DATE 08.04.21

0580

IGCSE Maths

Contens

Trigonometry Notes-1

Suresh Goel
(Former Director)
Alliance World School,
Noida - Delhi N.C.R
INIDA.
(+91 9810444804)

Note: Please find the following topics in trig - notes - 2
(i) Three - figure Bearing.
(ii) Trig in 3D.

Scanned with CamScanner

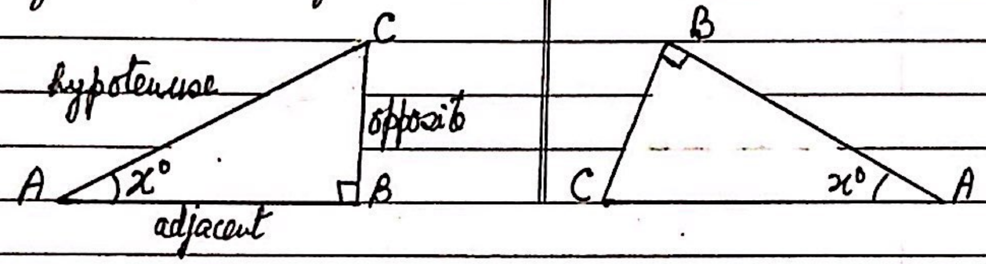
IG 0580

Trigonometry

Trigonometry is the study of the relationship between the sides and angles of a triangle.

§ Trigonometric Ratios:

In a right angled triangle ABC,
right angled at B, Let angle BAC = x°



We define:

(i) tangent x° or $\tan x^\circ = \frac{opp}{adj} = \frac{BC}{AB}$

(ii) Sine x° or $\sin x^\circ = \frac{opp}{hyp} = \frac{BC}{AC}$

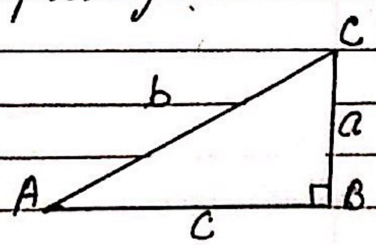
(iii) Cosine x° or $\cos x^\circ = \frac{adj}{hyp} = \frac{AB}{AC}$

§ Pythagoras theorem:

In a right angled triangle the square of the length of hypotenuse is equal to the sum of the squares of the lengths of other two sides (perp. sides).

$$AC^2 = BC^2 + AB^2$$

or $b^2 = a^2 + c^2$



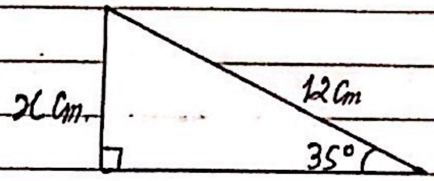
§ Values of certain particular angles:

x°	0°	30°	45°	60°	90°
$\sin x^\circ$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos x^\circ$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan x^\circ$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined

Note: $\sin 30^\circ = \frac{1}{2}$
$\Leftrightarrow \sin^{-1}\frac{1}{2} = 30^\circ$
(Sin inverse $\frac{1}{2}$ is 30°)

Scanned with CamScanner

Example 1: The diagram shows a right triangle.
Calculate the value of x. ...[2]

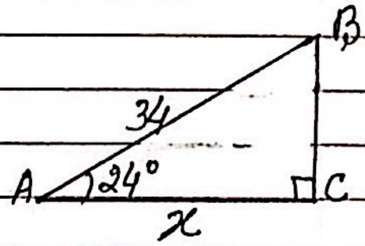


Solution: $\frac{x}{12} = \frac{opp}{hyp} = \sin 35^\circ$

$\Rightarrow \quad x = 12 \times \sin 35^\circ$

$= 12 \times 0.5735 = \underline{6.88}$ ✓ (6.882)

Example 2: Find x.

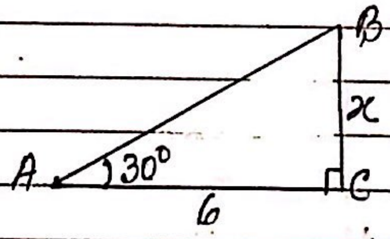


Solution: $\frac{x}{34} = \frac{adj}{hyp} = \cos 24^\circ$

$\Rightarrow \quad x = 34 \cos 24^\circ = 34 \times 0.9134$

or $x = \underline{31.1}$ ✓ (31.06)

Example 3: Find the length x



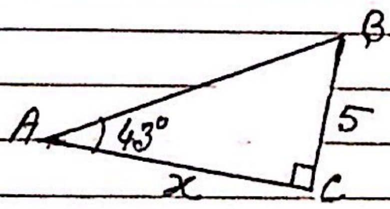
Solution: $\frac{x}{6} = \frac{opp}{adj} = \tan 30^\circ$

$\Rightarrow \quad x = 6 \times \tan 30^\circ$

$= 6 \times \frac{1}{\sqrt{3}} = 3.4641$

or $x = \underline{3.46}$ ✓

Example 4: Find the length x.



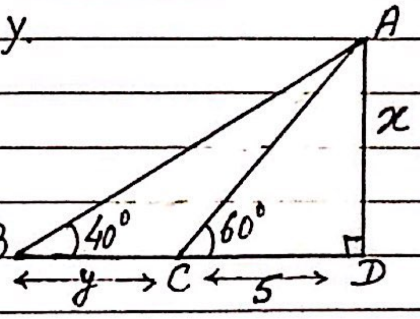
Solution: In rt $\triangle ACB$

$\frac{5}{x} = \frac{opp}{adj} = \tan 43^\circ$

$\Rightarrow \quad x \cdot \tan 43 = 5$

$\Rightarrow \quad x = \frac{5}{\tan 43^\circ} = \frac{5}{0.9325} = \underline{5.36}$ ✓ (5.3619)

Example 5: Find the lengths of x and y.



Solution: In rtΔ ACD

$$\frac{x}{5} = \frac{opp}{adj} = \tan 60 = \sqrt{3}$$

$$\Rightarrow x = 5 \times \sqrt{3} = \underline{8.66} \quad \text{---} (1)$$

Now In Δ ABD.

$$\frac{x}{(y+5)} = \frac{opp}{adj} = \tan 40° = 0.84$$

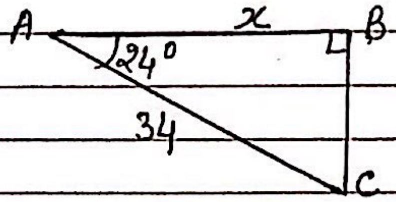
$$\Rightarrow \frac{8.66}{(y+5)} = 0.84 \qquad (\because \text{from } (1)\ x = 8.66)$$

$$\Rightarrow 8.66 = 0.84 \times (y+5)$$

$$\Rightarrow (y+5) = \frac{8.66}{0.84} = 10.3$$

$$\therefore y = 10.3 - 5 = \underline{5.3}$$

Example 6: Find x.



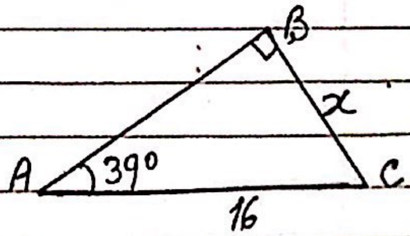
Solution: In rtΔ ABC

$$\frac{x}{34} = \frac{adj}{hyp} = \cos 24$$

$$\Rightarrow x = 34 \times \cos 24° = 34 \times 0.9135 = 31.06$$

$$\therefore x = \underline{31.1}$$

Example 7: Find the value of x



Solution: In rtΔ ABC

$$\frac{x}{16} = \frac{opp}{hy} = \sin 39°$$

$$\Rightarrow x = 16 \times \sin 39°$$

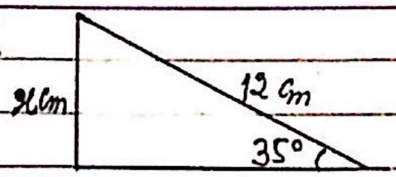
$$= 16 \times 0.6293$$

$$= 10.069$$

$$\therefore x = \underline{10.1}$$

Scanned with CamScanner

Example 8: The diagram shows a right-angled triangle. Calculate the value of x.



