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Mechanics-1

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Notes

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Forces and Equilibrium.

Forces

- **Force:** Force is anything which changes or tends to change, the state of rest or uniform motion of a body.
- **Mass:** The mass a body is the quantity of matter in the body.
- **Newton's First Law of Motion:**
  A body at rest will remain at rest unless an outside force acts on it, and a body in motion at a constant velocity will remain in motion in a straight line unless acted upon by an outside force.
- **Newton's Second Law of Motion:** It gives us information about the resultant force and the change in motion, i.e. force is proportional to the acceleration.

$$F = ma \qquad \left\{ \begin{array}{l} F = \text{force} \\ m = \text{mass of the body} \\ a = \text{acceleration/or deceleration produced.} \end{array} \right.$$

- **Unit of force is** 'Newton', $m = 1\,\text{Kg}$, $a = 1\,\text{ms}^{-2}$
  $\Rightarrow F = 1\,N$

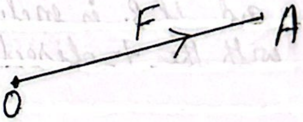
or A force of 1N produces an acceleration of $1\,\text{ms}^{-2}$ in a mass of 1Kg.

- **Weight of body of mass m**, is the force of attraction by the earth.

$$W = mg \qquad \left\{ \begin{array}{l} m = 1\,\text{kg} \\ g = 10\,\text{ms}^{-2} \\ W = 10\,N. \end{array} \right.$$

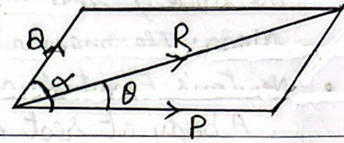
where g is the acceleration due to gravity.

- **Resultant Force:** It is a single force 'R', whose effect on a rigit body is same as that of two or more force P, Q, -- etc. and the forces P, Q, -- are called components of R.
- **Note:** Force is a vector quantity and it has
  (I) megnitude
  (II) direction
  (III) point of application.
  represented by $\overrightarrow{OA}$.



Composition and Resolution of Forces

- Resultant, R, of two forces P and Q acting an angle α, is given by



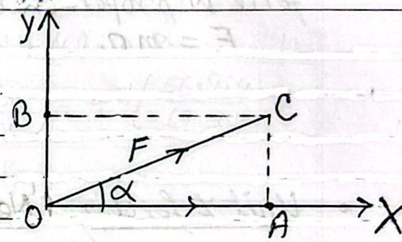
$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

and $$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

- If P and Q are at right angle, $(\alpha = 90°)$

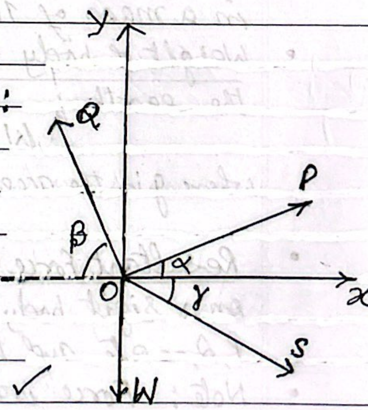
$$R = \sqrt{P^2 + Q^2} \text{ and } \tan\theta = Q/P,$$

- To Resolve a Force into two components at right angle:



Given a force F inclined at an angle α with the + direction of x-axis. Then the component of F along X-axis. $F_x = F\cos\alpha$ and along Y-axis $F_y = F\sin\alpha$.

- To find the resultant of two or more forces in plance acting upon a particle:



Let 'R' is the resultant of the forces P, Q, S and W

Let R_x and R_y are the components of the resultant along x-axis and y-axis

$$R_x = P\cos\alpha - Q\cos\beta + S\cos\gamma \quad \text{——— (1)}$$

$$R_y = P\sin\alpha + Q\sin\beta - S\sin\gamma + W \quad \text{——— (2)}$$

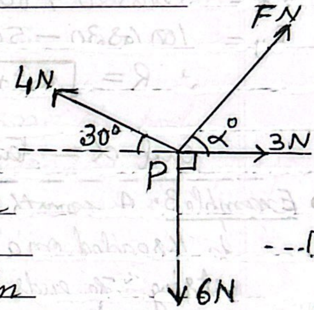
Then the Resultant $R = \sqrt{R_x^2 + R_y^2}$ ✓

and if R is inclined at an angle 'θ' with the + director of x-axis, then $\tan\theta = \dfrac{R_y}{R_x}$ ✓

Equilibrium / component of forces

- **Equilibrium:** When two or more forces act upon a body such that the body remains at rest, the forces are said to be in equilibrium.
  or the sum of components of all these forces in any direction is zero.
  Conversely: The sum of components of all forces in the horizontal and vertical directions (or parallel and perpendicular to a slope) is zero, then the body will be in the equilibrium.

- **Example 1:** Coplanar forces, of magnitudes F N, 3N, 6N and 4N act at a point P, as shown in the diagram.



(a) Given that $\alpha = 60°$, and that the resultant of the four forces is in the direction of the 3N force, Find F. ---[3]

(b) Given instead that the four forces are in equilibrium, find the value of F and α. -[5]

M-20/42/Q5

Solution (a) (The sum of) components of forces perpendicular to 3N force will be zero.

$\Rightarrow 4\sin 30° + F\sin 60° - 6 = 0 \Rightarrow \underline{F = 4.62}$ ✓

(b) Resolving forces vertically and horizontally:

$F\sin\alpha + 4\sin 30 - 6 = 0 \Rightarrow F\sin\alpha = +4$ —①

$F\cos\alpha + 3 - 4\cos 30 = 0 \Rightarrow F\cos\alpha = 0.464$ —②

Square and add ① & ② $F^2 = 4^2 + 0.464^2 = 16.2152$

$\Rightarrow \underline{F = 4.03}$ ✓

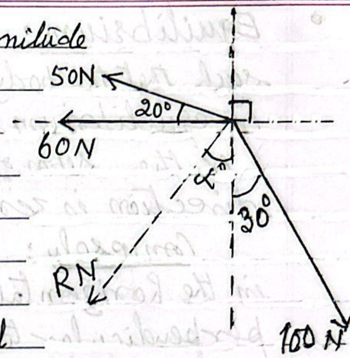
from ① & ② $\tan\alpha = \dfrac{4}{0.464}$

$\Rightarrow \alpha = \tan^{-1}\left(\dfrac{4}{0.464}\right) = 83.383 = \underline{83.4°}$ ✓

To find the resultant of forces and its inclination.

