DATE: 28.11.23

M

Mechanics

Forces and Equilibrium

Exercise 1. Solution (Revision)

SP-20	S-22	M-20	S-20	W-20	M-23
W-22	S-23	M-21	S-21	W-21	M-22

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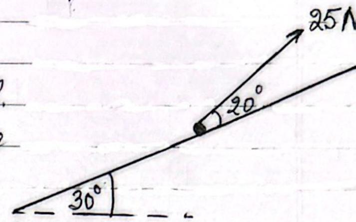
M1

Forces and Equilibrium (Revision)

1. A particle of mass 20 Kg is on a rough plane inclined at an angle of 30° to the horizontal. A force of magnitude 25N, acting at an angle of 20° above the line of greatest slope of the plane, is used to prevent the particle from sliding down the plane. The coefficient of friction between the particle and the plane is μ.



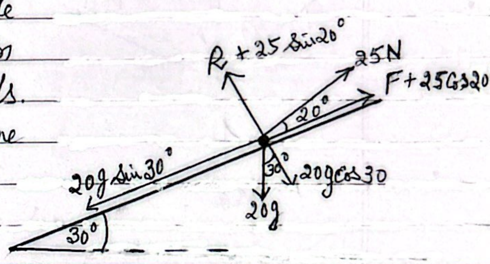
(a) Complete the diagram to show all the forces acting on the particle. --[1]

(b) Find the least value of μ. --[5]

/SP-20/04/Q4/

Solution (a) The particle tries to slide down, so the force of friction ($F = \mu R$) will act upwards.

(b) Resolving the force along the plane (and perp. to the plane)



Along the plane:

$$F + 25\cos 20^\circ = 20g\sin 30^\circ$$

$$\Rightarrow F = 200\sin 30^\circ - 25\cos 20^\circ = 76.5\ N \quad (1)$$

Perpendicular to the plane; Let 'R' is normal reaction of the particle.

$$R + 25\sin 20^\circ = 20g\cos 30^\circ$$

$$\Rightarrow R = 200\cos 30^\circ - 25\sin 20^\circ = 164.65\ N \quad (2)$$

$\therefore$ coefficient of friction $\mu = \frac{F}{R}$

$$= \frac{76.5}{164.65} = 0.46462$$

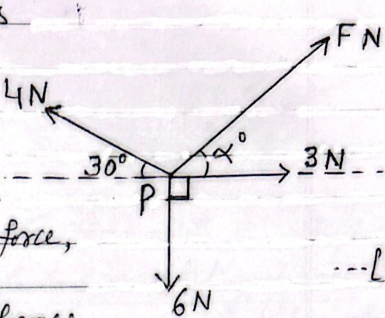
$$\therefore \mu = 0.465$$

2. Coplanar forces, of magnitudes FN, 3N, 6N and 4N, act at a point P.

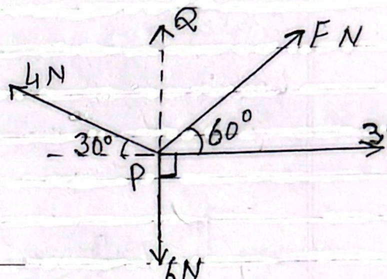
(a) Given that $\alpha = 60°$, and the resultant of the four forces is in the direction of the 3N force, Find F. ---[3]

(b) Given instead that the four forces are in equilibrium, find the values of F and α. --[5]

[M-20/42/Q 5]



Solution (a) The resultant of the four forces is along the 3N force. hence the sum of the components of the forces perpendiculare to the 3N force will be zero.



$\Rightarrow \quad 4\sin 30° + F\sin 60° - 6 = 0$

$\Rightarrow 4\times\frac{1}{2} + F\times\frac{\sqrt{3}}{2} - 6 = 0$

$\Rightarrow \quad F = 4\times\frac{2}{\sqrt{3}} = 4.6188$

$\therefore \; F = \underline{4.62\ N}$ ✓

(b) Given that the four forces are in equilibrium,
Resolving the forces vertically and horizontally.

Horizontally:

$F\cos\alpha + 3 - 4\cos 30 = 0 \Rightarrow F\cos\alpha = 0.464$ —— ①

Vertically:

$F\sin\alpha + 4\sin 30° - 6 = 0 \Rightarrow F\sin\alpha = 4$ —— ②

Square and add ① & ② $F^2 = 4^2 + 0.464^2 = 16.2152$

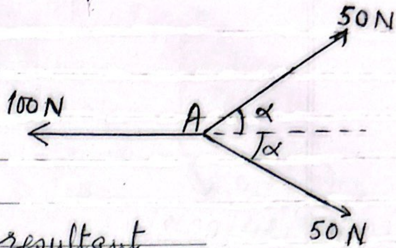
$\Rightarrow \; F = \underline{4.03}$ ✓

Also from ① & ② $\tan\alpha = \frac{4}{0.464} = 8.62$

$\Rightarrow \alpha = \tan^{-1} 8.62 = 83.383$

$\therefore \quad \underline{\alpha = 83.4°}$ ✓

3. Three force of magnitude 100 N, 50 N and 50 N act at a point A, as shown in the diagram.



The value of $\cos\alpha = 4/5$

Find the magnitude of the resultant of the three fores and state its direction. ---[3]

[S-20/41/Q1]

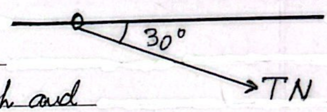
<u>Solution:</u> Resolving the forces horizontally.

Resultant $= 100 - 2\times 50\cos\alpha = 100 - 100\times\frac{4}{5} = 100-80$
$= \underline{20\text{ N}}$ ✓

($\because$ the vertical component $(50\sin\alpha - 50\sin\alpha = 0)$)

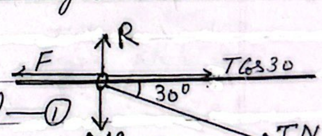
$\therefore$ The resultant $= \underline{20\text{ N}}$ towards the left (or in the direction of 100 N force)

4. The diagram shows a ring of mass 0.1 kg threaded on a fixed horizontal rod. The rod is rough and the coefficient of friction between the ring and the rod is 0.8. A force of magnitude T N acts on the ring in a direction at 30° to the rod, downwards in the vertical plane containing the rod. Initially the ring is at rest. [S-20/41/Q4(a)]



Find the greatest value of T for which the ring remains at rest. [4]

<u>Solution:</u> <u>Vertically</u>. $R = 0.1g + T\sin 30^\circ$



$\therefore$ force of friction $F = \mu R = 0.8(0.1g + T\sin 30^\circ)$ — ①

<u>Horizontally</u>: $T\cos 30 = F$

$\Rightarrow T\cos 30 = 0.8(0.1g + T\sin 30)$

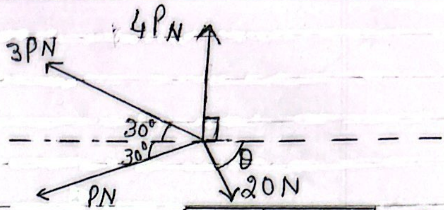
$T(\cos 30 - 0.8\times\sin 30) = 0.8\times 0.1g$ $(0.1g = 1)$

$0.466\,T = 0.8$

$T = \frac{0.8}{0.466} = 1.716$

$\therefore T = \underline{1.72\text{ N}}$ ✓

5. Coplanar forces of magnitudes 20 N, PN, 3PN and 4PN act at a point in the directions shown in the diagram. The system is in equilibrium. Find P and θ.



