DATE 09.04.21

0580

IGCSE Maths

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Trigonometry Notes 2

Suresh Goel
(Former Director)
Alliance World School
Noida. Delhi - N.C.R.
INDIA

(+91_ 9810444804)

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Trigonometry

Date P-1

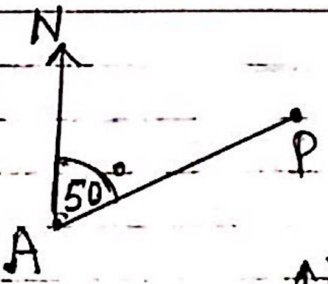
§

Three-Figure Bearings:

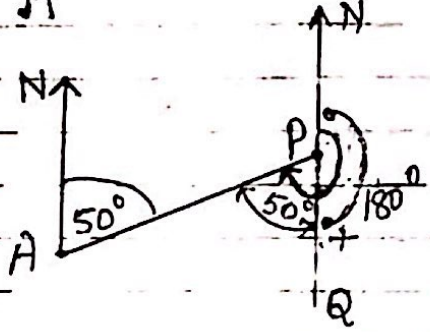
(000° to 360° in clockwise direction)

Compass indicates the 'North' direction.

If AP is inclined at an angle of 50° from North 'AN' in the clock-wise direction. We say "Bearing of P from A" is 050°



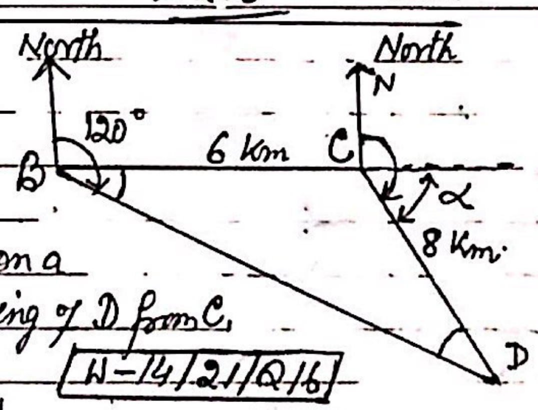
Back Bearing:



Given Bearing of P from A is 050°.

Then Bearing (Back Bearing) of A from P = 180° + angle APQ
= 180 + 50° (= Alt angle NAP)
= 230°

Example 1: A helicopter flies its base B to deliver supplies to two oil rigs at C and D. ---[5]
C is 6 Km due east of B and the distance C to D is 8 Km. D is on a bearing of 120° from B. Find the bearing of D from C.



Solution: angle CBD = 120 − 90° = 30° [W-14/21/Q16]

Using Sine rule in ΔBCD

$\frac{\sin D}{6} = \frac{\sin B}{8}$

$\sin D = \frac{6 \times \sin 30}{8} = 3/8$

$\angle D = \sin^{-1}(3/8) = 22°$

Now Bearing of D from C
= angle NCD
= $(90 + \alpha)$ — ①

$\alpha = \angle CBD + \angle D = 30 + 22 = 52°$
(Ext. angle)

∴ from ① Bearing of D from C.
= 90 + 52 = 142°

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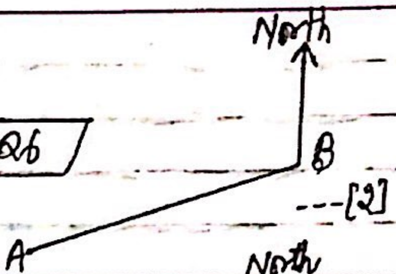
Example 2: The bearing of A from B is 227°.
Find the bearing of B from A.

[W-18/21/Q6] ---[2]



Solution: Let angle $ABC = \theta$

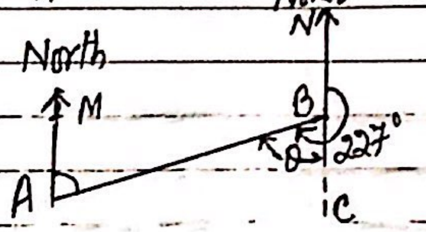
Bearing A from B $= 180 + \theta = 227^\circ$

$\Rightarrow \theta = 47^\circ$ --- (1)

Bearing of B from A = angle MAB

$= \theta$ / Alt. angle

$= 047^\circ$ ✓ from (1)



Example 3: The bearing of Alexandria from Paris is 128°. --[2]
Calculate the bearing of Paris from Alexandaria. [M-19/22/Q7]