- **Example 2:** Three coplanar forces of magnitude 50N, 60N and 100N act at a point. The resulant of the forces has magnitude R N. The directions of these forces are shown in the diagram. Find the values of R and α. ----[6]

  [W-19/43/Q3]



Solution: Resolving the forces, horizontally and vertically;

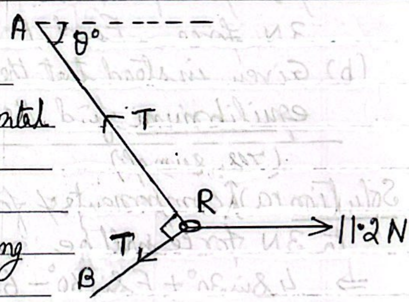
$R_x = 50\cos 20 + 60 - 100\sin 30^\circ = ---- = 56.984$

$R_y = 100\cos 30 - 50\sin 20 \quad = ---- = 69.501$

$\therefore R = \sqrt{R_x^2 + R_y^2} = \sqrt{(56.984)^2 + (69.501)^2} = \underline{89.9}$ ✓

and $\alpha = \tan^{-1}\left(\frac{R_x}{R_y}\right) = \tan^{-1}\left(\frac{R_x}{R_y}\right) = \tan^{-1}\left(\frac{56.984}{69.501}\right) = \underline{39.3}^\circ$ ✓

- **Example 3:** A smooth ring R of mass 0.16 kg is threaded on a light inextensible string. The ends of the string are attached to fixed points A and B. A horizontal force of magnitude 11.2N acts on R, in the same vertical plane as A and B. The ring is in equilibrium. The string is taut with angle ARB = 90°, and the part AR of the string makes an angle θ° with the horizontal. The tension in the string is T N.



(i) Find two simultaneous equations involving $T\sin\theta$ and $T\cos\theta$ --[3]

(ii) Hence find T and θ. ---[3]

[S-12/43/Q2]

Solution: (i) $T\cos\theta + T\sin\theta = 11.2$ — ①

Vertically; $-T\cos\theta + T\sin\theta = 0.169$ — ②

(ii) Solving ① & ② $T\cos\theta = 4.8$ — ③

and $T\sin\theta = 6.4$ — ④

Sq and add ③ & ④ $T^2 = 64$

$\therefore \underline{T = 8}$ ✓

from ③ & ④

$\tan\theta = \frac{6.4}{4.8} = 1.33\ldots$

$\theta = \tan^{-1}(1.33\ldots) = \underline{53.1^\circ}$

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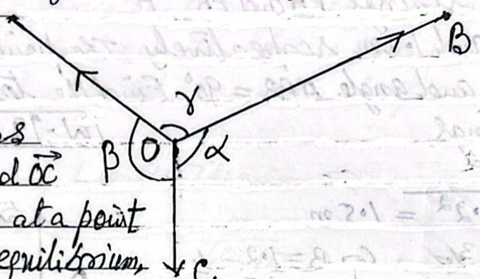
Forces and Equilibrium.

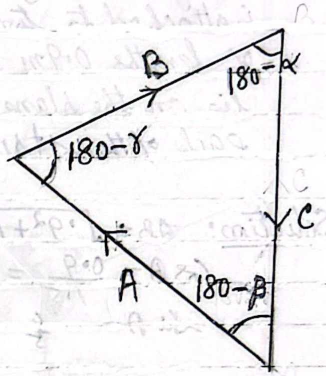
Three forces in equilibrium.

(i) Triangle law of Vector addition (ii) Lami's Theorem.

- Three forces in equilibrium acting at a point, then their resultant is zero, hence can be represented by a triangle-vector diagram:

Three forces $\vec{OA}$, $\vec{OB}$ and $\vec{OC}$ are acting at a point O, are in equilibrium,





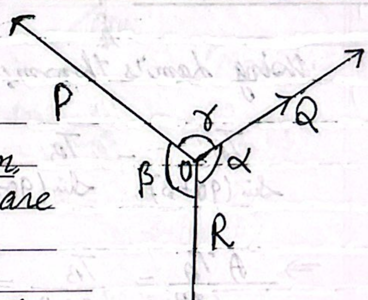
Using Sine Rule:

$$\frac{A}{\sin(180-\alpha)} = \frac{B}{\sin(180-\beta)} = \frac{C}{\sin(180-\gamma)}$$

or $$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

- **Lami's Theorem:**

Given three forces, P, Q and R acting at a point O are in equilibrium, and the angle between them are as shown in the figure,



then,

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}.$$

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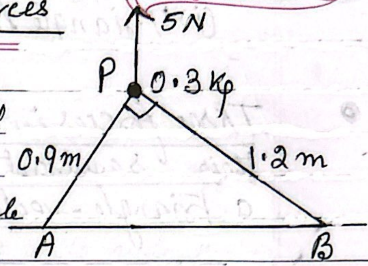
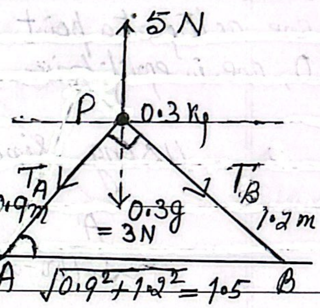
Application of Lami's Theorem, for three forces



- **Example 4:** A particle P of mass 0.3 kg is held in equilibrium above a horizontal plane by a force of magnitude 5N, acting vertically upwards. The particle is attached to two strings PA and PB of lengths 0.9m and 1.2m respectively. The points A and B lie on the plane and angle APB = 90°. Find the tension in each of the strings. [W-19/42/Q3]



Solution: $AB = \sqrt{.9^2 + 1.2^2} = 1.5\,m$

$\cos A = \frac{0.9}{1.5} = 3/5$, $\cos B = \frac{1.2}{1.5} = \frac{4}{5}$

$\sin A = \frac{4}{5}$; $\sin B = 3/5$

Vertical components:

$T_A \times \sin A + T_B \sin B + 0.3g = 5$

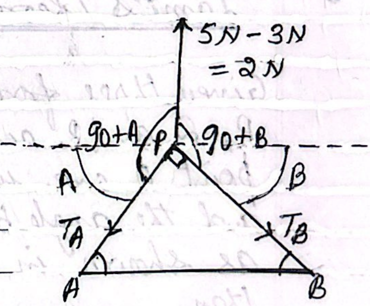
$\frac{4}{5} \times T_A + \frac{3}{5} T_B = 2$ —— ①

Horizontal components. $T_A \cdot \cos A = T_B \cos B$

$\frac{3}{5} \times T_A = T_B \times \frac{4}{5} \Rightarrow 3T_A = 4T_B$ —— ②

Solving ① and ② $\underline{T_A = 1.6N}$ ✓ and $\underline{T_B = 1.2N}$ ✓

Alternate Method:



Using Lami's theorm:

$$\frac{T_A}{\sin(90+B)} = \frac{T_B}{\sin(90+A)} = \frac{2}{\sin 90^0}$$

$$\Rightarrow \frac{T_A}{\cos B} = \frac{T_B}{\cos A} = 2$$

$\Rightarrow$ $T_A = 2 \cdot \cos B$ and $T_B = 2 \cdot \cos A$

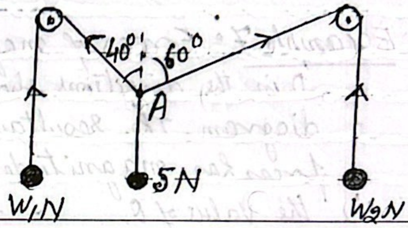
$= 2 \times \frac{4}{5} = 1.6N$ $\qquad = 2 \times \frac{3}{5} = 1.2N$

$\Rightarrow \underline{T_A = 1.6N}$ ✓ and $\underline{T_B = 1.2N}$ ✓

Forces and Equilibrium.