S-20/42/Q2 --- [6]

Solution: Resolving the forces horizontally:

$20\cos\theta = 3P\cos 30 + P\cos 30 \Rightarrow 20\cos\theta = 4P\cos 0^\circ \Rightarrow \cos\theta = \frac{\sqrt{3}}{10}P$ — (1)

Vertically: $4P + 3P\sin 30^\circ - P\sin 30^\circ = 20\sin\theta$

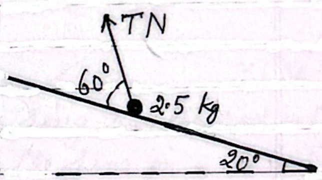
$4P + P = 20\sin\theta \Rightarrow \sin\theta = \frac{1}{4}P$ — (2)

Sq and add (1) & (2) $\left(\frac{3}{100} + \frac{1}{16}\right)P^2 = 1 \Rightarrow P^2 = \frac{1600}{148} = 10.81$

$\Rightarrow P = 3.29$ ✓

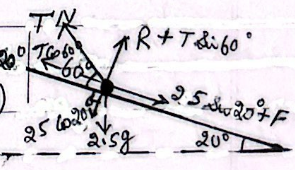
from (2) $\sin\theta = \frac{1}{4}P = \frac{1}{4} \times 3.2879 = 0.822 \Rightarrow \theta = \sin^{-1} 0.822 = 55.3^\circ$ ✓

6. A particle of mass 2.5 kg is held in equilibrium on a rough plane inclined at 20° to the horizontal by a force of magnitude T N making an angle of 60° with a line of greatest slope of the plane. The coefficient of friction between the particle and the plane is 0.3. Find the greatest and the least possible values of T. --- [8]



S-20/42/Q 3

Solution: Case I when the particle is just trying to move upwards



$T\sin 60 + R = 25\cos 20^\circ$

$R = (25\cos 20^\circ - T\sin 60)$

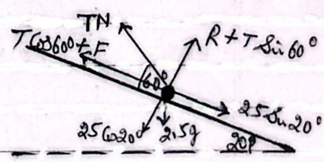
$F = \mu R$

$T\cos 60 = F + 25\sin 20^\circ$

$\Rightarrow T\cos 60 = 25\sin 20 + 0.3(25\cos 20^\circ - T\sin 60^\circ)$

$\Rightarrow T(\cos 60^\circ + 0.3\sin 60^\circ) = 25\sin 20 + 0.3 \times 25\cos 20$

$\Rightarrow T = \frac{15.56}{0.76} = 20.5$ ✓



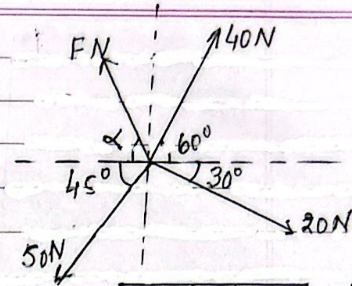
Case II: The particle is just trying to move down

$T\cos 60^\circ + F = 25\sin 20^\circ$

$\Rightarrow T\cos 60 = 25\sin 20 - 0.3(25\cos 20 - T\sin 60^\circ)$ $(F = \mu R)$

$T(\cos 60^\circ - 0.3\sin 60) = 25\sin 20 - 0.3 \times 25\cos 20 \Rightarrow T = \frac{1.5}{0.24} = 6.26$ ✓

7. Four coplanar forces of magnitudes 40 N, 20 N, 50 N and F N act at a point in the directions shown in the diagram. The four forces are in equilibrium.
Find F and α



[S-20/43/Q3] ---[6]

Solution: Resolving the forces horizontally:

$F\cos\alpha = 40\cos 60^\circ + 20\cos 30^\circ - 50\cos 45^\circ = 1.965$ — ①

$F\sin\alpha = 50\sin 45 + 20\sin 30 - 40\sin 60 = 10.714$ — ②

Sq. and add ① & ② $F = \sqrt{(1.965)^2 + (10.714)^2} = \underline{10.9}$ ✓

and $\alpha = \tan^{-1}\left(\frac{10.714}{1.965}\right) = 79.6^\circ$ ✓

$\underline{F = 10.9 \text{ and } \alpha = 79.6^\circ}$

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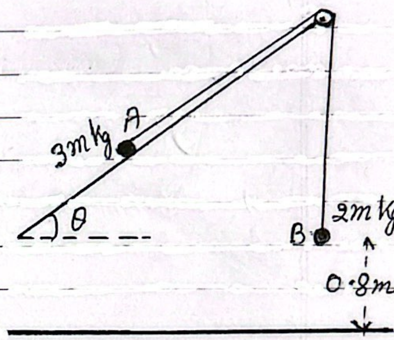
8. Two particles A and B, of masses 3m kg and 2m kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pully which is attached to the edge of a plane. The plane is inclined at an angle θ to the horizontal. A lies on the plane and B hangs vertically, 0.8 m above the floor, which is horizontal. The string between A and the pully is parallel to a line of greatest slope of the plane. Initially A and B are at rest. Given that the plane is smooth, find the value of θ for which A remains at rest.



[S-20/43/Q7(a)] --[3]

Solution: for Partical B, (A and B remain at rest)

$T - 2mg = 0 \Rightarrow T = 2mg$ — ①

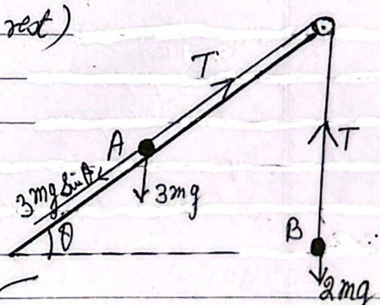
for A, $3mg\sin\theta - T = 0$ — ②

$\Rightarrow 3mg\sin\theta - 2mg = 0$ from ① & ②

$\Rightarrow \sin\theta = 2/3$

$\Rightarrow \theta = \sin^{-1}\left(\frac{2}{3}\right) = 41.81$

$\therefore \theta = \underline{41.8^\circ}$ ✓



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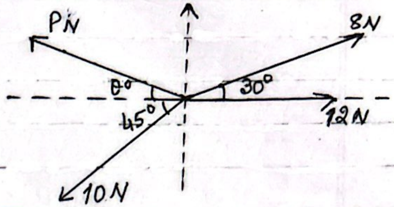
9. Coplanar forces of magnitude 8N, 12N, 10N and PN act at a point in the directions shown in the diagram. The system is in equilibrium.
Find P and θ.



