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M

# Mechanics

# Enery, work and power.
# Notes

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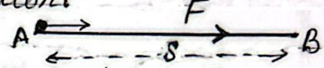
§ Workdone: A force is said to do work when it moves its point of application.




The workdone is measured by the product of the force 'F' and the distance 's' through which the point of application is moved in the direction of the force.

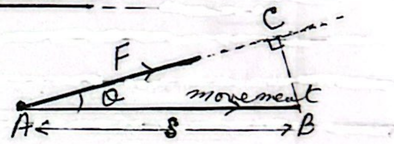
$W = F \times S$ [AB = S]

Unit of Work is "Joule"

A Joule is the workdone by a force of 1 newton to move an object by 1 metre in the direction of force.

"1 Joule = 1 newton metre"

§ Note: If the object is moved by a distance 'S' making an angle 'θ' with the direction of force 'F'




Then, Work done $W = F \cdot S \cos\theta$ [∵ $W = F \times AC$ $= F \times S\cos\theta$ ]

(Special Case: Workdone by a force perpendicular to the direction of motion is zero.)

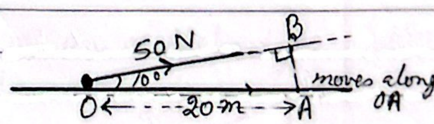
Example 1. A block of mass 25 Kg is pulled along horizontal ground by a force of magnitude 50N inclined at 10° above the horizontal. The block starts from rest and travels a distance of 20 m. Find the work done by the pulling force. [W-16/41/Q6(i)] --[2]

Solution:

$W = F \times OB$

$= 50 \times 20\cos 10°$

$= 984.8\,J$ ✓




[∵ $\frac{OB}{OA} = \cos 10°$

$\Rightarrow OB = OA \cdot \cos 10°$

$= 20\cos 10°$ ]

(M₁) Energy, Work and Power — Notes — P-2

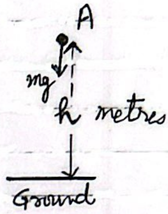
§ **Energy:** The energy of a body is its capacity for doing work, it is of two kinds, Kinetic Energy (K.E) and Potential Energy (P.E).

§ **Kinetic Energy (K.E):** K.E of a body is by virtue of it motion.

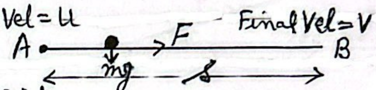
$$K.E = \frac{1}{2}mv^2 \quad \left[\begin{array}{l} m = \text{mass} \\ v = \text{velocity} \end{array}\right.$$

§ **Potential Energy (P.E):** If a particle of mass 'm' is at rest, at a height of 'h' metre above the ground, Then it possess:

$$P.E = mgh.$$




§ **Work-Energy Relation:**




If a particle of mass 'm' moves from point A to B by applying a force 'F', distance AB = s.
(and there is no resistance to motion)
Vel at A = u
Vel at B = V

Then Work done by force = gain in K.E

$$\text{or } W = F.s = \frac{1}{2}mV^2 - \frac{1}{2}mu^2$$

**Note:** If there is a resistance 'R' to motion,
Then Work done by F − Work done against resistance R = gain in K.E

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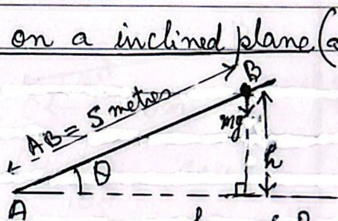
§ **Potential Energy of a body of mass 'm' on a inclined plane (at 'B') at an 'θ' with the horizontal and at a distance 'S' on the plane from the ground at A.**

Given AB = S.




P.E. of body at B = mgh

$$\frac{h}{S} = \sin\theta$$

$$\therefore h = S.\sin\theta$$

P.E lost when body moves down a distance 'S' (AB). $= mg.(S.\sin\theta)$ ✓

§ **Conservation of Energy:** The sum of P.E and K.E is constant for an object, if there is no work done by external forces other than the force of gravity.

or

$$\text{Gain in P.E.} = \text{Loss of K.E.} \quad (\text{or Gain in K.E.} = \text{Loss of P.E})$$

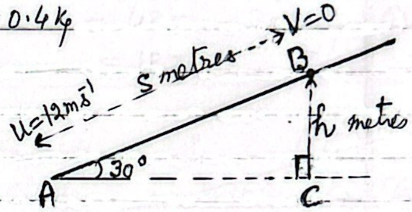
**Example 2:** A particle of mass 0.4 kg is projected with a speed of $12\,ms^{-1}$ up a line of greatest slope of a smooth plane inclined at 30° to the horizontal.

