DATE 05.01.24

M.1

Mechanics-1

Energy, Work and Power

Revision.

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(M.1) Energy, work and power. Revision

1. A constant resistance of magnitude 1350N acts on a car of mass 1200 Kg.

(a) The car is moving along a straight level road at a constant speed of $32\,ms^{-1}$. Find, in kW, the rate at which the engine of the car is working. ---[2]

(b) The car travels at a constant speed down a hill inclined at an angle of $\theta°$ to the horizontal, where $\sin\theta° = \frac{1}{20}$, with the engine working at 31.5 kW.
Find the speed of the car. --[3]

SP-20/04/Q2

Solution (a) DF (resistance) $F = 1350N$

Speed $v = 32\,ms^{-1}$

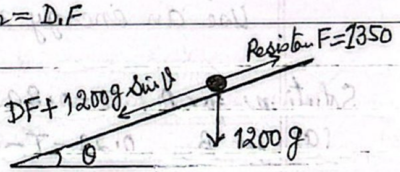
$\therefore$ rate of working; Power $= F \times v = 1350 \times 32$ W
$= \underline{43.2 \text{ kW}}$.

(b) Let Driving Force of the engine of the Car = D.F

along the Plane: $DF + 1200g \sin\theta = 1350$ (Resistance)

$DF + 1200 \times 10 \times \frac{1}{20} = 1350$

$DF = 1350 - 600 = 750N$. —— ①



Engine Power $P = 31.5$ kW $= 31500$ W

Now $P = DF \times v$

$31500 = 750 \times v$ from ①

$\therefore v = \frac{31500}{750} = 42\,ms^{-1}$,

Speed of the Car, $v = \underline{42\,ms^{-1}}$.

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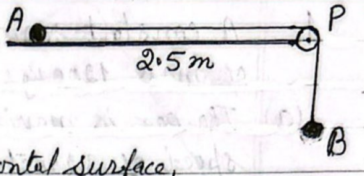
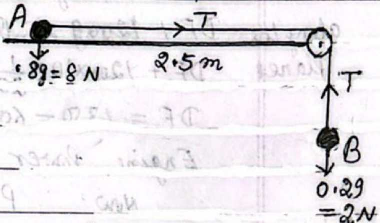
2. Two particles A and B, of masses 0.8 kg and 0.2 kg respectively, are connected by a light inextensible string. Particle A is placed on a horizontal surface.



The string passes over a small smooth pully P fixed at the edge of the surface, and B hangs freely. The horizonal section of the string, AP, is of length 2.5 m. The particles are released from rest with sections of the string taut.

(a) Given that the surface is smooth, find the time taken for A to reach the pulley. ---[5]

(b) It is given instead that the surface is rough and that the speed of A immediately before it reaches the pulley is $v\ ms^{-1}$. The work done against friction as A moves from rest to the pulley is 2 J.
Use an energy method to find v. [SP-20/04/Q7] ---[4]



Solution: for A, $T = 0.8a$ —— ①

(a) for B, $0.2g - T = 0.2a$ —— ②

for ① & ② $0.2g = 0.8a + 0.2a$

$\Rightarrow a = 2$

Now $s = ut + \frac{1}{2}at^2 \Rightarrow 2.5 = 0 + \frac{1}{2} \times 2 \times t^2$

$\Rightarrow t^2 = 2.5 \Rightarrow t = \underline{1.58\,s}$ ✓

(b) P.E loss by B = K.E gain by A and B + Work done against resistance

$\Rightarrow 0.2 \times g \times 2.5 = \frac{1}{2}(0.8+0.2) \times V^2 + 2 \quad \left[m_2 g h = \frac{1}{2}(m_1+m_2)V^2 + F_r\right]$

$5 = \frac{1}{2}V^2 + 2$

$\Rightarrow V^2 = 6$

$\underline{V = \sqrt{6}}$ (or 2.45) ✓

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3. A lorry of mass 16000 kg is travelling along a straight horizontal road. The engine of the lorry is working at constant power. The work done by the driving force in 10s is 750 000 J.

(a) Find the power of the lorry's engine. --[1]

(b) There is a constant resistance force acting on the lorry of magnitude 2400 N. Find the accelaration of the lorry at an instant when its speed is $25\,ms^{-1}$. --[3]

[M-20 | 42 | Q1]

<u>Solution</u>: (a) Power $= \frac{W}{t} = \frac{750\,000}{10} = 75\,000$ W (or 75 KW)

(b) Driving Force $= \frac{P}{v}$ $(\because P = F \times v)$

$= \frac{75000}{25}$ $(\because v = 25\,ms^{-1})$

$\Rightarrow$ D.F. $= 3000$ N.

Now DF − Resistance $= ma$

$3000 - 2400 = 16000 \times a$

$\Rightarrow a = \frac{600}{16,000} = \underline{0.0375\,ms^{-1}}$

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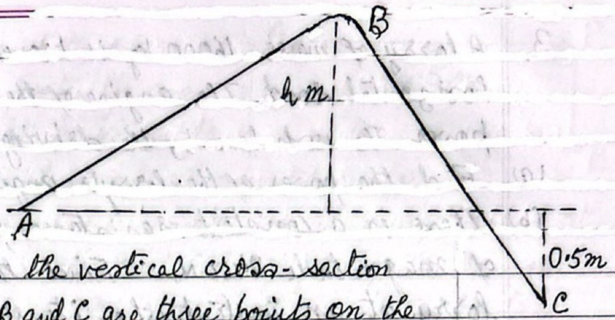
4.



The diagram shows the vertical cross-section of a surface. A, B and C are three points on the cross-section. The level of B is h m above the level of A. The level of C is 0.5 m below the level of A.
A particle of mass 0.2 kg is projected up the slope from A with initial speed 5 ms^{-1}. The particle remains in contact with the surface as it travels from A to C.

(a) Given that the particle reaches B with a speed of 3 ms^{-1} and that there is no resistance force, find h. ---[3]

(b) It is given instead that there is a resistance force and the particle does 3.1 J work against the resistance force as it travels from A to C.
Find the speed of the particle when it reaches C. ---[3]

[M-20/42/Q3]

Solution: Initial $K.E = \frac{1}{2}mv^2 = \frac{1}{2}\times 0.2\times 5^2$

(a) $\left\{\begin{array}{l}\text{Final } K.E = \frac{1}{2}\times 0.2\times 3^2 \\ \text{Gain in } P.E = mgh = 0.2gh.\end{array}\right.$

Using Energy equation:

$\frac{1}{2}\times 0.2\times 5^2 = 0.2gh + \frac{1}{2}\times 0.2\times 3^2$

$2.5 = 2h + 0.9$

$\Rightarrow h = 0.8m.$

(b) Applying work-energy equation from A to C.

$\frac{1}{2}\times 0.2\times 5^2 - 3.1 + 0.2g\times 0.5 = \frac{1}{2}\times 0.2v^2$

$\Rightarrow 2.5 - 3.1 + 1 = 0.1v^2$

$v^2 = \frac{.4}{.1} = 4 \Rightarrow v = 2$

$\therefore v = 2\,ms^{-1}$ ✓

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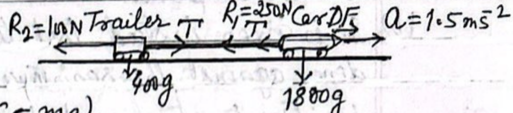
5. A car of mass 1800 kg is towing a trailer of mass 400 kg along a straight horizontal road. The car and trailer are connected by a light rigid tow-bar. The car is accelerating at $1.5\,ms^{-2}$. There are constant resistance forces of 250 N on the car and 100 N on the trailer.

(a) Find the tension in the tow-bar. ---[2]

(b) Find the power of the engine of the car at the instant when the speed is $20\,ms^{-1}$. ---[3]

[S-20/41/Q2]

Solution:



(a) For Trailer

$$T - 100 = 400 \times 1.5 \quad (F = ma)$$

$$T = \underline{700\ N}$$

(b) Let the driving force of the engine = D.F

$$DF - 250 - 100 = 2200 \times 1.5 \quad (F = ma,\ m = \underline{1800 + 400})$$

$$\Rightarrow DF = 3300 + 350 = 3650\,N.$$

Power of the engine $P = F.v$ Given $[v = 20\,ms^{-1}]$

$$= 3650 \times 20$$

$$= 73000\ W$$

$$\text{or } \underline{P = 73\ kW}$$

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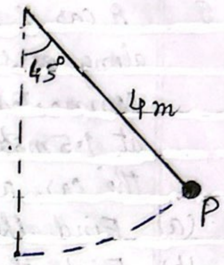
6. A child of mass 35 kg is swinging on a rope. The child is modelled as a particle P and the rope is modelled as a light inextensible string of length 4 m. Initially P is held at an angle of $45°$ to the vertical.



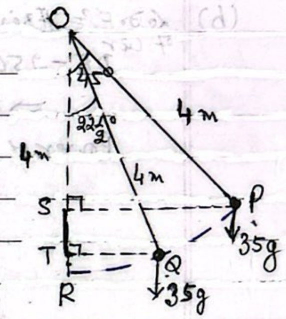
(a) Given that there is no resistance force, find the speed of P when it has travelled half way along the circular arc from its initial position to its lowest point. --[4]

(b) It is given instead that there is a resistance force. The work done against the resistance force as P travels from its initial position to its lowest point is X J. The speed of P at its lowest point is $4\,ms^{-2}$. --[3]

Find X

[S-20/41/Q5]

"Let Q is midway of PR"



Solution (a) P.E lost = mgh ($h = ST$)

$= 35g(4\cos 22.5 - 4\cos 45)$ — ①

Let the velocity gain at Q = v

K.E gained $= \frac{1}{2}mv^2$

$= \frac{1}{2}35 \times v^2$ — ②

from ① & ② $\frac{1}{2} \times 35 \times v^2 = 35g \times 4(\cos 22.5 - \cos 45)$

$v^2 = 2 \times 40 \times (0.9238 - 0.707)$

$= 80 \times 0.21688 = 17.35$

$v = \sqrt{17.5} = 4.16 \Rightarrow$ $\underline{v = 4.16\,ms^{-1}}$ ✓

(b) Using work-energy equation.

P.E. Lost = K.E. gain + Work done against resistance ③

Travel from P to R, P.E Lost $= 35g(4 - 4\cos 45)$ — ④

Gain in K.E. $= \frac{1}{2} \times 35 \times 4^2$ — ⑤

from ④ and ⑤ in ③ (X = work done against resistance)

$35g(4 - 4\cos 45) = \frac{1}{2} \times 35 \times 4^2 + X$

$35 \times 10 \times 4(1 - \cos 45) = 35 \times 8 + X$

$1400 \times 0.293 = 280 + X \Rightarrow X = 410 - 280 = 130\,J$

$\underline{X = 130\,J}$ ✓

7. A car of mass 1250 kg is moving on a straight road.

(a) On a horizontal section of the road, the car has a constant speed of $32\,ms^{-1}$ and there is a constant force of 750 N resisting the motion.