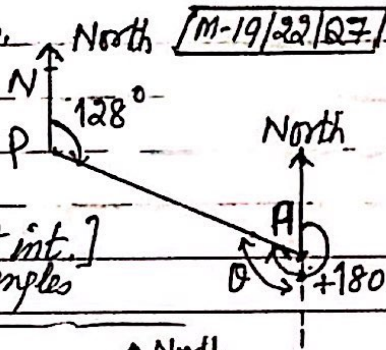
'A' → Alexandria 'P' → Paris

Solution: Bearing of A from P = angle $NPA = 128^\circ$

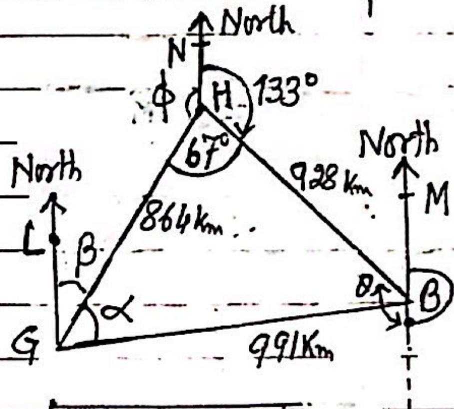
Bearing of P from A $= 180 + \theta$

$= 180 + 128^\circ$ [$\theta = 128^\circ$ Alt. int. angles]

$= 308^\circ$



Example 4: The diagram shows positions of three cities, Geneva (G), Budapest (B) and Hamburg (H).
The bearing of Budapest from Hamburg is 133°.
(i) Find the bearing of Hamburg from Budapest.
(ii) Calculate the bearing of Budapest from 'G'.



[W-18/42/Q8] -- [2+4]

Solution: (i) $\angle NHB = 133^\circ$ (Given) --- (1)

the Bearing of Hamburg from Budapest $= 180 + \theta$ ($\theta = 133^\circ$ Alt angle)

$= 180 + 133^\circ = 313^\circ$ ✓ from (1)

(ii) In ΔHGB using cosine rule.

$$\cos\alpha = \frac{864^2 + 991^2 - 928^2}{2 \times 864 \times 991} = 0.5065$$

$\alpha = \cos^{-1}(0.5065) = 59.5^\circ$ ✓ --- (2)

$\beta = 180 - \phi = 180 - [360 - (133 + 67)] = 20^\circ$ --- (3)

$\therefore$ Bearing Budapest from Geneva $= \beta + \alpha$ from (2) and (3)

$= 20 + 59.5$

$= 079.5^\circ$ ✓

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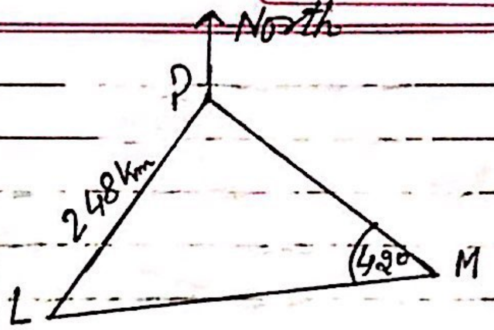
Bearings:



Example 5: The diagram shows two ports, L and P and buoy, M.
The bearing of L from P is 201°, and LP = 248 Km.
The bearing of M from P is 127°.
Angle PML = 42°

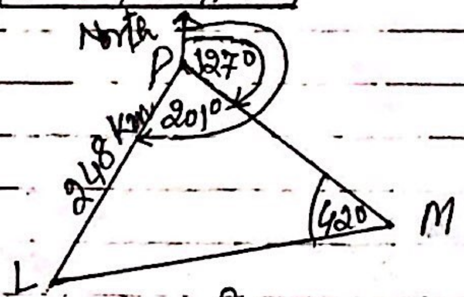
(a) Use sine rule to calculate LM. ---[4]

(b) A Ship sails directly from L to P.

(i) Calculate the shortest distance from M to LP. ---[3]

(ii) The ship leaves 'L' at 2045 and travels at a speed of 40 Km/h. Calculate the time the next day, that the ship arrives at P. ---[3]

