9cm, and OM = 5cm.
Find ON. [W-18/22/Q2] ---[1]



Solution: ON = OM = $\underline{5cm}$ ✓ (Equal chords are equidistant from the centre). Given AB = DE.

Example 13: A, B, P and Q lie on the circle, centre O, angle APB = 56°.
Find the value of x and the value of y. ---[2]
✓ [S-16/21/Q17]

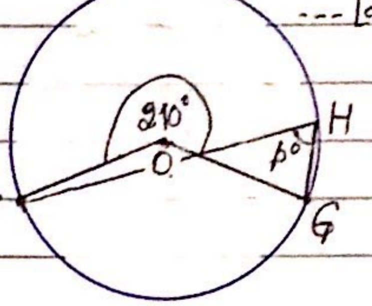


Solution: $x = 2 \times 56 = \underline{116}°$ ($\because$ angle subtended by an arc at the centre is double the angle at the remaining part of the circle)

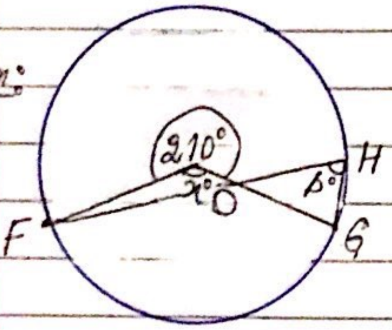
and

$y = \underline{56}°$ ✓ (angles in the same segment of the circle are equal)

Example 14. F, G and H are points on the circle, centre O.
Find the value of p. [S-17/21/Q21] --- [2]



Solution:



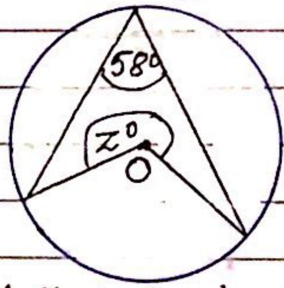
Let angle FOG = x°

$x^\circ + 210 = 360^\circ$ (angles at a point)

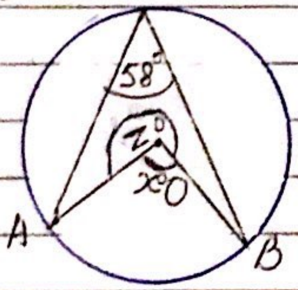
$\Rightarrow x = 150^\circ$ --- (i)

Now $p = \frac{x}{2} = \frac{150}{2} = \underline{75^\circ}$ (from (i) and for arc $\overset{\frown}{FG}$ angle at the centre is double the angle at the remaining circle)

Example 15: The diagram shows a circle, centre O.
Find the value of z. [M-16/22/Q18] --- [2]



Solution:



Let angle AOB = x°

$x = 2 \times 58^\circ$ (angle at the centre is double at the remaining part of the circumference)

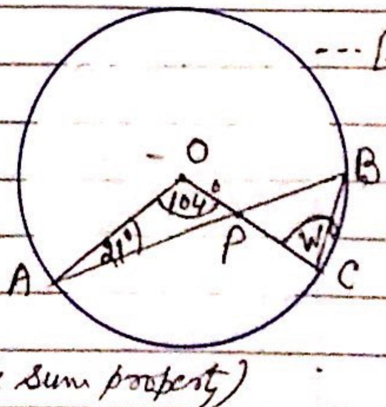
$= 116^\circ$ --- (i)

Now $z + x = 360^\circ$ (Angles at point)

$\Rightarrow z + 116^\circ = 360^\circ$ (from (i) $x = 116^\circ$)

$\underline{z = 244^\circ}$

Example 16: A, B and C are the points of the circle, centre O. AB and OC intersect at P.
Find the value of w. [M-19/22/Q15] --- [3]



Solution: angle ABC = $\frac{104}{2} = 52^\circ$ ---- (i)

angle BPC = angle OPA (Vertical angles)

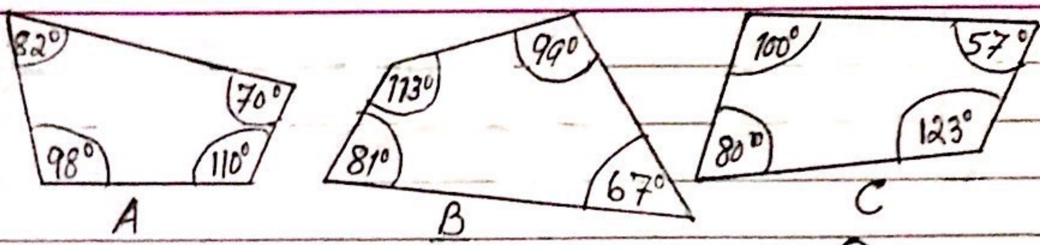
$= 180 - (104 + 21) = 55^\circ$ --- (ii)

Now in ΔPBC, $w + \angle PBC + \angle BPC = 180$ (Angle sum property)

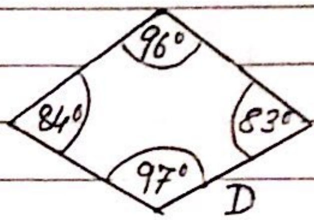
$w + 52 + 55 = 180^\circ \Rightarrow w = 73^\circ$

Cyclic quadrilateral

Example 17:



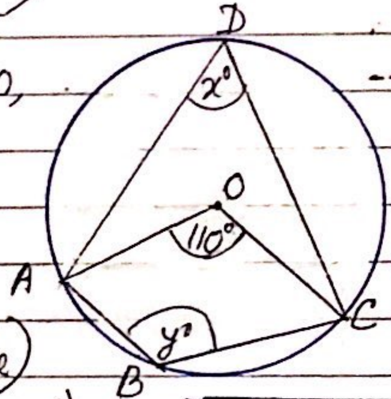
The diagram shows four quadrilaterals, A, B, C and D, which one of these could be a **cyclic** quadrilateral. ---[1]



[W-16/23/Q3]

Solution: Quadrilateral 'B' is cyclic as sum of opp. angles ($99+81° = 180°$, $113°+67 = 180$) is $180°$ (or opp. angles are supplementary)

Example 18: A, B, C and D lie on the circle, centre O, Find the value of x and the value of y. ---[2]



Solution: $x = \frac{110°}{2} = 55°$ ---(i) (angle at the centre is double the angle on the circumference)

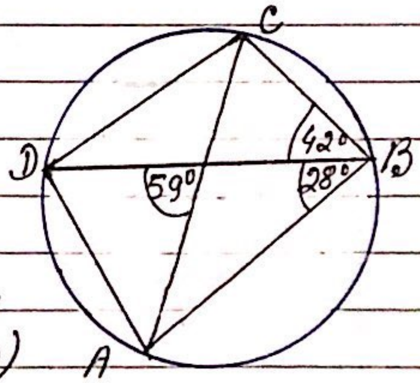
Now $x + y° = 180°$ (opp. angles of the **cyclic** quad).

frm (i) $55 + y = 180 \Rightarrow\ = \underline{125°}$ ✓

[W-16/21/Q9]