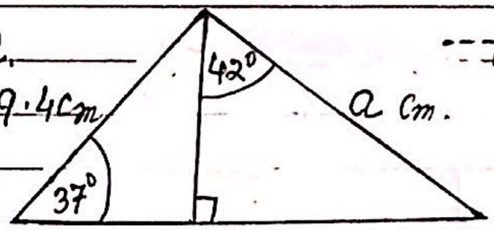
[S-19/21/Q21] --[2]

Solution: In rt Δ, $\dfrac{x}{12} = \dfrac{opp}{hyp} = \sin 35^\circ$

$\Rightarrow x = 12 \times \sin 35^\circ = 12 \times 0.5735 = 6.882$

$\therefore x = \underline{6.88}$ cm.

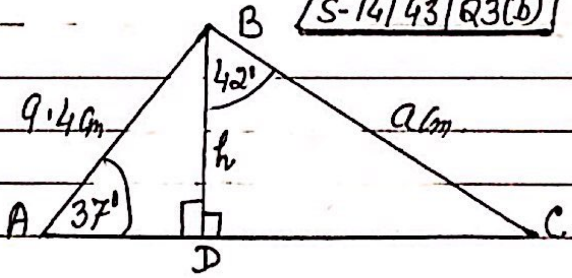
Example 9: Calculate the value of a. --[4]



Solution: In rt ΔBAD

$\dfrac{h}{9.4} = \dfrac{opp}{hyp} = \sin 37^\circ$ $(BD = h)$

[S-14/43/Q3(b)]



$\Rightarrow h = 9.4 \times \sin 37^\circ = 9.4 \times 0.6018$

$h = 5.657$ ✓ ---(1)

Now in rt ΔBDC

$\dfrac{h}{a} = \dfrac{adj}{hyp} = \cos 42^\circ$

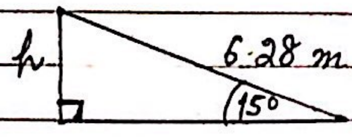
$h = a \cdot \cos 42^\circ \Rightarrow a = \dfrac{h}{\cos 42}$

$= \dfrac{5.657}{0.7431}$ (from (1) $h = 5.657$)

$= 7.612$

$\therefore a = \underline{7.61}$ cm.

Example 10: Calculate the length h. Give your answer correct to 2 significant figures.



--[3]

[W-13/23/Q10]

Solution: $\dfrac{h}{6\,28} = \dfrac{opp}{hyp} = \sin 15^\circ$

$\Rightarrow h = 628 \times \sin 15^\circ = 628 \times 0.2588 = 162.53$

$\therefore h = \underline{160}$ ✓ (2 significant figures)

Scanned with CamScanner

§ **Inverse trigonometric functions:**

Given $\sin\theta = a$ $\left\{ \begin{array}{ll} \text{for} & 0 \leq a \leq 1 \\ \text{and} & 0 \leq \theta \leq 90^\circ \end{array} \right.$

Then (sin inverse of a) $\sin^{-1} a = \theta$ (θ is the angle so that $\sin\theta = a$)

Example (i) $\sin 30^\circ = \frac{1}{2} \Leftrightarrow \sin^{-1}\frac{1}{2} = 30^\circ$

(ii) $\cos^{-1}\frac{1}{2} = 60^\circ \Leftrightarrow \cos 60 = \frac{1}{2}$

(iii) $\tan^{-1} 1 = 45^\circ \Leftrightarrow \tan 45^\circ = 1$

Using calculator find:

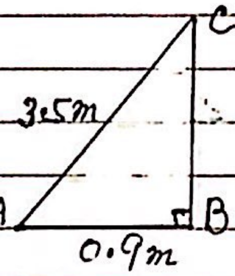
(i) $\sin^{-1} 0.6 = 36.87^\circ$ [or $\sin 36.87^\circ = 0.6$]

(ii) $\cos^{-1} 0.2 = 78.46^\circ$ [$\because \cos 78.46^\circ = 0.2$]

(iii) $\tan^{-1} 0.1 = 5.71^\circ$ [$\because \tan 5.71^\circ = 0.1$]

Example 11: Calculate angle BAC. ---[2]

[M-16/22/Q3]



Solution: In right triangle ABC,

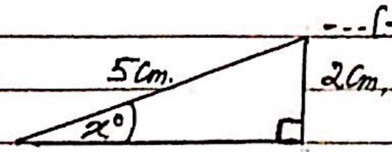
$$\cos BAC = \frac{adj}{hy} = \frac{0.9}{3.5} = 0.2571$$

$\therefore$ angle $BAC = \cos^{-1} 0.2571 = 75.099^\circ$

$\therefore$ angle $BAC = \underline{75.1^\circ}$ ✓

Example 12: Calculate the value of x. ---[2]

[W-15/23/Q9]



Solution: In rt Δ.

$$\sin x^\circ = \frac{opp}{hyp} = \frac{2}{5} = 0.4$$

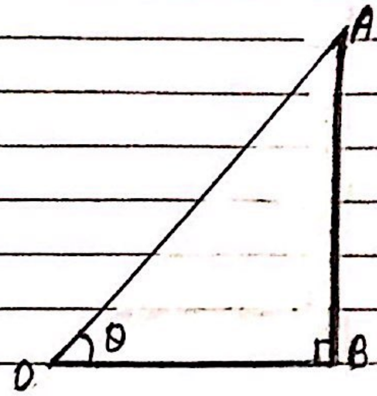
$\therefore x^\circ = \sin^{-1} 0.4 = 23.578$

$\therefore x = \underline{23.6^\circ}$ ✓

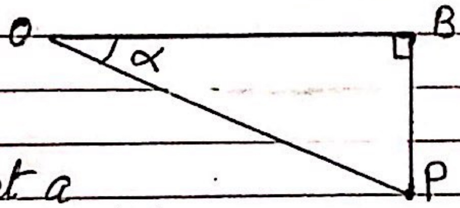
§ Heights and distances:

§ Angle of Elevation:

Given OB is a horizontal line, and 'A' is a point at a higher level (above OB). Then angle AOB(θ) is the angle of elevation of the point A from O.

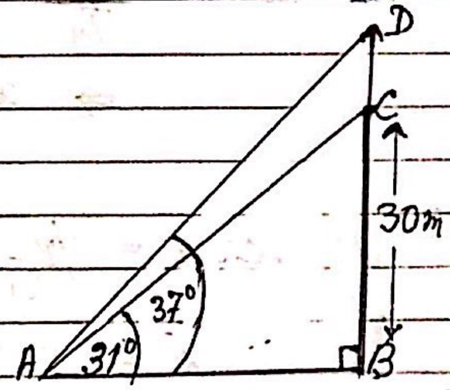


§ Angle of Depression:



OB is a horizontal line through the point O, and P is a point at a lower level (below OB). Then angle POB(α) is the angle of depression of P as observed from O.

Example 13: In the diagram, BC represents a building 30m tall.
A flag pole, DC, stands on the top of the building. From a point A, the angle of elevation of the top of the building is 31°.
The angle of elivation of the top of the pole flag is 37°. Calculate the height, DC, of the flag pole. --- [5]
[W-14/41/Q7(b)]



Solution: In rt Δ CAB,

$$\frac{BC}{AB} = \frac{opp}{adj} = \tan 31°$$

$$\Rightarrow AB \cdot \tan 31° = BC$$

$$AB = \frac{BC}{\tan 31°} = \frac{30}{0.6} = 50m$$

$$AB = 50m \quad \text{—— (1)}$$

Now in rt Δ DAB,

$$\frac{DB}{AB} = \frac{opp}{adj} = \tan 37°$$

$$\Rightarrow DB = AB \times \tan 37°$$

$$= 50 \times 0.7535 \quad \left(\text{from (1) } AB = 50m\right)$$

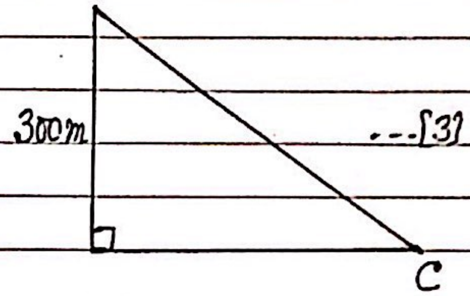
$$= 37.6 \quad \text{-- (2)}$$

Height of the Flagpole $DC = DB - BC$

$$= 37.6 - 30 = \underline{7.6\,m}$$

Example 14:

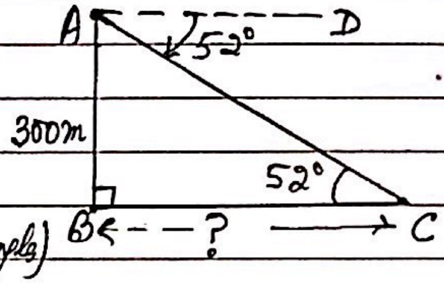
From the top of a building, 300 m high, the angle of depression of a Car C, is 52°. Calculate the horizontal distance from the car to the base of the building.