M-18/42/Q8



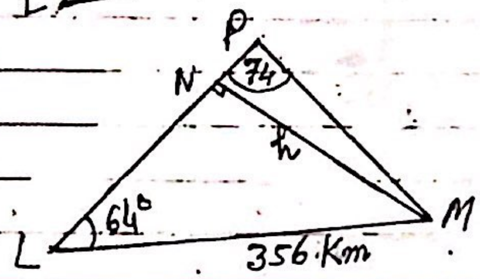
Solution: In Δ LPM

(a) $\angle LPM = 201° - 127° = 74°$

Using Sine rule in Triangle LPM,

$$\frac{LM}{\sin P} = \frac{LP}{\sin 42°}$$

$$\therefore LM = \frac{248 \times \sin 74°}{\sin 42°} = 356 \text{ Km}$$



(b)(i) $\angle PLM = 180 - (74 + 42) = 64°$

Draw $MN \perp PL$

$\therefore$ MN is the req. shortest distance

In rt Δ MNL: $\frac{MN}{LM} = \sin L$

$$\Rightarrow MN = LM \sin L$$
$$= 356 \times \sin 64°$$
$$= 320 \text{ Km}$$

(ii) Time of Travel from L to P $= \frac{248}{40}$

$= 6.2 h$

$= 6 h\ 12 \text{ min}$

$\therefore$ Time of arrival the next day $= 2045 + 6 h\ 12 \text{ Min}$

$= 0257$

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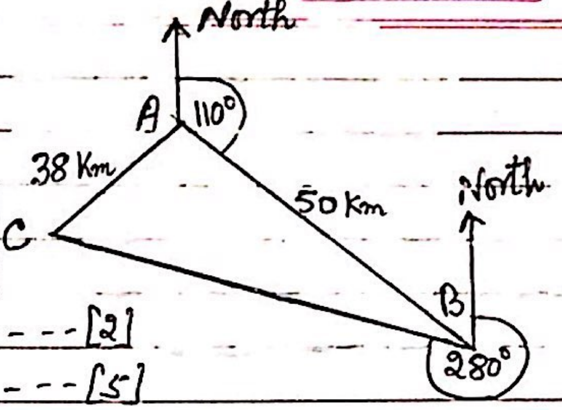
Bearings:

Example 6(a): A, B and C are three towns. The bearing of B from A is 110°. The bearing of C from B is 280°. AC = 38 Km and AB = 50 Km



(i) Find the bearing of A from B. ---- [2]

(ii) Calculate angle BAC. --- [5]

(iii) A road is built from A to join the straight road BC. Calculate the shortest possible length of this new road. --- [3]

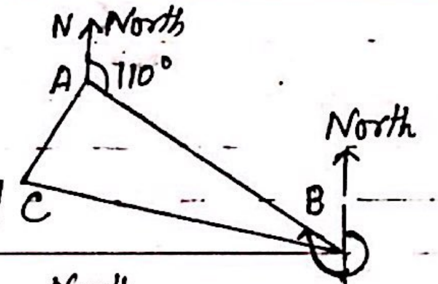
S-17/41/Q 8

Solution: (i) Bearing of A from B

$= 180 + \text{angle } ABP$

$= 180° + 110°$ [angle ABP = angle NAB Alt angle]

$= 290°$ ✓



(ii) angle $ABC = 290 - 280° = 10°$ ✓ —— ①

In Triangle ABC, Using Sine Rule,

$$\frac{\sin C}{AB} = \frac{\sin B}{AC}$$

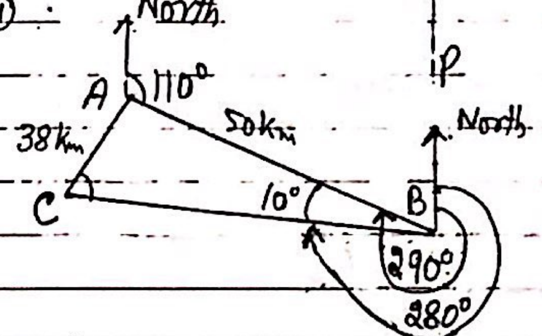
$$\Rightarrow \frac{\sin C}{50} = \frac{\sin 10°}{38}$$

or $\sin C = \dfrac{50 \times \sin 10°}{38} = 0.2285$

$\therefore$ angle $C = \sin^{-1}(0.2285) = 13.2°$ ✓ —— ②

$\therefore$ angle $BAC = 180 - (10 + 13.2)$ fr ① & ②

$= 156.8°$ ✓✓



(iii) Draw $AN \perp BC$.

Shortest distance from A to BC = AN.