Application of Lami's theorem for three forces

• **Example 5:** Each of the three light strings has a particle attached to one of its ends. The other ends of the strings are tied together to a point A. The strings are in equilibrium with two of them passing over fixed smooth horizontal pegs, and with the particles hanging freely. The weights of the particles, and the angles between the sloping parts of the strings and the vertical, are shown in the diagram. Find the values of W_1 and W_2. [W-05/04/Q3]



Solution: Using Lami's Theorem.

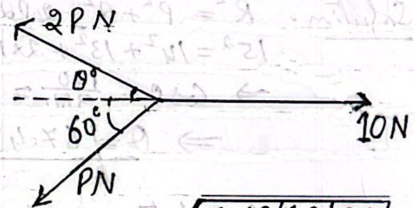
$$\frac{W_1}{\sin 120^0} = \frac{W_2}{\sin 140} = \frac{5}{\sin 100}$$

$$\Rightarrow \frac{W_1}{0.866} = \frac{W_2}{0.6427} = \frac{5}{0.9848}$$

$$\Rightarrow W_1 = \frac{5 \times 0.866}{0.9848} = \underline{4.40} \checkmark$$

$$\text{and } W_2 = \frac{5 \times 0.6427}{0.9848} = \underline{3.26} \checkmark$$

• **Example 6:** Three coplanar forces shown in the diagram are in equilibrium. Find the values of θ and P. ....[4]



[M-18/42/Q2]

Solution: Using Lami's theorem,

$$\frac{P}{\sin(180-\theta)} = \frac{10}{\sin(60+\theta)} = \frac{2P}{\sin 120^0}$$

$$\Rightarrow \frac{P}{\sin\theta} = \frac{10}{\sin(60+\theta)} = \frac{4}{\sqrt{3}}.P$$

$$\Rightarrow \sin\theta = \frac{\sqrt{3}}{4} = 0.433$$

$$\Rightarrow \theta = \sin^{-1} 0.433 = 25.658$$

$$\therefore \underline{\theta = 26.7^0} \checkmark$$

Now $\frac{P}{\sin\theta} = \frac{10}{\sin(60+\theta)}$

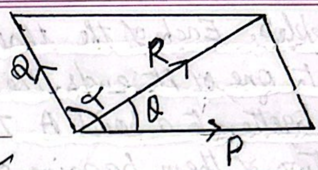
$$\Rightarrow P = \frac{\sin\theta}{\sin(60+\theta)} \times 10$$

$$= \frac{\sin 26.7}{\sin 86.7} \times 10$$

$$\text{or } P = \underline{4.34\,N} \checkmark$$

Resultant of two forces (Cosine Rule)

- Resultant 'R' of two forces P and Q inclined at an angle α with each other is,



$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

and $\tan\theta = \dfrac{Q\sin\alpha}{P + Q\cos\alpha}$, where θ is the angle of inclination of R with P.

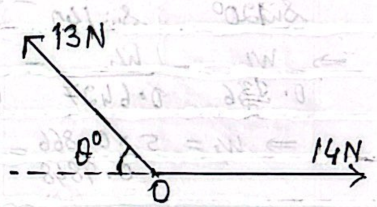
Note:

If $P = Q$ $\Rightarrow$ $R = 2P\cos\alpha/2$ and $\theta = \frac{1}{2}\alpha$

- **Example 7:** Forces of magnitude 13N and 14N act at a point O in the directions shown in the diagram. The resultant of these forces has magnitude 15N. Find,

(i) the value of θ, ---- [3]

(ii) the component of the resultant in the direction of the force of magnitude 14N. [S-12 / 41 / Q2] ---- [2]



Solution: $R^2 = P^2 + Q^2 + 2PQ\cos(180 - \theta)$

$15^2 = 14^2 + 13^2 + 2 \times 14 \times 13 \times \cos(180 - \theta)$

$\Rightarrow \cos\theta = \dfrac{140}{364} = 0.3846 \Rightarrow \theta = \cos^{-1} 0.3846 = 67.38$

$\Rightarrow \underline{\theta = 67.4^\circ}$

Component of R along the force 14N.

$R_x = 14 - 13\cos\theta$

$= 14 - 13\cos 67.38$

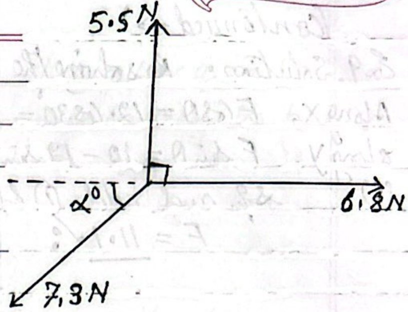
$= 14 - 5$

$= 9$

$\therefore$ Component of R along force 14N

$= \underline{9\ N}$

- **Example 8:** Three coplanar forces act at a point. The magnitudes of the forces are 5.5 N, 6.8 N and 7.3 N, and the direction in which the forces act are as shown in the diagram. Given that the resultant of the three forces is in the same direction as the force of magnitude 6.8 N, find the value of α and the magnitude of the resultant. [S-10/43/Q1] ...[4]



Solution: component of the resultant perp. to 6.8 N force.