W-20/41/ Q3 -- [6]

<u>Solution:</u> Resolving the forces:

Horizontally: $P\cos\theta = 12 + 8\cos 30° - 10\cos 45°$

$\Rightarrow P\cos\theta = 11.857$ ——— (1)

Vertically: $P\sin\theta = 10\sin 45 - 8\sin 30°$

$\Rightarrow P\sin\theta = 3.071$ ——— (2)

square and add (1) & (2) $P = \sqrt{11.857^2 + 3.071^2} = 12.2$ ✓

(2) ÷ (1) $\tan\theta = \frac{3.071}{11.857} = 0.259$

$\Rightarrow \theta = \tan^{-1} 0.259 = 14.5°$

$\therefore$ $P = \underline{12.2}$ ✓ and $\theta = 14.5°$ ✓

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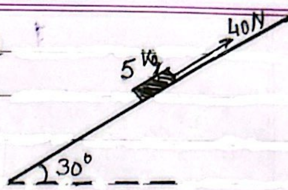
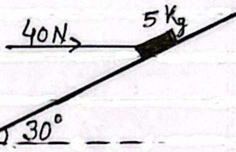
10. A block of mass 5 Kg is placed on a plane inclined at 30° to the horizontal. The coefficient of friction between the block and the plane is μ.



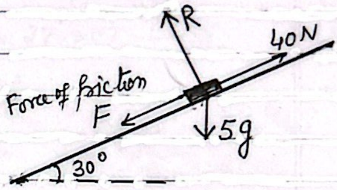
(a) When a force of magnitude 40 N is applied to the block, acting up the plane to a line of greatest slope, the block begins to slide up the plane. Show that $\mu < \frac{1}{5}\sqrt{3}$ ---[4]

(b) When a force of magnitude 40 N is applied horizontally, in a vertical plane containing a line of greatest slope, the block does not move. Show correct to three decimal places, the least value of μ is 0.152. [W-20/42/Q6] ---[4]



Solution (a) $R = 5g\cos 30° = 25\sqrt{3}$ ——— ①

For the block to start moving up, along the plane: $40 - 5g\sin 30° - F > 0$ — ②



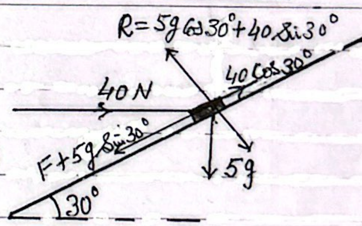
$F = \mu R = 25\sqrt{3}\cdot\mu$ from ①

from ②

$$40 - 5\times 10\times\frac{1}{2} - 25\sqrt{3}\cdot\mu > 0$$

$$\Rightarrow 25\sqrt{3}\,\mu < 15 \Rightarrow \mu < \frac{15}{25\sqrt{3}} \Rightarrow \mu < \frac{1}{5}\cdot\sqrt{3} \checkmark$$

(b) Resolving the forces parallel and perpendicular to the inclined plane.



Perp to the plane: $R = 5g\cos 30° + 40\sin 30°$

$R = 25\sqrt{3} + 20 = 63.3$ — ③

Parallel to the plane: $F + 5g\sin 30° \geq 40\cos 30°$ (for the block does not move up)

$\Rightarrow F \geq 40\cos 30° - 5g\sin 30°$

$\Rightarrow 63.3\,\mu \geq 20\sqrt{3} - 25 = 9.64$ ($F = \mu R = 63.3\mu$ from ③)

$\Rightarrow \mu \geq \frac{9.64}{63.3} = 0.152$

$\Rightarrow \underline{\mu \geq 0.152} \checkmark$

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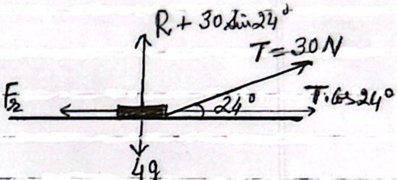
11. A string is attached to a block of mass 4 kg which rests in limiting eqnilibrium on a rough horizontal table. The string makes an angle of 24° above the horizontal and the tension in the string is 30 N. [W-20/43/Q3]

(a) Draw a diagram showing all the forces acting on the block. --- [1]

(b) Find the coefficient of friction between the block and the table. --- [5]

Solution (a)



(b) Resolving the forces,

Horizontally: $F = 30 \cos 24°$ (F_r = Force of friction)

$\Rightarrow F = 27.406$ — ①

Vertically: $R + 30 \sin 24 = 4g = 40$

$R = 40 - 30 \sin 24° = 27.797$ — ②

Now Force of friction $F = \mu R$

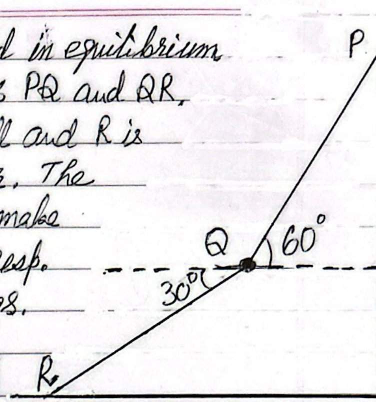
or $\mu = \frac{F}{R} = \frac{27.406}{27.797} = 0.9859$

$\therefore$ Coefficient of friction $\mu = 0.986$

12 A particle Q of mass 0.2 Kg is held in equilibrium by two light inextensible strings PQ and QR. P is a fixed point on a vertical wall and R is a fixed point on a horizontal floor. The angles which strings PQ and QR make with the horizontal are 60° and 30° resp. Find the tensions in the two strings.