--[3]

[W-16/22/Q9]

Solution: Let AD is a horizontal line through the point A. Then, the angle of depression of the Car, C, from A, is angle DAC $= 52^\circ$



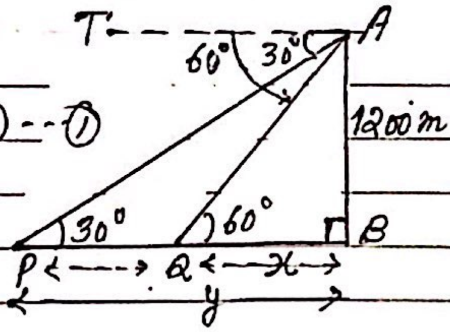
Now angle ACB = angle DAC ($\because$ Alt angle)
$= 52^\circ$ ✓

$\therefore$ In rt ΔABC, $\frac{300}{BC} = \frac{opp}{adj} =$

$\Rightarrow BC \cdot \tan 52^\circ = 300 \Rightarrow BC = \frac{300}{\tan 52^\circ} = 234.38\,m$

Distance of car from the base of building $= BC = 234\,m$ ✓

Example 15: An aeroplane at an altitude of 1200 metres, finds that two ships are sailing towards it in the same direction. The angles of depression of the ships as observed from the aeroplane are 60° and 30° respectively. Find the distance between the ships.



Solution: Distance between the two ships $PQ = (y - x)$ --- ①

In rt ΔABQ

$\frac{1200}{x} = \frac{opp}{adj} = \tan 60 = \sqrt{3}$

$\Rightarrow \sqrt{3}x = 1200$

or $x = \frac{1200}{\sqrt{3}} = \frac{400 \times 3}{\sqrt{3}} = 400\sqrt{3}$ --- ①

Now in rt ΔAPB

$\frac{1200}{y} = \frac{opp}{adj} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow y \times \frac{1}{\sqrt{3}} = 1200$

$\Rightarrow y = 1200\sqrt{3}$ ---- ②

From ① & ④

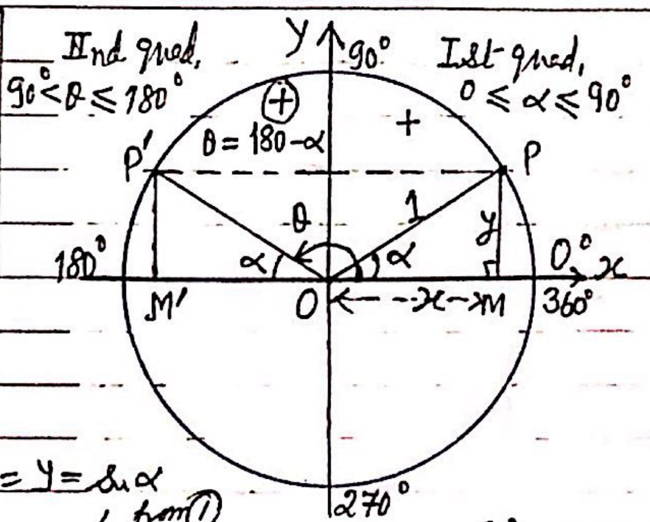
$PQ = 1200\sqrt{3} - 400\sqrt{3} = 800\sqrt{3}$
$= 1385.64\,m$

$\therefore$ Req. distance $= 1385.6\,m$ ✓

Scanned with CamScanner

§ Extending trig. function as circular functions: $0 \leq \theta \leq 360°$

Draw a circle with centre 'O'.
and radius $r = 1$ unit.
P is a point in the Ist quad
$PM \perp x$-axis.
$0 \leq \alpha \leq 90$ [α is the basic angle]



(i) $\sin\alpha = \frac{opp}{hyp} = \frac{PM}{OP} = \frac{y}{1} = y$ --- (1)

(ii) $90° < \theta \leq 180°$

Express $\theta = 180 - \alpha$

$\sin\theta = \sin(180 - \alpha) = \frac{P'M'}{OP'} = \frac{PM}{1} = y = \sin\alpha$ ✓ from (1)

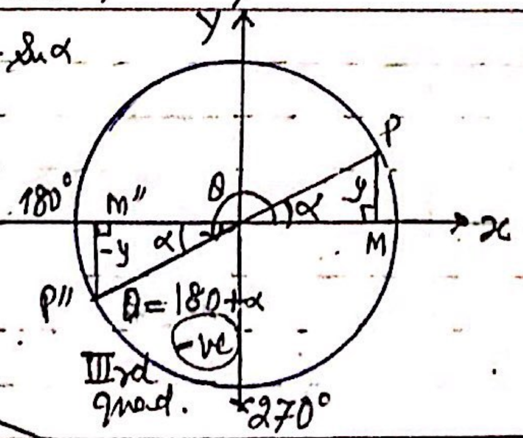
$\therefore \sin(180 - \alpha) = \sin\alpha$ · (2) [α is the basic angle]

Example: $\sin 150° = \sin(180 - 30°) = \sin 30° = \frac{1}{2}$ ✓

§ (iii) $180° < \theta \leq 270°$ (θ lies in the IIIrd quadrant)

$\sin\theta = \sin(180 + \alpha) = \frac{P''M''}{OP''} = \frac{-y}{1} = -\sin\alpha$

$\sin(180 + \alpha) = -\sin\alpha$ ✓ --- (3)

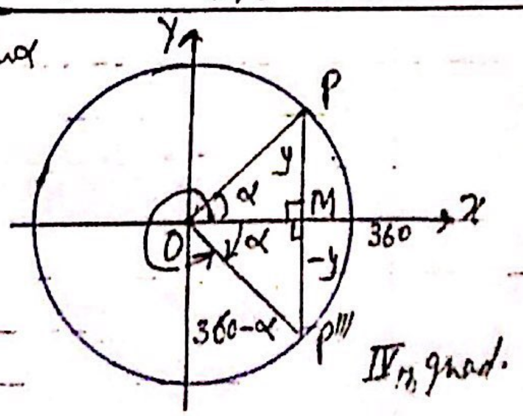


Example: $\sin 225 = \sin(180 + 45)$
$= -\sin 45° = -\frac{1}{\sqrt{2}}$ ✓

§ (iv) $270 < \theta \leq 360°$

$\sin\theta = \sin(360 - \alpha) = \frac{P'''M}{OP'''} = \frac{-y}{1} = -\sin\alpha$

$\sin(360 - \alpha) = -\sin\alpha$ ✓



Example:
$\sin 300 = \sin(360 - 60) = -\sin 60$
$= -\frac{\sqrt{3}}{2}$ ✓

Scanned with CamScanner

Note:

(i) $\sin\alpha = \sin(180-\alpha) = +$

(ii) $\sin(180+\alpha) = \sin(360-\alpha)$
$= -\sin\alpha$

$-1 \leq \sin\theta \leq 1$

IInd quad
$90^\circ < \theta \leq 180^\circ$
$\theta = (180-\alpha)$
180° $\sin\theta$ $+$

Ist quad.
y 90°
$\sin\theta = \sin\alpha$ is $+$
$0 \leq \alpha \leq 90$
α is basic angle
0° x

$(180+\alpha)$ 0
$\sin\theta$ is $-ve$
$180^\circ < \theta \leq 270^\circ$
IIIrd quad

$\theta = (360-\alpha)$
$\sin\theta$ is $-ve$
$270^\circ < \theta \leq 360^\circ$
IVth quad

§ **Graph of $\sin\theta$:** $0 \leq \theta \leq 360^\circ$

	α			$\sin(180-\alpha) = \sin\alpha$		$\sin(180+\alpha) = -\sin\alpha$		$\sin(360-\alpha) = -\sin\alpha$	
θ	0°	30°	90°	150	180°	210	270	330°	360°
$\sin\theta$	0	0.5	1	0.5	0	-0.5	-1	-0.5	0