In rt Δ ANC

$$\frac{AN}{AC} = \sin C$$

$$\Rightarrow AN = AC \times \sin C$$

$$= 38 \times \sin 13.2 = 8.68 \text{ km}$$ ✓

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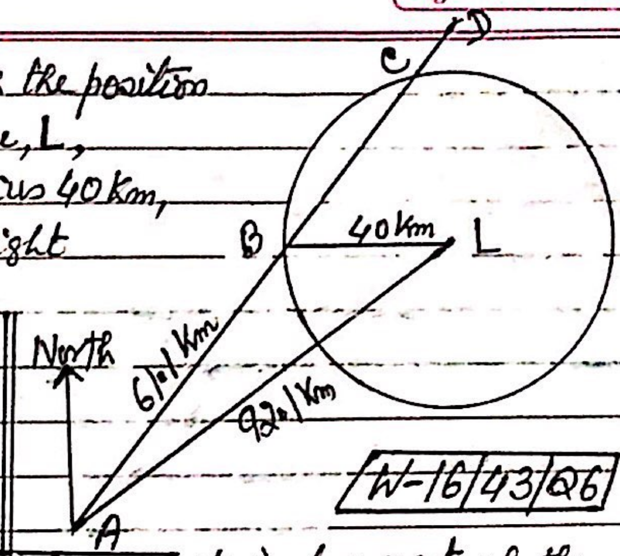
Bearings:

Example 7: The diagram shows the position of a port, A, and a light house, L. The circle, centre L and radius 40 Km, shows the region, where the light from the light house can be seen. The straight line ABCD, represents the course taken by a ship after leaving the port.



W-16/43/Q6

When the ship reaches position B, it is due west of the light house. AL = 92.1 Km, AB = 61.1 Km and BL = 40 Km.

(a) Use cosine rule to show that angle ABL = 130.1°, correct to one decimal place. ---[4]

(b) Calculate the bearing of lighthouse 'L', from the port, A. ---[4]

(c) The ship sails at a speed of 28 Km/hr. Calculate the length of time for which the light from the light house can be seen from the ship. Give your answer correct to the nearest minute. - [5]

Solution: (a) Using Cosine Rule:

$$\cos(ABL) = \frac{AB^2 + BL^2 - AL^2}{2 \times AB \times BL}$$

$$= \frac{(61.1)^2 + 40^2 - (92.1)^2}{2 \times 61.1 \times 40}$$

$$= -0.6443$$

$\therefore$ angle $ABL = \cos^{-1}(-0.6443)$

$= 130.11°$

(b) In right $\triangle ANL$.

The bearing of L from A = 90 − angle ALB --- ①

~~In $\triangle$ ABL,~~ $\cancel{\frac{\sin ALB}{61.1}} = \cancel{\frac{\sin 130.11}{92.1}}$

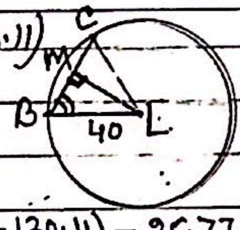
$$\Rightarrow \sin ALB = \frac{61.1 \times \sin 130.11}{92.1} = 0.507$$

$\therefore$ angle $ALB = \sin^{-1}(0.507) = 30.49°$

(c) $\angle CBL = (180 - 130.11)$

$LM \perp BC$

$BC = 2BM$ — ②



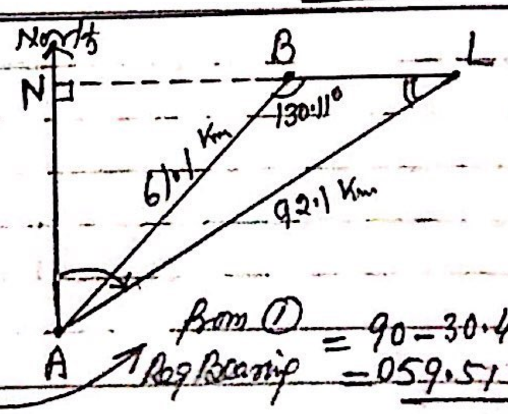
In rt $\triangle LMB$,

$BM = 40 \cos(180 - 130.11) = 25.77$

$\therefore BC = 2 \times BM = 2 \times 25.77 = 51.54$

$\therefore$ Time $= \frac{51.54}{28} = 1.84$ hours

$= 1$ hr 50 min.



from ① $= 90 - 30.49$

Req Bearing $= 059.51°$

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Trigonometry

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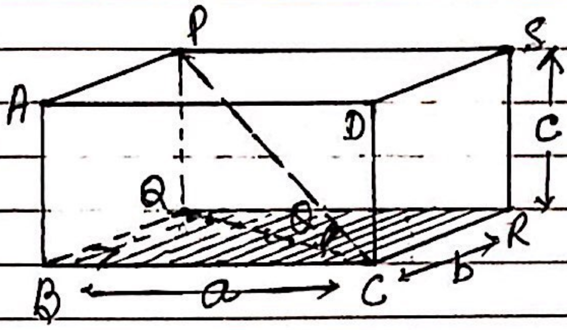
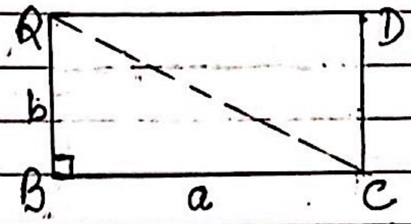
Application of Trig in three dimension:

Given a cuboid, the lengths of sides a, b and c,



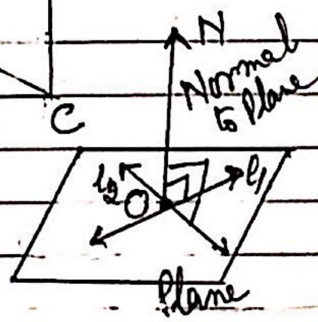
(i) Lenth of the diagonal of the base QBCR

$QC = \sqrt{BC^2 + QB^2}$

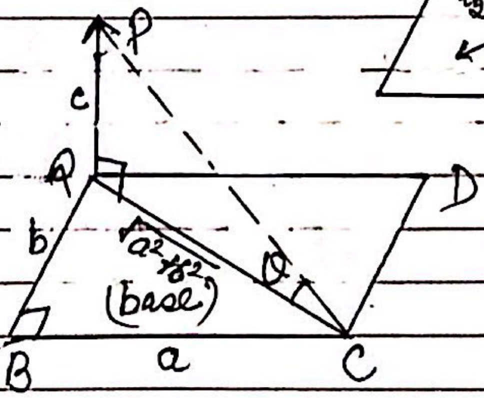


$\therefore QC = \sqrt{a^2 + b^2}$ ✓

(Using Pythagoras Theo. in right ΔQBC)



(ii) Lenth of the diagonal of the cuboid.



$PC = \sqrt{QC^2 + QP^2}$

(In rt ΔPQC)

$\therefore PC = \sqrt{(a^2+b^2) + c^2}$

$= \sqrt{a^2+b^2+c^2}$ ✓

(iii) Angle between line (diagonal of the cuboid) and the plane (base QBCD).

Here $PQ \perp$ Base (QBCD)

$\therefore$ In right triangle PQC. $[\angle PQC = 90°]$

$\tan\theta = \tan(\angle PCQ) = \frac{PQ}{QC} = \frac{c}{\sqrt{a^2+b^2}}$

or angle $PCQ = \tan^{-1}\left(\frac{c}{\sqrt{a^2+b^2}}\right)$

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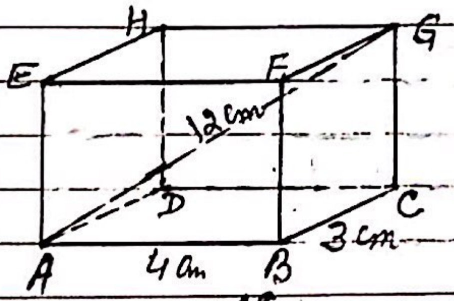
Application of Trigonometry in 3D.

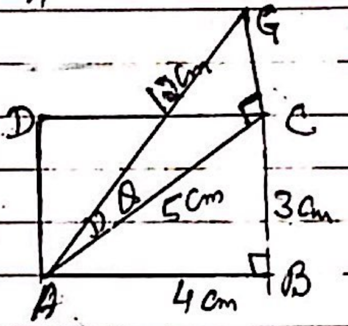
Example 8: ABCDEFGH is a cuboid. AB = 4 cm, BC = 3 cm, and AG = 12 cm.
Calculate the angle that AG makes with the base ABCD. ----[4]

[S-14/23/Q16]



Solution: In rt ΔABC

$AC = \sqrt{4^2+3^2} = 5$ --- ①



Now GC is perp. to base ABCD

$\therefore$ angle $ACG = 90^\circ$

Let Required angle $GAC = \theta$

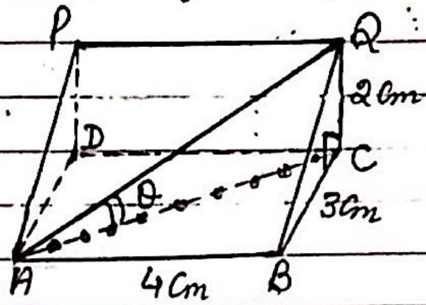
In rt ΔGCA

$$\cos\theta = \frac{5}{12}$$

$$\therefore \theta = \cos^{-1}\frac{5}{12} = 65.4^\circ$$

Example 9: The diagram shows a prism of length 4 cm. ---[4]
The cross-section is a right-angled triangle, BC = 3 cm and CQ = 2 cm.
Calculate the angle between the line AQ and the base, ABCD, of the prism.