--- [5]

| M-21 | 42 | Q 3 |



Solution: Resolving the forces:

Horizontally: $T_P \cos 60 = T_R \cdot \cos 30 \Rightarrow T_P \cdot \frac{1}{2} = \frac{\sqrt{3}}{2} \cdot T_R$

$\Rightarrow T_P = \sqrt{3}\, T_R$ — (1)

Vertically: $T_P \sin 60 = T_R \sin 30^\circ + 0.2g$

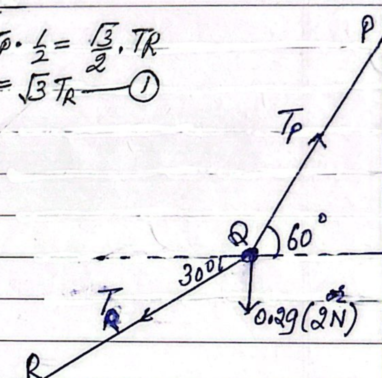
$\Rightarrow \frac{\sqrt{3}}{2} T_P = \frac{1}{2} T_R + 2$ — (2)

from (1) & (2)

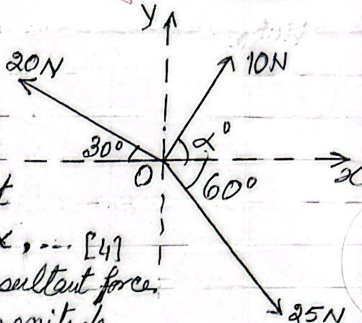
$\frac{\sqrt{3}}{2} \times \sqrt{3} \cdot T_R = \frac{1}{2} T_R + 2$

$\Rightarrow 3 T_R = T_R + 4 \Rightarrow T_R = 2N$ ✓

from (1) $T_P = \sqrt{3} \times 2 = 3.46N$ ✓



13 Three coplanar forces of magnitudes 10 N, 25 N and 20 N act at a point O in the directions shown in the diagram.



(a) Given that the component of the resultant force in the x-direction is zero, find α, ... [4]
and hence find the magnitude of the resultant force.

(b) Given instead that $\alpha = 45°$, find the magnitude and direction of the resultant of the three forces. ---[5]

|S-21|41|Q6|

Solution: Resolving the forces:

(a) Horizontally: $20\cos 30° = 25\cos 60 + 10\cos\alpha$ [$\because$ component of Resultant in X-axis = zero]

$\Rightarrow 20 \times \frac{\sqrt{3}}{2} = 25 \times \frac{1}{2} + 10\cos\alpha \Rightarrow \cos\alpha = 0.4821 \Rightarrow \alpha = 61.2°$ ✓

$\therefore$ Resultant is along Y-axis $= 20\sin 30° + 10\sin 61.2° - 25\sin 60°$

$\therefore$ Magnitude of Resultant force $= 10 + 8.761 - 21.651 = 2.89\,N$ ✓

(b) Now for $\alpha = 45°$

Horizontal Component $X = 25\cos 60 + 10\cos 45 - 20\cos 30$

$X = 12.5 + 7.07107 - 17.32051 = 2.25056$ — (1)

Vertical Component $Y = 20\sin 30 + 10\sin 45° - 25\sin 60°$

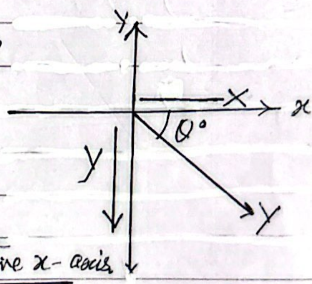
$Y = 10 + 7.07107 - 21.65064 = -4.57957$ — (2)

$\therefore$ Magnitude of the result $R = \sqrt{X^2 + Y^2}$

$= \sqrt{(2.25056)^2 + 4.57957^2}$

$R = 5.10\,N$ ✓

Now let the resultant makes angle $\theta°$ below the X-axis:

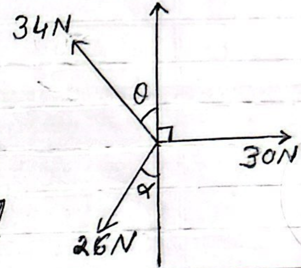


$\tan\theta = \frac{Y}{X} = \frac{4.57957}{2.25056}$

$\tan\theta = 2.03484$

$\theta = 63.8°$ below positive x-axis

14. Coplanar forces of magnitudes 34N, 30N and 26N act at a point in the directions shown in the diagram. Given that $\sin\alpha = \frac{5}{13}$ and $\sin\theta = 8/17$, find the magnitude and direction of the Resultant of the three forces. ---[6]



[S-21/42/Q2]

Solution: Resolving the forces; ($\cos\theta = \frac{15}{17}$; $\cos\alpha = 12/13$)

Along X-axis; $X = 30 - 34\sin\theta - 26\sin\alpha$

$X = 30 - 34 \times 8/17 - 26 \times 5/13 = 4$ ✓

Along Y-axis; $Y = 34\cos\theta - 26\cos\alpha$

$= 34 \times \frac{15}{17} - 26 \times \frac{12}{13} = 6$ ✓

∴ Magnitude of the Resultant $R = \sqrt{X^2 + Y^2}$

$= \sqrt{4^2 + 6^2} = \sqrt{52}$

$= 7.21$ N.

and the direction of the Resultant

$\tan\theta = \frac{Y}{X} = \frac{6}{4} = 1.5$

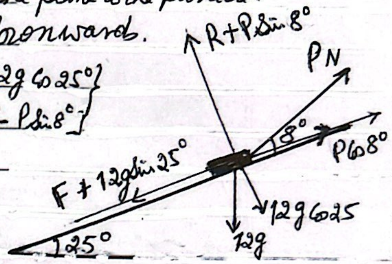
$\theta = \tan^{-1} 1.5 = 56.3°$ ✓

above 30N force.

15. A particle of mass 12 Kg is stationary on a rough plane inclined at an angle of 25° to the horizontal. A pulling force of magnitude PN acts at an angle 8° above a line of greatest slope of the plane. This force is used to keep the particle in equilibrium. The coefficient of friction between the particle and the plane is 0.3.
Find the greatest value of P. ---[6]

[S-21 | 42 | Q4]

Solution: The pulling force PN is trying to pull up the plane to the particle. Hence the force of friction F will act downward.
Resolving the forces perp. to plane $\{R + P\sin 8^\circ = 12g\cos 25^\circ\}$
$F = \mu R = 0.3[12g\cos 25^\circ - P\sin 8^\circ]$ --- (1) $\{\Rightarrow R = 12g\cos 25^\circ - P\sin 8^\circ\}$



Now resolving the force along the plane.