$\sin 330 = \sin(360-30)$
$= -\sin 30 = -.5$

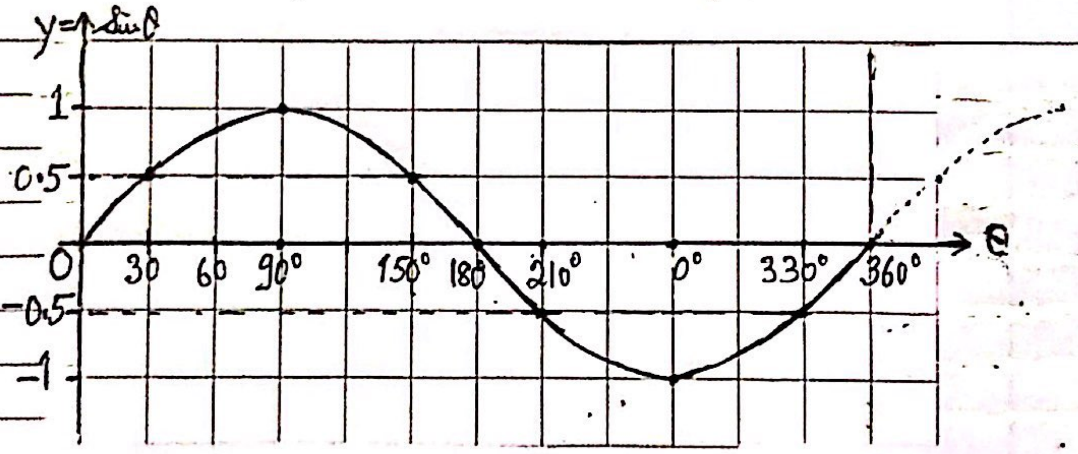
$\sin 360^\circ = \sin(360-0)$
$= -\sin 0$
$= 0$

$\sin 150 = \sin(180-30) = \sin 30^\circ = 0.5$

$\sin 180^\circ = \sin(180-0) = \sin 0^\circ = 0$

$\sin 210 = \sin(180+30) = -\sin 30 = -0.5$

$\sin 270^\circ = \sin(180+90) = -\sin 90 = -1$



(i) Period $= 360^\circ$

(ii) Amplitude $= 1$

Range $-1 \leq \sin\theta \leq 1$

Scanned with CamScanner

§ $y = \cos\theta \quad 0 \le \theta \le 360^\circ$

Note:

$\cos(180-\alpha) = \cos(180+\alpha) = -\cos\alpha$

$\cos(360-\alpha) = +\cos\alpha$

90°

IInd quad
$90 < \theta \le 180^\circ$
$\cos\theta = \cos(180-\alpha) = -\cos\alpha$

Ist quad
$0 \le \theta \le 90^\circ$
$\cos\theta = \cos\alpha$ (+)

180° — 0, 360° x

$\cos\theta = \cos(180+\alpha) = -\cos\alpha$
$180^\circ < \theta \le 270^\circ$
IIIrd quad

$270^\circ < \theta \le 360^\circ$
$\cos\theta = \cos(360-\alpha) = \cos\alpha$ (+)

Example:

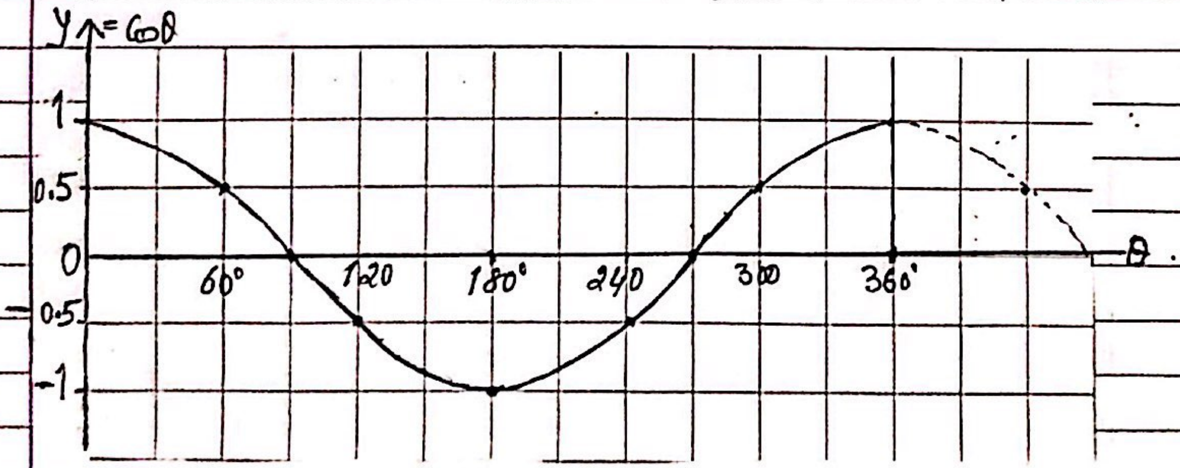
$\left.\begin{aligned} \cos 120 &= \cos(180-60) = -\cos 60 = -\tfrac{1}{2} \\ \cos 180^\circ &= \cos(180-0) = -\cos 0 = -1 \end{aligned}\right\}$

$\left.\begin{aligned} \cos 240 &= \cos(180+60) = -\cos 60 = -\tfrac{1}{2} \\ \cos 270^\circ &= \cos(180+90) = -\cos 90 = 0 \end{aligned}\right\}$

$\left.\begin{aligned} \cos 300 &= \cos(360-60) = \cos 60 = 0.5 \\ \cos 360 &= \cos(360-0) = \cos 0 = 1 \end{aligned}\right\}$

Graph of $\cos\theta$ $\quad 0 \le \theta \le 360$

	α		+	$\cos(180-\alpha) = -\cos\alpha$		$\cos(180+\alpha) = -\cos\alpha$		$\cos(360-\alpha) = \cos\alpha$	
θ	0°	60°	90°	120	180	240	270	300	360
$\cos\theta$	1	0.5	0	-0.5	-1	-0.5	0	0.5	1



$y = \cos\theta \quad 0 \le \theta \le 360^\circ$

(i) Period $= 360^\circ$
(ii) Amplitude $= 1$
Range: $-1 \le \cos\theta \le 1$

Scanned with CamScanner

§ $y = \tan\theta \qquad 0 \le \theta \le 360°$

$\tan(180+\alpha) = +\tan\alpha$

$\tan(180-\alpha) = -\tan\alpha$

$\tan(360-\alpha) = -\tan\alpha$

IInd quad. $90 < \theta \le 180°$
$\tan\theta = \tan(180-\alpha) = -\tan\alpha$

y $90°$

$0 \le \alpha \le 90°$
$\tan\theta = \tan\alpha \; +$

$180°$ — $0°$ x

$\tan\theta = \tan(180+\alpha) = +\tan\alpha$
$180° < \theta \le 270°$
IIIrd quad. $270°$

$\tan\theta = \tan(360-\alpha) = -\tan\alpha$ $360°$
$270° < \theta \le 360°$
IVth quad.

Example:

$\tan 135° = \tan(180-45) = -\tan 45 = -1$

$\tan 180 = \tan(\quad) = -\tan 0 = 0$

$\tan 225 = \tan(180+45) = +\tan 45 = +1$

$\tan 270^- = \tan(180+90^-) = \tan 90^- \rightarrow +\infty$

$\tan 270^+ = \tan(180+90^+) = \tan 90^+ \rightarrow -\infty$

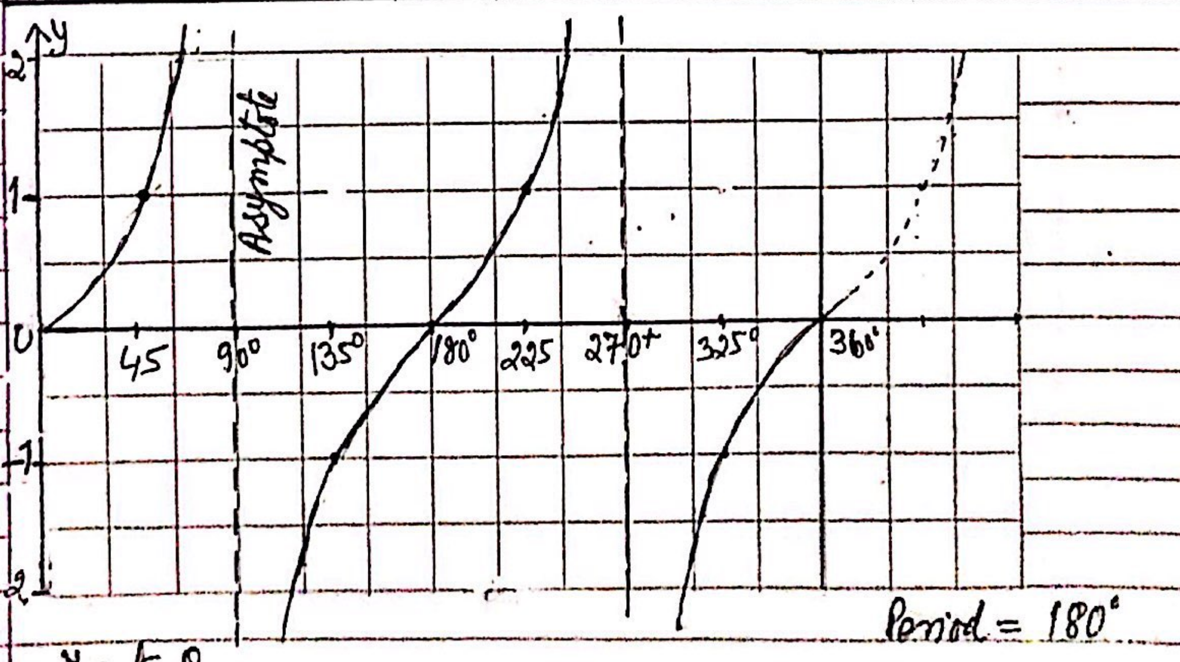
$\tan 325 = \tan(360-45) = -\tan 45 = -1$