(i) Calculate, in kW, the power developed by the engine of the car. --[2]

(ii) Given that this power is suddenly decreased by 8 kW, find the intantaneous deceleration of the car. --[3]

(b) On a section of the road inclined at $\sin^{-1} 0.096$ to the horizontal, the resistance to the motion of the car is $(1000+8v)$ N, when the speed of the car is $v\,ms^{-1}$. The car travels up this section of the road at constant speed with the engine working at 60 kW. Find this constant speed. --[5]

[S-20/42/Q5]

Solution (a)(i) $DF = 750\text{ N}$, $\because$ speed $= 32\,ms^{-1}$

$\therefore$ Power $P = F \times v = 750 \times 32 = 24000\,W = \underline{24\,kW}$

(a)(ii) Now Power $= 24\,kW - 8\,kW = 16\,kW = 16000\,W$.

or $16,000 = DF \times 32$

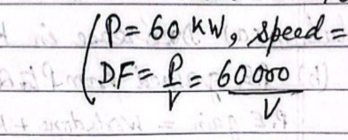
$\therefore$ now $DF = 500\,N$

$\therefore$ $500 - 750 = 1250a$ [DF − Resistance = ma]

$\Rightarrow a = -0.2$ or Deceleration $= \underline{0.2\,ms^{-2}}$

(b)

$DF = 1250g\sin\theta + (1000+8V)$



$\therefore 1250g \times 0.096 + (1000+8V) = \frac{60,000}{V}$

$\Rightarrow 1200 + 1000 + 8V = \left(\frac{60000}{V}\right)$

$\because \theta = \sin^{-1} 0.096$

$\Rightarrow \sin\theta = 0.096$

$P = 60\,kW$, speed $= v$

$DF = \frac{P}{V} = \frac{60000}{V}$

$\Rightarrow 8V^2 + 2200V - 60000 = 0$

$V^2 + 275V - 7500 = 0$

$(V+300)(V-25) = 0$

$V = 25$ or $V = -300$

$\therefore V = \underline{25\,ms^{-1}}$

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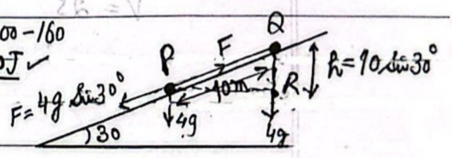
8. A minibus of mass 4000 kg is travelling along a straight horizontal road. The resistance to motion is 900 N.
(a) Find the driving force when the acceleration of the minibus is $0.5\,ms^{-2}$. ---[2]
(b) Find the power required for the minibus to maintain a constant speed of $25\,ms^{-1}$. [S-20/43/Q2] ---[2]

Solution: Driving force $= F$, mass $m = 4000$ kg, Resistance $= 900$ N
(a) $F - 900 = 4000 \times 0.5$ $[F = ma]$ $(a = 0.5\,ms^{-1})$
$\Rightarrow F = \underline{2900\,N}$ ✓
(b) $P = F \times v$ = resistance $\times v$
$= 900 \times 25 = 22500$ $(v = 25\,ms^{-1})$
$\therefore P = \underline{22.5\,kW}$ ✓

9. A block B of mass 4 kg is pushed up a line of greatest slope of a smooth plane inclined at 30° to the horizontal by a force applied to B, acting in the direction of motion of B. The block passes through points P and Q with speeds $12\,ms^{-1}$ and $8\,ms^{-1}$ respectively. P and Q are 10 m apart with P below the level of Q.
(a) Find the decrease in kinetic energy of the block as it moves from P to Q. ---[2]
(b) Hence find the work done by the force pushing the block up the slope as the block moves from P to Q. --[3]
(c) At the instant block reaches Q, the force pushing the block up the slope is removed. Find the time taken, after this instant, for the block to return to P. [S-20/43/Q5] --[4]

Solution (a) Decrease in K.E $= \frac{1}{2}4\times(12^2 - 8^2) = 160$ J.
(b) P.E. gained from P to Q $= mgh = 40g \times h = 40g\times 10\sin 30^\circ = \underline{200\,J}$ ✓
P.E gain = Workdone + K.E loss
200 = Workdone + 160 $\Rightarrow$ Workdone = 200 − 160 = $\underline{40\,J}$ ✓



(c) $-4g\sin 30 = 4a \Rightarrow a = -5$
$s = ut + \frac{1}{2}at^2 \Rightarrow -10 = 8t - \frac{1}{2}5t^2$
$\Rightarrow \underline{t = 4.16\,s}$ ✓

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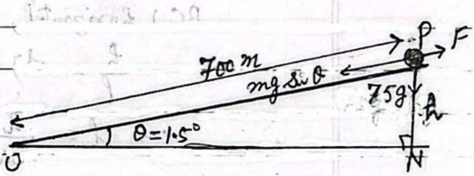
10. A crane is lifting a load of 1250 kg vertically at a constant speed $V\,ms^{-1}$. Given that the power of the crane is a constant 20 kW, Find the value of V. [W-19/41/Q1] --[2]

Solution: Power $P = F \times v \Rightarrow 20000 = 1250g \times V$.

$$\Rightarrow V = \frac{20000}{1250 \times 10} = 1.6\,ms^{-1}$$

11. The total mass of a cyclist and her bicycle is 75 kg. The cyclist ascends a straight hill of length 0.7 km inclined at 1.5° to the horizontal. Her speed at the bottom of the hill is $10\,ms^{-1}$ and at the top it is $5\,ms^{-1}$. There is a resistance to the motion, and the work done against this resistance as the cyclist ascends the hill is 2000 J. The cyclist exerts a constant force of magnitude F N in the direction of motion. Find F. [W-19/41/Q2] --[5]



Solution: $h = 700 \sin 1.5°$

P.E gained from O to P $= mgh$

$= 75g \times 700 \sin 1.5°$ —①

Loss in K.E $= \frac{1}{2} \times 75(10^2 - 5^2)$

Work done by the force F = $W = F \times$ distance $= F \times 700$ J.

Now Loss in K.E + Work done W = P.E gain + Work against resistance

$$\frac{1}{2} \times 75 \times (10^2 - 5^2) + 700F = 75g \times 700 \sin 1.5° + 2000$$

$$\frac{1}{2} \times 75 \times 75 + 700F = 750 \times 700 \times 0.02617$$

$$2812.5 + 700F = 13742.9 + 2000$$

$$700F = 15742.9 - 2812.5 = 12930.4$$

$$F = \frac{12930.4}{700} = 18.472$$

$$F = 18.5\,N$$

12. A Lorry of mass 25000 kg travels along a straight horizontal road, There is a constant force of 3000 N resisting the motion.

(i) Find the power required to maintain a constant speed of $30\,ms^{-1}$. --[2]

The lorry comes to a straight hill inclined at 2° to the horizontal. The driver switches off the engine of the lorry at the point A which is at the foot of the hill. Point B is further up the hill. The speeds of the lorry at A and B are $30\,ms^{-1}$ and $25\,ms^{-1}$ respectively. The resistance force is still 3000 N,

(ii) Use energy method to find the height of B above the level of A. [W-19/42/Q4] --[5]

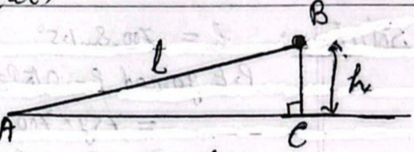
Solution: (i) $P = F \times v = 3000 \times 30 = 90{,}000\,W = \underline{90\,kW}$

(ii) Let distance along hill $AB = l$

$BC \perp$ horizontal, $BC = h$ (let)

$\frac{h}{l} = \sin 2^\circ$

$\Rightarrow l = \frac{h}{\sin 2^\circ}$ —①



Workdone by force 3000 N in moving $AB = F \times l$

$= 3000 \times \frac{h}{\sin 2^\circ}$ —②

P.E. gained $= mgh = 25000\,gh$.

Change in K.E. $= \frac{1}{2}m(v^2 - u^2) = \frac{1}{2} \times 25000(30^2 - u^2)$

Now Change in K.E = P.E gain + workdone against resistance

$\frac{1}{2} \times 25000(30^2 - 25^2) = 25000 \times g \times h + 3000 \times \frac{h}{\sin 2^\circ}$

$12500 \times 275 = h\left[250000 + \frac{3000}{\sin 2^\circ}\right]$

$3437500 = 335961\,h$

$\Rightarrow h = \frac{3437500}{335961} = 10.2318$

$\therefore h = \underline{10.2\,m}$

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13. A train of mass 150 000 kg ascends a straight slope inclined at α° to the horizontal with a constant driving force of 16 000 N. At a point A on the slope the speed of the train is $45 ms^{-1}$. Point B on the slope is 500 m beyond A. At B the speed of the train is $42 ms^{-1}$. There is a resistance force acting on the train and the train does 4×10^6 J of work against the resistance force between A and B. Find the value of α. [W-19/43/Q2] ---[5]

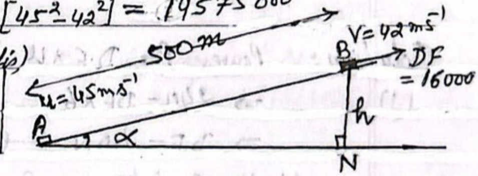
Solution: Height at the point B above A, $h = 500 \sin\alpha$ $\left[\frac{h}{500} = \sin\alpha\right]$

P.E. gain $= mgh = 150\,000 g \times 500 \sin\alpha$ — ①

K.E. Loss from A to B $= \frac{1}{2} \times 150000 [45^2 - 42^2] = 19575000$ ②

Work done by DF $= 16000 \times 500$ (F×dis)

Resistance against the motion $= 4 \times 10^6$



Gain in P.E = Loss in K.E + Work done by DF − Resistance force

$150000g \times 500 \sin\alpha = 19575000 + 16000 \times 500 - 4 \times 10^6$

$750000000 \sin\alpha = 27575000 - 4 \times 10^6$

$750000000 \sin\alpha = 23575000$

$\therefore \sin\alpha = \frac{23575000}{750000000} = 0.03433$

$\alpha = 1.801$

$\therefore \alpha = 1.8^\circ$

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14. A cyclist is travelling along a straight horizontal road. The total mass of the cyclist and his bicycle is 80 kg. His power output is a constant 240 W. His acceleration when he is travelling at $6\,ms^{-1}$ is $0.3\,ms^{-2}$.

(i) Show that the resistance to the cyclist's motion is 16 N. --[3]

(ii) Find the steady speed that the cyclist can maintain if his power output and the resistance force are both unchanged. --[2]

(iii) The cyclist later ascends a straight hill inclined at 3° to the horizontal. His power output and the resistance force are still both unchanged. Find the acceleration when he is travelling at $4\,ms^{-1}$. [W-19/43/Q5] --[3]

Solution: Power $P = D.F \times v$

(i)

$\Rightarrow 240 = DF \times 6$

$\Rightarrow D.F. = 40N$ --- (1)

Let the Resistance $= R$

$DF - R = ma$

$40 - R = 80 \times 0.3$ $\left[\because \begin{matrix} m = 80 \\ a = 0.3 \end{matrix}\right]$

$\Rightarrow R = \underline{16N}$ ✓

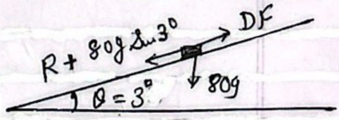
(ii) Now Resistance force $F = 16N$

$P = F \times v$

$240 = 16 \times v$

$\Rightarrow v = \underline{15\,ms^{-1}}$ ✓

(iii) Now $D.F = \frac{P}{v} = \frac{240}{4} = 60N.$



$DF - R - 80g \sin 3^\circ = 80a$

$60 - 16 - 800 \times 0.052336 = 80a$ $\because [R = 16]$

$60 - 16 - 41.8687 = 80a$

$2.131 = 80a \Rightarrow a = \underline{0.0266\,ms^{-2}}$ ✓

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15. A car of mass 1400 kg is moving along a straight horizontal road against a resistance of magnitude 350 N.

(a) Find, in kW, the rate at which the engine of the car is working when it is travelling at a constant speed of $20\,ms^{-1}$. --- [2]

(b) Find the acceleration of the car when its speed is $20\,ms^{-1}$ and the engine is working at 15 kW. W-20/41/Q2 --- [3]

Solution: (a) Power, $P = F \times v = 350 \times 20 = 7000\,W = \underline{7\,kW}$.

(b) $DF \times v = P \Rightarrow DF \times 20 = 15000 \Rightarrow DF = 750$

Now $DF - \text{Resistance} = ma \Rightarrow 750 - 350 = 1400 \times a$

$\Rightarrow a = \frac{2}{7}\,ms^{-2}$

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16. A car of mass 1500 kg is pulling a trailer of mass 750 kg up a straight hill of length 800 m inclined at an angle of $\sin^{-1} 0.08$ to the horizontal. The resistances to the motion of the car and trailer are 400 N and 200 N respectively. The car and the trailer are connected by a light rigid tow-bar. The car and trailer have speed $30\,ms^{-1}$ at the bottom of the hill and $20\,ms^{-1}$ at the top of the hill.