[W-17/22/Q26]



Solution: In rt ΔABC, using Pythagoras thm.

$AC = \sqrt{4^2+3^2} = 5$ --- ①

Let angle between AQ and Plane $ABCD = \theta = \angle QAC$.

In rt ΔQCA ($QC \perp$ Plane ABCD) $\angle QCA = 90^\circ$

$$\tan\theta = \frac{QC}{AC} = \frac{2}{5} \quad (AC = 5 \text{ from ①})$$

$$\therefore \theta = \tan^{-1}\left(\frac{2}{5}\right) = 21.8^\circ$$

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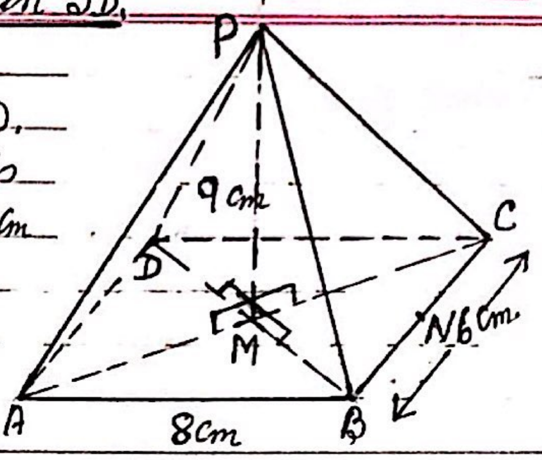
Application of Trigonometry in 3D.

Example 10. The diagram shows a pyramid on a rectangular base ABCD. AC and BD intersect at M and P is vertically above M. AB = 8 cm, BC = 6 cm and PM = 9 cm.



(a) N is the mid point of BC. Calculate angle PNM. ---[2]

(b) Show that BM = 5 cm --[1]

(c) Calculate the angle between the edge PB and base ABCD. ---[2]

S-17/42/Q8

(d) A point X is on PC, so that PX = 7.5 cm, Calculate BX. ---[6]

Solution:

(a) $MN = \frac{1}{2} AB$

$MN = \frac{1}{2} \times 8 = 4 \text{ cm}$

Let angle $PNM = \theta$

In rt $\triangle PMN$

$\tan\theta = \frac{PM}{MN} = \frac{9}{4}$

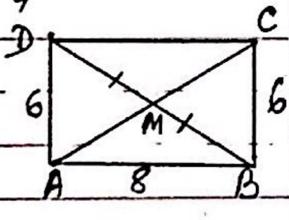
$\therefore$ angle $\theta = \tan^{-1}\frac{9}{4} = 66°$

(b) $BM = \frac{1}{2} BD$

$= \frac{1}{2}\sqrt{AB^2 + AD^2}$

$= \frac{1}{2}\sqrt{8^2 + 6^2}$

$= \frac{1}{2} \times 10 = 5 \text{ cm}$



(c) $PB^2 = PM^2 + MB^2$

$= 9^2 + 5^2$

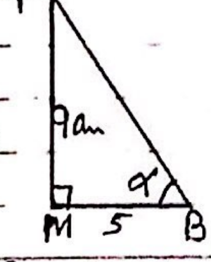
$= 106$

$PB = \sqrt{106}$

Let angle between PB and base ABCD $= \angle PBM = \alpha$

$\tan\alpha = \frac{PM}{MB} = 9/5$

$\therefore \alpha = \tan^{-1}\frac{9}{5} = 60.9°$



PM is perp to plane ABCD, hence PM is perp to every line in the plane ABCD, passing through M

(d) $PC = PB = \sqrt{106}$

$= 10.3$

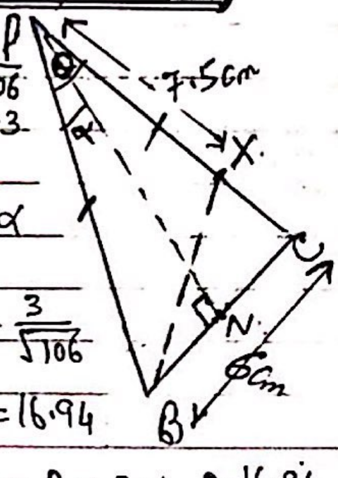
$PX = 7.5 \text{ cm}$

Let angle $BPN = \alpha$

In rt $\triangle PNB$

$\sin\alpha = \frac{BN}{PB} = \frac{3}{\sqrt{106}}$

$\therefore \alpha = \sin^{-1}\frac{3}{\sqrt{106}} = 16.94$



Now angle $BPC = \theta = 2\alpha = 2 \times 16.94$

or $\theta = 33.88°$

In $\triangle BPX$ using Cosine Rule.