$\tan 360 = \tan(360-0) = -\tan 0 = 0$

Note: $\tan 90^- \rightarrow +\infty$
$\tan 90^+ \rightarrow -\infty$

Graph of $\tan\theta \qquad 0 \le \theta \le 360°$

	$\tan\alpha$		+	$\tan(180-\alpha) = -\tan\alpha$			$\tan(180+\alpha) = +\tan\alpha$		$\tan(360-\alpha) = -\tan\alpha$		
θ	$0°$	$45°$	90^-	90^+	$135°$	$180°$	$225°$	270^-	270^+	325	360
$\tan\theta$	0	1	∞	$-\infty$	-1	0	1	∞	$-\infty$	-1	0



Period $= 180°$

$y = \tan\theta$

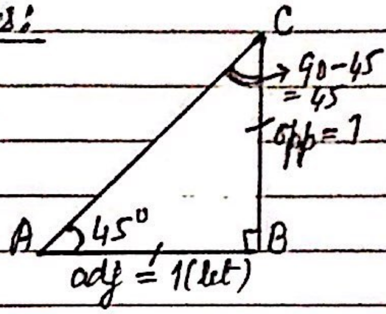
§ Values of trig-ratios of some particular angles:

In rt ΔABC

(i) for 45°: Given $\angle BAC = 45^\circ \Rightarrow \angle ACB = 90-45 = 45^\circ$

Let $AB = 1$, ΔABC is an isosceles Δ

$BC = AB = 1$



Now using Pythagoras theorem,

$AC^2 = AB^2 + BC^2 = 1^2 + 1^2 = 2$

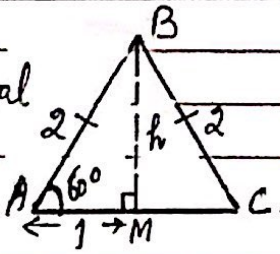
$\Rightarrow AC = \sqrt{2}$

$\therefore \sin 45^\circ = \frac{opp}{hyp} = \frac{BC}{AC} = \frac{1}{\sqrt{2}}$ ✓ || $\cos 45^\circ = \frac{adj}{hyp} = \frac{AB}{AC} = \frac{1}{\sqrt{2}}$ ✓

and $\tan 45^\circ = \frac{opp}{Adj} = \frac{BC}{AB} = \frac{1}{1} = 1$ ✓

(ii) for 60°

Consider an equilateral ΔABC.



Let

$AB = BC = AC = 2$

Draw $BM \perp AC \Rightarrow AM = \frac{AC}{2} = 1$ unit

Let $BM = h$

using Pythagoras theorem in rt ΔABM

$AM^2 + BM^2 = AB^2 \Rightarrow h^2 + 1^2 = 2^2 \Rightarrow h^2 = 3 \Rightarrow h = \sqrt{3}$

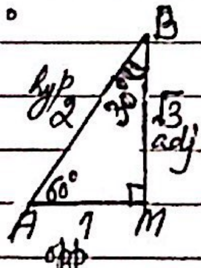
$\therefore \sin 60^\circ = \frac{opp}{hyp} = \frac{BM}{AB} = \frac{\sqrt{3}}{2}$ ✓

$\cos 60^\circ = \frac{adj}{hyp} = \frac{AM}{AB} = \frac{1}{2}$ ✓

$\tan 60^\circ = \frac{opp}{adj} = \frac{BM}{AM} = \frac{\sqrt{3}}{1} = \sqrt{3}$ ✓

(iii) for 30° from Part (ii)

$\angle ABM = 90 - 60^\circ = 30^\circ$



for angle ABM

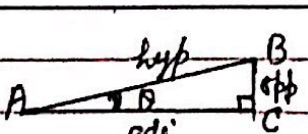
$\sin 30^\circ = \frac{opp}{hyp} = \frac{AM}{AB} = \frac{1}{2}$ ✓

$\cos 30^\circ = \frac{adj}{hyp} = \frac{BM}{AB} = \frac{\sqrt{3}}{2}$ ✓

and $\tan 30^\circ = \frac{opp}{adj} = \frac{AM}{BM} = \frac{1}{\sqrt{3}}$ ✓

(iv) for 0°

$(B \to C)$



as $\theta \to 0$
$BC \to 0$
and $AB \to AC$ $\Big\} \Rightarrow \sin 0^\circ = \frac{BC}{AB} = \frac{0}{AB} = 0$ ✓

$\cos 0^\circ = \frac{AC}{AB} = \frac{AC}{AC} = 1$ ✓

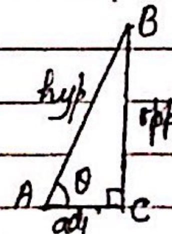
$\tan 0^\circ = \frac{BC}{AC} = \frac{0}{AC} = 0$ ✓

(v) for 90°

$\theta \to 90^\circ$ $(A \to C)$

$AC \to 0$

$AB \to BC$



$\sin 90^\circ = \frac{opp}{hyp} = \frac{BC}{AB} = \frac{BC}{BC} = 1$ ✓

$\cos 90^\circ = \frac{adj}{hyp} = \frac{AC}{AB} = \frac{0}{AB} = 0$ ✓

$\tan 90 = \frac{opp}{adj} = \frac{BC}{AC} = \frac{BC}{0} =$ Not defined ✓

Scanned with CamScanner

§

θ	0°	30°	45°	60°	90	180°	270	360°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	−1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	−1	0	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not def (∞)	0	not def (∞)	0

Complementary angles

$\sin(90-\theta) = \cos\theta$

$\cos(90-\theta) = \sin\theta$

$0 \leq \alpha \leq 90°$

§

$\sin(180-\alpha) = \sin\alpha = +$

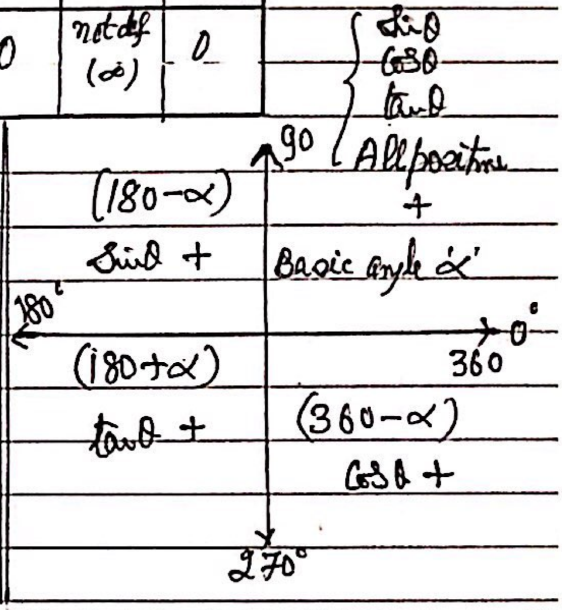
$\sin(180+\alpha) = \sin(360-\alpha) = -\sin\alpha$

$\cos(360-\alpha) = \cos\alpha = +$

$\cos(180-\alpha) = \cos(180+\alpha) = -\cos\alpha$

$\tan(180+\alpha) = \tan\alpha = +$

$\tan(180-\alpha) = \tan(360-\alpha) = -\tan\alpha$



Example 16: Find the values of (a) $\sin\theta$ (b) $\cos\theta$ (c) $\tan\theta$ for the following angles; 120°, 135°, 180°, 210°, 270°, 330°, 360°

(a)