$P\cos 8 = F + 12g\sin 25^\circ$

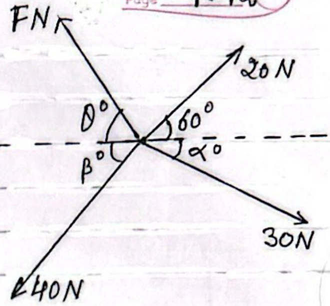
$\Rightarrow P\cos 8^\circ = 0.3[12g\cos 25^\circ - P\sin 8^\circ] + 12g\sin 25^\circ$

$\Rightarrow P[\cos 8^\circ + 0.3 \times \sin 8^\circ] = 12 \times 10 \times \sin 25^\circ - 0.3 \times 12 \times 10 \times \cos 25^\circ$

$\rightarrow P(1.032) = 50.714 + 32.627$

$P = \dfrac{83.341}{1.032} = 80.756$

$\therefore P = \underline{80.8}$ ✓



16 Four coplanar forces act at a point. The magnitudes of the forces are 20N, 30N, 40N and FN. The direction of the forces are as shown in the diagram, where $\sin\alpha^\circ = 0.28$ and $\sin\beta^\circ = 0.6$. Given that the forces are in equilibrium, Find F and θ.

S-21/43/Q3 ---[6]

$\cos\alpha = 0.96$, $\cos\beta = 0.8$

Solution: Resolving the forces: (In equilibrium)

Vertically: $F\sin\theta + 20\sin 60 - 30\sin\alpha - 40\sin\beta = 0$

$\Rightarrow F\sin\theta + 20\times\frac{\sqrt{3}}{2} - 30\times 0.28 - 40\times 0.6 = 0$

$\Rightarrow F\sin\theta = 15.07949$ ---- (1)

Horizontally: $F\cos\theta + 40\cos\beta - 30\cos\alpha - 20\cos 60 = 0$

$\Rightarrow F\cos\theta + 40\times 0.8 - 30\times 0.96 - 20\times\frac{\sqrt{3}}{2} = 0$

$\Rightarrow F\cos\theta = 6.8$ ---- (2)

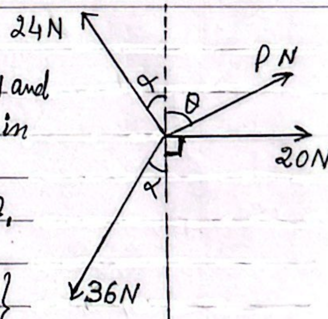
Square and add (1) & (2)

$F = \sqrt{15.07942^2 + 6.8^2} = \underline{16.5}$

(1) ÷ (2) $\Rightarrow \tan\theta = \frac{15.07949}{6.8} = 2.2175$

$\theta = \tan^{-1} 2.2175$

$\theta = 65.7^\circ$



17. Coplanar forces of magnitudes 24N, PN, 20N and 36N act at a point in the directions shown in the diagram. The system is in equilibrium. Given that $\sin\alpha = 3/5$, find the values of P and θ.

W-21/41/Q3 ---[6]

Solution: Resolving the forces: $\sin\alpha = \frac{3}{5} \Rightarrow \cos\alpha = \frac{4}{5} = .8$

Vertically: $P\cos\theta + 24\cos\alpha - 36\cos\alpha = 0$

$P\cos\theta + (24-36)\times 0.8 = 0$

$\Rightarrow P\cos\theta = (36-24)\times .8 = 9.6$ ---- (1)

Horizontally: $P\sin\theta + 20 = 24\sin\alpha + 36\sin\alpha$

$\Rightarrow P\sin\theta = (24+36)\sin\alpha - 20$

$P\sin\theta = 60\times\frac{3}{5} - 20 = 16$ ---- (2)

Square and add (1) & (2)

$P = \sqrt{9.6^2 + 16^2} = 18.7$

$\underline{P = 18.7}$ ✓

(2) ÷ (1) $\Rightarrow \tan\theta = \frac{16}{9.6} = 1.6667$

$\theta = \tan^{-1} 1.6667$

$\theta = 59^\circ$ ✓

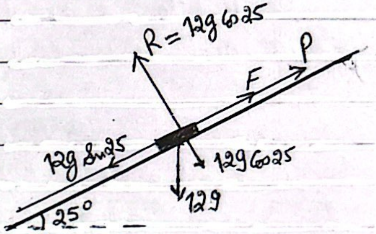
18 A particle of mass 12 kg is stationary on a rough plane inclined at an angle of 25° to the horizontal. A force of magnitude P N acting parallel to a line of greatest slope of the plane is used to prevent the particle sliding down the plane. The coefficient of friction between the particle and the plane is 0·35.

(a) Draw a sketch showing the forces acting on the particle. ---[1]

(b) Find the least possible value of P. ---[5]

[W-21/41/Q4]

Solution: (a)



(b) Resolving the force perp. to plane:

$R = 12g\cos 25^\circ = 108.8\ N$

Force of friction $F = \mu R = 0.35 \times 108.8 = 38.1\ N$ --- (1)

Resolving the forces along the plane,

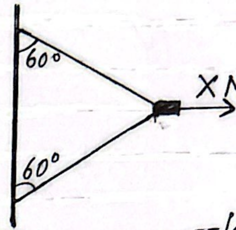
$P + F = 12g\sin 25^\circ$

$\Rightarrow P + 38.1 = 50.71$

$\Rightarrow P = 50.71 - 38.1$

$P = \underline{12.6}$ ✓

19. A block of mass 5 kg is held in equilibrium near a vertical wall by two light strings and a horizontal force of magnitude X N. The two strings are both inclined at 60° to the vertical.

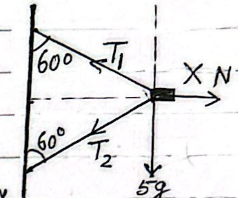


(a) Given that $X = 100$, find the tension in the lower string. ---[4]

(b) Find the least value of X for which the block remains in equilibrium in the position shown. ---[4]

[W-21/42/Q6]

Solution (a) Resolving the forces: $X = 100$ N



Horizontally: $100 - T_1 \sin 60° - T_2 \sin 60° = 0$

$\Rightarrow T_1 + T_2 = \frac{200}{\sqrt{3}}$ ——(1)

Vertically: $T_1 \cos 60 - T_2 \cos 60 - 5g = 0$

$\Rightarrow T_1 - T_2 = 100$ ——(2) Tension in lower string

(1) − (2) $\Rightarrow 2 \cdot T_2 = \frac{200}{\sqrt{3}} - 100 = 15.47 \Rightarrow T_2 = 7.74$ (7.35)

(b) Resolving the forces; $X = ?$ (For the lowest value of X, $T_2 = 0$)

$X - T_1 \sin 60 = 0 \Rightarrow T_1 = \frac{2}{\sqrt{3}} X$ — (3)

Vertically, $T_1 \cos 60 - 5g = 0 \Rightarrow T_1 = 100$ — (4)

from (3) & (4) $\frac{2}{\sqrt{3}} X = 100 \Rightarrow X = \frac{100\sqrt{3}}{2} = 86.6$

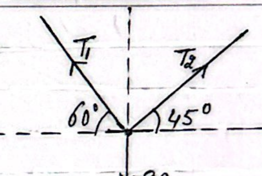
20. A particle of mass 8 kg is suspended in equilibrium by two inextensible strings which make angles 60° and 45° above the horizontal.