$BX^2 = PB^2 + PX^2 - 2 \times PB \times PX \times \cos\theta$

$= (\sqrt{106})^2 + 7.5^2 - 2 \times \sqrt{106} \times 7.5 \times \cos 33.88°$

$= 34.04$

$\therefore$ Req Distance $BX = \sqrt{34.04}$

$= 5.83 \text{ cm}$

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Trigonometry

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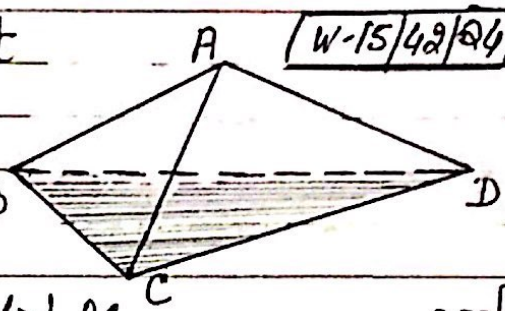
Application of Trig in 3D.

Example 11: The diagram shows a tent ABCD. The front of the tent is an isosceles triangle ABC, with AB = AC. The sides of the tent are congruent triangles ABD and ACD. [W-15/42/Q4]



(a) BC = 1.2 m and angle ABC = 68° find AC. ---[3]

(b) CD = 2.3 m and AD = 1.9 m Find angle ADC. --[4]

(c) The floor of the tent, triangle BCD, is also an isosceles triangle with BD = DC. Calculate the area of the floor of the tent. --[4]

(d) When the tent is on horizontal ground, A is vertical distance 1.25 m above the ground. Calculate the angle between AD and the ground. ---[3]

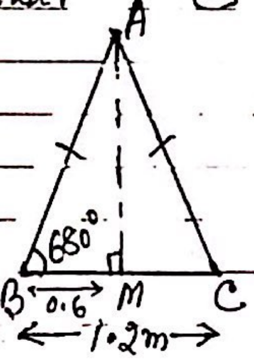
Solution:

(a) AB = AC

Draw $AM \perp BC$

M is mid point of BC,

$BM = \frac{BC}{2} = \frac{1.2}{2} = 0.6\,m$



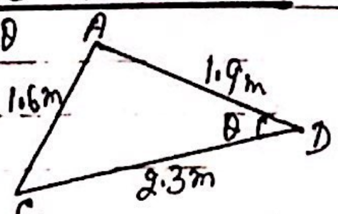
In rt $\triangle AMB$, $\cos B = \frac{BM}{AB}$

or $\cos 68^\circ = \frac{0.6}{AB}$

or $AB = \frac{0.6}{\cos 68^\circ} = 1.6\,m$

$\therefore$ Required $AC = AB = 1.6\,m$

(b) Let angle ADC = θ

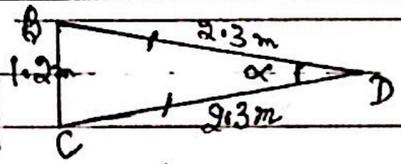


Using cosine rule in $\triangle ADC$.

$\cos\theta = \frac{(2.3)^2 + (1.9)^2 - (1.6)^2}{2 \times 2.3 \times 1.9} = 0.7254 \Rightarrow$ angle $ADC = \theta = \cos^{-1} 0.7254$

$= 43.50^\circ$

(c)



$DB = DC = 2.3\,m$

Let $\angle BDC = \alpha$, using cosine in $\triangle BDC$,

$\cos\alpha = \frac{(2.3)^2 + (2.3)^2 - (1.2)^2}{2 \times 2.3 \times 2.3}$

$= 0.8624$

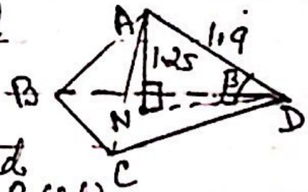
$\therefore \alpha = \cos^{-1} 0.8624 = 30.24^\circ$

$\therefore$ Area $\triangle BDC = \frac{1}{2} \times BD \times DC \times \sin\alpha$

$= \frac{1}{2} \times 2.3 \times 2.3 \times \sin 30.24^\circ$

$= 1.33\,m^2$

(d) Let $AN \perp$ ground BCD.