Let $R_y = 5.5 - 7.3 \sin\alpha = 0$ ($\because$ Resultant is along 6.8 N force)

$\Rightarrow \sin\alpha = \frac{5.5}{7.3} = 0.7534$

$\Rightarrow \alpha = \sin^{-1} 0.7435 = 48.887°$

or $\alpha = \underline{48.9}$ ✓

Now as the Resultant is along 6.8 N force

$\therefore R = 6.8 - 7.3 \cos\alpha$

$= 6.8 - 7.3 \cos 48.887$

$= 6.8 - 4.8$

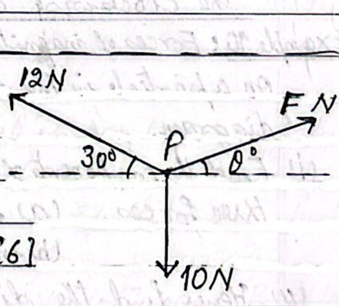
$\therefore R = \underline{2 N}$ ✓

- **Example 9:**

The three coplanar forces shown in the diagram act at a point P and are in the equilibrium,

(i) Find the value of F and θ. ---[6]

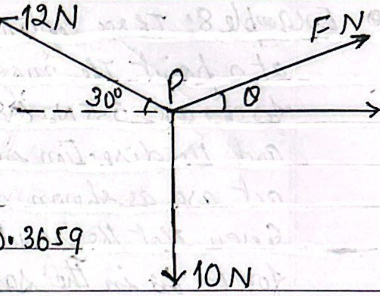
(ii) State the magnitude and direction of the resultant force at P when the force of magnitude 12 N is removed. [S-11/42/Q4] ---[2]



Solution: (Continued→)

(Continued →)

Ex9. Solution: Resolving the forces



Along X, $F\cos\theta = 12\cdot\cos 30^\circ = 10.932$ — ①

along y, $F\sin\theta = 10 - 12\sin 30^\circ = 4$ — ②

(i) sq. and add ① & ②

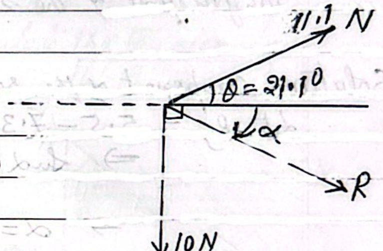
$F = \underline{11.1}$ ✓; ② ÷ ①

$\tan\theta = \dfrac{4}{10.932} = 0.3659$

$\Rightarrow \theta = \tan^{-1} 0.3659 = 20.097$

$\therefore \underline{\theta = 20.1^\circ}$ ✓

(ii) When the force of 12N is removed.



Now $R_x = 11.1\cos 21.1 = 10.3557$ — ①

$R_y = 11.1\sin 21.1 - 10 = -6$ — ②

$R = \sqrt{R_x^2 + R_y^2} = 11.96 = 12$ ✓

$\therefore \underline{R = 12}$ ✓

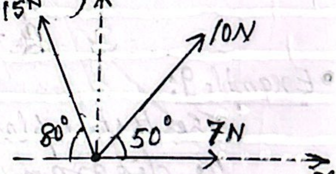
$\tan\alpha = \dfrac{R_y}{R_x} = \dfrac{6}{10.3557}$

$\tan\alpha = 0.58$

$\alpha = \tan^{-1} 0.58 = \underline{30^\circ}$ ✓ clockwise.

∴ Resultant is inclined at 30° with X-axis in the clockwise direction.

Example 10: Forces of magnitude 7N, 10N and 15N act on a particle in the directions shown in the diagram.



(i) Find the components of the resultant of the three forces (a) in the X-direction. (b) in the Y-direction.

[S-09/04/Q3] ---[3]

(ii) Hence find the direction of the resultant. --[2]

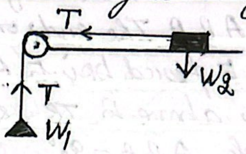
Solution: (i) $R_x = 7 + 10\cos 50 - 15\cos 80 = \underline{10.8N}$ ✓

$R_y = 10\sin 50 + 15\sin 80 = \underline{22.4N}$ ✓

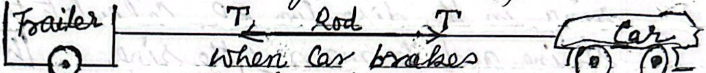
(ii) $\tan\theta = \dfrac{22.4}{10.8} = 2.074$

$\therefore \theta = \tan^{-1} 2.074 = \underline{64.2^\circ}$ ✓ anticlockwise from X-axis

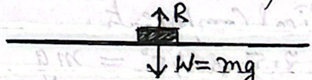
- **Tension:** The tension in a string (or a rod) connecting two bodies has a tendency to bring the bodies together (pull).
It acts inwards at the two ends.



- **Thrust:** The trust is applied by a rod which keeps the two particles (or bodies) together. Thurst acts outwards at the two ends of the rod along the direction of the rod.

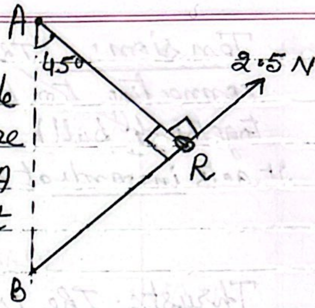


- **Normal contact force (or Normal Reaction)**
If one body is pressed against another body, each body experiences a force at the point of contact. Such a force, R, is called "Normal Reaction" or Normal contact force.
$R = mg.$

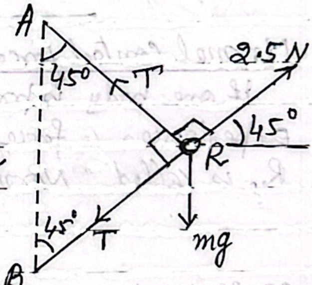


- **Friction:** If two bodies are in contact with one another, the property of the two bodies, by virtue of which a force is exerted between them at their point of contact to prevent one body sliding on the other, is called friction, and the force exerted is called the force of friction.
(i) The direction of friction on one of them at its point of contact is opposite to the direction in which this point of contact would try to move.
(ii) The magnitude of the friction is, when there is equilibrium, just sufficient to prevent the body from moving.
- **Coefficient of Friction μ:** If 'F' is the force of friction and 'R' is the normal contact force (or Normal Rection), between the two bodies, when equilibrium is just on the point of being destroyed, we have $\mu = \dfrac{F}{R}$ or $F = \mu R$

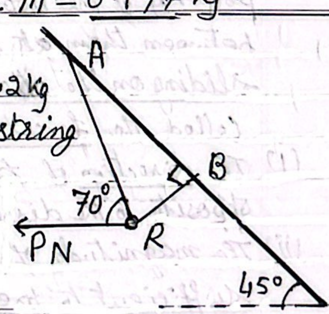
Equilibrium - Tension in string



Example 11: A smooth ring R of mass m kg is treaded on a light inextensible string ARB. The ends of the string are attached to fixed points A and B, with A vertically above B. The string is taut and angle ARB = 90°.
The angle between the part AR of the string and the vertical is 45°. The ring is held in equilibrium in this position by a force of magnitude 2.5 N, acting on the ring in the direction BR. Calculate the tension in the string and the mass of the ring. [W-18/42/Q1] --[4]