$\sin 120 = \sin(180-60) = \sin 60 = \frac{\sqrt{3}}{2}$ ✓

$\sin 135° = \sin(180-45) = \sin 45 = \frac{1}{\sqrt{2}}$

$\sin 180° = \sin(180-0) = \sin 0 = 0$

$\sin 210° = \sin(180+30) = -\sin 30° = -\frac{1}{2}$

$\sin 270° = \sin(180+90) = -\sin 90° = -1$

$\sin 330° = \sin(360-30) = -\sin 30 = -\frac{1}{2}$

$\sin 360 = \sin(360-0) = -\sin 0 = 0$

(b)

$\cos 120° = \cos(180-60) = -\cos 60 = -\frac{1}{2}$

$\cos 135° = \cos(180-45) = -\cos 45 = -\frac{1}{\sqrt{2}}$

$\cos 180 = \cos(180-0) = -\cos 0 = -1$

$\cos 210° = \cos(180+30) = -\cos 30° = -\frac{\sqrt{3}}{2}$

$\cos 270° = \cos(180+90) = -\cos 90 = 0$

$\cos 330 = \cos(360-30) = +\cos 30° = \frac{\sqrt{3}}{2}$

$\cos 360 = \cos(360-0) = \cos 0 = 1$

(c)

$\tan 120° = \tan(180-60)$
$= -\tan 60 = -\sqrt{3}$

$\tan 135° = \tan(180-45)$
$= -\tan 45 = -1$

$\tan 180 = \tan(180-0)$
$= \tan 0 = 0$

$\tan 210 = \tan(180+30) = \tan 30°$
$= \frac{1}{\sqrt{3}}$

$\tan 270 = \tan(180+90) = +\tan 90°$
$=$ not def.

$\tan 330 = \tan(360-30)$
$= -\tan 30° = -\frac{1}{\sqrt{3}}$

$\tan 360 = \tan(360-0)$
$= -\tan 0 = 0$

Scanned with CamScanner

Example 17: Express each of the following in terms of Sine of another angle between 0° and 360°

(a) $\sin 30^\circ$ (b) $\sin 70^\circ$ (c) $\sin 115^\circ$ (d) $\sin 140^\circ$ (e) $\sin 210^\circ$.
(f) $\sin 225^\circ$ (g) $\sin 300$ (h) $\sin 320^\circ$

Solution:

(a) $\sin 30^\circ = \sin(180-30) = \sin 150^\circ$ ✓
(b) $\sin 70^\circ = \sin(180-70) = \sin 110^\circ$ ✓
(c) $\sin 115^\circ = \sin(180-115^\circ) = \sin 65^\circ$
(d) $\sin 140^\circ = \sin(180-140^\circ) = \sin 40^\circ$

$0 \leq \alpha \leq 180^\circ$
$\underline{\sin(180-\alpha) = \sin\alpha}$

(e) $\sin 210^\circ = \sin(180+30) = \sin(360-30) = \sin 330^\circ$ ✓
(f) $\sin 225^\circ = \sin(180+45^\circ) = \sin(360-45) = \sin 315^\circ$ ✓
(g) $\sin 300 = \sin(360-60^\circ) = \sin(180+60) = \sin 240^\circ$ ✓
(h) $\sin 320^\circ = \sin(360-40^\circ) = \sin(180+40) = \sin 220^\circ$ ✓

$180^\circ < \theta \leq 360^\circ$
"$\sin(180+\alpha) = \sin(360-\alpha)$" $= -\sin\alpha$

Example 18: Express each of the following in terms of cosine of another angle between 0° and 360°.

(a) $\cos 60^\circ$ (b) $\cos 80^\circ$ (c) $\cos 110^\circ$ (d) $\cos 145^\circ$ (e) $\cos 225^\circ$
(f) $\cos 300^\circ$ (g) $\cos 350^\circ$

Solution:

(a) $\cos 60 = \cos(360-60) = \cos 300^\circ$
(b) $\cos 80 = \cos(360-80) = \cos 280^\circ$

$\cos\alpha = \cos(360-\alpha)$

(c) $\cos 110^\circ = \cos(180-70) = \cos(180+70) = \cos 250$
(d) $\cos 145^\circ = \cos(180-35^\circ) = \cos(180+35) = \cos 215^\circ$
(f) $\cos 225^\circ = \cos(180+45) = \cos(180-45^\circ) = \cos 135^\circ$
(e) $\cos 210^\circ = \cos(180+30) = \cos(180-30) = \cos 150^\circ$

"$\cos(180-\alpha) = \cos(180+\alpha)$" $= -\cos\alpha$

(g) $\cos 300^\circ = \cos(360-60) = \cos 60$.
(h) $\cos 320 = \cos(360-40) = \cos 40^\circ$

$\underline{\cos(360-\alpha) = \cos\alpha}$

Scanned with CamScanner

Solution of Trig. equations.

Example 19: x° is an obtuse angle and $\sin x^\circ = 0.43$. ---[2]
Find the value of x. [W-18/22/Q12]

Solution: $\sin x^\circ = 0.43$

$\Rightarrow x = \sin^{-1} .43 = 25.5^\circ$ or $(180 - 25.5)$

$\therefore x^\circ = 180 - 25.5^\circ = \underline{154.5^\circ}$ ✓ ($\because x$ is obtuse)

Example 20: Solve the equation $3\cos x = 1$ for $0 \leq x \leq 360^\circ$
Give your answer correct to 1 decimal place. ---[4]
[SP-20/04/Q8(a)]

Solution: $3\cos x = 1$ $\quad 0 \leq x \leq 360^\circ$

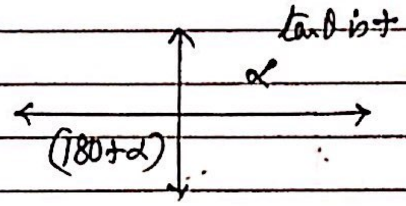
$\Rightarrow \cos x = \frac{1}{3} \Rightarrow x = \cos^{-1}\frac{1}{3} = 70.5^\circ$

$\therefore x = 70.5^\circ; (360 - 70.5^\circ)$ $\quad [\cos(360-\alpha) = \cos\alpha]$

$= \underline{70.5^\circ}; \underline{289.5^\circ}$ ✓

Example 21: Solve $\tan\theta = \sqrt{3}$ $\quad 0 \leq \theta \leq 360^\circ$

Solution: $\tan\theta = \sqrt{3} \Rightarrow \theta = \tan^{-1}\sqrt{3} = 60^\circ$

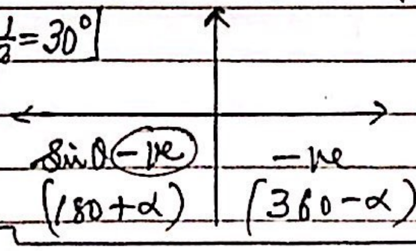


$\therefore \theta = 60^\circ; 180 + 60^\circ$

$\theta = \underline{60^\circ}; \underline{240^\circ}$ ✓

Example 22: Solve $\sin\theta = -\frac{1}{2}$ $\quad [\sin^{-1}\frac{1}{2} = 30^\circ]$

$= -\sin 30^\circ$



$\theta = 180 + 30; 360 - 30^\circ$

$= \underline{210^\circ}; \underline{330^\circ}$ ✓

Example 23: Solve $\cos x = -\frac{1}{4}$

$\Rightarrow \cos x = -\cos 75.5^\circ$ $\quad \left[\cos^{-1}\frac{1}{4} = 75.5^\circ\right]$

$x = (180 - 75.5); (360 - 75.5)$

$x = \underline{104.5^\circ}; \underline{284.5^\circ}$

$\cos x$ is $-$ve $(180-\alpha)$	α
$(180+\alpha)$ $-$ve	

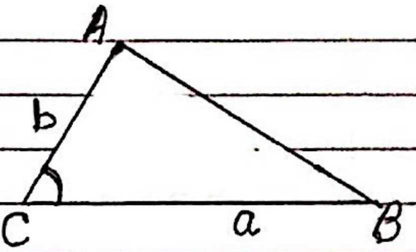
Scanned with CamScanner

Area of Triangle

§ **Area of Triangle:**

Given two sides $BC = a$, $AC = b$ and angle ACB. (SAS)

Area of Triangle $= \frac{1}{2}ab\sin C$

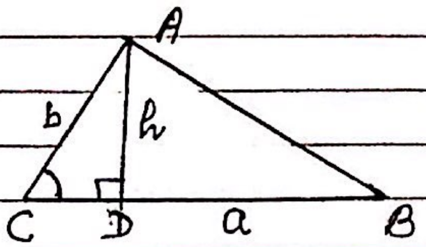


Proof: Draw $AD \perp BC$.