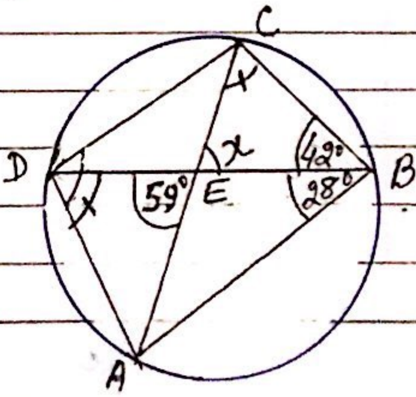
Example 19: A, B, C and D lie on the circle,
Find (a) angle ADC ---[1]
(b) angle ADB ---[2]

[N-13/21/Q12]

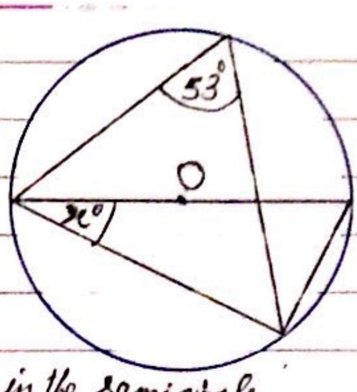


Solution (a) angle ADC + angle ABC = 180° (opp. angles of cyclic quad ABCD)
$\Rightarrow$ angle ADC + (42+28) = 180
$\Rightarrow$ angle ADC = 180 − 70 = $\underline{110°}$ ✓

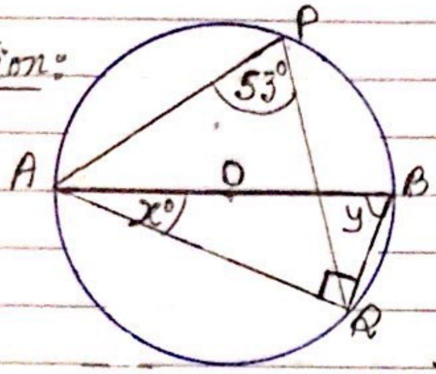
(b) $x = 59°$ (Vertical angles)
angle ADB = angle ACB (angles in the same segment of circle)
$= 180 - (x + 42)$ (Δ CEB)
$= 180 - (59+42) = 180 - 101 = 79$
$= \underline{79°}$ ✓



Example 20: The diagram shows a circle, centre O. Find the value of x. [M-15/22/Q7] ---[2]

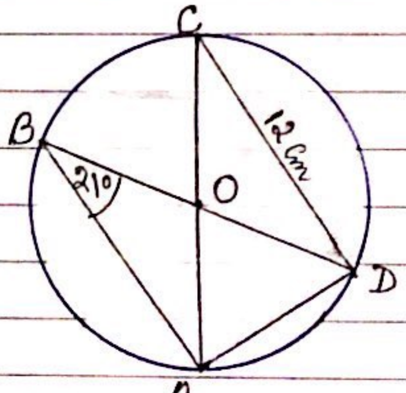


Solution:

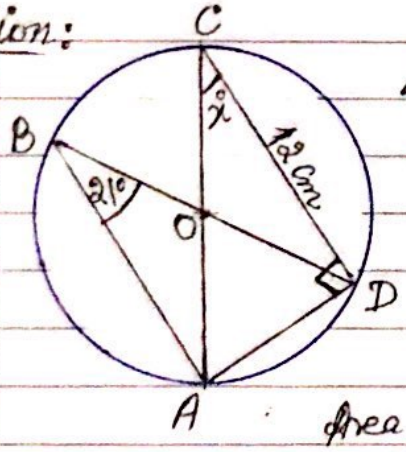


AB is diameter.
angle AQB = 90° (angle in the **semicircle**)
angle $y = 53°$ (angles in the same segment of circle APBQ)
In Δ ABQ
$$x + y + 90° = 180°$$
$$x + 53 + 90 = 180° \Rightarrow x = 37°$$

Example 21. A, B, C and D are the points on the circle, centre O. Angle ABD = 21°. and CD = 12 cm.
Calculate the area of the circle. ---[5]
[W-20/41/Q4(C)]



Solution:



$\angle ACD = x° = 21°$ (angles in the same segment of circle)
$\angle ADC = 90°$ (AC is diameter, angle in **semicircle** is right angle)
In rt Δ ACD, $\frac{12}{AC} = \cos 21° \Rightarrow AC = \frac{12}{\cos 21°} = 12.853$
$\therefore$ radius $r = \frac{AC}{2} = \frac{12.853}{2} = 6.4267$ cm.
Area of circle $= \pi r^2 = \pi (6.4267)^2 = 130 \text{ cm}^2$ (or 129.8)

Example 22 (a) Show that each interior angle of a regular pentagon is 108°. --- [2]

(b) The diagram shows a regular pentagon ABCDE. The vertices of the pentagon lies on a circle, Centre O, radius 12cm.

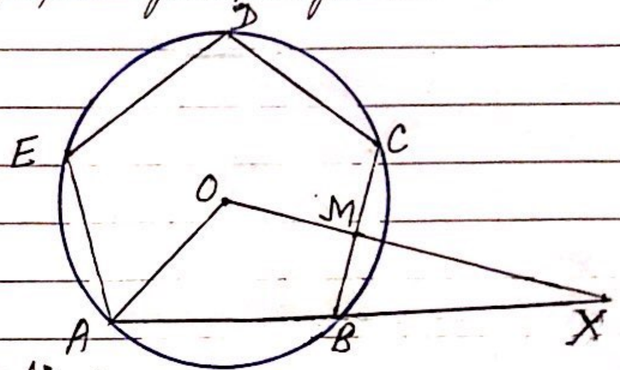
M is is the midpoint of BC.

(i) Find BM ---- [3]

(ii) OMX and ABX are straight lines.

(a) Find BX --- [3]

(b) Calculate the area of triangle AOX. [S-19/42/Q7] -- [3]



Solution (a) Each angle of a regular pentagon $= \frac{(5-2)\times 180^\circ}{5}$ $\left[\theta = \frac{(n-2)180^\circ}{n}\right]$

$= \underline{108^\circ}$ ✓

(b) M is the mid point of BC ⇒ OM ⊥ BC.