Solution: Let the mass of the ring is m kg.
Resolving the forces at the point R.
Horizontally; $T\cos45^\circ + T\cos45^\circ = 2.5\cos45$
$\Rightarrow 2T = 2.5 \Rightarrow T = \underline{1.25\text{ N}}$ ✓
Vertical Components;
$2.5\sin45^\circ = mg$
$\therefore m = \dfrac{2.5\times\sin45^\circ}{10} = 0.1767$
$\therefore m = \underline{0.177\text{ kg}}$ ✓



Example 12: A small smooth ring R of mass 0.2 kg is threaded onto a light inextensible string ARB. The two ends of the string are attached to points A and B on a sloping roof inclined at 45° to the horizontal.
A horizontal force of magnitude P N, acting in the plane ARB, is applied to the ring. The section BR of the string is perpendicular to the roof and the section AR of the string is inclined at 70° to the horizontal. The system is in equilibrium. Find the tension in the string and the value of P. [W-18/43/Q1] --[4]
(continued→)

(Continued→)

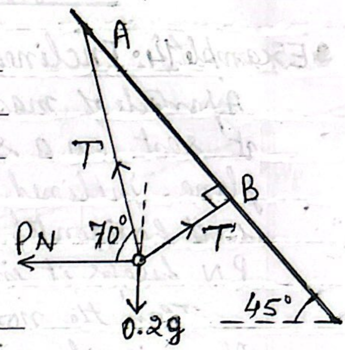
Ex-12, Solution: Resolving the forces,

→ Vertical component;

$T\sin 70^\circ + T\sin 45 = 0.2g$

$T = \dfrac{2}{\sin 70^\circ + \sin 45^\circ}$ $(g = 10)$

or $T = \dfrac{2}{1.6468} = 1.21\,N$ ✓

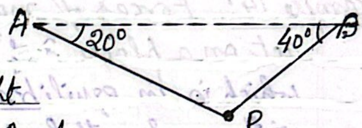


→ Horizontal Components;

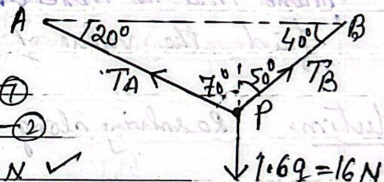
$P + T\cos 70^\circ = T\cos 45^\circ$

$P = T(\cos 45^\circ - \cos 70^\circ) = 1.21 \times 0.3420 = 0.443$ ✓

Example 13: A particle P of mass 1.6 kg is suspended in equilibrium by two light inextensible strings attached to points A and B. The strings make angles of 20° and 40° respectively with the horizontal. Find the tensions in the two strings. [M-17/42/Q2] --[6]



Solution: Resolving the forces,



Horizontally; $T_A \cos 20^\circ = T_B \cdot \cos 40$ —(1)

Vertically; $T_A \sin 20^\circ + T_B \sin 40 = 16$ —(2)

Solving (1) & (2) $\begin{cases} T_A = 14.2\,N \\ T_B = 17.4\,N. \end{cases}$ ✓

Alternate method:

Using Lami's Theorem;

$\dfrac{T_A}{\sin 130^\circ} = \dfrac{T_B}{\sin 110^\circ} = \dfrac{16}{\sin 120}$,

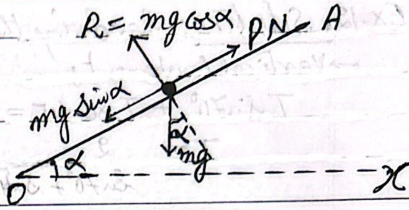
$\Rightarrow T_A = \dfrac{\sin 130}{\sin 120} \times 16 = 14.2\,N$ ✓

and $T_B = \dfrac{\sin 110^\circ}{\sin 120^\circ} \times 16 = 17.4\,N$ ✓

Forces and Equilibrium

'Smooth' Horizontal and Inclined Planes

- **Smooth Inclined Plane:**

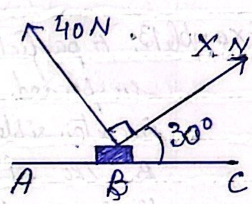


A particle of mass m kg is at rest on a smooth inclined plane, inclined at an angle α, with horizontal, a force of P N keeps it in equilibrium.

Then the normal reaction (contact force) $R = mg\cos\alpha$
(Normal)

Along the plane, force $P = mg\sin\alpha$

Example 14: Forces of magnitude X N and 40N act on a block B of mass 15 kg, which is in equilibrium in contact with a smooth horizontal surface, between points A and C on the surface. The forces act in the same vertical plane and the directions shown in the diagram. Find the value of X.



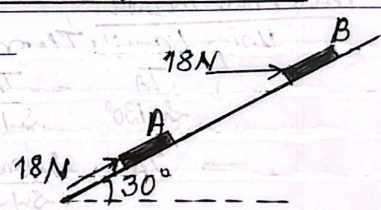
Solution: Resolving along AC.

$$X\cos 30^\circ = 40\cos 60$$

$$\Rightarrow x = \frac{40 \cdot \cos 60}{\cos 30^\circ} = 23.1\,N \checkmark$$

Example 15: Small blocks A and B are held at rest on a smooth plane inclined at 30° to the horizontal. Each is held in equilibrium by a force of magnitude 18 N. The force on A acts upwards parallel to a line of greatest slope of the plane, and the force on B acts horizontally in a vertical plane containing a line of greatest slope. Find the weight of A and weight of B.

[W-14/41/Q2] --[4]



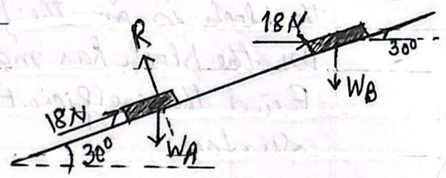
(continued →)

(Continued example 15)

Solution: Resolving horizontally

for A, $W_A \cdot \sin 30^\circ = 18$

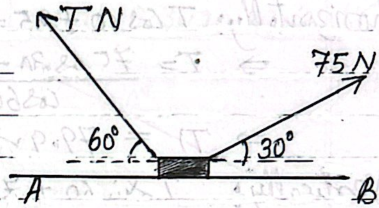
$\Rightarrow W_A = \dfrac{18}{\sin 30^\circ} = \underline{36\,N}$



for B, $W_B \sin 30^\circ = 18 \cos 30^\circ$

$W_B = \dfrac{18 \cos 30}{\sin 30} = \underline{31.2\,N}$

Example 16: Two light strings are attached to a block of mass 20 kg. The block is in equilibrium on a horizontal surface AB with the strings taut. The string make angles 60° and 30° with horizontal, on either side of the block, and the tensions in the strings are T N and 75 N respectively.