(a) Use an energy method to find the constant driving force as the car and trailer travel up the hill. --- [5]

After reaching the top of the hill the system consisting of the car and trailer travels along a straight level road. The driving force of the car's engine is 2400 N and the resistances to motion are unchanged.

(b) Find the acceleration of the system and the tension in the tow-bar. --- [4]

W-20/41/Q6

Solution: Initial K.E. $= \frac{1}{2} \times 1500 \times 30^2 + \frac{1}{2} \times 750 \times 30^2$

(a) Final K.E. $= \frac{1}{2} \times 1500 \times 20^2 + \frac{1}{2} \times 750 \times 20^2$

P.E. Gain $= 2250 \times 10 \times 800 \times 0.08$

WD against friction $= 600 \times 800$

$\therefore \frac{1}{2} \times 2250 \times 30^2 + DF \times 800 = 600 \times 800 + \frac{1}{2} \times 2250 \times 20^2 + 2250 \times 10 \times 800 \times 0.08$

$\Rightarrow DF = \underline{1700\,N}$. (continued →)

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(continued→)

16(b). Solution

$DF - \text{Resistance} = ma \Rightarrow 2400 - 600 = 2250a \Rightarrow \underline{a = 0.8\,ms^{-2}}$ ✓

for Trailer $T - 200 = 750a$

$\Rightarrow T - 200 = 750 \times 0.8 \Rightarrow \underline{T = 800\,N}$ ✓

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17. A car of mass 1800 kg is travelling along a straight horizontal road. The power of the car's engine is constant. There is a constant resistance to motion of 650 N.

(a) Find the power of the car's engine, given that the car's acceleration is $0.5\,ms^{-2}$ when its speed is $20\,ms^{-1}$. --[3]

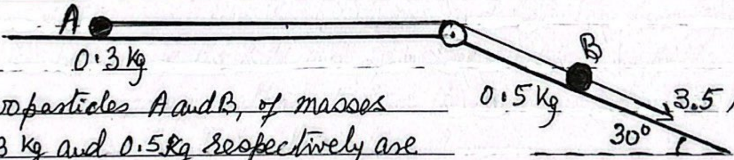
(b) Find the steady speed which the car can maintain with the engine working at this power. [W.20 | 42 | Q2] --[2]

Solution(a) $DF - 650 = 1800 \times 0.5 \Rightarrow DF = 1550\,N.$

$\therefore$ Power $P = DF \times v = 1550 \times 20 = 31000\,W = \underline{31\,kW}$ ✓

(b) $P = F \times v \Rightarrow 31000 = 650 \times v \Rightarrow v = \underline{47.7\,ms^{-1}}$ ✓

18.



Two particles A and B, of masses 0.3 kg and 0.5 kg respectively are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley which is attached to a horizontal plane and to the top of an inclined plane. The particles are initially at rest, with A on the horizontal plane and B on the inclined plane, which makes an angle of 30° with the horizontal. The string is taut and B can move on a line of greatest slope of the inclined plane. A force of magnitude 3.5 N is applied to B acting down the plane.

It is given that the two planes are rough. When each particle has moved a distance of 0.6 m from rest, total amount of work done against friction is 1.1 J.

(Continued→)

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(Continued →)

18. Use energy method to find the speed of B when it has moved this distance down the plane. [W-20/42/Q8(b)] --[4]

Solution: for particle B. Let the speed of B is $v\ ms^{-1}$

$$0.5g\ \sin 30° \times 0.6 + 3.5 \times 0.6 = \frac{1}{2} \times (0.3 + 0.5)v^2 + 1.1$$

$$\Rightarrow 1.5 + 2.1 = 0.4v^2 + 1.1 \Rightarrow v = 2.5\ ms^{-1}$$ ✓

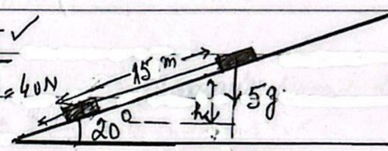
19. A box of mass 5 kg is pulled at a constant speed a distance of 15 m up a rough plane inclined at angle of 20° to the horizontal. The box moves along a line of greatest slope against a frictional force of 40 N. The force pulling the box is parallel to the line of greatest slope.

(a) Find the work done against friction. --[1]

(b) Find the change in gravitational potential energy of the box. --[2]

(c) Find the work done by the pulling force. [W-20/43/Q2] --[1]

Solution (a) WD against friction $F_r \times s = 40 \times 15 = \underline{600\ J}$ ✓



(b) $PE = mgh = 5 \times 10 \times 15 \cdot \sin 20°$

$= \underline{257\ J}\ (256.515)$

$\left\{ \frac{h}{15} = \sin 20° \Rightarrow h = 15 \times \sin 20° \right.$

(c) $WD = 40 \times 15 + 5 \times 10 \times 15 \sin 20 = \underline{857\ J}$ ✓

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20. A car of mass 1600 kg is pulling a caravan of mass 800 kg. The car and the caravan are connected by a light rigid tow-bar. The resistances to the motion of the car and caravan are 400N and 250 N respectively.

(a) The car and caravan are travelling along a straight horizontal road.

(i) Given that the car and caravan have a constant speed of $25 ms^{-1}$, find the power of the car's engine. --[2]

(ii) The engine's power is now suddenly increased to 39 kW. Find the instantaneous acceleration of the car and caravan and find the tension in the tow-bar. --[5]

(b) The car and caravan now travel up a straight hill, inclined at an angle of $\sin^{-1} 0.05$ to the horizontal, at a constant speed of $v\ ms^{-1}$. The car's engine is working at 32.5 kW. Find v. [W-20/43/Q6] --[3]

Solution (a) (i) $P = (400+250)\times 25 = 16250 = 16.25\ kW$ $(P = F\times v)$

(ii) $DF = \frac{P}{v} = \frac{39000}{25} = 1560\ N$

$1560 - 650 = (1600+800)a$ (Newton's second law)

$\Rightarrow a = \frac{910}{2400} = 0.379\ ms^{-2}$

also $T - 250 = 800a$

$\Rightarrow T = 250 + 800\times 0.379 = 553\ N$ (553.33)

$\therefore T = 553N$

(b) Now $DF = 650 + 2400\times 10\times 0.05$

$P = F\times v \Rightarrow 32500 = (650 + 2400\times 10\times 0.05)\times v$

$\Rightarrow v = 17.6\ ms^{-1}$

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21 A car of mass 1400 kg is travelling at constant speed up a straight hill inclined at α to the horizontal, where $\sin\alpha = 0.1$. There is a constant resistant force of magnitude 600 N. The power of the car's engine is 22500 W.

(a) Show that the speed of the car is $11.25\ ms^{-1}$. --[3]

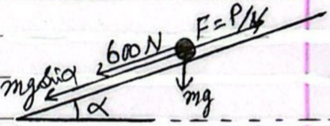
The car moving with speed $11.25\ ms^{-1}$, comes to a section of the hill which is inclined at $2°$ to the horizontal.

(b) Given that the power and the resistance force do not change, find the initial acceleration of the car up this section of the hill. --[3]

[M-21/42/Q2]

Solution: Power of Car $P = 22500\,W$, mass of car $= 1400\,kg$.
Let the forward speed of the Car $= V\ ms^{-1}$, $\sin\alpha = 0.1$
Upward force $F = P/V = \frac{22500}{V}$; V is constant → acceleration $a = 0$

(a) along the plane.

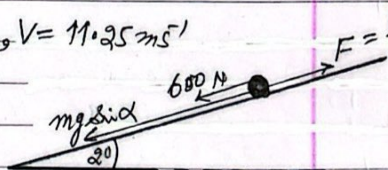


$F - mg\sin\alpha - 600 = m \times a$

$\Rightarrow \frac{22500}{V} - 1400 \times 10 \times 0.1 - 600 = m \times 0$

$\Rightarrow \frac{22500}{V} = 1400 + 600 \Rightarrow V = \frac{22500}{2000} = \underline{11.25\ ms^{-1}}$ ✓

(b) Now $\alpha = 2°$, acceleration $a = ?$, $V = 11.25\ ms^{-1}$



$F - mg\sin\alpha - 600 = ma$

$\frac{P}{V} - mg\sin\alpha - 600 = ma$

$\Rightarrow \frac{22500}{11.22} - 1400 \times 10 \times \sin 2° - 600 = 1400a$

$\Rightarrow 2000 - 14000 \times 0.34899 - 600 = 1400a$

$2000 - 488.6 - 600 = 1400a$

$\Rightarrow 1400a = 911.4$

$a = \frac{911.4}{1400} = \underline{0.651\ ms^{-2}}$ ✓

$\underline{a = 0.651\ ms^{-2}}$

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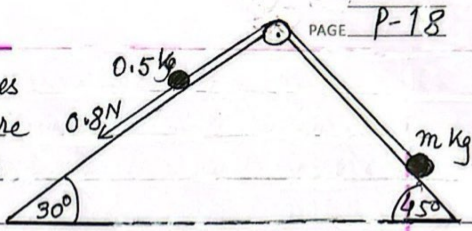


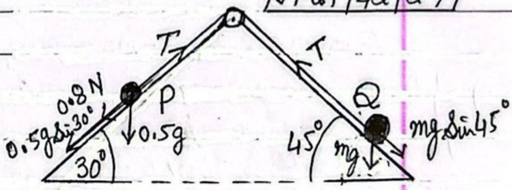
22. Two particles P and Q of masses 0.5 kg and m kg respectively are attached to the ends of a light inextensible string.
The string passes over a smooth pulley which is attached to the top of two inclined planes. The particles are initially at rest with P on a smooth plane inclined at 30° to the horizontal and Q on a plane inclined at 45° to the horizontal. The string is taut and the particles can move on lines of greatest slope of the two planes.
A force of magnitude 0.8 N is applied to P acting down the plane, causing P to move down the plane.

(a) It is given that $m = 0.3$ and that the plane on which Q rests is smooth. Find the tension in the string. --[5]

(b) It is given instead that the plane on which Q rests is rough, and that after each particle has moved a distance of 1 m, their speed is $0.6\,ms^{-1}$. The work done against friction in this part of the motion is 0.5 J.
Use energy method to find the value of m. ---[5]

M-21/42/Q7

Solution (a) Given $m = 0.3$ kg, Plane on which Q rests is smooth.