(i) Join OB, angle $BOM = \frac{108}{2} = 54^\circ$

In rt △ OBM, $\frac{BM}{12} = \cos 54$

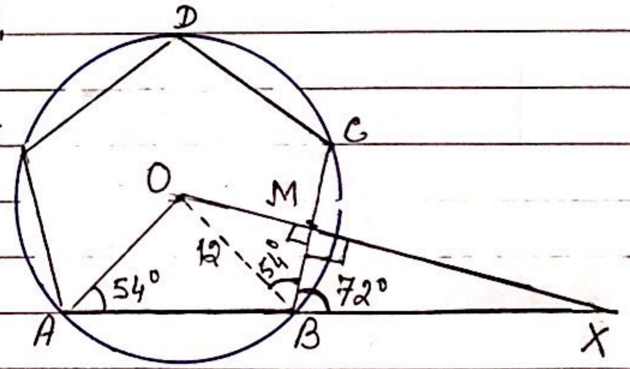
$\Rightarrow BM = 12\times\cos 54 = 7.05\,cm$ ✓

(ii) (a) angle $MBX = 180 - 108 = 72^\circ$

In rt △ MBX

$\frac{MB}{BX} = \cos 72^\circ \Rightarrow BX = \frac{MB}{\cos 72^\circ}$

$\Rightarrow BX = \frac{7.05}{\cos 72} = \underline{22.8^\circ}$ ✓



(b) Area of △ AOX $= \frac{1}{2} OA\times AX\cdot \sin 54^\circ$

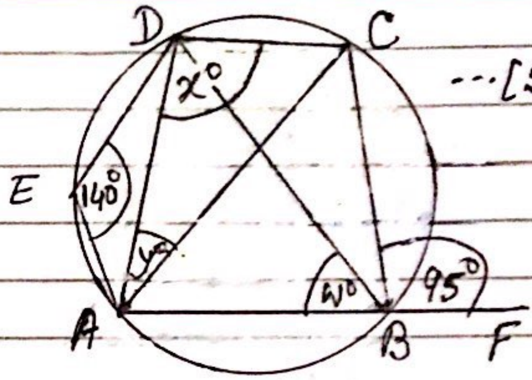
$= \frac{1}{2}\times 12\times 36.91\times \sin 54^\circ$

$= \underline{179.1\ cm^2}$ ✓

$\because AX = AB + BX$
$= BC + 22.81$
$= 2\times MB + 22.81$
$= 2\times 7.05 + 22.81$
$= 36.91\,cm$

Circles and cyclic quadrilaterals.

Example 23: A, B, C, D and E lie on a circle. AB is extended to F.
angle AED = 140° and angle CBF = 95°
Find the value of w, x and y. ...[5]



S-17/22/Q26

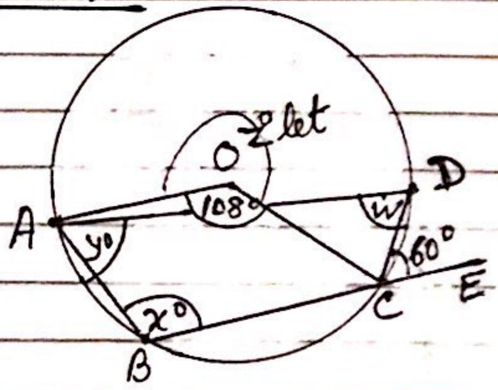
Solution:

$x = 95°$ ✓ [Cyclic quad. ABCD, AB is ext. to F.
∴ Exterior angle CBF = x° opp. int angle.]

$\angle W + 140 = 180$ [opp. angles of cyclic quad ABDE, §18-2(X)]
$\angle W = 40°$ ✓

Now $\angle ABC + \angle CBF = 180°$ [Linear pair or angles on a line]
$W + \angle DBC + 95 = 180$
$40 + \angle DBC + 95 = 180$ $(W = 40°)$
$\angle DBC = 45°$ --- (i)
$\angle Y = \angle DBC$ [Angles in the same segment of circle. or angle subtended by $\widehat{DC}$]
$= 45°$ ✓

Example 24: A, B, C, D are points on the circle, centre O. BCE is a straight line.
angle ADC = 108° and Angle DCE = 60°
Calculate the values of W, x and y. ---[3]

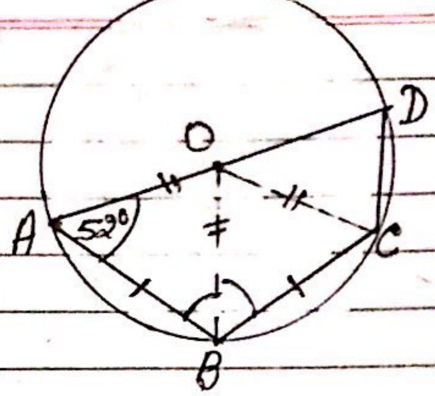


W-17/22/Q22

Solution: $\angle Z + 108 = 360°$
∴ $Z = 252$; $x = \frac{1}{2}\angle Z = \frac{1}{2} \times 252 = 126$ ✓
Cyclic quad DABC, BC is produced
Ext angle DCE = opp. int angle
$60° = y° \Rightarrow y = 60°$ ✓
For cyclic quad ABCD, $x + W = 180$
$126 + W = 180$
$W = 54°$ ✓

Circles and cyclic quadrilaterals

Example 25: The diagram shows points A, B, C and D on the circumference of a circle, centre O, AD is a straight line. AB = BC, angle OAB = 52°. Find angle ADC ---[3]



Solution: Join OB and OC,

$OA = OB = OC =$ radius ---(i)

In ΔOAB, $\angle OBA = \angle OAB = 52$ (isosceles Δ)

Now $\Delta OBC \cong \Delta OAB$ (SSS)

$\therefore \angle OBC = \angle OAB = 52$ ---(ii)

$\therefore \angle ABC = \angle ABO + \angle OBC$ (from (i) & ii)

$= 52 + 52 = 104°$ --- (iii)

Now In cyclic quad. ABCD,

$\angle ADC + \angle ABC = 180$ (opp. angle of cyclic quad)

$\angle ADC + 104 = 180$ [from iii]

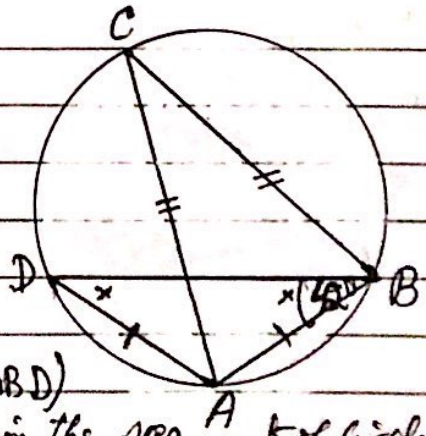
$\therefore \angle ADC = 76°$ ✓

[M-17/42/Q6(b)]

Example 26: The points A, B, C and D lie on the circumference of a circle. AB = AD and AC = BC and angle ABD = 42° ---[3]

Find angle CAB.

[S-17/43/Q2(b)]



Solution: $\angle ADB = \angle ABD = 42°$ (In Iso Δ ABD)

Now $\angle ACB = \angle ADB = 42°$ --(1) (Angles in the segment of circle)

Now In isosceles Δ CAB.

$\angle CAB = \angle CBA = x$ (let)

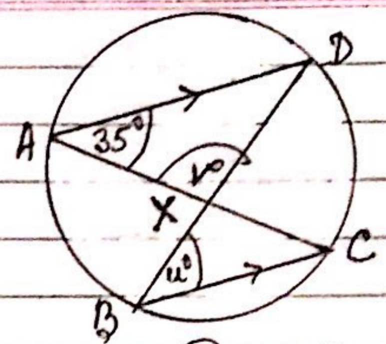
$\therefore$ Angle ACB + angle CAB + angle CBA = 180 (Angle sum Prop.)