In rt ΔACD

$$\frac{h}{b} = \sin C$$

$$\Rightarrow h = b\sin C \quad \text{---} (1)$$

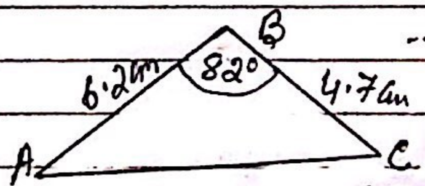


$\therefore$ Area of triangle $ABC = \frac{1}{2} \cdot \text{base} \times \text{Alt}$

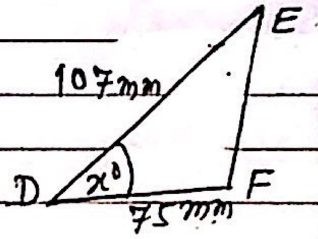
$$= \frac{1}{2} \times a \times h = \underline{\frac{1}{2}ab\sin C} \quad (\text{from } (1)\ h = b\sin C)$$

$$\left(\text{or} = \underline{\tfrac{1}{2} \times BC \times AC \times \sin C}\right)$$

Example 24(a) Calculate the area of triangle ABC. --- [2]



(b) The area of triangle DEF is 2050 mm². Work out the value of x. --- [2]



W-16/23/Q21

Solution (a) Area of $\Delta ABC = \frac{1}{2} \times AB \times BC \times \sin 82°$

$$= \frac{1}{2} \times 6.2 \times 4.7 \times$$

$$= \underline{14.4\ cm^2} \checkmark$$

(b) Area of triangle $DEF = \frac{1}{2} \times DF \times DE \sin x° = 2050$ (Given)

$$= \frac{1}{2} \times 75 \times 107 \times \sin x° = 2050$$

$$= 4012.5 \sin x° = 2050$$

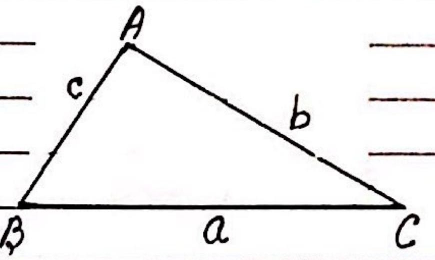
$$\Rightarrow \sin x° = \frac{2050}{4012.5} = 0.5109$$

$$\Rightarrow x = \sin^{-1} 0.5109 = 30.72°$$

$$x = \underline{30.7°} \checkmark$$

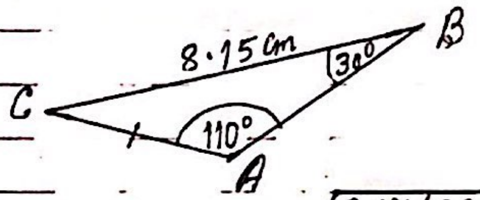
Sine Rule

§ **Sine Rule:**
In any triangle the sides (lengths) are proportial to the sine of the opposite angles.



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$ $\left\{ \begin{array}{l} BC = a,\ AC = b \\ AB = c \end{array} \right.$

Example 25: Calculate AC. ---[3]



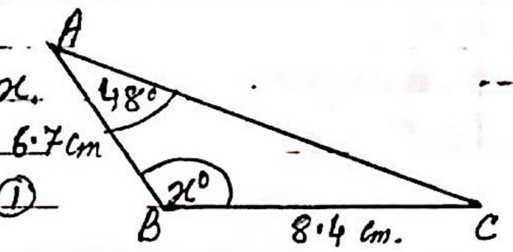
[S-17/22/Q17]

Solution: Using Sine Rule:

$$\frac{AC}{\sin 30^\circ} = \frac{BC}{\sin 110^\circ}$$

$$\Rightarrow AC = \frac{8.15 \times \sin 30^\circ}{\sin 110^\circ} = \frac{8.15 \times 0.5}{0.9397} = \underline{4.34}\ \text{cm.}$$

Example 26: Calculate the value of x. ---[4]



[M.15/42/Q5(b)]

Solution: $x = 180 - (48° + C)$ ---(1)

Using Sine Rule;

$$\frac{\sin C}{6.7} = \frac{\sin 48^\circ}{8.4}$$

$$\Rightarrow \sin C = \frac{6.7 \times \sin 48^\circ}{8.4} = \frac{6.7 \times 0.7431}{8.4} = 0.5927$$

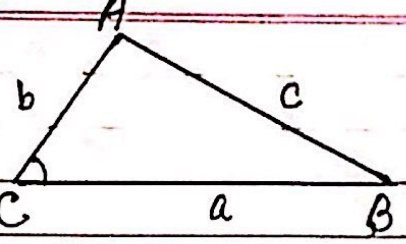
$\therefore$ angle $C = \sin^{-1} 0.5927 = 36.4°$

$\therefore$ from (1) $x = 180 - (48 + 36.4°) = 95.64°$

$$x = \underline{95.64°}$$

§ Cosine Rule

In any triangle ABC.



Given any two sides and included angle, we can work out the third side (SAS)

(i) $c^2 = a^2 + b^2 - 2ab\cos C$

(ii) $b^2 = c^2 + a^2 - 2ca\cos B$

(iii) $a^2 = b^2 + c^2 - 2bc\cos A$

or Given all three sides (SSS) we can work out any angle.

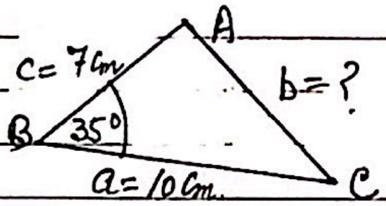
or $\cos C = \dfrac{a^2+b^2-c^2}{2ab}$

or $\cos B = \dfrac{c^2+a^2-b^2}{2ca}$

or $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$

Example 27. Calculate the length AC. [S-16/21/Q26(b)] --- [4]

Solution: Using cosine rule:



AC^2 or $b^2 = c^2 + a^2 - 2ca\cos B$

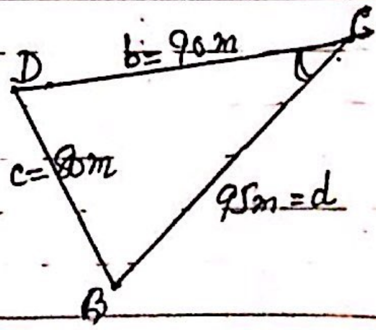
$= 7^2 + 10^2 - 2\times7\times10\times\cos 35^\circ$

$= 149 - 114.68$

$b^2 = 34.32$

$\therefore AC = b = \sqrt{34.32} = 5.86$ ✓

Example 28: The diagram shows a triangle BCD, BD = 80m, BC = 95m and CD = 90m. Calculate angle BCD.



Solution: Here DC = b = 90m, BC = d = 95m and BD = c = 80m.

Using Cosine Rule. $\cos C = \dfrac{b^2+d^2-c^2}{2bd}$

$= \dfrac{90^2+95^2-80^2}{2\times90\times95}$

$= \dfrac{10{,}725}{17{,}100} = .6271$

$\therefore$ angle $C = \cos^{-1} 0.6271$

$\therefore$ angle $BCD = 51.2^\circ$ ✓

[S-14/42/Q3(b)]

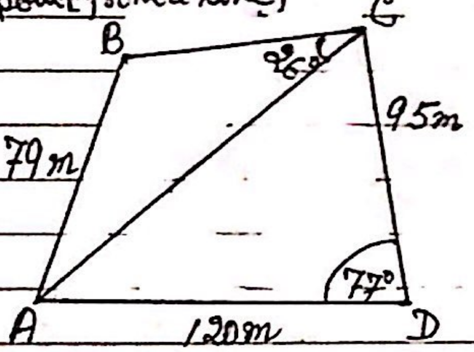
Scanned with CamScanner

IG-Maths 0580

Trigonometry

Date P-19
Page No.

§ Sine Rule, Cosine Rule and Area of Triangle;
(Also to find the shortest distance of a point from a line)



Example 29. The quadrilateral ABCD, represents an area of land. There is a straight road from A to C.
AB = 79m, AD = 120m and CD = 95m, Angle BCA = 26° and angle CDA = 77°

(a) Show that the length of the road, AC is 135m correct to the nearest metre. --- [4]

(b) Calculate the size of the obtuse angle ABC. --- [4]

(c) A straight path is to be built from B to the nearest point on the road AC. Calculate the length of this path. --- [3]

(d) Houses are to be built on the land in triangle ACD. Each house needs at least 180 m^2 of land. Calculate the maximum number of houses which can be built. Show all your working. - [4]

S-15/04/Q6

Solution: (a) Use cosine rule in triangle ACD.