(i) Find the initial kinetic energy of the projectile. --- [1]

(ii) Use an energy method to find the distance of the particle moves up the plane before coming to instantaneous rest. [M-17/42/Q1] --- [3]

**Solution:**

(i) At point A, $u = 12\,ms^{-1}$, $m = 0.4\,kg$




Initial $K.E = \frac{1}{2}mu^2$

$= \frac{1}{2} \times 0.4 \times 12^2$

$= 28.8\,J$ ✓

(ii) Let particle moves up point B,

Let AB = S metres.

Gain in P.E $= mgh$

$= mg(S \sin 30°)$

$= 0.4 \times g \times S \times \frac{1}{2}$

$= 2S\ J$ ——— (1)

[BC ⊥ horizontal, Let BC = h

$\frac{h}{AB} = \sin 30° \Rightarrow h = AB \cdot \sin 30$

$\Rightarrow h = S \cdot \sin 30°$]

Loss of when K.E when particle stops at B ($V=0$)

$\Rightarrow$ Loss of $K.E = \frac{1}{2}mu^2 - \frac{1}{2}mv^2$

$= \frac{1}{2} \times 0.4 \times 12^2 - 0$

$= 28.8\,J$ ——— (2)

as no external force is working.

∴ Gain in P.E = Loss of K.E (Conservation of Energy)

$\Rightarrow 2S = 28.8$ (from (1) & (2))

$\Rightarrow$ distance up the plane. $S = 14.4\,m$ ✓

§ Work done by a force − Work done against resistance R = gain in K.E.

**Example 3:** A particle of mass 0.6 kg is dropped from a height of 8 m above the ground. The speed of the particle at the instant before hitting the ground is $10 ms^{-1}$. Find the work done against air resistance. [S-17/41/Q1] --[3]

**Solution:** $m = 0.6K$, $h = 8m$

$P.E\ Loss = mgh = 0.6 \times g \times 8 = 48J$ —— (1)

$K.E.\ gain = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.6 \times 10^2$   $[v = 10 ms^{-1}]$

$= 30J$ —— (2)

Now P.E. Loss = gain in KE + Work done against air Resistance

⇒ Work done against air resistance = P.E. Loss − gain of K.E

$= 48J - 30J$ [from (1) and (2)]

$= \underline{18J}$ ✓

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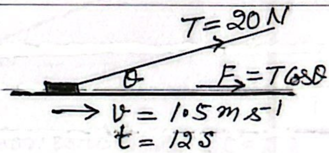
**Example 4:** One end of a light inextensible string is attached to a block. The string makes an angle θ with horizontal. The tension in the string is 20 N. The string pulls the block along a horizontal surface at a constant speed of $1.5 ms^{-1}$ for 12 s. The work done by the tension in the string is 50 J. Find θ.




**Solution:** Distance moved $s = vt = 1.5 \times 12 = 18m$ —— (1)

Work done $W = F \times s = T\cos\theta \times s$

$W = 20\cos\theta \times 18 = 360\cos\theta$ —— (2)

Given $W = 50J$ —— (3)

$\therefore 360\cos\theta = 50$   from (2) & (3)

$\cos\theta = 50/360 = 0.1389$

$\Rightarrow \theta = \underline{82^\circ}$ ✓

**Example 5:** A particle of mass 8 kg is projected with a speed of $5 ms^{-1}$ up a line of greatest slope of a <u>rough plane</u> inclined at an angle $\alpha$ to the horizontal, where $\sin\alpha = 5/13$. The motion of the particle is resisted by a constant frictional force of magnitude 15 N. The particle comes to instantaneous rest after travelling a distance of '$x$' m up the plane.

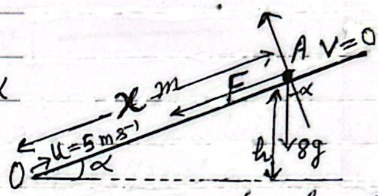
(i) Express the change in the gravitational potential energy of the particle in terms of $x$. --- [2]

(ii) Use an energy method to find '$x$'. [S-16/42/Q3] --- [4]

**Solution:** (i) $\sin\alpha = 5/13$

Let the particle comes to rest at A.