$42° + x + x = 180°$

$2x = 138° \Rightarrow x = 69°$ or angle CAB = 69° ✓

Circles and Cyclic Quadrilateral — Geometry

Example: 7(a) A, B, C and D are the points on the circle. AD is parallel to BC. The chords AC and BD intersect at X. Find the value of u and the value of v. ---[3]



Solution: $u = 35°$ ✓ --(i) [∵ angles subtended by arc $\overset{\frown}{DC}$ on the remaining part of circle are equal]

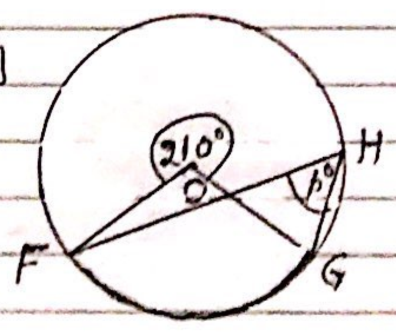
Now angle $ACB = 35°$ [AD ∥ BC cut by transversal AC] Alt. interior angles.
or $XCB = 35$ --(ii)

Now angle $AXB = u + \angle XCB$ [Exterior angle of △XBC is equal to sum of opp. int. angles]
$= 35° + 35°$
angle $AXB = 70°$ ---(iii)

Now $v° + \angle AXB = 180$ (linear pairs, angles on a line)
$v + 70 = 180°$ from (iii)
or $v = 110°$ ✓

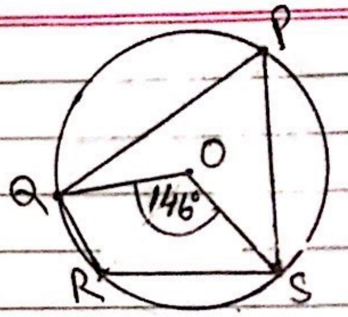
[S-17/21/Q21]

(b) F, G and H are points on a circle, Centre O. Find the value of p. ---[2]



Solution: $\angle FOG + 210° = 360°$ (Angles at a point)
∴ $\angle FOG = 150°$ ---(i)
$\angle FOG = 2\angle FHG$
or $\angle FHG = \frac{1}{2}$ angle FOG
or $p = \frac{1}{2} \times 150°$ (from i) [∵ Angle subtends by an arc FG at the centre is double the angle FHG, subtended by it in the remain part of the circle). § 18:2(VI)(a)
or $p = 75°$

Example 28: The points P, Q, R and S lie on the circumference of the circle, centre O. Angle QOS = 146°; find angle QRS. ---[2]

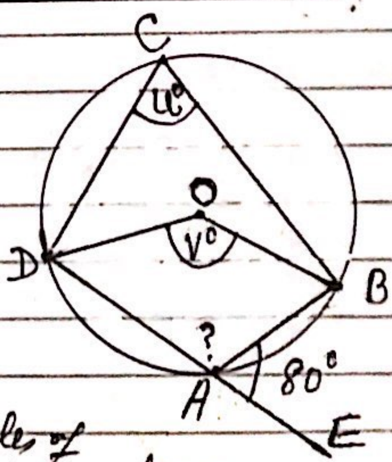


S-17/43/Q2(c)

Solution: angle QPS = $\frac{1}{2}$ angle QOS = $\frac{1}{2}$ × 146° = 73° [§18-2(vi)]

In cyclic quad. angle QRS + angle QPS = 180 [opp. angles of a cyclic quad]
or angle QRS + 73° = 180°
or angle QRS = 107° ✓

Example 29(a) A, B, C and D lie on the circle, centre O, DAE is a straight line. Find the value of u and the value of v ---[2]



Solution: Angle DAB + ∠BAE = 180° [∵ linear pair]

∴ angle DAB = 100 —— ①

Now in cyclic quad ABCD.

∠DAB + ∠DCB = 180 [opp. angles of a cyclic quad are supplementary)

100 + u = 180 from ①

u = 80° ✓

Now V = 2u° (angle at the centre subtended by arc $\widehat{DAB}$ is double the angle at the remaining circle)

or V = 2×8

V = 160° ✓

W-16/42/Q8

(b)

The diagram shows a circle, centre O, radius 8cm, GH is a chord of length 10cm, calculate the length of the perpendicular from O to GH. ---[3]

Solution: Draw OM perp. GH and join OH,

MH = $\frac{1}{2}$ GH = $\frac{1}{2}$ × 10 = 5cm (∵ perp. from centre to a chord bisects the chord).

and OH = radius = 8cm

OM = ?

Using Pythagoras theo. $OM^2 + MH^2 = OH^2 \Rightarrow OM^2 + 5^2 = 8^2$

∴ OM = 6.24 ✓ $OM^2 = 39$

Circles and Cyclic Quad.

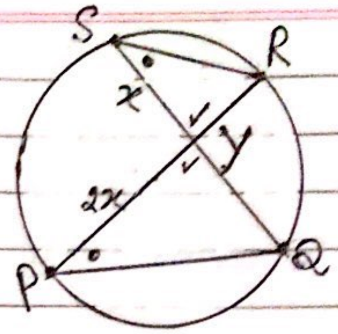
Example 30 (a) All lengths are in centimetres.

P, Q, R and S lie on the circle. PR and QS intersect at Y. $PY = 2x$ and $YS = x$

The area of triangle $YRS = \frac{5}{12}x(x-1)$

The area of triangle $YQP = x(x+1)$

Find the value of x. --[4]



Solution: Triangle $YSR \sim$ Triangle YPQ (Similar) (AA Similarity
{ angle Y - Vertically opp.
angle YSR = angle YPQ
angles in the same segment of circle

$\therefore \dfrac{\text{Area of Triangle } YSR}{\text{Area of Triangle } YPQ} = \left(\dfrac{YS}{YP}\right)^2$

[Areas of similar triangles are proportional to ratio of sq. of corresponding sides]

or $\dfrac{\frac{5}{12}x(x-1)}{x(x+1)} = \left(\dfrac{x}{2x}\right)^2$

$\dfrac{5(x-1)}{12(x+1)} = \dfrac{1}{4} \Rightarrow 20(x-1) = 12(x+1)$

$\Rightarrow 8x = 32$

$\Rightarrow x = 4$ ✓

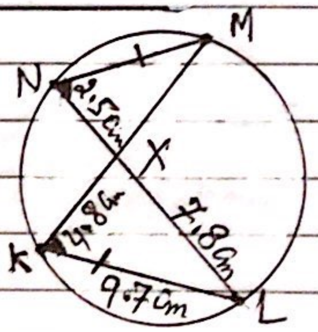
(b) K, L, M and N lie on a circle.

KM and NL intersect at X.

$KL = 9.7$ cm, $KX = 4.8$ cm,

$LX = 7.8$ cm $NX = 2.5$ cm.