(a) Draw the diagram showing the forces acting on the particle. ---[1]

(b) Find the tensions in the strings. ---[6]

[W-21/43/Q2]

Solution: (a) 3 forces diagram, including the angles shown.



(b) Resolving the forces:

Horizontally: $T_1 \cos 60° = T_2 \cos 45 \Rightarrow \frac{1}{2} T_1 = \frac{1}{\sqrt{2}} T_2$

$\Rightarrow T_1 = \sqrt{2}\, T_2$ -------(1)

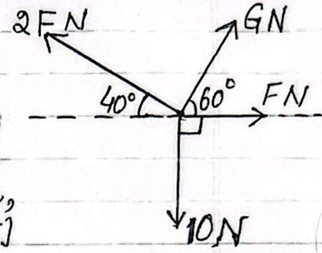
Vertically: $T_1 \sin 60° + T_2 \sin 45° = 8g$

$\frac{\sqrt{3}}{2} T_1 + \frac{1}{\sqrt{2}} T_2 = 80 \Rightarrow \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} T_2 + \frac{1}{\sqrt{2}} T_2 = 80$ [from (1) $T_1 = \sqrt{2}\, T_2$]

$\Rightarrow T_2 \left(\frac{\sqrt{3}+2}{\sqrt{2}} \right) = 80 \Rightarrow T_2 = \frac{80\sqrt{2}}{(\sqrt{3}+2)} = 41.4$ N (41.444)

$T_1 = \sqrt{2} \times T_2 = \sqrt{2} \times 41.444\ldots = 58.6$ N

2.1. Four coplanar forces act at a point. The magnitude of the forces are 10 N, F N, G N and 2F N. The direction of forces are as shown in the diagram.



(a) Given that the forces are in equilibrium, find the values of F and G. ---- [5]

(b) Given instead that F = 3, find the value of G for which the resultant of the forces is perpendicular to the 10 N force. ---[2]

M-22/42/Q5

Solution (a) Resolving the forces horizontally:

$F + G\cos 60° - 2F\cos 40 = 0$

$0.5G + F(1 - 2\times 0.766) = 0$

$\Rightarrow 0.5G - 0.532F = 0$ --- (1)

Resolving the forces vertically:

$G\sin 60° + 2F\sin 40° - 10 = 0$

$\Rightarrow 0.866G + 1.2855F = 10$ -- (2)

Solving (1) & (2), $F = 4.53$; $G = 4.82$

(b) Now given F = 3

The resultant is perpendicular to 10N force. Hence the sum of resolved part of vertical forces = 0

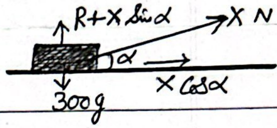
$\Rightarrow G\sin 60° + 2\times 3\sin 40° - 10 = 0$

$\Rightarrow G = \dfrac{10 - 6\sin 40}{\sin 60°} = 7.0$

22. A crate of mass 300 kg is at rest on rough horizontal ground. The coefficient of friction between the crate and the ground is 0.5. A force of magnitude X N, acting at an angle α above the horizontal, is applied to the crate, where $\sin\alpha = 0.28$.
Find the greatest value of X for which the crate remains at rest. ---[5]

S-22/41/Q3

Solution: Resolving Vertically: $R + X\sin\alpha = 300g$



$\Rightarrow R = 300g - 0.28X$ --- ①

Resolving Horizontally:

$X\cos\alpha - F = 0$

$\Rightarrow 0.96X - 0.5(300g - 0.28X) = 0$

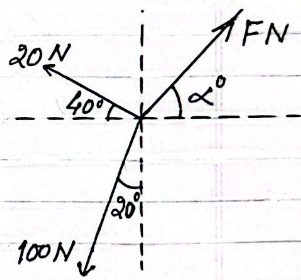
$\left\{ \begin{array}{l} \sin\alpha = 0.28 \\ \cos\alpha = 0.96 \end{array} \right.$

$\because F = \mu R,\ \mu = 0.5 \Rightarrow F = 0.5(300g - .28X)$

$\Rightarrow 0.96X + 0.5 \times 0.28X = 0.5 \times 300g$

$\Rightarrow 1.1X = 1500 \Rightarrow X = \underline{1363.6\text{ N}}$ ✓ (1363.63)

23. Three coplanar forces of magnitude 20N, 100N and FN act at a point. The directions of these forces are shown in the diagram. Given that the three forces are in equilibrium, find F and α.



S-22/41/Q4 --[6]

Solution: Resolving the forces horizontally;

$F\cos\alpha - 20\cos40° - 100\sin20 = 0 \Rightarrow F\cos\alpha = 2\cos40° + 100\cdot\sin20° = 49.5229$ --- ①

Resolving the forces vertically:

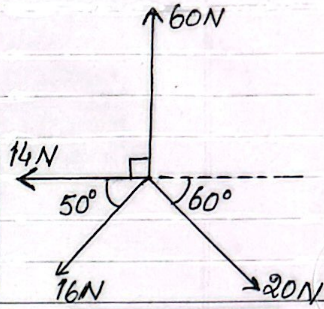
$F\sin\alpha + 2\sin40° - 100\cos20° = 0 \Rightarrow F\sin\alpha = 100\cos20° - 2\sin40° = 81.1135$ --- ②

from ① and ② $F = \sqrt{(49.5229)^2 + (81.1135)^2} = \underline{95\text{ N}}$ ✓ (95.0364)

from ① $F\cos\alpha = 49.5229 \Rightarrow \cos\alpha = \dfrac{49.523}{95.0364} = 0.521$

$\alpha = \cos^{-1}(0.521) = \underline{58.6°}$ (58.59°)

24. Coplanar forces of magnitudes 60N, 20N, 16N and 14N act at a point in the directions shown in the diagram.
Find the magnitude and direction of the resultant force. ---[6]



[S-22/42/Q2]

Solution: Resolving the forces along x-axis;

$R_x = 20\cos 60 - 14 - 16\cos 50° = 14.2846$ --- ①

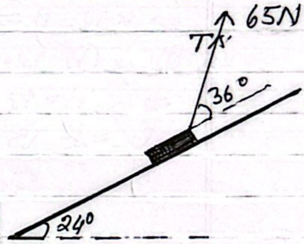
Resolving the forces along y-axis:

$R_y = 60 - 20\sin 60 - 16\sin 50° = 30.42278$ --- ②

Square and add ① and ② $R = \sqrt{(14.2846)^2 + (30.42278)^2} = \underline{33.6N}$