$AC^2 = AD^2 + CD^2 - 2 \times AD \times CD \times \cos 77$

$= 120^2 + 95^2 - 2 \times 120 \times 95 \times \cos 77 = 18296$

$\therefore AC = \sqrt{18296} = 135\,m$ ✓

(b) Using Sine Rule in ΔABC,

$\frac{\sin ABC}{AC} = \frac{\sin ACB}{AB}$

or $\sin ABC = \frac{AC \times \sin ACB}{AB} = \frac{135 \times \sin 26°}{79}$

or $\sin ABC = 0.749$

Let $\alpha = \sin^{-1} 0.749 = 48.5$

$\therefore$ Obtuse angle $ABC = 180 - \alpha$

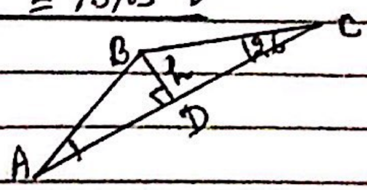
$= 180 - 48.5$

$= 131.5°$ ✓

(C)

Draw $BD \perp AC$

Req dis $BD = h$.



(c) continued →

In ΔABC,

$\angle BAC = 180 - (131.5 + 26)$

$= 22.5°$

$\therefore$ In rt ΔBAD,

$\frac{h}{AB} = \sin 22.5$

or $h = 79 \times \sin 22.5$

Req dis $= 30.2\,m$ ✓

(d) Area of $\Delta ACD = \frac{1}{2} \times AD \times CD \times \sin D$

$= \frac{1}{2} \times 120 \times 95 \times \sin 77°$

$= 5553.9\,m^2$

$\therefore$ Area of one house $= 180\,m^2$

$\therefore$ No of houses $= \frac{5553.9}{180} = 30.85$

$\therefore$ Number of houses $= 30$ ✓

Scanned with CamScanner

IG-Maths 0580

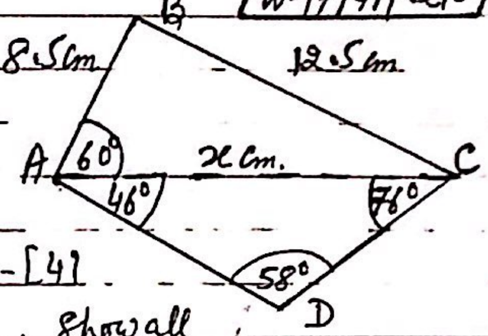
Trigonometry

Date P-20 Page No.

Sine Rule, Cosine Rule and Area of Triangle:

W-17/41/Q10.

Example 30: The diagram shows a quadrilateral ABCD.



(a) The length of AC is x cm, Use cosine rule in triangle ABC to Show that $2x^2-17x-168=0$ ---[4]

(b) Solve the equation $2x^2-17x-168=0$, Show all your working and give your answer to 2 decimal places. --[4]

(c) Use Sine rule to calculate the length CD. ---[3]

(d) Calculate the area of the quadrilateral ABCD. --[3]

Solution. (a) Using Cosine rule in triangle ABC.

$BC^2 = AB^2 + AC^2 - 2\times AB\times AC\times \cos 60°$

$(12.5)^2 = (8.5)^2 + x^2 - \cancel{2}\times 8.5\times x\times \frac{1}{\cancel{2}}$

$156.25 = 72.25 + x^2 - 8.5x$

or $x^2 - 8.5x - 84 = 0$

or $2x^2 - 17x - 168 = 0$ ✓

(b) To Solve $2x^2-17x-168=0$

Using quad formula

$$x = \frac{17 \pm \sqrt{(-17)^2 - 4\times 2\times(-168)}}{2\times 2}$$

$$= \frac{17 \pm \sqrt{1633}}{4}$$

$$= \frac{17 \pm 40.41}{4}$$

$$= 14.35,\ -5.85^{\times}$$

(c) In triangle ACD, Using Sine Rule,

$$\frac{CD}{\sin 46} = \frac{AC}{\sin 58}$$

$$\Rightarrow CD = \frac{AC\cdot \sin 46°}{\sin 58°} = \frac{14.35\times \sin 46°}{\sin 58°}$$

$= 12.2$ cm (or 12.17 cm)

(d) Area of quad ABCD

$= ar\,\Delta ABC + ar\,\Delta ADC$

$= \frac{1}{2} AB\times AC \sin 60° + \frac{1}{2} AC\times CD\times \sin 76°$

$= \frac{1}{2}[8.5\times 14.35\times \sin 60 + 14.35\times 12.2\times \sin 76°]$

$= \frac{1}{2}[105.63 + 169.87]$

$= 137.75\ cm^2$

Scanned with CamScanner

Trigonometry

Sine Rule, Cosine Rule and Area of a Triangle and Bearing

Example 31: The diagram shows a field ABCD.

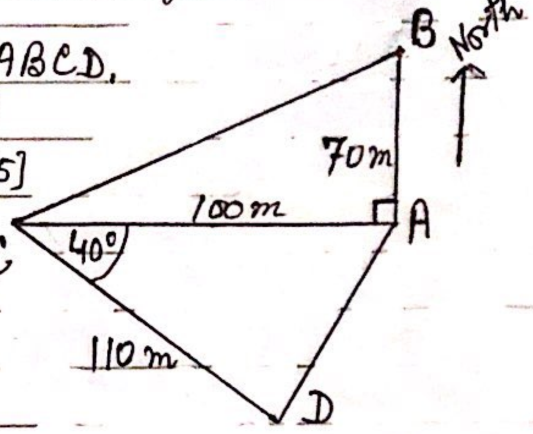
(a) Calculate the area of field ABCD ---[3]

(b) Calculate the perimeter of the field ABCD; --[5]

(c) Calculate the shortest distance from A to CD. ---[2]

(d) B is due North of A. Find the bearing of C from B. ---[3]

W-17/42/Q3



Solution:

(a) Area of field ABCD

$= \text{area of } \Delta BAD + \text{ar of } \Delta ACD$

$= \frac{1}{2} \times AC \times AB + \frac{1}{2} AC \times CD \times \sin ACD.$

$= \frac{1}{2} \times 100 \times 70 + \frac{1}{2} \times 100 \times 110 \times \sin 40^\circ$

$= 3500 + 3535.33$

$= 7035.33 \, m^2$ ✓

(b) Perimeter of the field ABCD.

$P = AB + BC + CD + AD$ —— ①

Now in rt ΔBAC, using Pythagoras theorem, $BC^2 = 100^2 + 70^2 = 14900$

$\therefore BC = \sqrt{14900} = 122$ ✓

To find AD in triangle ACD, using Cosine rule:

$AD^2 = AC^2 + CD^2 - 2 \times AC \times CD \times \cos 40$

$= 100^2 + 110^2 - 2 \times 100 \times 110 \times \cos 40$

$= 22100 - 16853$

$= 5247$

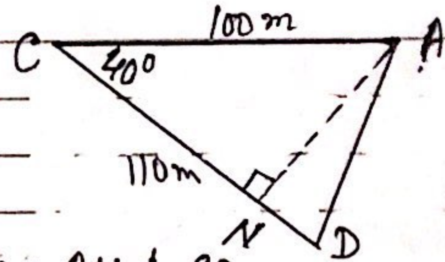
$AD = \sqrt{5247} = 72.43$ ✓

from ① $P = 70 + 122 + 110 + 72.43$

$= 374.43$

$\therefore$ Req. Perimeter $= 374 \, m.$ ✓

(c)



Draw $AN \perp CD$

Req shortest distance from A to CD = AN.

In right triangle ANC

$\frac{AN}{AC} = \sin 40^\circ$

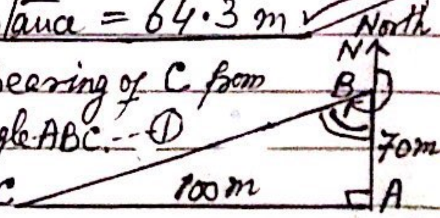
$\therefore AN = AC \times \sin 40$

$= 100 \times \sin 40$

$= 100 \times 0.643$

$\therefore$ Req distance $= 64.3 \, m$ ✓

(d) The bearing of C from B $= 180^\circ + \text{angle } ABC$ --- ①



In rt ΔABC,

$\tan ABC = \frac{100}{70} \Rightarrow \text{angle } ABC = \tan^{-1} \frac{10}{7}$

$= 55^\circ$

$\therefore$ from ① The bearing of C from B

$= 180 + 55 = 235^\circ$ ✓