Particle P is moving downwards

$0.8 + 0.5g\sin 30° - T = 0.5a$ --- (1)

Particle Q is moving upwards:

$T - 0.3g\sin 45° = 0.3a$ --- (2)

add (1) and (2) $\quad 0.8 + \frac{5}{2} - \frac{3}{\sqrt{2}} = 0.8a$

$\Rightarrow 0.8a = 0.8 + 2.5 - 2.12132 = 1.178679$

$\Rightarrow a = \frac{1.178679}{0.8} = 1.47335$

from (1) $\quad 0.8 + 2.5 - T = 0.5 \times 1.47335$

$\Rightarrow T = 0.8 + 2.5 - 0.736675 = 2.5633$

$\therefore T = 2.56$ N

(Continued →)

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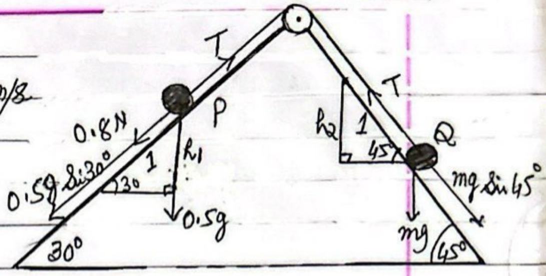
22(b) Now Q rest on rough plane.
Distance travelled = 1 m, $v = 0.6$ m/s
Work done against friction = 0.5 J.
$m = ?$ (Using Energy method)



Work-energy equation:
P.E. Loss + Work done by 0.8N force
= K.E gain + Work done against friction ---- 1

$\left\{ \frac{h_1}{1} = \sin30^\circ \Rightarrow h_1 = 1 \times \sin30 = 0.5\,m \right.$ ✓

$\left\{ \frac{h_2}{1} = \sin45^\circ \Rightarrow h_2 = \frac{1}{\sqrt{2}}\,m \right.$ ✓

Now K.E and P.E for m Kg particle:

$$\left. \begin{aligned} K.E &= \tfrac{1}{2} m \times (.6)^2 = 0.18m \checkmark \\ P.E &= mgh_2 = m \times 10 \times \tfrac{1}{\sqrt{2}} = 5\sqrt{2}m \end{aligned} \right\}$$

and K.E and P.E for 0.5 kg particle:

$$\left. \begin{aligned} K.E &= \tfrac{1}{2} \times 0.5 \times (0.6)^2 = 0.09 \checkmark \\ P.E &= mgh_1 = 0.5 \times 10 \times 0.5 = 2.5 \checkmark \end{aligned} \right\}$$

from 1

$$2.5 - 5\sqrt{2}m + 0.8 \times 1 = 0.18m + 0.09 + 0.5$$
$$\Rightarrow (5\sqrt{2} + 0.18)m = 2.5 + 0.8 - 0.59$$
$$7.251\,m = 2.71$$
$$m = \frac{2.71}{7.251} = 0.374 \quad (0.37374)$$

$\therefore m = \underline{0.374}$ kg. ✓

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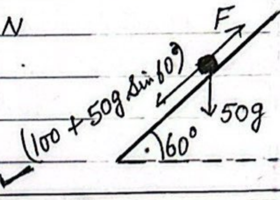
23. A winch operates by means of a force applied by a rope. The winch is used to pull a load of mass 50 kg up a line of greatest slope of a plane inclined at 60° to the horizontal. The winch pulls the load a distance of 5 m up the plane at constant speed. There is a constant resistance to motion of 100 N. Find the work done by the winch. ---[3]

[S-21/41/Q1]



Solution: Force exerted by winch $F = (100 + 50g \sin 60^\circ)$ N

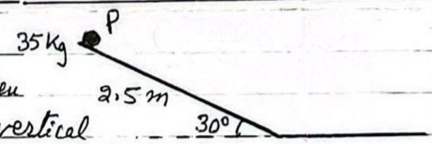
distance = 5 m

Work done = Force × distance

$= (100 + 50g \sin 60^\circ) \times 5 = 533 \times 5$

$= \underline{2665\ J}$

24. A slide in a playground descends at a constant angle of 30° for 2.5 m. It then has a horizontal section in the same vertical plane as the sloping section. A child of mass 35 kg, modelled as a particle P, starts from rest at the top of the slide and slides straight down the sloping section. She then continues along the horizontal section until she comes to rest. There is no instantaneous change in speed when the child goes from sloping section to the horizontal section. The child experiences a resistance force on the horizontal section, and the work done against the resistance force on the horizontal section is 250 J per metre.



(a) It is given that the sloping section of the slide is smooth.

(i) Find the speed of the child when she reaches the bottom of the sloping section. ---[3]

(ii) Find the distance that the child travels along the horizontal section of the slide before she comes to rest. ---[2]

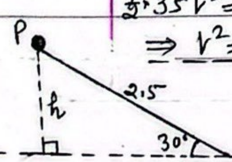
[S-21/41/Q7]

Solution (a) (i) $P.E = 35g \times 2.5 \sin 30^\circ$ (at the top)

K.E at the bottom of slope $= \frac{1}{2} 35 v^2$

$\frac{1}{2} \cdot 35 v^2 = 35g \times 2.5 \sin 30^\circ$

$\Rightarrow v^2 = 25 \Rightarrow v = 5\ ms^{-1}$



$h = 2.5 \sin 30^\circ$
$P.E. = mgh$

a(ii) Let distance travelled on horizontal $= d$

Work done $= 250 \cdot d$ J.

Work done = K.E $= \frac{1}{2} mv^2$

$\Rightarrow 250d = \frac{1}{2} \times 35 \times 5^2$

$d = 1.75$ m.

(Continued →)

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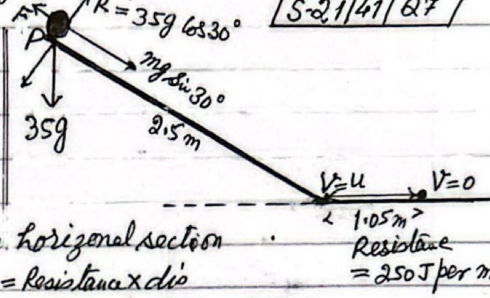
24(b) It is given instead that the sloping section of the slide is rough and that the child comes to rest on the slide 1.05m after she reaches the horizontal section. Find the coefficient of friction between the child and the sloping section of the slide. ---[6]

S-21/41/Q7



Solution: $R = 35g\cos 30°$ --- ①

Let V is the velocity at the point of sloping section & Horizontal section

$K.E = \frac{1}{2}mv^2 = \frac{1}{2}35V^2$ J - ②

this K.E make child to go 1.05 m on the horizonal section and Work done = Resistance × dis

$= 250 \times 1.05$ J -- ③

from ② & ③

$\frac{1}{2} \times 35 \times V^2 = 250 \times 1.05 \Rightarrow V^2 = 15$ --- ④

Now on the sliding plane let the acceleration is 'a',

$V^2 = 0 + 2 \times a \times 2.5 = 15$ from ④ $[V^2 = u^2 + 2aS,\ u = 0,\ S = 2.5m)$

$\Rightarrow a = 3\ ms^{-2}$ — ⑤

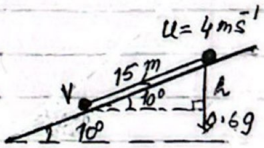
Now along the sloping section: $35g\sin 30° - F = 35a$

$F = 35g\sin 30 - 35a = 175 - 35 \times 3 = 70$; $F = \mu R \Rightarrow \mu = F/R$

$\Rightarrow \mu = \frac{70}{35g\cos 30°} = 0.231$ ✓

25. A particle of mass 0.6 kg is projected with a speed of $4ms^{-1}$ down a line of greatest slope of a smooth plane inclined at 10° to the horizontal. Use an energy method to find the speed of the particle after it has moved 15m down the plane. ---[3]

S-21/42/Q1



Solution: Initial $K.E = \frac{1}{2}mu^2 = \frac{1}{2} \times 0.6 \times 4^2 = 4.8$ J -- ①

Final $K.E = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.6 V^2 = 0.3V^2$ J - ②

Loss in $P.E = mgh = 0.6g \times 15\sin 10°$

$= 15.628$ -- ③

Initial K.E + Loss in P.E = Final K.E

$4.8 + 15.628 = 0.3V^2 \Rightarrow V^2 = \frac{20.428}{0.3} = 68 \Rightarrow V = 8.25\ ms^{-1}$

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26. A car of mass 1250 kg is pulling a caravan of mass 800 kg along a straight road. The resistances to the motion of the car and caravan are 440 N and 280 N respectively. The car and caravan are connected by a light rigid tow-bar.

(a) The car and caravan move along a horizontal part of the road at a constant speed of $30\,ms^{-1}$.

(i) Calculate in, KW, the power developed by the engine of the car. ---[2]

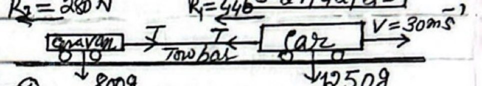
(ii) Given that this power is suddenly decreased by 8 KW, find the instantaneous deceleration of the car and caravan and the tension in the tow-bar. ---[4]

(b) The car and caravan now travel along a part of the road inclined at $\sin^{-1}0.06$ to the horizontal. The car and caravan travel up the incline at a constant speed with the engine of the car working at 28 kW.

(i) Find this constant speed. --[3]

(ii) Find the increase in the potential energy of the caravan in one minute. --[2]

S-21/42/Q5



Solution (a)(i) $P = F \times v = (440+280) \times 30 = 720 \times 30$

$= 21.6$ KW — (1)

(ii) Now $P = 21600 - 8000 = 13600$ W

$DF = \frac{P}{V} = \frac{13600}{30} = 453.33$ N — (2)

Car; $DF - 440 - T = 1250a$ — (3)

Caravan; $T - 280 = 800a$ — (4)

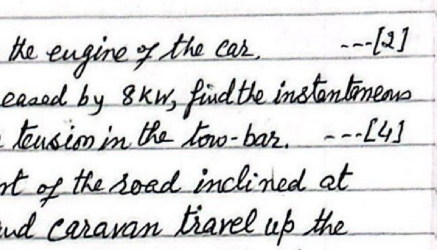
add (3) & (4) $DF - (440+280) = 205$

$\Rightarrow a = \frac{453.33 - 720}{2050} = \frac{-266.33}{2050} = -0.13$

Put $a = -0.13$ in (4) $\Rightarrow T - 280 = 800 \times (-0.13)$

$\Rightarrow T = 176$ N ✓

(b)



$\alpha = \sin^{-1}0.06 \Rightarrow \sin\alpha = 0.06$

B(i) for Car; $DF - T - 440 - 1250g \times 0.06 = 0$ --- (5)

Caravan; $T - 280 - 800g \times 0.06 = 0$ --- (6)

add (5) and (6) $DF - 720 - 2050g \times 0.06 = 0$

$DF = 720 + 2050 \times 10 \times 0.06 = 1950$

Now $P = DF \times v$

$28000 = 1950 \times V \Rightarrow V = 14.4\,ms^{-1}$ ✓

B(ii) distance travelled in one minute along the plane upward $= V \times t$

$d = (14.4 \times 60)$ m

∴ Increase in P.E of the caravan $= mgh$

$= 800g \times d \sin\alpha$

$= 800g \times (14.4 \times 60) \times 0.06$

$= 414000$ J

$= 414$ KJ.

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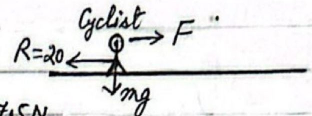
27. A cyclist is travelling along a straight horizontal road. She is working at a constant rate of 150W. At an instant when her speed is $4\,ms^{-1}$, her acceleration is $0.25\,ms^{-2}$. The resistance to motion is 20N.

(a) Find the total mass of the cyclist and her bicycle. ---[3]

The cyclist comes to a straight hill inclined at an angle θ above the horizontal. She ascends the hill at constant speed $3\,ms^{-1}$. She continues to work at the same rate as before and the resistance force is unchanged.

(b) Find the value of θ. S-21/43/Q2 ---[2]

Solution (a) $u = 4\,ms^{-1}$; $a = 0.25\,ms^{-2}$

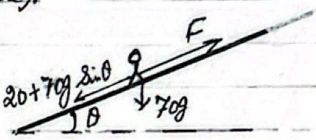


$P = F \cdot V \Rightarrow 150 = F \times 4 \Rightarrow F = \frac{150}{4} = 37.5N$

$37.5 - 20 = m \times 0.25$ ($\because F - R = ma$)

$\Rightarrow m = \underline{70\,kg}$ (mass of cyclist and Bicycle)

(b) Now $u = 3\,ms^{-1}$, $P = 150$; $a = 0$



$\therefore F = \frac{P}{u} = \frac{150}{3} = 50N$.

$\therefore F - 20 - 70g\sin\theta = m \times 0$ $(a = 0)$

$50 - 20 - 700\sin\theta = 0 \Rightarrow \sin\theta = 30/700 = 0.4285 \Rightarrow \theta = \sin^{-1} 0.4285$

$\theta = \underline{2.5°}$ (2.456)

27'. A crane is used to raise a block of mass 600 kg vertically upwards at a constant speed through a height of 15m. There is a resistance to the motion of the block, which the crane does 10000 J of work to overcome.