$OA = x \Rightarrow$ height at A, $h = x\sin\alpha$




$\left(\frac{h}{x} = \sin\alpha\right)$

P.E. at O = 0

∴ Change in P.E $= (mgh - 0)$

$= 8g \cdot x\sin\alpha$

P.E Gain $= 8g \cdot x \times \frac{5}{13} = \underline{30.8x}$ ✓ —— ①

(ii) Force of friction $F = 15$ N

Work done against the force of friction $= F \cdot x = 15x$ —— ②

Loss of K.E. $= \frac{1}{2}mu^2 - \frac{1}{2}mv^2$ [$u = 5 ms^{-1}$, $v = 0$]

$= \frac{1}{2} \times 8 \times 5^2 - 0$

$= 100$ J —— ③

Now Using Energy equation:

P.E gain + Work done against friction = Loss in K.E

$30.8x + 15x = 100$ [from ①, ② and ③]

$\Rightarrow 45.8x = 100$

or $x = 100/45.8 = 2.18$

∴ $\underline{x = 2.18 m}$ ✓

§ **Power:** Power of an engine (or a person) is defined as the rate of working.

$$P = \text{Work}/\text{Time} \quad \text{——— (1)}$$

Unit of Power is Watt (or W)

$1\,W = 1\,J s^{-1}$ [1 KW = 1000 Watt]

Again

$$P = \frac{\text{Work}}{\text{Time}} = \frac{F \times \text{distance}}{\text{Time}} = F \times v$$

If by applying a constant force 'F' a particle moves with a velocity v. Then

$$P = F \cdot v \quad \text{——— (2)}$$

**Example 6:** A weight lifter performs an exercise in which he raises a mass of 200 kg from rest, vertically through a distance of 0.7 m and holds it at that height.

(i) Find the workdone by the weightlifter. ---[2]

(ii) Given that the time taken to raise the mass is 1.2 s, find the average power developed by the weightlifter. [SP-17/04/Q1] [2]

**Solution:** (i) Work done $= F \times s = mg \times h$

$= 200 \times 10 \times 0.7$

$\therefore$ Workdone $= 1400\,J$ ✓

(ii)

$$\text{Power} = \text{Work}/\text{Time} = \frac{1400}{1.2} \quad (\text{Given } t = 1.2\,s)$$

$\therefore P = 1170\,W$ ✓

**Example 7:** A block is pulled along a horizontal floor by a horizontal rope. The tension in the rope is 500 N and the block moves at a constant speed of $2.75\,ms^{-1}$. Find the workdone by the tension in 40 s, and find the power applied by the tension. [S-15/43/Q1] [4]

**Solution:** Distance $= v \times t = (2.75 \times 40)\,m$

and Force = Tension = 500 N.

$\therefore$ Workdone $= F \times s = (2.75 \times 40) \times 500$

$= 55{,}000\,J$ ✓

and Power $= F \times v = 500 \times 2.75 = 1375\,W$ ✓

**Example 8:** A car of mass 900 kg is moving on a straight horizontal road ABCD. There is a constant resistance of magnitude 800 N in the sections AB and BC, and a constant resistance of magnitude R N in the section CD. The power of car's engine is constant 36 kW.

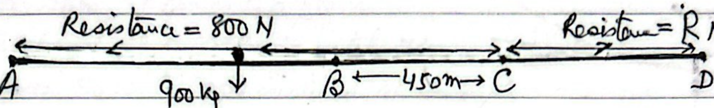
(i) The car moves from A to B at a constant speed in 120 s. Find the speed of the car and the distance AB. --- [3]

Now the car's engine is switched off at B.

(ii) The distance BC is 450 m. Find the speed of the car at C. --- [3]

(iii) The car comes to rest at D. The distance AD is 6637.5 m. Find the deceleration of the car and the value of R. [M-17 | 42 | Q4] --- [4]

**Solution:**




(i) for distance AB,

$P = 36000 W$, Resistance $= 800 N$

and moves with a constant speed $v = ?$

Now $P = F \times v \Rightarrow 36000 = 800 \times v$

$\Rightarrow$ speed $v = 45 ms^{-1}$ ✓ --- (1)

and distance $AB = v \times t = 45 \times 120$

$\Rightarrow AB = 5400 m$ ✓

(ii) Given $BC = 450 m$

Now as the car's engine is switched off at B (or $P = 0$), there will be a retardation 'a' (because of Resistance) $R = 800 N$

$\therefore -800 = 900a \quad [\because F = ma]$

$\Rightarrow a = -8/9$

Now speed of the car at B, $u = 45 ms^{-1}$ from (1)

Let the speed at C is v

$v^2 = 45^2 - 2 \times \frac{8}{9} \times 450 \quad [\because v^2 = u^2 + 2as]$

$\Rightarrow v = 35 ms^{-1}$ ✓

(iii) $u = 35 ms^{-1}$ at C, $v = 0$ at D

Given Total distance $AD = 6637.5 m$

$\therefore CD = AD - (AB + BC)$

$= 6637.5 - (5400 + 450)$

$\Rightarrow CD = 787.5 m$

for CD.