$\tan\theta = \frac{R_y}{R_x} = \frac{30.42278}{14.2846} = 2.1297 \Rightarrow \theta = \tan^{-1}(2.1297) = \underline{64.8°}$

25. A block of mass 12 kg is placed on a plane which is inclined at an angle of 24° to the horizontal. A light string, making an angle of 36° above a line of greatest slope, is attached to the block. The tension in string is 65N. The coefficient of friction between the block and plane is μ. The block is in limiting equilibrium is on the point of sliding up the plane. Find μ. [S-22/42/Q5] --[6]



Solution: Resolving perpendicular to the plane at P (Block)

$R + 65\sin 36 = 12g\cos 24$

$\Rightarrow R = (12g\cos 24 - 65\sin 36°)$ --- ①

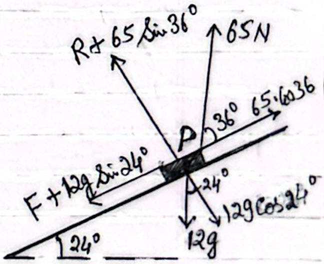
Resolving parallel to the plane (at P)

$F + 12g\sin 24 = 65\cos 36°$

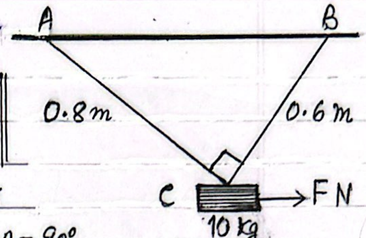
$\Rightarrow F = (65\cos 36 - 12g\sin 24)$ --- ②

$\mu = \frac{F}{R} = \frac{(65\cos 36° - 12g\sin 24°)}{(12g\cos 24 - 65\sin 36°)}$ $\left\{ \begin{array}{l} F = \mu R \\ \Rightarrow \mu = F/R \end{array} \right.$

$\mu = \underline{0.0529}$



26. The diagram shows a block of mass 10 kg suspended below a horizontal ceiling by two strings AC and BC, of lengths 0.8 m and 0.6 m respectively, attached to the fixed points on the ceiling. Angle $ACB = 90°$.

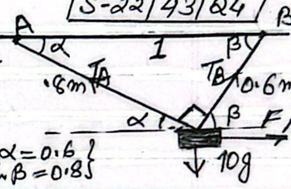


There is a horizontal force of magnitude F N acting on the block. The block is in the equilibrium.

(a) In the case where $F = 20$, find the tensions in each of the strings --[5]

(b) Find the greatest value of F for which the block remains in equilibrium in the position shown. --[3]

[S-22/43/Q4]



Solution (a) $F = 20$, Tension in string $CA = T_A$, " " " $CB = T_B$

$AB = \sqrt{.8^2 + .6^2} = 1$

$\cos\alpha = 0.8$, $\cos\beta = .6$; $\sin\alpha = 0.6$, $\sin\beta = 0.8$

Resolving horizontally;

$T_A \cos\alpha - T_B \cos\beta - 20 = 0$

$\Rightarrow 0.8 T_A - 0.6 T_B = 20$ —— ①

Resolving vertically,

$T_A \sin\alpha + T_B \sin\beta - 10g = 0$

$0.6 T_A + 0.8 T_B = 100$ —— ②

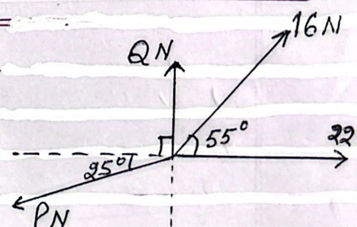
Solving ① and ② $T_A = \underline{76 N}$; $T_B = \underline{68 N}$

(b) $T_B = 0$, Horizontally, $T_A \times 0.8 - F = 0$ ③

Vertically, $T_A \times 0.6 - 10g = 0 \Rightarrow T_A = \frac{500}{3}$ ④

From ③ & ④ $F = \underline{\frac{400}{3}}$

27 Coplanar forces of magnitudes P N, Q N, 16 N and 22 N act at a point in the directions shown in the diagram. The forces are in equilibrium. Find the values of P and Q.



[W-22/41/Q1] --[5]

Solution: Resolving the forces horizontally and vertically:

Horizontally: $22 + 16\cos 55 = P\cos 25 \Rightarrow 0.9063P = 22 + 9.1772$

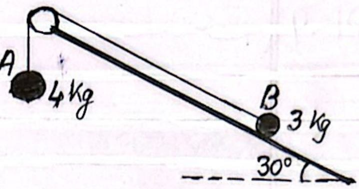
$\Rightarrow P = \frac{31.1772}{0.9063} = \underline{34.4}$

Vertically: $Q + 16\sin 55 = P\sin 25$

$\Rightarrow Q + 16 \times 0.8191 = 34.4 \times 0.4226 \Rightarrow Q = 14.5380 - 13.1056 = 1.43$

$P = \underline{34.4}$ and $Q = 1.43$

28. Figure shows particles A and B, of masses 4 kg and 3 kg respectively, attached to the ends of a light inextensible string that passes over a small smooth pully. The pully is fixed at the top of a plane which is inclined at an angle of 30° to the horizontal. A hangs freely below the pully and B is on the inclined plane. The string is taut and the section of the string between B and the pulley is parallel to a line of greatest slope of the plane. It is given that the plane is rough and the particles are in limiting equilibrium.



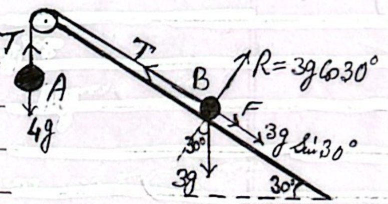
Find the coefficient of friction between B and the plane. ---[6]

W-22/41/Q6(a)

Solution:

At A: $T = 4g$ ----①

At B, The normal reaction $R = 3g\cos 30$ ---②



Resolving the forces along the force:

Force of friction = F

$T = F + 3g\sin 30°$ --- ③

from ① & ③ ⇒ $4g = F + 3g\sin 30° \Rightarrow F = 4g - 3g\sin 30° = 25\,N$ --④ ($g = 10N$)

Now $F = \mu R \Rightarrow \mu = \frac{F}{R} = \frac{25}{3g\cos 30} = \frac{25}{25.98} = 0.962$

from ② & ④

$\Rightarrow \mu = 0.962$ ✓

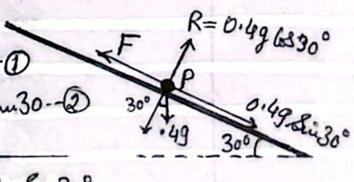
29. A particle P of mass 0.4 kg is in limiting equilibrium on a plane inclined at 30° to the horizontal.