(a) Find the total work done by the crane. --[2]

(b) Given that the average power exerted by the crane is 12.5 kW, Find the total time for which the block is in motion. M-22/42/Q1 -[2]

$m = 600\,kg$, $h = 15m$

Solution (a) Work done to raise the mass $= mg.h$
$= 600g \times 15 = 90,000J$

work for Resistance $= 10000J$

Total work $= (90000 + 10,000) = \underline{100,000\,J}$.

(b) Work $= P \times t$

$10,000 = 12500 \times t$

$t = \underline{8\,s}$

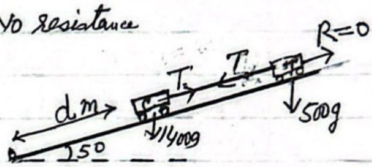
28. A car of mass 1400 kg is towing a trailer of mass 500 kg down a straight hill inclined at an angle of 5° to the horizontal. The car and the trailer are connected by a light rigid tow-bar. At the top of the hill the speed of the car and trailer is $20 ms^{-1}$ and at the bottom of the hill their speed is $30 ms^{-1}$.

(a) It is given that as the car and trailer descend the hill, the engine of the car does 150000 J of work, and there are no restistance forces.
Find the length of the hill.

(b) It is given instead that there is a resistance force of 100 N on the trailer, the length of hill is 200 m, and the acceleration of the car and trailer is constant.
Find the tension in the tow-bar between the car and trailer. --[4]

S-21/43/Q5

Solution: $u = 20 ms^{-1}$, $V = 30 ms^{-1}$, $W = 150000 J$, No resistance



(a) Increase in KE $= \frac{1}{2} \times 1900 \times 30^2 - \frac{1}{2} \times 1900 \times 20^2$
$= 475000 J$. —— ①

Loss in P.E $= 1900 g \times d \sin 5^\circ$ (mgh) --②
$(h = d \sin 5^\circ)$

$\therefore$ Loss in PE + Work done by car engine = gain in K.E

$1900 g \times d \sin 5^\circ + 150000 = 475000 J$ (from ①&②)

$d = \frac{325000}{1900 g \times \sin 5^\circ} = \frac{325000}{1655.96} = \underline{196 m}$

(b) $u = 20 ms^{-1}$, $V = 30 ms^{-1}$, acceleration $= a \, ms^{-2}$, S (distance) $= 200 m$, Resistance $= 100 N$ (In trailer)

$V^2 = u^2 + 2as \Rightarrow 30^2 = 20^2 + 2 \times a \times 200 \Rightarrow a = 1.25 ms^{-1}$

Now for trailer: $T - 100 + 500 g \sin 5^\circ = 500 a$

$\Rightarrow T = 100 + 500 \times 1.25 - 500 g \sin 5^\circ = 289.22$

$T = \underline{289 \text{ N}}$.

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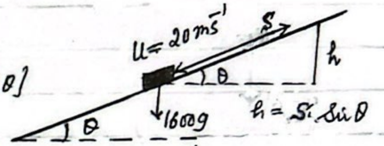
29. A car of mass 1600 kg travels at constant speed $20\,ms^{-1}$ up a straight road inclined at an angle of $\sin^{-1}0.12$ to the horizontal.

(a) Find the change in potential energy of the car in 30s. ---[3]

(b) Given that the total work done by the engine of the car in this time is 1960 kJ, find the constant force resisting the motion. ---[3]

(c) Calculate, in kW, the power developed by the engine of the car. ---[2]

(d) Given that this power is suddenly decreased by 15%, find the instantaneous deceleration of the car. $\boxed{W\text{-}21/41/Q5}$ ---[3]



$\theta = \sin^{-1}0.12 \Rightarrow \sin\theta = 0.12$

Solution: (a) distance travelled in 30s $= 30\times20 = 600\,m$

P.E change $= mgh = 1600g\times600\times0.12$ [$h = S\sin\theta$]

$= \underline{1152\,000\,J}$ ✓

(b) Total work done = Work done against resistance + gain in PE

$\Rightarrow 1960\,000 = WD_{res} + 1152000 \Rightarrow WD_{resistance} = 808000\,J.$

$R = \text{Resistance force} = \frac{WD_{resistance}}{\text{Distance}} = \frac{808000}{600} = \underline{1350\,N}$ ✓ $\left\{\begin{array}{l} W = F\times d \\ F = W/d \end{array}\right.$

(c) $\text{Power} = \frac{\text{Work done}}{\text{Time}} = \frac{1960\,000}{30} = \underline{65.3\ kW}$

(d) Power reduced by 15% $\rightarrow$ New $P = 0.85\times\frac{1960\,000}{30} = DF\times20$ [$P = F\times t$]

$\Rightarrow DF = \frac{8300}{3}\,N$

Now $DF - R - 1600g\sin\theta = 1600a$

$\frac{8300}{3} - \frac{4040}{3} - 1920 = 1600a \Rightarrow a = \underline{-0.306\,ms^{-2}}$ ✓

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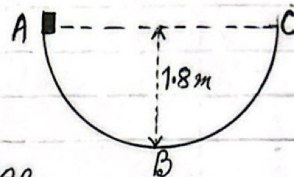
30. The diagram shows a semi-circular track ABC of radius 1.8 m which is fixed in a vertical plane. The points A and C are at the same horizontal level and the point B is at the bottom of the track. The section AB, is smooth and the section BC is rough. A small is released from rest at A.



(a) Show that the speed of the block at B is $6\,ms^{-1}$. ---[2]

The block comes to instantaneous rest for the first time at a height of 1.2 m above the level of B. The work done against the resistance force during the motion of the block from B to this point is 4.5 J.

(b) Find the mass of the block. [W-21/42/Q3] ---[3]

Solution (a) from A to B, $KE = \frac{1}{2}mv^2$, Loss in $P.E = mg \times 1.8$ $\Rightarrow \frac{1}{2}mv^2 = mg \times 1.8 \Rightarrow v^2 = 36 \Rightarrow v = \underline{6\,ms^{-1}}$ ✓

(b) Loss in K.E = Resistance + gain in P.E $\Rightarrow \frac{1}{2}m.6^2 = 4.5 + mg \times 1.2$

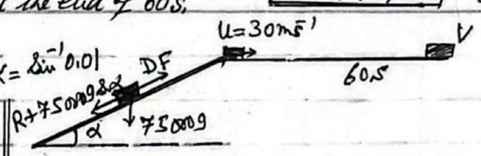
$\Rightarrow 18m - 6m = 4.5 \Rightarrow 6m = 4.5 \Rightarrow \underline{m = 0.75\,kg}$ ✓

---

31. A railway engine of mass 75000 kg is moving up a straight hill inclined at an angle α to the horizontal, where $\sin\alpha = 0.01$. The engine is travelling at a constant speed of $30\,ms^{-1}$. The engine is working at 960 kW. There is a constant force resisting the motion of the engine. (a) Find the resistance force. --[3]

The engine comes to a section of track which is horizontal. At the start of the section the engine is travelling at $30\,ms^{-1}$ and the power of the engine is now reduced to 900 kW. The resistance to motion is no longer constant, but in the next 60 s the work done against the resistance force is 46500 kJ.

(b) Find the speed of the engine at the end of 60 s. [W-21/42/Q5] --[4]



Solution (a) $DF = \frac{960000}{30} = 32000$ N.

$DF - 75000g\sin\alpha - R = 0$

Resistance force $R = 32000 - 75000 \times 10 \times 0.01$
$= \underline{24500\,N}$

(b) WD by Engine in $60s = 900000 \times 60$

$K.E. = \frac{1}{2} \times 75000 \times 30^2$ (Initial)

$K.E\,(final) = \frac{1}{2} \times 75000 \times v^2$

work done by Engine + Initial KE = Resistance + final K.E

$\Rightarrow 900000 \times 60 + \frac{1}{2} 75000 \times 30^2 = 46500000 + \frac{1}{2} \times 75000 \times v^2$

$\Rightarrow v = \underline{33.2\,ms^{-1}}$ ✓ (Velocity after 60 s)

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32. A ball of mass 1.6 kg is released from rest at a point 5 m above horizontal ground. When the ball hits the ground it instantaneously loses 8 J of kinetic energy and start to move upwards.

(a) Use an energy method to find the greatest height that the ball reaches after hitting the ground. ---[3]

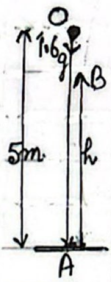
(b) Find the total time taken, from the initial release of the ball until it reaches this greatest height. [W-21/43/Q3] ---[3]

Solution (a) $m=1.6$, $H=5m$,

K.E when reaches the ground

$=\frac{1}{2}mv^2 = \frac{1}{2}\times1.6\times100 = 80J$

$\left(v^2 = u^2+2gh = 0+2\times10\times5 = 100\right)$



$\therefore$ K.E at A = 80 J

When rebound K.E = 80 − 8 = 72 J.

Let Height is h: $P.E = mgh = KE = 72$

$1.6\times10\times h = 72 \Rightarrow \underline{h = 4.5\,m}$

(b) Time t_1 for OA,

$5 = 0+\frac{1}{2}\times10\times t_1^2$ $\quad(s=ut+\frac{1}{2}at^2)$

$\Rightarrow t_1 = 1$ --- (1)

for AB, time t_2, AB = 4.5 m

$4.5 = 0-\frac{1}{2}(-10)t_2^2$ $\quad(s=vt-\frac{1}{2}at^2)$

$\Rightarrow t_2^2 = 0.9 \Rightarrow t_2 = \sqrt{0.9} = 0.95$

$\therefore$ Total time $= t_1+t_2 = 1+0.95 = \underline{1.95\,s}$

33. A car of mass 1400 kg is moving on a straight road against a constance force of 1250 N resisting the motion.

(a) The car moves along a horizonal section of the at a constant speed of $36\,ms^{-1}$.

(i) Calculate the work done against the resisting force during first 8 seconds. [2]

(ii) Calculate, in kW, the power developed by the engine of the car. --[2]

(iii) Given that this power is suddenly increased by 12 kW, find the instantaneous acceleration of car. --[3]

(b) The car now travels at a constant speed of $32\,ms^{-1}$ up a section of the road inclined at $\theta°$ to the horizonal, with engine working at 64 kW. Find the value of θ. [W-21/43/Q4] --[2]

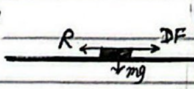
Solution (a)(i) $m = 1400$ kg, Resistance = 1250

$WD = F\times d = 1250\times(36\times8) = \underline{360000\,J}$.

(ii) $P = F\times v = 1250\times36 = \underline{45000\,J}$ (or $\frac{WD}{t}$)

(iii) $P = 45000+12000 = 57000\,J$.

$DF = \frac{P}{v} = \frac{57000}{36}\,N$



DF − Resistance = ma

$\frac{57000}{36} - 1250 = 1400a \Rightarrow a = 0.238\,ms^{-2}$

(b) $DF = \frac{P}{v} = \frac{64000}{32}\,N$

$DF = R + mg\sin\theta$

$\Rightarrow \frac{64000}{32} = 1250 + 1400g\sin\theta$

$\Rightarrow \sin\theta = 0.5357$

$\theta = \sin^{-1}0.5357 = \underline{3.1°}$ (3.0708)

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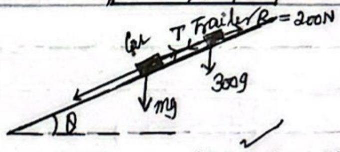
34. A car of mass m kg is towing a trailer of mass 300 kg down a straight hill inclined at $3°$ to the horizontal at a constant speed. There are resistance forces on the car and on the trailer, and the total work done against the resistance forces in a distance of 50 m is 40000 J. The engine of the car is doing no work and the tow-bar is light and rigid.