$v^2 = u^2 + 2as$

$0 = 35^2 + 2(-d)787.5 \quad$ [deceleration $a = -d$]

$\Rightarrow d = 7/9 = 0.778 ms^{-2}$ ✓ --- (2)

Given Resistance force in section CD $= R$ N.

$F = ma \quad [F = ma]$

$\Rightarrow R = 900 \times \frac{7}{9}$ from (2) $\{d = 7/9\}$

$R = 700 N$ ✓

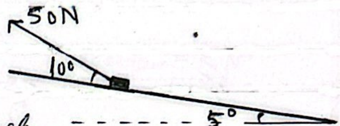
**Example 9:** A block of mass 25 is pulled along horizontal ground by a force of magnitude 50 N inclined at $10^\circ$ above the horizontal. The block starts from rest and travels a distance of 20 m, there is a constant resistance force of magnitude 30 N opposing motion.

(i) Find the workdone by the pulling force. ---[2]

(ii) Use an energy method to find the speed of the block when it has moved a distance of 20 m. [2]

(iii) Find the greatest power exerted by the 50 N force. [2]

After the block has travelled the 20 m, it comes to a plane inclined $5^\circ$ to the horizontal. The force 50 N is now inclined at an angle of $10^\circ$ to the plane and pulls the block directly up the plane. The resistance force remains 30N.

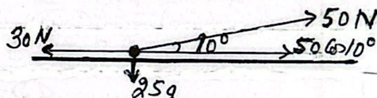



(iv) Find the time it takes for the block to come to rest from the instant when it reaches the foot of the inclined plane. [W-16/41/Q6] --[4]

**Solution:** (i) work done $= F \times s$

$= 50\cos 10^\circ \times 20$

$= 984.8\,J$ ✓ —— (1)




(ii) Energy Equation:

Work done by Force = K.E gain + Workdone against resistance

from (1)

$\Rightarrow 984.8 = 30 \times 20 + \frac{1}{2} \times 25 \times v^2$

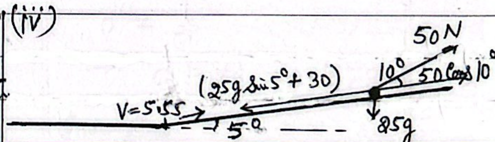
$\Rightarrow v = 5.55\,m\,s^{-1}$ ✓

(iii) Max. Power $= DF \times v$

$= 50\cos 10^\circ \times 5.55$

$= 273\,W$ ✓

(iv)




Components of forces along the inclined plane

$50\cos 10^\circ - 30 - 25g\sin 5^\circ = 25a$

$\Rightarrow a = -0.102\,m\,s^{-2}$

Now block comes to rest, $V = 0$

$u = 5.55\,m\,s^{-1}$, Let Time $= t$ s

$0 = 5.55 - 0.102t$ $[V = u + at]$

$\Rightarrow t = 54.4\,s$ ✓

**Example 10:** A straight hill AB has length 400m with A at the top and B at the bottom and is inclined at an angle of 4° to the horizontal. A straight horizontal road BC has length 750m, A car of mass 1250kg has a speed of $5ms^{-1}$ at 'A' when starting to move down the hill, while moving down the hill the resistance to the motion of the car is 2000 N and the driving force is constant. The speed of the car on reaching 'B' is $8ms^{-1}$.