Calculate MN. ---[2]



Solution; Triangle MNX ~ (Similar) Triangle LKX [∵ AA Similarity theo.
{ at angle X, Vertically opp. angles.
angle MNX = angle LKX angles in the same segment of a circle

Corresponding sides are in the same ratio:

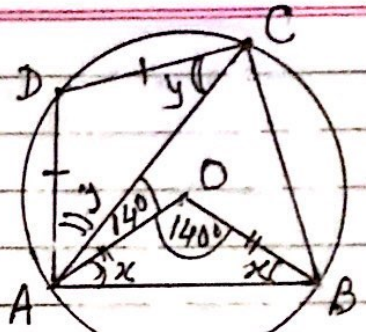
$\dfrac{MN}{KL} = \dfrac{NX}{KX}$

or $\dfrac{MN}{9.7} = \dfrac{2.5}{4.8} \Rightarrow MN = \dfrac{2.5}{4.8} \times 9.7 = 5.05$ cm ✓

[W-16/42/Q8].

Circles and Cyclic Quadrilaterals

Example 31: A, B, C and D are points on the circle, centre O.
Angle $AOB = 140°$ and angle $OAC = 14°$.
$AD = DC$.
Calculate angle ACD. ---[5]



S-14/42/Q6(C)

Solution: In ΔOAB, $OA = OB$ = radius

$\therefore \angle OAB = \angle OBA = x$ (let)

$\therefore 140° + x + x = 180$ (In ΔOAB, angle sum)

$\therefore x = 20°$

$\therefore \angle CAB = 14 + x = 14 + 20 = 34°$ —(i)

$\angle ACB = \frac{1}{2} \times 140 = 70°$ --(ii) (Angle at the centre is double the angle at the remaing circle)

In ΔABC

$\angle ABC + \angle CAB + \angle ACB = 180$

$\angle ABC + 34° + 70° = 180$ from (1) & (2)

$\therefore \angle ABC = 76$ —(iii)

In cyclic quad. ABCD, opp. angles are supplementary,

$\therefore \angle ADC + \angle ABC = 180$

$\angle ADC + 76 = 180$ from (iii)

$\angle ADC = 104$ —(iv)

In ΔADC, $AD = AC$

$\angle DAC = \angle DCA = y$ (let) (isosceles triangle theo)

Now In ΔADC

$\angle ADC + \angle DAC + \angle DCA = 180$ (Angle sum prop)

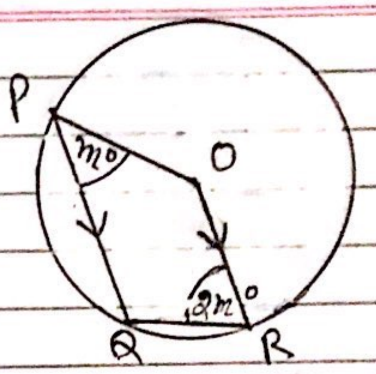
$104 + y + y = 180$ from (iv)

$2y = 76$

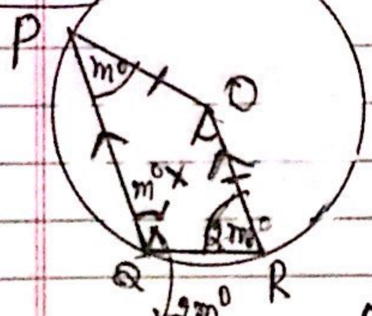
$y = 38$

$\therefore \angle ACD = 38°$ ✓

Example 32: In the diagram, P, Q and R lie on the circle, centre O. PQ is parallel to OR. Find the value of m. ---[5]



Solution:



Join OQ,
OP = OQ = radius.
∴ angle OQP = m° ---(i)

[W-15/43/Q8(C)]

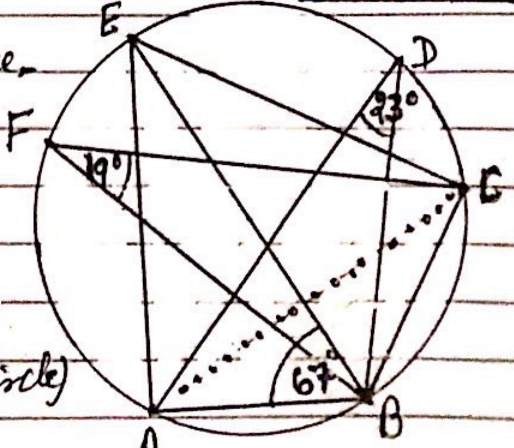
angle QOR = ∠OQP (Alt angle, PQ ∥ QR cut by OQ)
∴ angle QOR = m° ---(ii)

Now In isosceles Triangle OQR (QO = RO = r.)
∠OQR = ∠ORQ = 2m (given)
∠OQR = 2m° ---(iii)

Now In Triangle OQR
angle QOR + angle OQR + angle ORQ = 180° {angle sum Prop in Triangle}
or m + 2m + 2m = 180 (fr (ii) & (iii))
or 5m = 180° ⇒ m = 36° ✓

[W-17/23/Q22]

Example 33: In The diagram, points A, B, C, D, E and F lie on the circumference of the circle, angle BFC = 19° and angle ADB = 23° and angle ABE = 67°
Work out:



(a) angle BEC ---[1]

Solution: angle BEC = angle ADB (∵ angles in the same segment of circle)
= 23° ✓

(b) Work out angle ABC. ---[3]

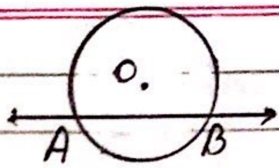
Solution: angle ABC + angle AEC = 180 ---(i) opp. angles of a cyclic quad ABCE.
Now angle AEC = ∠AEB + ∠BEC
= ∠ADB + ∠BFC (Same seg of circle)
= 23° + 19°
= 42° ---(ii)
from (i) angle ABC + 42 = 180 fr (ii)
∴ angle ABC = 138° ✓

(C) Work out angle BCE ---[2]

Join AC.
angle BCE = angle BCA + ∠ACE
= angle ADB + ∠EBA
= 23° + 67°
= 90° ✓

Circle and Tangent.

§19. (i) Secant of circle: A line which intersects a circle in two distinct point.

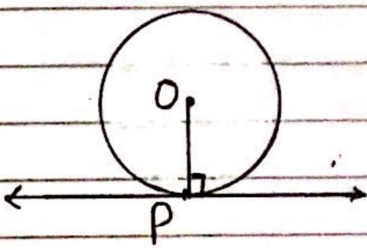


(ii) Tangent line: A line which intersect the circle at exact (one and only one) one point. 'P' and,
The point 'P' is called the point of contact.

§20. Properties of a tangent to a circle;

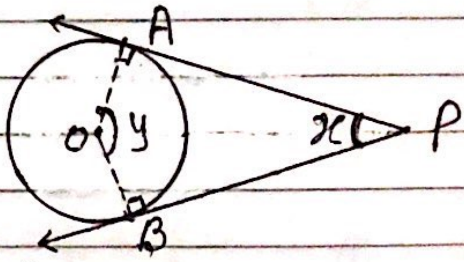
(i) Tangent to a circle is perpendicular to the radius the point of contact.

$OP \perp AB.$



Conversely;
(ii) A line drawn perpendicular to end point 'P' of a radius is tangent to the circle.

(iii) The lengths of two tangents (PA and PB) drawn from an external point 'P' are equal.