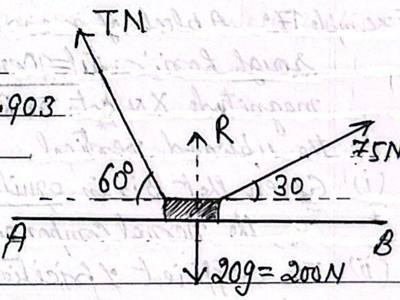
(i) Given that the surface is smooth, find the value of T and the magnitude of the contact force acting on the block. ---[5]
$\boxed{\text{S-07/04/Q7(i)}}$

Solution: Resolving the forces.

horizontally: $T \cos 60 = 75 \cos 30^\circ$

$\Rightarrow T = \dfrac{75 \cdot \cos 30}{\cos 60} = 129.903$

$\therefore \underline{T = 130}$



Vertically:

$T \sin 60 + 75 \sin 30^\circ + R = 20g$

$\Rightarrow 130 \sin 60^\circ + 75 \sin 30^\circ + R = 200$ $\quad [\because T = 130]$

$R = 200 - (130 \cdot \sin 60 + 75 \sin 30^\circ)$

$= 200 - 150 = 50$

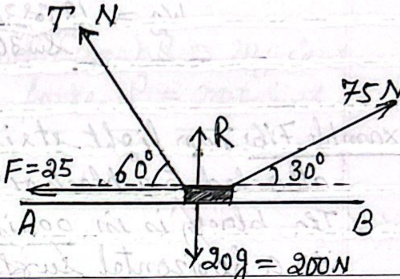
$\therefore$ Magnitude of contact force $R = 50\,N$.

Forces and Equilibrium

"Friction"

(Continued)

Example 16 (ii) It is given instead that surface is rough and the block is on the point of slipping, the frictional force on the block has magnitude 25 N and acts towards A. Find the coefficient of friction between the block and the surface; [S-07/04/Q7(ii)] --- [6]



Solution: Resolving the forces.

Given force of friction $F = 25\,N$

horizontally: $T\cos 60 + 25 = 75\cos 30°$

$\Rightarrow T = \dfrac{75\cos 30 - 25}{\cos 60}$

$\Rightarrow T = 79.9$ ✓

Vertically: $T\sin 60 + 75\sin 30° + R = 200$

$\Rightarrow 79.9 \times \sin 60 + 75\sin 30° + R = 200$

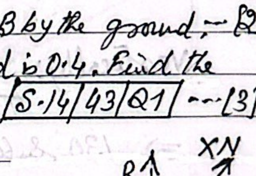
$\Rightarrow R = 93.3$ ✓

$\therefore$ Coefficient of Friction $\mu = \dfrac{\text{Force of friction}}{\text{Normal Contact force}}$

or $\mu = \dfrac{F}{R} = \dfrac{25}{93.3}$ $\;\;$ [$\because$ Force of friction $F = \mu R$

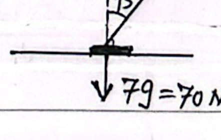
$\therefore \mu = 0.268$ ✓

- **Example 17:** A block of mass 7 kg is at rest on rough horizontal ground. A force of magnitude X N acts on B at an angle of 15° to the upward vertical.



(i) Given that B is in equilibrium, find in terms of X, the normal component of the force exerted on B by the ground; --- [2]

(ii) The coefficient of friction between B and the ground is 0.4. Find the value of X for which B is in limiting equilibrium. [S-14/43/Q1] --- [3]



Solution: (i) Normal Component $= R$,

Resolving Vertically: $R + X\cos 15° = 70 \Rightarrow R = (70 - X\cos 15°)$

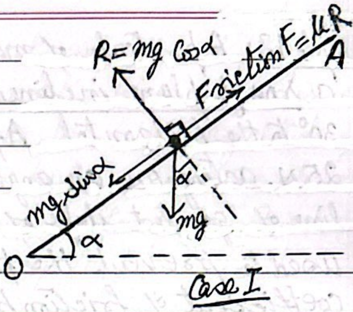
(ii) Force of friction $F = \mu R = 0.4(70 - X\cos 15°)$ —①

Horizontal Component: $F = X\cdot\sin 15°$

from ① $\Rightarrow 0.4(70 - X\cos 15°) = X\sin 15° \Rightarrow \underline{X = 43.4}$ ✓

Forces and Equilibrium
Rough Inclined Plane _ Friction

- **Rough Inclined Plane:**
If a body is placed upon a rough inclined plane (inclined at an angle α, with the horizontal) Coefficient of friction between the particle and plane is μ. mass of the particle is m.



Case I The particle is in equilibrium just before sliding down the plane.

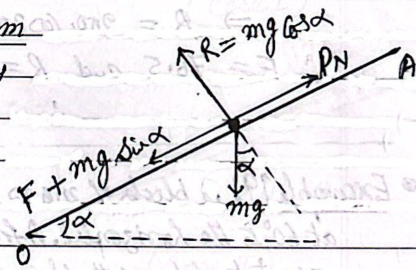
Normal contact force $R = mg\cos\alpha$

Particle exerts a force downward $= mg\sin\alpha$

The force of friction, F, will act upwards along the plane to keep the particle at rest, and $F = \mu R$

or $F = \mu \cdot mg\cos\alpha$. ✓

Case II. When the particle is in equilibrium just before sliding up the plane by applying a force of PN upwards along the plane,



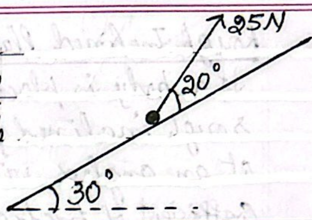
Then $R = mg\cos\alpha$

and $P = F + mg\sin\alpha$

$= \mu R + mg\sin\alpha$

or $P = \mu mg\cos\alpha + mg\sin\alpha$.

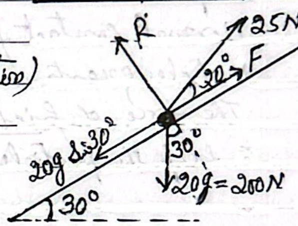
• **Example 18:** A particle of mass 20Kg is on a rough plane inclined at an angle of 30° to the horizontal. A force of magnitude 25 N, acting on an angle of 20° above a line of greatest slope of the plane, is used to prevent the particle from sliding down the plane. The coefficient of friction between the particle and the plane is μ.



(a) Complete the diagram to show all the forces acting on particle. ---[1]

(b) Find the least possible value of μ. [SP-20/04/Q4] ---[5]

Solution: Resolving forces parallel to the plane and perp. to the plane, (F is the force of friction)



Along the plane: $F + 25\cos 20^\circ = 20g\sin 30^\circ$

or $F = 200\sin 30 - 25\cos 20 = 76.5$ ✓

Perp. to the Plane:

$R + 25\sin 20 = 20g\cos 30$

$\Rightarrow R = 200\cdot\cos 30 - 25\sin 20^\circ = 164.65$ ✓

$\therefore F = 76.5$ and $R = 164.65$

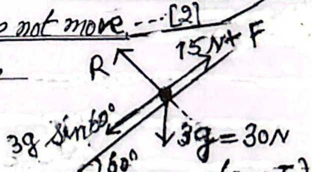
$\therefore \mu = \frac{F}{R} = \frac{76.5}{}$

• **Example 19:** A block of mass 3kg is at rest on a rough plane inclined at 60° to the horizontal. A force of magnitude 15 N acting up a line of greatest slope of the plane is just sufficient to prevent the block from sliding down the plane.