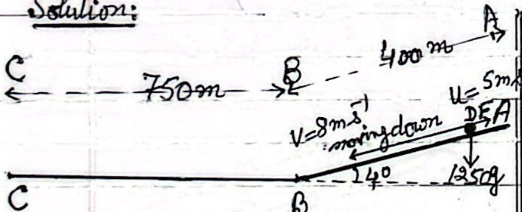
(i) By using work and energy, find the driving force of the car. [5]

On reaching B the car moves along the road BC, the driving force is constant and is twice that when the car was on the hill. The resistance to the motion of the car continues to be 2000N. Find,

(ii) the acceleration of the car while moving from B to C. [3]

(iii) the power of the car's engine as the car reaches C. [W-15/43/Q7] [3]

**Solution:**




(i) Downward motion from A→B

Gain in K.E $= \frac{1}{2}m(V^2 - u^2)$

$= \frac{1}{2} \times 1250(8^2 - 5^2)$ —①

Loss in P.E $= mgh = mg.d\sin\theta$

$= 1250g \times 400 \sin 4°$ —②

Let the Driving force = D.F

Work done by DF = DF × Dis = 400 DF —③

Work against resistance = 2000 × 400 —④

Now Work-Energy Equation:

Work done by DF + Loss in P.E

= gain in K.E + Work against Resistance

$\Rightarrow 400(DF) + 1250g \times 400\sin 4°$

$= \frac{1}{2} \times 1250(8^2 - 5^2) + 2000 \times 400$

$\Rightarrow$ D.F = **1189 N** ✓

(ii) Motion along Road BC.

Now DF = 2×1189, Resistance = 2000

Let the accelation along BC is 'a'

∴ Eqn of motion: DF − Resistance = ma

$\Rightarrow 1189 \times 2 - 2000 = 1250a$

$\Rightarrow$ **$a = 0.302\ ms^{-2}$** ✓

(iii) Now let Velocity at C = $V_C$

$V_C^2 = V_B^2 + 2as$

$\Rightarrow V_C^2 = 8^2 \times 2 \times 0.302 \times 750$

$\Rightarrow V_C = 22.75\ ms^{-1}$ —⑤

Now let Power of Engine at C = P

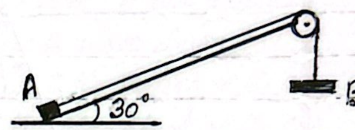
$P = DF \times V_C$

$= (2 \times 1189) \times 22.75$ [from ⑤]

$= 54100$ W

$\Rightarrow$ **P = 54.1 KW** ✓

**Example 11:** A light inextensible rope has a block 'A' of mass 5 kg attached to one end, and a block 'B' of mass 16 kg attached at the other end. The rope passes over a smooth pully, which is fixed at the top of a rough plane inclined at an angle of 30° to the horizontal.




Block 'A' is held at rest at the bottom of the plane and block 'B' hangs below the pulley. The coefficient of friction between 'A' and plane is $\frac{1}{\sqrt{3}}$. Block 'A' is released from rest and the system starts to move, when each of the blocks has moved a distance of $x$ m each has speed $v$ ms$^{-1}$.

(i) Write down the gain in K.E of the system in terms of $v$. ---[1]

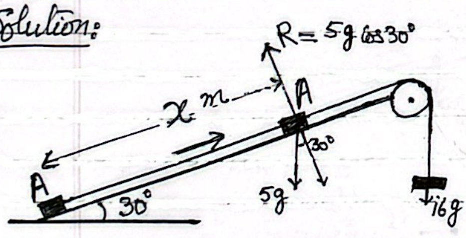
(ii) Find, in terms of $x$,

(a) the loss of gravitational potential energy of the system. ---[2]

(b) the work done against the frictional force. [3]

(iii) Show that $21v^2 = 220x$ [S-14/42/Q5] [2]

**Solution:**




(i) Gain in K.E $= \frac{1}{2}mv^2$ $\left[ m = 5+16 = 21 \text{ kg} \right.$

$= \frac{1}{2} \times 21 \times v^2$

$= \underline{10.5v^2}$ ✓

(ii)(a) P.E. loss of system

= Loss of P.E of 'B' − Gain in P.E. by 'A'

$= 16g \cdot x - 5g \cdot x \sin 30°$

$= \underline{135x \text{ J}}$ ✓

(ii)(b) Normal Reaction $R = m_A g \cos 30$

$= 5g \times \sqrt{3}/2$

∴ Force of friction $F = \mu R = \frac{1}{\sqrt{3}} \times \left(50 \times \frac{\sqrt{3}}{2}\right)$

$\Rightarrow F = 25 \text{ N}$,

∴ Work done against friction $= F \times$ dist.

$= \underline{25x \text{ J}}$ ✓

(iii) Energy Equation:

Loss in P.E = Work done against friction + gain in K.E

$\Rightarrow 135x = 25x + 10.5v^2$

$\Rightarrow \underline{21v^2 = 220x}$ ✓