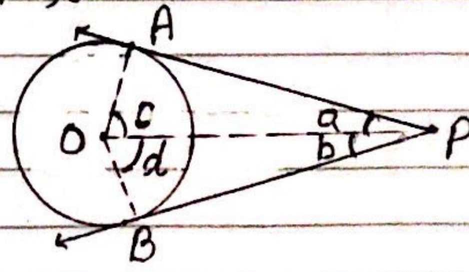
(a) $PA = PB$

(b) angle $OAP = 90°$ and angle $OBP = 90°$

(c) $OA^2 + AP^2 = OP^2$

(d) angle x + angle $y = 180$

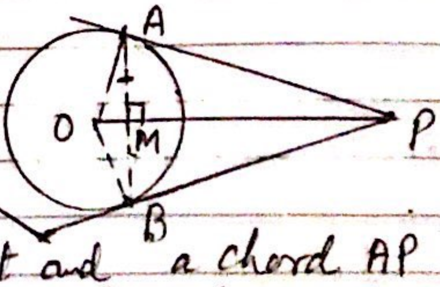
(iv) PA and PB are tangents drawn from an external point 'P' to a circle, centre O. Than line OP bisect angles APB and angle AOB; $a = b$ and $c = d$.



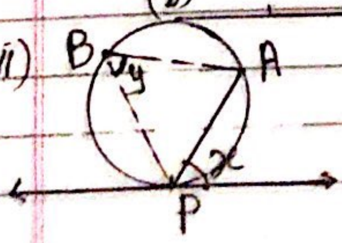
(v) OP is the perp. bisector of AB.

(a) $AM = BM$

(b) $AB \perp OP$.



(vi) $\angle x = \angle y$



Angle between the tangent and a chord AP is equal to the angle in the alternate segment of circle.

IG-Maths
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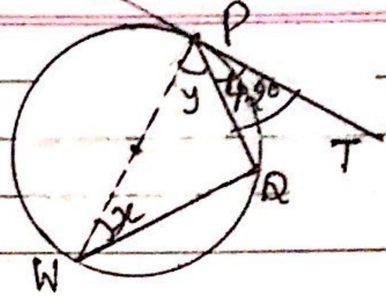
Tangents to a Circle.

Geometry   Date P-24

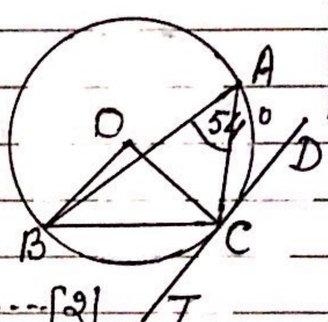
Example 34: In the diagram, PT is tangent to the circle at P. PW is a diameter and angle $TPQ = 42°$. ----[2]
Find angle PWQ. [W-16/21/Q6]



Solution $\angle PWQ = \angle QPT$ (angle between the tangent and chord is equal to the angle in alternate segment of a circle)
$= 42°$

$x + y = 90$ --(i)
$\because \angle Q = 90°$
$y + 42 = 90$ --(ii)
from (i) & (ii) $y = 48°$ ✓

Example 35: A, B and C are the points on a circle, centre O, TCD is a tangent to a circle. Angle $BAC = 54°$



(a) Find angle BOC, giving a reason for your answer....[2]

Solution: angle $BOC = 2 \cdot$ angle BAC
$= 2 \times 54 = 108°$ ✓ ($\because$ angle subtended by an arc at the centre is double the angle at the remaining part of the circle).

(b) When O is the origin, the position vector of C is $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$. ---[1]
Work out the gradient of radius OC.

Solution: C(3, −4). Gradient of OC $(m_1) = \frac{y}{x} = \frac{-4}{3}$ ✓ ---(i)

(c) D is point (7, K), find the value of K.

Solution: Given C(3, −4) and D(7, K).

Gradient of CD, $m_2 = \frac{(K+4)}{(7-3)} = \frac{K+4}{4}$ ---(ii)

Now tangent CD is perp to radius OC.

$m_1 \times m_2 = -1$

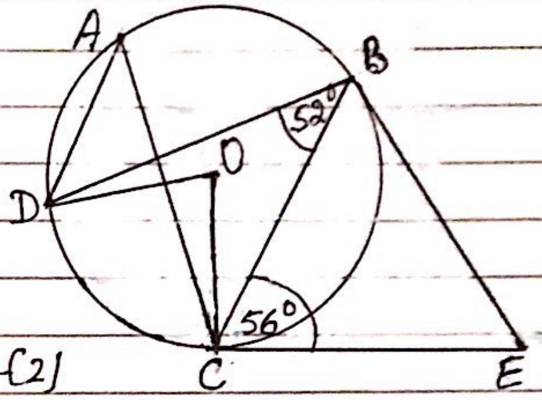
$-\frac{4}{3} \times \left(\frac{K+4}{4}\right) = -1$

or $K + 4 = +3$

$K = -1$ ✓

Tangent to a circle — Geometry

Example 36: A, B, C and D are points on a circle, centre O.
CE is tangent to the circle at C.



(a) Find the sizes of following angles and give a reason for each answer.

(i) angle DAC = --- [2]

Solution: angle DAC = angle DBC (Angles in the same segment of circle)
$= 52°$ ✓

(ii) Angle DOC = --- [2]

Solution: Angle DOC $= 2 \times \angle DBC$
$= 2 \times 52 = 104°$

{∵ Angle subtended by arc $\widehat{DC}$ at the centre is double angle on the remaining part of the circle.

(iii) Angle BCO = --- --- [2]

Solution: OC is perp to the tangent at C
Radius

$\therefore \angle OCE = 90°$

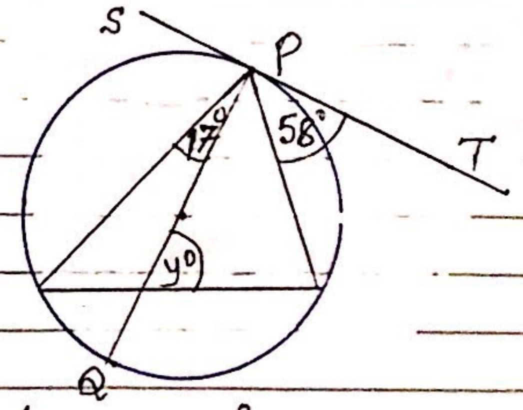
$\angle BCO + \angle BCE = 90°$

$\angle BCO + 56° = 90° \Rightarrow \angle BCO = 34°$ ✓

[W-14 | 43 | Q3(a)]

Example 37: The diagram shows a circle with diameter PQ.
SPT is a tangent to the circle at P. Find the value of y. --- [5]

[S-19 | 42 | Q2]



Solution: Let angle PRA $= x°$

In Δ PRA, RA is produced to S.