(i) Find the coefficient of friction between the block and the plane. ---[5]

The force of magnitude 15 N is now replaced by a force of magnitude X N acting up the line of greatest slope. [W-19/41/Q3]

(ii) Find the greatest value of X for which the block does not move. ---[2]



Solution: (i) $R = 3g\cos 60 = 30\cos 60$ ✓

$F = \mu R = \mu \times 30\cos 60$

∴ Along the plane $15 + F = 3g\sin 60^\circ$

$\Rightarrow 15 + \mu \times 30\cos 60 = 30\cdot\sin 60$

$\Rightarrow \mu = \frac{30\times\sin 60 - 15}{30\cdot\cos 60} = 0.732$ ✓

(ii) The force X N is trying to pull up.

$X = 3g\sin 60 + F$

$= 30\sin 60 + \mu \times 30\cos 60$

$X = 30\sin 60 + 0.732 \times 30\cos 60 = 36.96 = \underline{37}$ ✓

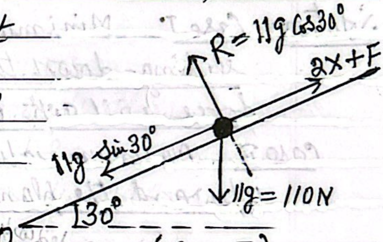
Force and Equilibrium
Rough Inclined Plane

Example 20: A block of mass 11 kg is at rest on a rough plane inclined at 30° to the horizontal. A force acts on the block in a direction up the plane to a line of greatest slope. When the magnitude of the force is 2X N the block is on the point of sliding down the plane, and when the magnitude of the force is 9X N the block is on the point of sliding up the plane. Find
(i) the value of X. ---[3]
(ii) the coefficient of friction between the block and the plane. --[4]

[S-11 / 41 / Q4]

Solution: Case I. The block is at the point of sliding down,

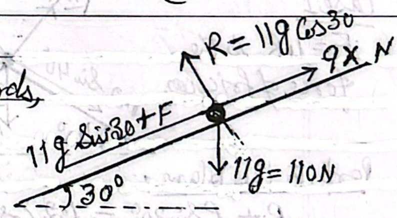
(i) So the force of friction, F, will act upwards.



(Case I)

$\therefore\ 2X + F = 11g\,\sin 30$
or $2X + F = 110\,\sin 30$ —— (i)

Case II The block is at the point of sliding up,
So force of friction will act downwards,



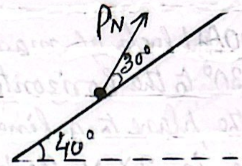
(Case II)

$\therefore\ 11g\,\sin 30° + F = 9X$
or $110\,\sin 30 + F = 9X$
or $9X - F = 110\,\sin 30°$ —— (ii)
Solving (i) and (ii) $\underline{X = 10}$ and $F = 35$

(ii) $R = 11g\,\cos 30 = 110\,\cos 30$ —— (iii)
$\therefore\ F = 35 = \mu R \Rightarrow \mu = \frac{35}{R} = \frac{35}{110\,\cos 30} = 0.367$ from (iii)

$\therefore$ The the coefficient of friction $\mu = \underline{0.367}$

Forces and Equilibrium
Rough Inclined plane



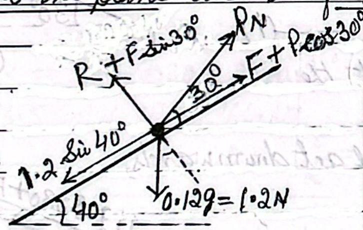
- **Example 21:** A particle of mass 0.12 kg is placed on a plane which is inclined at an angle of 40° to the horizontal. The particle is kept in equilibrium by a force of magnitude PN acting up the plane at an angle of 30°, above a line of greatest slope. The coefficient of friction between the particle and the plane is 0.32. Find the set of possible values of P. [S-17/42/Q5] ---[8]

Solution: Case I – Minimum value of P needed to stop the particle sliding down the plane and in this case the frictional force will act upward the plane.

Case II – Maximum value of P needed to start the particle moving upward the plane and the frictional force will act downwards.

Case I

F = μR is the force of friction.



Perp. to the plane:

$R + P\sin 30° = 1.2\cos 40$

$\Rightarrow R = (1.2\cos 40 - P\sin 30°)$

$F = \mu R = 0.32(1.2\cos 40 - P\sin 30)$ —①

Along the Plane

$P\cos 30 + F = 1.2\sin 40$

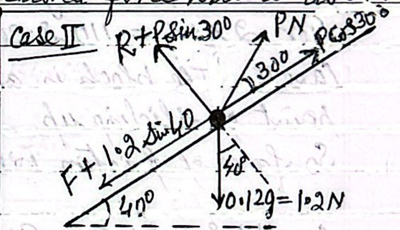
fr① $\Rightarrow P\cos 30 + 0.32(1.2\cos 40 - P\sin 30) = 1.2\sin 40$

$\Rightarrow \boxed{P(\cos 30 - 0.32\sin 30) = 1.2\sin 40 - 0.32\times 1.2\cos 40}$

$\Rightarrow P = \dfrac{1.2(\sin 40 - 0.32\cos 40)}{(\cos 30 - 0.32\sin 30)}$

$\Rightarrow P_{Min} = \underline{0.676}$ ✓

Case II



Same as in case I

$F = 0.32(1.2\cos 40 - P\sin 30°)$

along the Plane

$F + 1.2\sin 40 = P\cos 30$

or $0.32(1.2\cos 40 - P\sin 30) + 1.2\sin 40 = P\cos 30°$

$\Rightarrow P(0.32\sin 30 + \cos 30) = 0.32\times 1.2\cos 40 + 1.2\sin 40$

$\Rightarrow P = \dfrac{(0.32\times 1.2\cos 40 + 1.2\sin 40)}{(0.32\sin 30 + \cos 30)}$

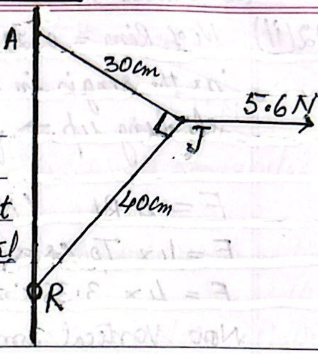
$\Rightarrow P_{Max} = \underline{1.04}$ ✓

∴ Possible value of P, $0.676 \leq P \leq 1.04$

M-1

Forces and Equilibrium
Rough wire (or rod)
Horizontal or Vertical

- **Example 22:** A small ring R is attached to one end of a light inextensible string of length 70 cm. A fixed rough vertical wire passes through the ring. The other end of the string is attached to a point A on the wire, vertically above R. A horizontal force of magnitude 5.6 N is applied to the point J of the string 30 cm from A and 40 cm from R. The system is in equilibrium with each of the parts AJ and RJ of the string taut and angle AJR equal to 90°.