(a) Find the value of m. ---[3]

(b) The resistance forces on the trailer is 200 N.
Find the tension in the tow-bar between the car and the trailer. --[2]

[M-22/42/Q3]

Solution (a) P.E lost in 50 m $= mgh = (m+300)g \cdot 50\sin 3°$



Workdone against resistances $= 40000$ J

$\therefore (m+300)g \times 50\sin 3° = 40000$

$\Rightarrow m+300 = \dfrac{40,000}{10 \times 50\sin 3°} = 1528.6 \Rightarrow m = 1226.8 = \underline{1230}$ (3 s.f)

(b) for Trailer: $T + 300g\sin 3° - 200 = 0$ (Constant speed $\to a = 0$)

$\Rightarrow T = 200 - 300g\sin 3° = \underline{43\text{ N}}$

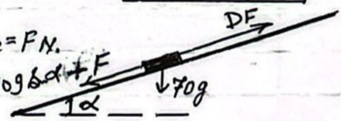
35. The total mass of a cyclist and her bicycle is 70 kg. The cyclist is riding with constant power of 180 W up a straight hill inclined at an angle α, where $\sin\alpha = 0.05$. At an instant when the cyclist's speed is 6 ms^{-1}, her acceleration is -0.2 ms^{-2}. There is a constant resistance to motion of magnitude F N.

(a) Find the value of F. ---[4]

(b) Find the steady speed that the cyclist could maintain up the hill when working at this power. --[2]

[M-22/42/Q4]

Solution (a) $V = 6\text{ ms}^{-1}$, $a = -0.2\text{ ms}^{-2}$, $P = 180$ W, Resistance $= F$ N.



$DF = \dfrac{P}{V} = \dfrac{180}{6} = 30\text{ N}$

$DF - F - 70g\sin\alpha = 70 \times (-0.2)$ (ma)

$\Rightarrow 30 - F - 70g \times 0.05 = 70 \times (-0.2)$

$\Rightarrow \underline{F = 9\text{ N.}}$

(b) Let Velocity $= V$ (for $a = 0$)

$DF - F - 70g\sin\alpha = m \times 0$

$\dfrac{180}{V} - 9 - 70g \times 0.05 = 0$

$\Rightarrow \dfrac{180}{V} - 44 = 0 \Rightarrow V = \dfrac{180}{44} = 4.09$

$\Rightarrow \underline{V = 4.09\text{ ms}^{-1}}$

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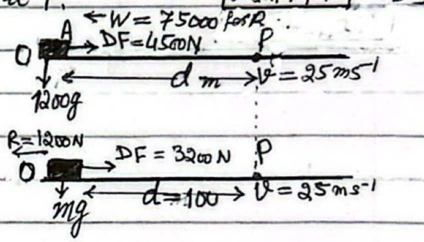
36. Two racing cars A and B are at rest alongside each other at a point O on a horizontal test track. The mass of A is 1200 kg. The engine of A produces a constant driving force of 4500N. When A arrives at a point P its speed is $25\,ms^{-1}$. The distance OP is d m. The work done against the resistance force experienced by A between O and P is 75000J.

(a) Show that $d = 100$ ---[3]

Car B starts off at the same instant as car A. The two cars arrive at P simultaneously and with the same speed. The engine of B produces a driving force of 3200N and the car experiences a constant resistance to motion of 1200N.

(b) Find the mass of B. ---[3]

(c) Find the steady speed which B can maintain when its engine is working at the same rate as it is at P. [S-22/41/Q5] ---[3]



<u>Solution:</u>

(a) $4500 \times d - 75000 = \frac{1}{2} \times 1200 \times 25^2$

$\underline{d = 100\ m}$ ✓

(b) $v^2 = u^2 + 2as \Rightarrow 25^2 = 0 + 2 \times a \times 100$

$\Rightarrow a = 3.125\ ms^{-2}$

$DF - R = ma \Rightarrow 3200 - 1200 = m \times 3.125 \Rightarrow \underline{m = 640}$ kg ✓

(C) At P, power $= DF \times v = 3200 \times 25 = 80000$ J

Now $DF - R = m \times 0$ [Speed is constant → $a = 0$]

$\frac{P}{v_1} - R = 0 \Rightarrow \frac{80000}{v_1} - 1200 = 0 \Rightarrow v_1 = \frac{80000}{1200} = 66.7$ ✓ (66.66)

$V = 66.7\ ms^{-1}$

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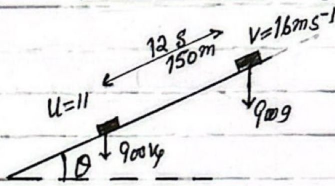
37. A car of mass 900 kg is moving up a hill inclined at $\sin^{-1}0.12$ to the horizontal. The initial speed of the car is $11\,ms^{-1}$. After 12 s, the car has travelled 150 m up the hill and has speed $16\,ms^{-1}$. The engine of the car is working at a constant rate of 24 kW.

(a) Find the work done against the resistive forces during the 12 s ---[5]

The car then travels along a straight horizontal road. There is a resistance to the motion of the car of $(1520+4v)$N, when the speed of the car is $v\,ms^{-1}$. The car travels at a constant speed with the engine working at a constant rate of 32 kW.

(b) Find this speed. S-22/42/Q6 ---[3]



Solution (a) $P = 24000\,W.,\ t = 12s$

Work done by Car Engine in $12s = P.t = 24000\times12$
$= 288000\,J$ --(1)

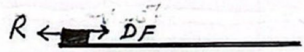
Gain in K.E $= \frac{1}{2}m(v^2-u^2)$
$= \frac{1}{2}\times900(16^2-11^2) = 60750\,J$

$\left\{\theta = \sin^{-1}0.12 \Rightarrow \sin\theta = 0.12\right.$

Gain in PE $= mg.d\sin\theta = 900\times10\times15\times0.12 = 162000\,J.$

Work done against the resistive force = Work done by car Engine − K.E. gain − P.E gain
$= 288000 - 60750 - 162000 = \underline{65250\,J}$ ✓

(b) $P = 32000\,W,\ R = (1520+4v)N,\ Vel = v$



$DF = \frac{P}{V} = \frac{32000}{V}\,N,$

Now DF = Resistive force

$\Rightarrow \frac{32000}{V} = 1520+4V \Rightarrow 4V^2+1520V-32000=0$

$\Rightarrow V^2+380V-8000=0$

$(V+400)(V-20)=0$

$V = \underline{20\,m\,s^{-1}}$ ✓ or $(V=-400)$ ✗

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38. A cyclist is riding along a straight horizontal road. The total mass of the cyclist and her bicycle is 70kg. At an instant when the cyclist's speed is $4ms^{-1}$, her acceleration is $0.3ms^{-2}$. There is a constant resistance to motion of magnitude 30N.

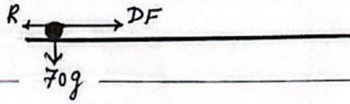
(a) Find the power developed by the cyclist. ---[3]

The cyclist comes to the top of a hill inclined at $5°$ to the horizontal. The cyclist stops pedalling and free wheels down the hill (so that the cyclist is no longer supplying any power). The magnitude of the resistive force remains at 30N. Over a distance of d m, the speed of the cyclist increases from $6ms^{-1}$ to $12ms^{-1}$.

(b) Find the change in kinetic energy. ---[2]

(c) Use an energy method to find d. $\boxed{S\text{-}22/43/Q5}$ ---[3]

Solution (a) $u = 4ms^{-1}$, $a = 0.3ms^{-2}$, $R = 30N$



$DF - R = ma$

$DF - 30 = 70 \times 0.3$

$DF = 51N$

$\text{Power} = DF \times u = 51 \times 4 = \underline{204W}$ ✓

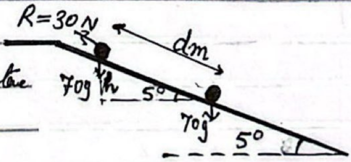
(b) $u = 6ms^{-1}$, $v = 12ms^{-1}$

$\text{Change in K.E} = \frac{1}{2}m(v^2 - u^2)$

$= \frac{1}{2} \times 70(12^2 - 6^2)$

$= \underline{3780J}$

(c) Work-energy method



Loss in P.E = gain KE + Work against Resistance

$70g \times d \cdot \sin 5° = 3780 + 30 \times d$

$31d = 3780$

$d = \frac{3780}{31} = 121.935$

$h = d \sin 5°$

$\therefore d = \underline{122m}$ ✓

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39. A constant resistance of magnitude 1400 N acts on a car of mass 1250 kg.

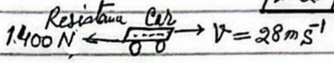
(a) The car is moving along a straight level road at a constant speed of $28\,ms^{-1}$. Find in kW, the rate at which the engine of the car is working. --- [2]

(b) The car now travels at a constant speed up a hill inclined at an angle of θ to the horizontal, where $\sin\theta = 0.12$, with the engine working at 43.5 kW. Find the speed. --- [3]

(c) On another occasion, the car pulls a trailer of mass 600 kg up the same hill. The system of the car and the trailer is modelled as particles connected by a light inextensible cable. The car's engine produces a driving force of 5000N and the resistance to the motion of the trailer is 300N. The resistance to the motion of the car remains 1400N.
Find the acceleration of the system and the tension in the cable. --- [4]

W-22/41/Q3

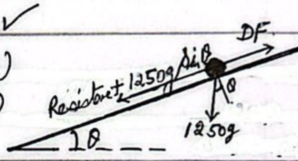
Solution: Power = Resistance × speed



(a) $= 1400 \times 28 = 39200\ W = \underline{39.2\ kW}$ ✓

(b) $DF = 1400 + 1250g\sin\theta \qquad (\because \sin\theta = 0.12)$

$DF = 1400 + 1250 \times 10 \times 0.12 = 2900\ N$ --- ①



Let the speed of the car $= V\,ms^{-1}$

Power of engine $= DF \times V = 43500\ W$ (Given $P = 43.5\ kW$)

$\Rightarrow 2900 \times V = 43500$ (from ①)

$V = 43500/2900 = \underline{15\,ms^{-1}}$ ✓

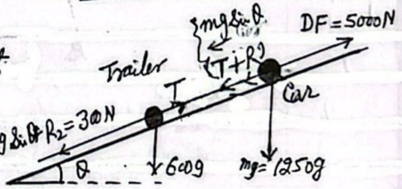
(c) Resolving the forces along the plane:

Car: $5000 - 1250g \times 0.12 - 1400 - T = 1250a$ (a = accelerat.)

$\Rightarrow 2100 - T = 1250a$ --- ②

Trailer: $T - 300 - 600g \times 0.12 = 600a$

$\Rightarrow T - 1020 = 600a$ --- ③



adding ② & ③ $2100 - 1020 = 1850a \Rightarrow a = \frac{1080}{1850} = 0.584\,ms^{-2}$ ✓

from ② $T = 600a + 1020 = 600 \times \frac{108}{185} + 1050$
$= 1370N$ ✓

$\therefore$ acceleration $= \underline{0.584\,ms^{-1}}$ and Tension $= \underline{1370\ N}$ ✓

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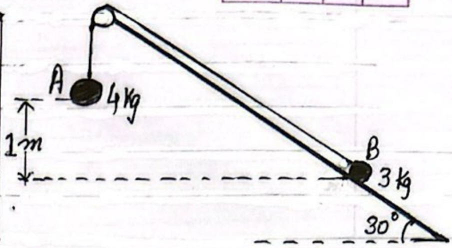
40. Particles A and B, of masses 4 kg and 3 kg respectively, attached to the ends of a light inextensible string that passes over a small smooth pully. The pully is fixed at the top a plane which is inclined at an angle of 30° to the horizonel. A hangs freely below the pully and B is on the inclined plane. The string is taut and the section of the string between B and the pulley is parallel to a line of greatest slope of the slope.



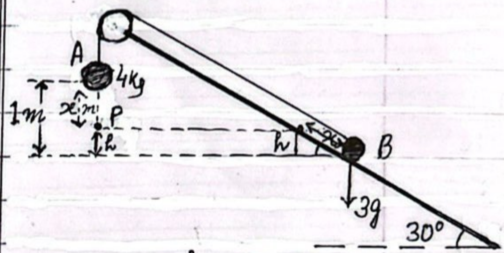
The plane is smooth and the particles are released from rest when the difference in the vertical heights of the particles is 1 m.