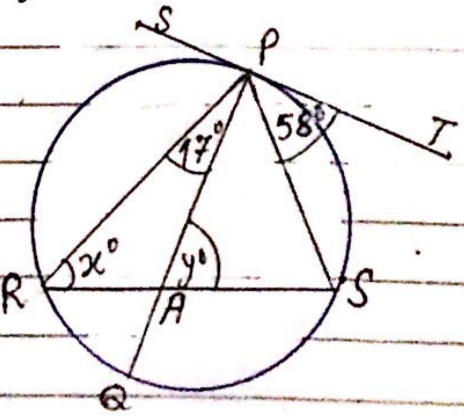
$\Rightarrow y = x + 17°$ --- (I) (Exterior angle Theorem)

Now

$x = 58°$ --- (II) ∵ Angle between tangent and chord is equal to the angle in the alternate segment of circle)

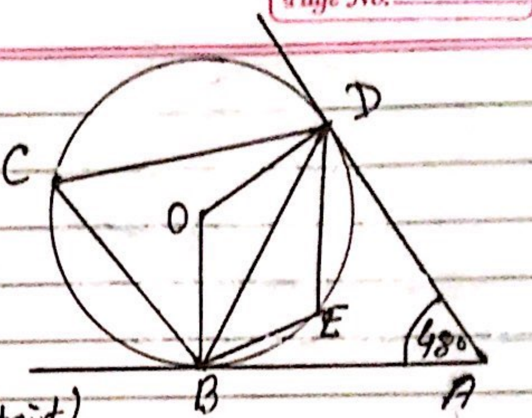
∴ from (I) and (II)

$y = 58 + 17° = 75°$ ✓



Tangent to a circle

Example 38: In diagram, B, C, D and E lie on the circle, centre O. AB and AD are tangents to the circle. Angle BAD = 48°



(a) (i) Find angle ABD ---[1]

Solution: AB = AD (tangents from Ext point)

In ΔABD, $\angle ABD = \angle ADB = x$

Now In ΔABD $\angle ABD + \angle ADB + \angle BAD = 180$ (angle sum of triangle)

$x + x + 48 = 180$

$2x = 132 \Rightarrow x = 66°$ ✓

$\therefore \angle ABD = 66°$ ✓ --- (1)

(ii) Find angle BCD, ---[1]

Solution:

Now required angle BCD = angle ABD ($\because$ angle between tangent and the chord is equal to angle in the alternate segment of circle)

$= 66°$ ✓ ——(2).. from (1)

(ii) Find angle OBD ---[2]

In isosceles ΔOBD (OB = OD = radius)

$\therefore \angle OBD = \angle ODB = y$ let

$\therefore \Delta OBD$, $\angle BOD + \angle OBD + \angle ODB = 180°$

$132° + y + y = 180$

$\Rightarrow y = 24°$ ✓ $= \angle OBD$

{$\angle BOD + 48° = 180°$ $\Rightarrow$ $\angle BOD = 132°$}

(iii) To find angle BED ----[1]

angle BED + $\angle BCD = 180°$ [opp. angles of a cyclic quad BCDE

angle BED + 66° = 180°

fr (2) angle BCD = 66°

angle BED = 114° ✓

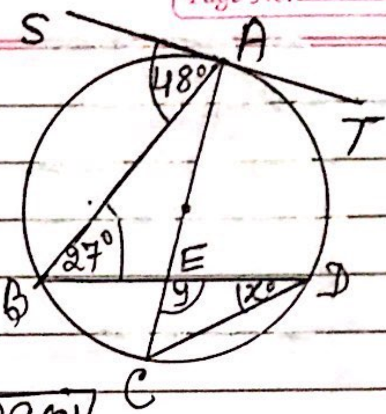
[S-15/42/Q2(a)]

Tangent to a circle.

Example 39: The points A, B, C and D lie on a circle, AC is a diameter of the circle. ST is the tangent to the circle at A. Find the value of

(a) x -- [2]

(b) y -- [2]



S-15/41/Q9(a)

Solution (a) $\angle SAC = 90°$ [angle between radius (AC is diameter) and tangent]

or $\angle SAB + \angle BAC = 90$

or $48° + x = 90°$ [$\because \angle BAC = \angle BDC = x$ In the same segment of circle]

$\therefore x = 42°$ ✓

(b) $\angle ACD = \angle ABD$ (Same segment of circle)

$= 27°$ —— (1)

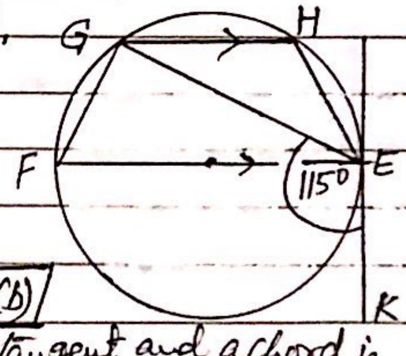
In ΔCDE, $x + y + \angle ACD = 180°$

$42 + y + 27 = 180$ (from (1))

$\therefore y = 111°$ ✓

Example 40: EFGH is a cyclic quadrilateral, EF is a diameter of the circle. KE is a tangent to the circle at E. GH is parallel to FE and angle KEG = 115°

Calculate angle GEH. —— [4]



S-14/42/Q6(b)

Solution: angle GEK = angle GHE (angle between tangent and a chord is equal to the angle in the alternate circle) §20 (VI) Page-40

$115° = $ angle GHE —— (1)

$\angle GEF + \angle FEK = 115$ given

$\angle GEF + 90 = 115$ [$\angle FEK = 90°$ angle between tangent and radius. §20(i)]

$\therefore \angle GEF = 25°$ —— (2)

$\angle HEF + \angle GHE = 180°$ [GH ∥ FE cut by transversal HE]

$\angle HEF + 115° = 180$ by (1)

$\angle HEF = 65°$ —— (3)

Now $\angle GEH + \angle GEF = \angle HEF$

$\angle GEH + 25 = 65$ from (2) & (3)

$\therefore \angle GEH = 40°$ ✓

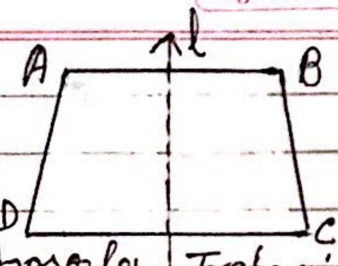
Symmetry

§20(i) Line of Symmetry:

(i) A line passing through a plane shape which divides it into two congruent figures (Reflection of each other in the line) is a line of symmetry.

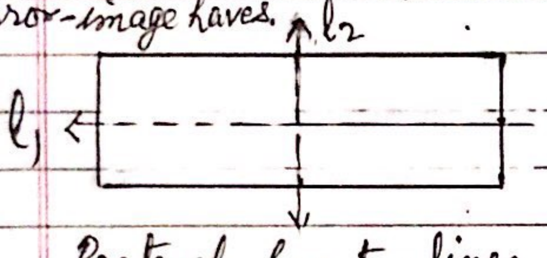
or

(ii) A line of Symm. divides a fig. into two mirror-image haves.