(i) Find the tension in the part AJ of the string and the tension in the part JR of the string. --[5]

The ring R has mass 0.2 kg and is in limiting equilibrium on the point of moving up the wire.

(ii) Show that the coefficient of friction between R and the wire is 0.341, correct to three significant figures. --[4]

A particle of mass m kg is attached to R and R is now in limiting equilibrium, on the point of moving down the wire.

(iii) Given that the coefficient of friction is unchanged, find the value of m. --[3]

S-15/42/Q7

Solution: Let angle RAJ = α, Angle AJK = $(90+\alpha)$

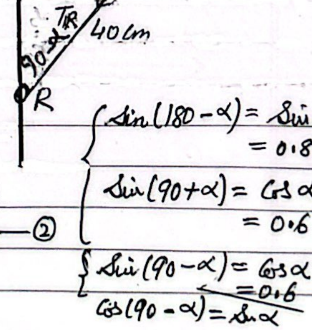
(i) $AR = \sqrt{30^2+40^2} = 50\text{cm}$

Using Lami's Theorem:

$$\frac{5.6}{\sin 90} = \frac{T_A}{\sin(180-\alpha)} = \frac{T_R}{\sin(90+\alpha)}$$

$$\Rightarrow \frac{5.6}{1} = \frac{T_A}{0.8} = \frac{T_R}{0.6}$$

$\Rightarrow T_A = 4.48$ ✓ and $T_R = 3.36$ ✓



$\sin(180-\alpha) = \sin\alpha = 0.8$

$\sin(90+\alpha) = \cos\alpha = 0.6$

Alternative (i) $T_A \sin\alpha + T_R \sin(90-\alpha) = 5.6$

$0.8T_A + 0.6T_R = 5.6$ —— ①

and $T_A\cos\alpha = T_R\cos(90-\alpha) \Rightarrow 0.06T_A + 0.8T_R$ —— ②

Solving ① & ② $T_A = 4.48$ ✓ & $T_R = 3.36$ ✓

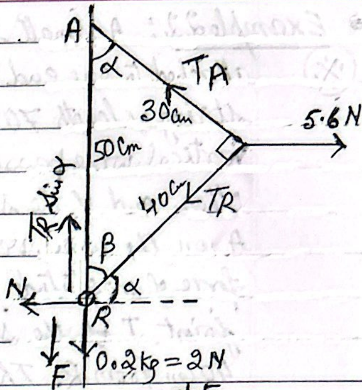
$\sin(90-\alpha) = \cos\alpha = 0.6$

$\cos(90-\alpha) = \sin\alpha$

(Continued→)

22(ii) Wt of Ring = 0.2g = 2N

as the ring is in the limiting position of sliding up, ⇒ force of friction F will act downwards.



$F = \mu N$ (N denotes the normal reaction)

$F = \mu \times T_R \cos\alpha$

$F = \mu \times 3.36 \times 0.6$ ——— ①

Now Vertical components at point R

$0.2g + F = T_R \sin\alpha$

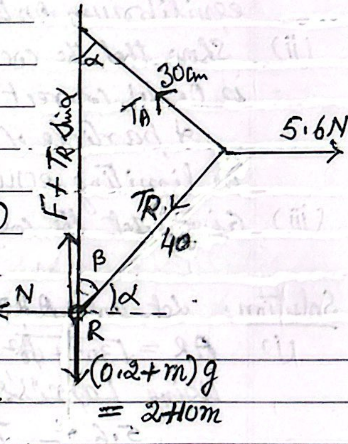
$2 + \mu \times 3.36 \times 0.6 = 3.36 \times 0.8$

$\Rightarrow \mu = \dfrac{3.36 \times 0.8 - 2}{3.36 \times 0.6} = 0.341$

$\therefore \underline{\mu = 0.341}$

(iii) Weight $= 0.2g + mg = (2N + 10m)N$

as the ring is in the limiting position of sliding down ⇒ force of friction 'F' will act downwards.



$F = \mu N = 0.341 \times T_R \cos\alpha$

$= 0.341 \times 3.36 \times 0.6 = 0.6874\ N$ ——— ①

Now taking vertical components at R,

$0.2g + mg = F + T_R \sin\alpha$

$\Rightarrow 2 + 10m = 0.6874 + 3.36 \times 0.8$

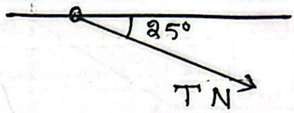
$\Rightarrow 10m = 0.6874 + 3.36 \times 0.8 - 2$

$10m = 1.3754$

$\Rightarrow m = 0.13754$

or $\underline{m = 0.137}$ or $\underline{0.138}$

- Example 23: A ring of mass 4kg is attached to one end of a light string. The ring is threaded on a fixed horizontal rod and the string is pulled at an angle of 25° below the horizontal. With a tension in the string of T N the ring is in equilibrium.



(i) Find, in terms of T, the horizontal and vertical components of the force exerted on the ring by the rod. ---[4]

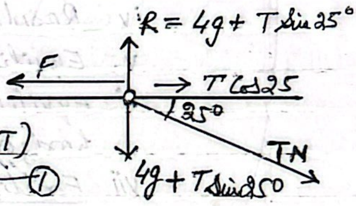
The coefficient of friction between the ring and the rod is 0.4.

(ii) Given that the equilibrium is limiting, find the value of T. --[3]

[S-12/42/Q4]

Solution: Horizontal component $= T.\cos 25^\circ = 0.906\,T.$

(i) Vertical Component $= 4g + T\sin 25^\circ = 40 + 0.423T$



(ii) Contact force $R = 4g + T\sin 25$
$= (40 + 0.423T)$

Force of Friction $= \mu R = 0.4(40 + 0.423T)$
$= (16 + 0.169T)$ — ①

Now Horizontally;

$$T\cos 25 = F$$

$0.906\,T = 16 + 0.169\,T$ from ①

$$T = \frac{16}{(0.906 - 0.169)} = 21.7$$

$\therefore$ $T = 21.7\,N$ ✓

← X — X — X →