$AN = 1.25\,m$

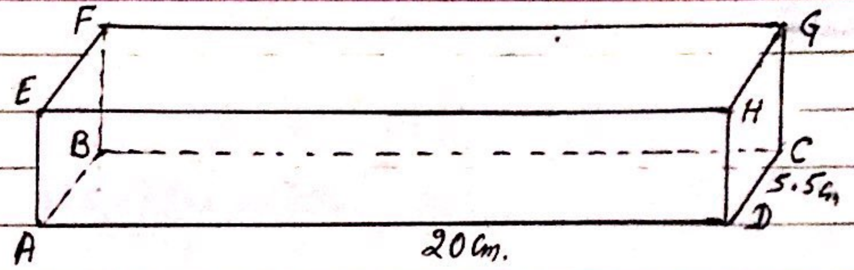
Angle between AD and ground $\angle ADN = \beta$ (let)

$\sin\beta = \frac{AN}{AD} = \frac{1.25}{1.9} = 0.6579$

$\therefore \beta = \sin^{-1}(0.6579) = 41.1^\circ$

Three dimensions problems.

12.



The diagram shows cuboid ABCDEFGH of length 20 cm and width 5.5 cm. The volume of the cuboid is 495 cm^3.
Find the angle between the line AG and the base of the cuboid ABCD.
[S-20/21/Q 19]

Solution: Volume of cuboid $= l \times b \times h$ [Let $GC = h$ cm]

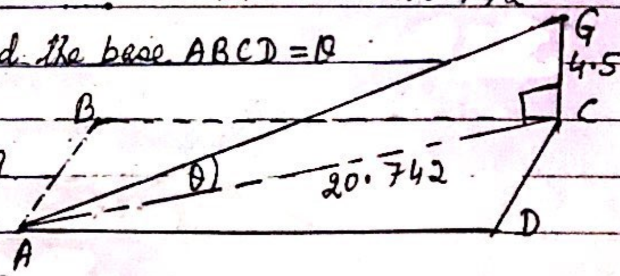
$= 20 \times 5.5 \times h = 495$ (Given)

$\Rightarrow h = \dfrac{495}{20 \times 5.5} = 4.5$ cm. —— ①

Now In rectangular base ABCD, $AC^2 = 20^2 + 5.5^2 = 430.25$

$\therefore AC = \sqrt{430.25} = 20.742$

Let the angle between AG and the base ABCD $= \theta$

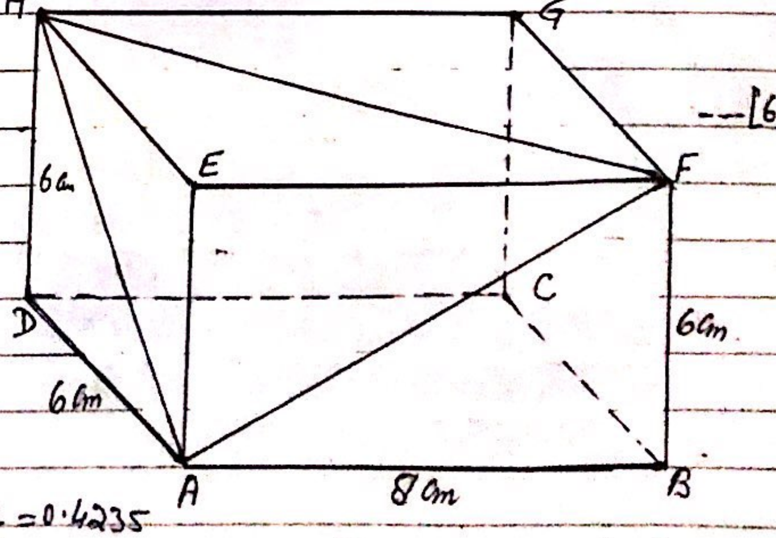


$\tan\theta = \dfrac{GC}{AC} = \dfrac{4.5}{20.742} = 0.2169$

$\therefore \theta = \tan^{-1} 0.2169$

Angle GAC $= \underline{12.24°}$ ✓

13. The diagram shows a cuboid. AB = 8cm. AD = 6cm, BE = 6cm
Calculate angle HAF. ---[6]



Solution: Using Cosine Rule:

$AF = \sqrt{8^2 + 6^2} = 10$

$FH = \sqrt{8^2 + 6^2} = 10$

$AH = \sqrt{6^2 + 6^2} = 6\sqrt{2}$

$\cos HAF = \dfrac{100 + 72 - 100}{2 \times 10 \times 6\sqrt{2}} = \dfrac{72}{170} = 0.4235$

$\left[\because \cos HAF = \dfrac{AF^2 + AH^2 - FH^2}{2 \times AF \times AH}\right]$

$\Rightarrow$ Angle HAF $= \cos^{-1}(0.4235) = \underline{64.9°}$ ✓

[S-20/22/Q27]