Use energy method to find the speed of the particles at the instant when the particles are at the same horizontal level. ...[6]

[W-22/41/Q6(b)]

Solution:

Let the particle A moves down by x m.
So the particle B move up the plane x m → height $= x\sin 30°$
for B: $h = x\sin 30°$



$\frac{h}{x} = \sin 30° \Rightarrow h = x\sin 30°$

Hence:
$x + x\sin 30° = 1$ (m)

$\Rightarrow x + \frac{x}{2} = 1 \Rightarrow x = 2/3$ m.

Change in Kinetic Energy $= \frac{1}{2}\times 4\times V^2 + \frac{1}{2}\times 3\times V^2$ [$K.E = \frac{1}{2}mv^2$]
$= \frac{1}{2}\times 7\times V^2$ ---- ①

Change in PE $= 4g x - 3g\cdot x\sin 30°$ [$P.E = mgh$]
$= 4\times 10\times 2/3 - 3\times 10\times \frac{2}{3}\times\frac{1}{2}$ [$x = 2/3$]
$= \frac{50}{3}$ ---- ②

Conservation of energy:
$\frac{1}{2}\cdot 7\cdot V^2 = \frac{50}{3}$ (from ① & ②)

$\Rightarrow V^2 = \frac{50}{3}\times\frac{2}{7} = \frac{100}{21} \Rightarrow V = \sqrt{\frac{100}{21}} = 2.18\ ms^{-1}$ ✓

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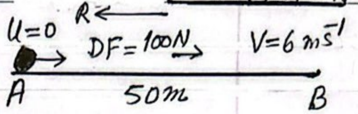
41. A cyclist is riding a bicycle along a straight horizontal road AB of length 50m. The cyclist starts from rest at A and reaches a speed of $6\,ms^{-1}$ at B. The cyclist produces a constant driving force of magnitude 100N. There is a resistance force, and the work done against the resistance force from A to B is 3560 J.
Find the total mass of the cyclist and bicycle. --- [3]

[W-22/42/Q1]

Solution: Work done against resistance $= 3560\,J$.
Work done by the driving force $= DF \times distance$
$= 100 \times 50 = 5000\,J$.



$\therefore$ Resultant work done $= K.E$

$5000 - 3560 = \frac{1}{2} \times m \times 6^2$ $\quad [K.E = \frac{1}{2}mv^2]$

$\Rightarrow 1440 = 18m \Rightarrow \underline{m = 80\,kg}$ ✓

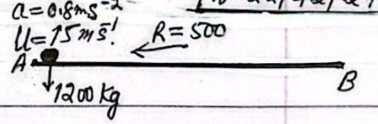
42. A car of mass 1200 kg is travelling along a straight horizontal road AB. There is a constant resistance force of magnitude 500N. When the car passes point A, it has a speed of $15\,ms^{-1}$ and an acceleration of $0.8\,ms^{-2}$.

(a) Find the power of car's engine at the point A. --- [3]

The car continues to work with this power as it travels from A to B. The car takes 53 seconds to travel from A to B and the speed of the car at B is $32\,ms^{-1}$.

(b) Show that the distance AB is 1362.6m. --- [3]

[W-22/42/Q4]

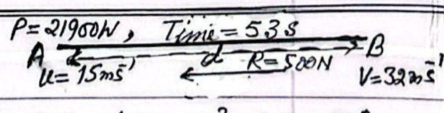


Solution: $P = DF \times u$ --- ①

(a) $DF - 500 = 1200 \times 0.8$ $\quad$ Newton Law, $[force = ma]$

$\Rightarrow DF = 960 + 500 = 1460\,N$ ✓

At A:
from ① Power $= 1460 \times 15$
$= \underline{21900\,W}$



(b) Work done by engine from $A \to B$
$= P \times t = 21900 \times 53$
$= 1160700\,J$ --- ②

Work against Resistance $= 500d\,J$ --- ③

Change in $K.E = \frac{1}{2} \times 1200 \times 32^2 - \frac{1}{2} \times 1200 \times 15^2$
$= 479400\,W$

Now
Work done by engine $-$ Work against Resistance $=$ Change in K.E

$\Rightarrow 1160700 - 500d = 479400$ (from ②, ③ and ④)

$\Rightarrow d = \underline{1362.6m}$ ✓

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43. A box of mass 5 kg is pulled at a constant speed of $1.8\ ms^{-1}$ for 15 s up a rough plane inclined at an angle of 20° to the horizontal. The box moves along a line of greatest slope against a frictional force of 40 N. The force pulling the box is parallel to the line of greatest slope.

(a) Find the change in gravitational potential energy of the box. --- [2]

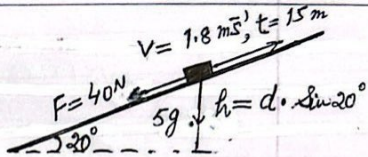
(b) Find the work done by pulling the force. --- [2]

W-22 / 43 / Q2

Solution:

(a) distance along the plane $= v \times t$

$$d = 1.8 \times 15$$



$\therefore$ Change in height $= d \sin\theta$

$$h = (1.8 \times 15 \times \sin 20^\circ)$$

$\therefore$ Change in P.E $= mgh = 5g \cdot (1.8 \times 15 \times \sin 20^\circ) = 461.72$

$$= \underline{462\text{ J.}} \checkmark \text{ ---- (1)}$$

(b) Work done by the pulling force $=$ Change in PE $+$ Work against the force of friction

$= 462 + 40 \times (1.8 \times 15) = 462 + 1080 = \underline{1542\text{ J}} \checkmark$

Scanned with CamScanner

44. A car of mass 1750 kg is pulling a caravan of mass 500 kg. The car and the caravan are connected by a light rigid tow-bar. The resistances to the motion of the car and caravan are 650 N and 150 N respectively.

(a) The car and caravan are moving along a straight horizontal road at a constant speed of $24\ ms^{-1}$.

(i) Find the power of the car's engines. --- [2]

(ii) The engine's power is now suddenly increased to 40 kW. Find the instantaneous acceleration of the car and caravan and find the tension in the tow-bar. --- [5]

(b) The car and caravan now travel up a straight hill, inclined at an angle $\sin^{-1}0.14$ to the horizontal, at a constant speed of $v\ ms^{-1}$. The car's engine is working at 31 kW. The resistance to the motion of the car and caravan are unchanged. Find v. --- [3]

W-22/43/Q6

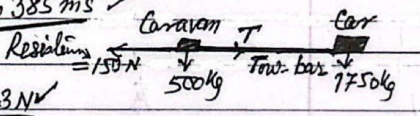
Solution: $P = DF \times v$

(a) (i) $= (650+150)\times 24 = 19200\ W = 19.2\ kW$

(ii) $P = DF \times v \Rightarrow 40{,}000 = DF \times 24 \Rightarrow DF = \frac{40{,}000}{24}\ N$

$\frac{40000}{24} - (650+150) = (1750+500)\cdot a$ --- (1)

$\Rightarrow 2250a = 866.7 \Rightarrow a = 0.385\ ms^{-2}$

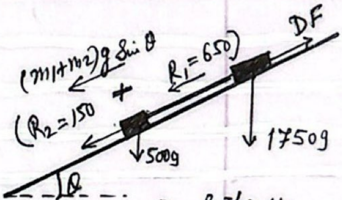


For Caravan: $T - 150 = 500a$

$\Rightarrow T - 150 = 500 \times 0.385 \Rightarrow T = 343\ N$

(b) $P = 31\ KW = 31000 = DF \times v$

$\Rightarrow DF = \frac{31000}{v}$ --- (1)



also $DF =$ Total resistance + Weight along the plane

$= (650+150) + (1750+500)g \times 0.14$

$= 800 + 2250 \times 10 \times 0.14 = 3950\ N$ --- (2)

$\theta = \sin^{-1}0.14$

$\sin\theta = 0.14$

from (1) & (2) $\frac{31000}{v} = 3950 \rightarrow v = \frac{31000}{3950} = 7.85$

$\therefore v = 7.85\ ms^{-1}$

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45. A crate of mass 200 kg is being pulled at constant speed along horizontal ground by a horizontal rope attached to a winch. The winch is working at a constant rate of 4.5 kW and there is a constant resistance to the motion of the crate of magnitude 600 N.

(a) Find the time that it takes for the crate to move a distance of 15 m. ... [2]

The rope breaks after the crate has moved 15 m.

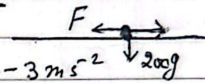
(b) Find the time taken, after the rope breaks, for the crate to come to rest. ... [3]

M-23/42/Q1

Solution: (a) $P = 4500\,W$, distance $d = 15\,m$, Resistance force $F = 600\,N$

Work done $= F \times d = 600 \times 15$, Let the time $= t$ s

Now $P = \frac{W}{t} \Rightarrow 4500 = \frac{600 \times 15}{t} \Rightarrow t = \frac{600 \times 15}{4500} = \underline{2\,s}$



(b) When the rope breaks $-F = ma \Rightarrow -600 = 200 \times a \Rightarrow a = -3\,ms^{-2}$

Now the initial speed of the crate $= \frac{\text{distance}}{t} = \frac{15}{2}\,ms^{-1}$

Now after the rope breaks, the crate comes to rest, when $V = 0$

$0 = \frac{15}{2} + (-3)t \qquad (V = u + at)$

$\Rightarrow t = \underline{2.5\,s}$

46. A toy railway locomotive of mass 0.8 kg is towing a truck of mass 0.4 kg on a straight horizontal track at a constant speed of $2 ms^{-1}$. There is a constant resistance force of magnitude 0.2 N on the locomotive, but no resistance force on the truck. There a light rigid horizontal coupling connecting the locomotive and the truck.

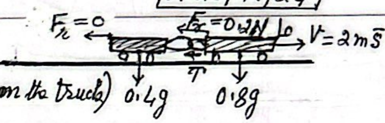
(a) State the tension in the coupling. ...[1]

(b) Find the power produced by the locomotive's engine. ..[1]

The power produced by the locomotive's engine is now changed to 1.2 W.

(c) Find the magnitude of the tension in the coupling at the instant that the locomotive begins to accelerates. ...[5]

M-23/42/Q4



Solution (a) Tension in the coupling $= 0$ N (as there is no resistance force on the truck)

(b) Power $= F_r \times v = 0.2 \times 2 = 0.4$ W

(c) Now Power Locomotive engine $= 1.2$ W

$$P = DF \times V = 1.2 N$$

$$DF = \frac{1.2}{V} = \frac{1.2}{2} = 0.6 N.$$

Now for locomotive $DF - F_r - T = 0.8a$

$$\Rightarrow 0.6 - 0.2 - T = 0.8a$$

$$T = 0.4 - 0.8a \quad \text{--- (1)}$$

For Truck $T = 0.4a$ -- (2)

$$\Rightarrow 2T = 0.8a \quad \text{--- (2)}$$

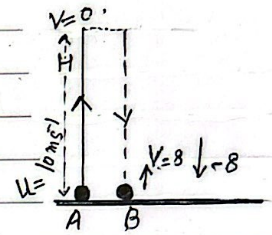
add (1) & (2) $3T = 0.4$

$$3T = \frac{4}{10}$$

$$T = \frac{2}{15} N$$

47. A particle P of mass 0.4 kg is projected vertically upwards from horizontal ground with speed $10 ms^{-1}$.
(a) Find the greatest height above the ground reached by P. ...[2]
When P reaches the ground again, it bounces vertically upwards. At the first instant that it hits the ground, P loses 7.2 J of energy.
(b) Find the time between the first and second instants at which P hits the ground. ...[4]

S-23/41/Q2



Solution (a) $U=10$, at greatest height 'H', $V=0$

$0=10^2+2(-g)H \qquad (V^2=U^2+2gh)$

$2\times10\times H=10^2 \Rightarrow H=\underline{5\,m}$ ✓

(b) mass of the particle $m=0.4$ kg

K.E before impact $=\frac{1}{2}mV^2$

$=\frac{1}{2}\times0.4\times10^2=20J.$

After impact energy lost $=7.2J$

∴ After impact the particle is going upward with energy $=20-7.2$
$=12.8J.$

∴ upward $K.E=\frac{1}{2}mV^2=12.8$ (let now velocity $=v$)

$\Rightarrow \frac{1}{2}\times0.4\times V^2=12.8 \Rightarrow V^2=\frac{12.8\times2}{.4}=64 \Rightarrow V=8\,ms^{-1}$ ✓

or for upward motion $U=8\,ms^{-1}$

It will reach the hightest point and again comes back at B, $V=-8\,ms^{-1}$

Let the total time $=t$

$-8=8+(-g)t \qquad (V=u+gt)$

$10t=16 \Rightarrow t=\underline{1.6\,s}$ ✓

48. A car of mass 1200 kg is travelling along a straight horizontal road. The power of the car's engine is constant and is equal to 16 kW. There is a constant resistance to motion of magnitude 500 N.