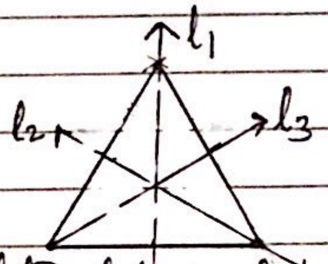


Isosceles Trapezium

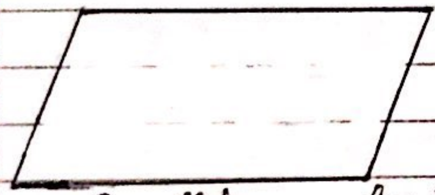
(Here 'l' is the line of symmetry of the isosceles trapezium)



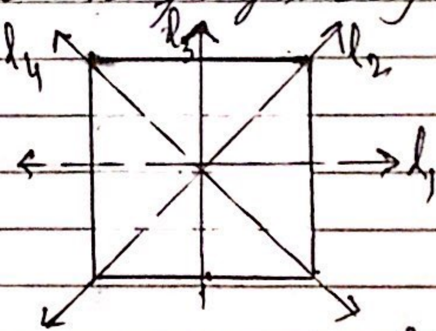
Rectangle has two lines of symmetry l_1 & l_2



Equilateral triangle has three line of Symmetry l_1, l_2, l_3



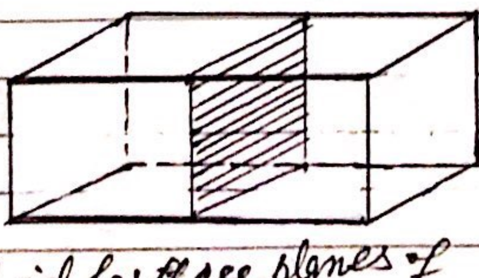
Parallelogram has 'no' line of symmetry



A Square has four lines of Symmetry.

§20 (ii) Plane of Symmetry:

A plane passing through a solid shape, which divides it in two congruent (reflective) solid shapes.



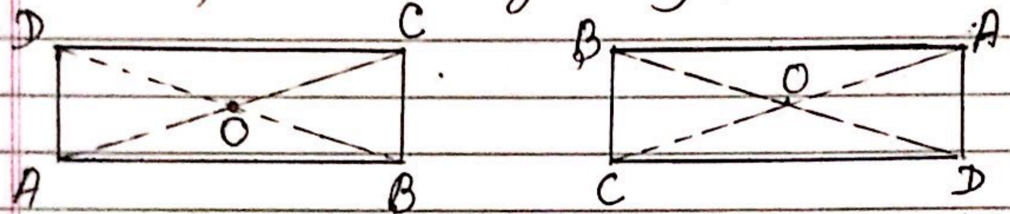
A cuboid has three planes of symmetry (One is shown here)

A Sphere has infinite planes of Symmetry.

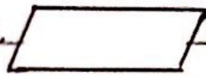
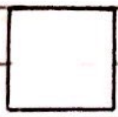
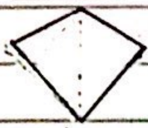
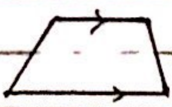
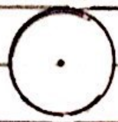
Symmetry

§20(iii) Rotational Symmetry:

If an object is rotated about a point (centre of rotation) then the image may look exactly the same, when it is rotated through 360°.
The number of times it looks the same is called order of rotational symmetry.



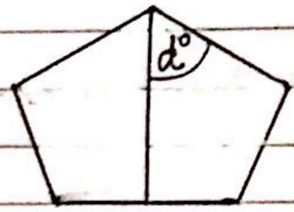
A rectangle is rotated through O (centre of rotation), the order of rotation is 2.

Geometric figure		Number of lines of Symmetry	Order of Rotational Symmetry
Parallelogram		0	2
Square		4	4
Kite		1	1
Isosceles Triangle		1	1
Trapezium		0	1
circle		Infinite	Infinite
I		2	2
N		0	2

Symmetry

Example 41: The diagram shows a regular pentagon. AB is a line of symmetry. Work out the value of d. ----[3]

W-17/21/Q11



Solution: Given a line of symmetry,

$$d = \frac{\text{Interior angle}}{2} \quad \text{----(i)}$$

Interior angle of n-sided regular polygon

$$= \frac{(n-2)\cdot 180^\circ}{n}$$

for $n=5$,

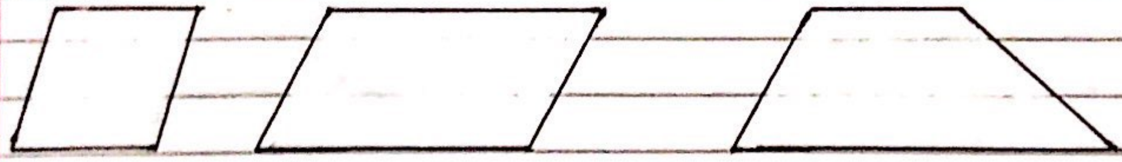
$$= \frac{(5-2)\times 180^\circ}{5} = 108^\circ$$

from (i) $d = \frac{1}{2}\times 108 = 54^\circ$ ✓

Example 42: A quadrilateral has a rotational symmetry of order 2 and no line of symmetry. Write down the mathematical name of the quadrilateral. S-16/22/Q4 ---[1]

Solution: Parallelogram.

Example 43:



Rhombus Parallelogram Trepezium.

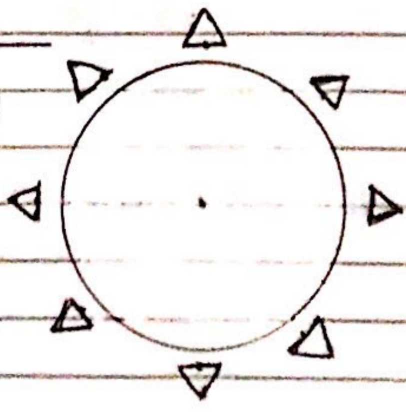
Write down: Which one of these shapes has: ---[1]

- Rotational symmetry of order 2
- No line of symmetry.

W-15/22/Q2

Solution: Parallelogram.

Example 44: Write down the order of rotational symmetry of this shape.

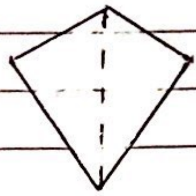


Solution: 8.

Example 45: A quadrilateral has one line of symmetry and no rotational symmetry. Write down the name of this quadrilateral. --- --[1]

[W-17/23/Q4]

Solution: Kite.



Example 46: Write down the letters in the word 'ZEBRA' that have:

(i) Exactly one line of symmetry. ---[1]

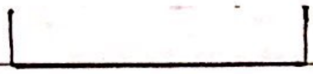
(ii) Rotational symmetry of order 2. [W-14/21/Q3] ---[1]

Solution (i) E, B, A

(ii) Z

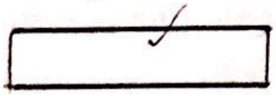
Example 47 (a) Add one line to the diagram so that it has two lines of symmetry. --[1]

(b) Add two line to the diagram so that it has rotational symmetry of order 2. --[1]



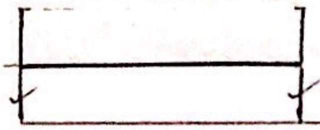
[W-13/22/Q5]

Solution: (a)

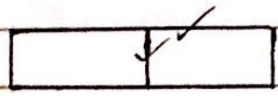


(b)

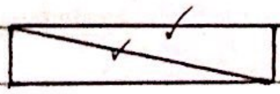
(i)



(ii)



(iii)



(iv)