(a) Find the acceleration of the car at an instant when its speed is $20\,ms^{-1}$. ---[3]

(b) Assuming that the power and the resistance forces remain unchanged, find the study speed at which the car can travel. ---[2]

The car comes to the bottom of a straight hill of length 316 m, inclined at an angle to the horizontal of $\sin^{-1}\left(\frac{1}{60}\right)$. The power remains constant at 16 kW, but the magnitude of the resistance is no longer constant and changes such that the work done against the resistance force in ascending the hill is 128 400 J. The time taken to ascend the hill is 15 s.

(c) Given that the car is travelling at a speed of $20\,ms^{-1}$ at the bottom of the hill, find the speed at the top of the hill. ---[6]

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Solution (a) $P = 16000\,W$, $V = 20\,ms^{-1}$, $m = 1200\,kg$

$P = DF \times V \Rightarrow DF = \frac{P}{V} = \frac{16000}{20} = 800\,N$ ✓

Using Newton's second law

$DF - \text{Resistance} = ma$

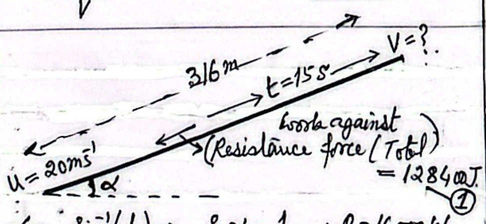
$\Rightarrow 800 - 500 = 1200a \Rightarrow a = 0.25\,ms^{-2}$ ✓

(b) for study speed acceleration = 0

$\Rightarrow DF - \text{Resistance} = 0$

$\frac{16000}{V} - 500 = 0 \Rightarrow V = 32\,ms^{-1}$ ✓

(c)



$\alpha = \sin^{-1}\left(\frac{1}{60}\right) \Rightarrow \sin\alpha = \frac{1}{60}$; $P = 16000\,W$, $t = 15\,s$

Work done by Engine in ascending the hill:

$W = P \times t = 16000 \times 15 = 240000\,J$. $\left(\because P = \frac{\text{Work}}{t}\right)$ ---②

(C) continued → Let the velocity at top = V

$\text{Loss in K.E} = \left(\frac{1}{2} \times 1200\,V^2 - \frac{1}{2} \times 1200 \times 20^2\right)$ ---③

$\text{Gain in P.E.} = mgh$

$= 1200 \times 10 \times 316 \sin\alpha$

$= 1200 \times 10 \times 316 \times \frac{1}{60}$

$= 63200\,J$ ---④

Using Work-energy equation:

Work done by engine − work done against the resistance

= (Change in KE) + Gain in P.E.

⇒ from

$240000 - 128400 = \frac{1}{2} \times 1200\,(V^2 - 400) + 63200$

$\Rightarrow 600V^2 - 240000 + 63200 = 240000 - 128400$

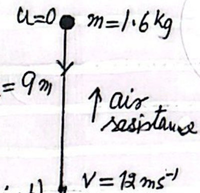
$\Rightarrow 600V^2 = 288400$

$V^2 = 480.66$

$V = 21.9\,ms^{-1}$ (21.924)

49. A particle of mass 1.6 kg is dropped from a height of 9 m above horizontal ground. The speed of the particle at the instant before hitting the ground is $12 ms^{-1}$. Find the work done against air resistance. ... [3]

S-23/42/Q1



Solution: $h = 9\,m$, $m = 1.6\,kg$, final $v = 12\,ms^{-1}$, $u = 0$

Loss in P.E $= mgh = 1.6 \times 10 \times 9 = 144\,J$ --- ①

Gain in K.E $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$= \frac{1}{2} \times 1.6 \times 12^2 - 0 = 115.2$

Using Energy equation

Loss in P.E = Gain in K.E + (work done against) air resistance

$144 = 115.2 + W \Rightarrow W = \underline{28.8\,J}$ ✓

(work done against the air resistance)

50. An athlete of mass 84 kg is running along a straight road.

(a) Initially the road is horizontal and he runs at a constant speed of $3ms^{-1}$. The athlete produces a constant power of 60W.

Find the resistive force which acts on the athlete. ---[1]

(b) The athlete then runs up a 150m section of the road which is inclined at 0.8° to the horizontal. The speed of the athlete at the start of this section of the road is $3ms^{-1}$ and he now produces a constant driving force of 24N. The total resistive force which acts on the athlete along this section of road has constance magnitude 13N.

Use an energy method to find the speed of the athlete at the end of the 150m section of road. ---[6]

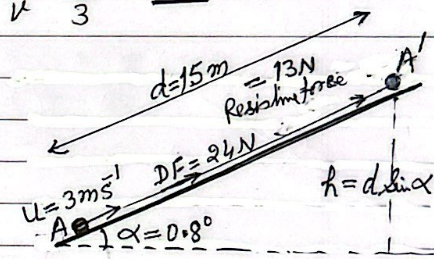
[S-23/42/Q4]

Solution (a) $m = 84kg$, $v = 3ms^{-1}$, $P = 60W$

$P = F \times v \Rightarrow$ Resistive force $F = \frac{P}{v} = \frac{60}{3} = \underline{20N}$

(b) Change in K.E $= \frac{1}{2}m(v^2 - u^2)$

$= \frac{1}{2} \times 84(v^2 - 3^2)$ --- (1)



Work done by DF $= F \times d$

$= 24 \times 150$ —— (2)

Work done against resistive force $= 13 \times 150$ -- (3)

Using Work-energy equation:

Gain in P.E + Gain in K.E = Work done by DF − Work against Resistance

$84g \times 150 \sin 0.8° + \frac{1}{2}84 \cdot v^2 - \frac{1}{2}84 \times 3^2 = 24 \times 150 - 13 \times 150$

$\Rightarrow 1759.23 + 42v^2 - 378 = 3600 - 1950$

$\Rightarrow 42v^2 = 268.765$

$\Rightarrow v = 2.53\ ms^{-1}$

51. A lorry of mass 15 000 kg moves on a straight horizontal road in the direction from A to B. It passes A and B with speeds $20\,ms^{-1}$ and $25\,ms^{-1}$ respectively. The power of the lorry's engine is constant and there is a constant resistance to motion of magnitude 6000 N. The acceleration of the lorry at B is 0.5 times the acceleration of the lorry at A.

(a) Show that the power of the lorry's engine is 200 kW, and hence find the acceleration of the lorry when it is travelling at $20\,ms^{-1}$. -- [5]

The lorry begins to ascend a straight hill inclined at $1°$ to the horizontal. It is given that the power of the lorry's engine and the resistance force do not change.

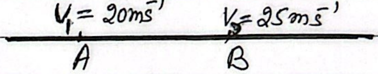
(b) Find the study speed up the hill that the lorry could maintain. -- [2]

S-23/43/Q4

Solution: $m = 15000$ kg., P is constant,

(a) resistance to the motion $= 6000$ N.

acc. at B = 0.5 acc at A.



using Newton's second law,

at A: $\frac{P}{20} - 6000 = 15000a$ --(1) $\left(\begin{matrix} P = F \times V \\ F = P/V \end{matrix}\right)$

and

at B. $\frac{P}{25} - 6000 = 15000 \times \left(\frac{1}{2}a\right)$ --(2)

multiply (2) by 2.

$\frac{2P}{25} - 12000 = 15000a$ --(3)

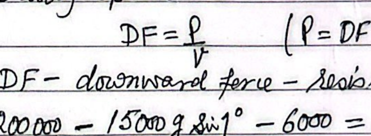
(3)-(1) $\Rightarrow \frac{2P}{25} - \frac{P}{20} - 6000 = 0$

$\frac{3P}{100} = 6000$

$\Rightarrow P = 200000 = \underline{200\,kW}$

for (1) $a = \underline{4/15\,ms^{-2}}$

acc. of lorry at A



up the hill, $\Rightarrow$ acc $a = 0$

Study speed $= V$

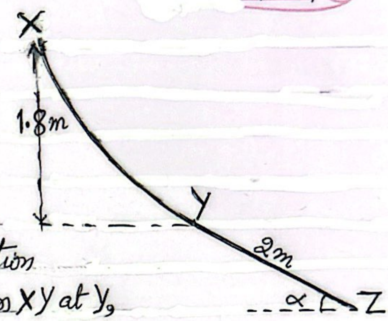
$DF = \frac{P}{V}$ $(P = DF \times V$

DF − downward force − resistance = 0

$\frac{200000}{V} - 15000g\sin 1° - 6000 = 0$

$\Rightarrow$

Study speed $V = \underline{23.2\,ms^{-1}}$



52. The diagram shows the vertical cross-section XYZ of a rough slide. The section YZ is a straight line of length 2m inclined at an angle of α to the horizontal, where $\sin\alpha = 0.28$. The section YZ is tangential to the curved section XY at Y, and X is 1.8m above the level of Y. A child of mass 25 Kg slides down the slide, starting from rest at X. The work done by the child against the resistance force in moving from X to Y is 50J.

(a) Find the speed of the child at Y. ---[4]

It is given that the child comes to rest at Z.

(b) Use an energy method to find the coefficient of friction between the child and YZ, giving your answer as a fraction in its simplest form. --[6]

S-23/43/Q7

Solution: $m = 25$ kg, $h = 1.8$ (from X to Y)

(a) Work done against resistance from X to Y = 50 N.

Using Work energy equation:

P.E. Lost − Work done against resistance = Gain in K.E

$\Rightarrow mgh - 50 = \frac{1}{2}mv^2$ (Speed at Y = v)

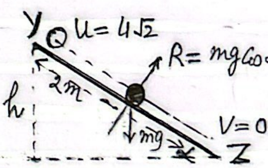
$\Rightarrow 25 \times 10 \times 1.8 - 50 = \frac{1}{2} \times 25 v^2$

$\Rightarrow v = 4\sqrt{2} = 5.66\ ms^{-1}$

Let the force of friction from Y to Z = F

(b) $\sin\alpha = 0.28$

$\cos\alpha = 0.96$



from Y to Z.

P.E Lost = $mgh = 25g \times 2 \cdot \sin\alpha$

$= 25 \times 10 \times 2 \times 0.28$

$= 140$ J. --- ①

K.E Loss = $\frac{1}{2}m(u^2 - 0) = \frac{1}{2} \times 25 \times (4\sqrt{2})^2$

$= 400$ J --- ②

For work energy equation:

Work done by Force = $F \times d = F \times 2$

Normal reaction $R = mg\cos\alpha = 25g \times 0.96$

$R = 240$ --- ③

Work done by force F = Loss K.E + P.E Lost

$F \times 2 = 400 + 140 = 540$

$F = 270$ N --- ④

fr ③ & ④ $\mu = \frac{F}{R} = \frac{270}{240} = \frac{9}{8}$