Show that the coefficient of friction between the particle and the plane is $\frac{1}{3}\sqrt{3}$. ---[3]

W-22/42/Q2(a)

Solution: Normal reaction at P, $R = 0.4g\cos 30°$

Force of friction $F = \mu R = \mu \times 0.4g\cos 30$ --①



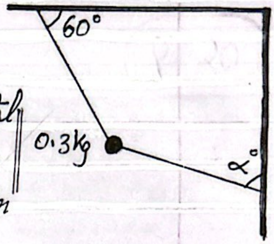
downward force along the plane $= 0.4g\sin 30$ --②

Under limiting equilibrium

for ① & ② $F = \mu \times 0.4g\cos 30° = 0.4g\sin 30°$

$\Rightarrow \mu = \frac{\sin 30°}{\cos 30°} = \frac{1}{2} \Big/ \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}} = \frac{1\sqrt{3}}{3}$ ✓

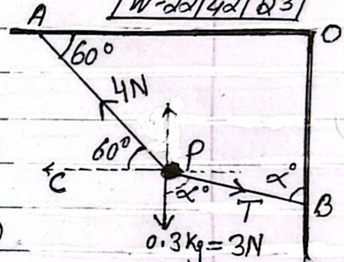
30. A partical of mass 0.3Kg is held at rest by two inextensible strings. One string is attached at an angle of 60° to a horizontal ceiling. The other string is attached at an angle $\alpha°$ to a vertical wall. The tension in the string attached to the ceiling is 4N. Find the tension in the string which is attached to the wall and find the value of α.



---[6]

[W-22/42/Q3]

Solution: Resolving the forces (in equilibrium):



Vertically: $3 + T\cos\alpha - 4\sin 60 = 0$

$\Rightarrow T\cos\alpha = 0.464 \cdots (1)$

Horizontally: $T\sin\alpha - 4\cos 60 = 0$

$\Rightarrow T\sin\alpha = 2 \quad --- (2)$

Sq. and add (1) & (2) $T^2 = (0.464^2 + 2^2) = 4.215$

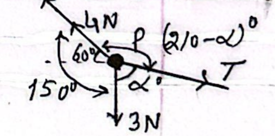
$T = 2.05\text{ N}$ ✓

from (1) & (2):

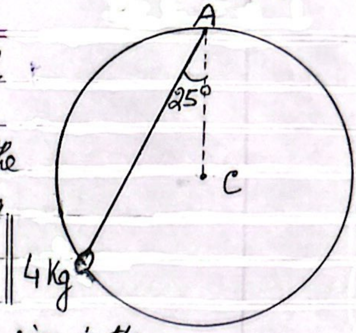
and $\tan\alpha = \dfrac{2}{0.464} = 4.310 \Rightarrow \alpha = \tan^{-1} 4.31 = 76.93°$

$\therefore$ $T = \underline{2.05\text{N}}$ ✓ and $\alpha = \underline{76.93°}$ ✓

Alternatively (Lami's Theo)



31. A ring of mass 4kg is threaded on a smooth circular rigid wire with centre C. The wire is fixed in a vertical plane and the ring is kept at rest by a light string connected to A, the highest point of the circle. The string makes an angle of 25° to the vertical. Find the tension in the string and the magnitude of the normal reaction of the wire on the ring.



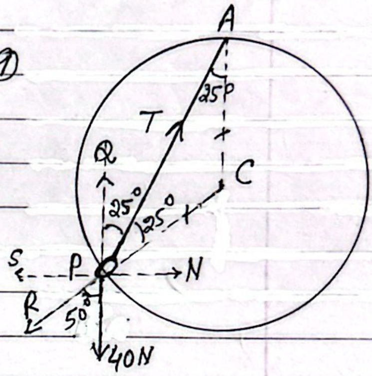
[W-22/43/Q3] -- [6]

Solution: Resolving the forces at P (ring)

Vertically: $T\cos 25° = 40 + R\cos 50°$ --- ①

Horizontally $R\sin 50 = T\sin 25$

$\Rightarrow R = \dfrac{T\sin 25°}{\sin 50°}$ --- ②



from ② & ①

$T.\cos 25 = 40 + T\dfrac{\sin 25}{\sin 50°} \times \cos 50°$

$\Rightarrow 0.9063T = 40 + T \times 0.3546$

$T(0.9063 - 0.3546) = 40$

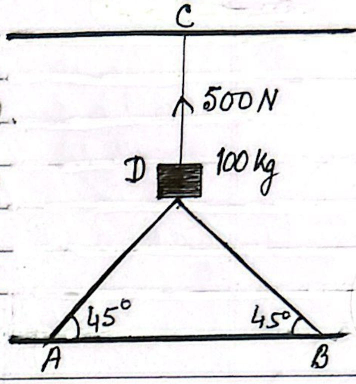
$T = \dfrac{40}{0.55168} = 72.5\,N$ ✓

from ② $R = \dfrac{\sin 25}{\sin 50} \times T = \dfrac{\sin 25°}{\sin 50} \times 72.5 = 39.90$

$= 40N$ ✓

Hence $T = \underline{72.5\,N}$ ✓ and $R = \underline{40\,N}$ ✓

32. The diagram shows a block D of mass 100 Kg supported by two sloping struts AD and BD, each attached at an angle of 45° to fixed points A and B respectively on a horizonal floor. The block is also held in place by a vertical rope CD attached to a fixed point C on a horizontal ceiling. The tension in the rope CD is 500 N and the block rests in equilibrium.



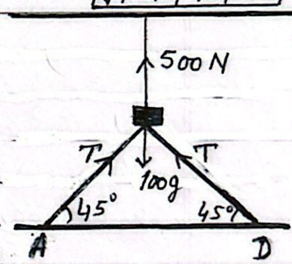
(a) Find the magnitude of the force in each of the struts AD and BD. ---[3]

A horizontal force of magnitude F N is applied to the block in a direction parallel to AB.

(b) Find the value of F for which the magnitude of the force in the strut AD is zero. ---[3]

[M-23/42/Q5]

Solution (a) Resolving vertically:



$500 + T_A \cos 45° + T_B \cos 45° - 100g = 0$

$[T_A = T_B = T]$

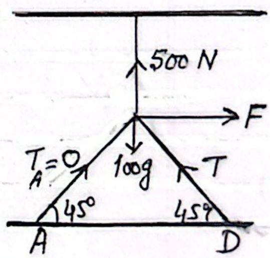
$500 + 2 \times \frac{1}{\sqrt{2}} T - 1000 = 0$

$\Rightarrow \sqrt{2}\,T = 500 \Rightarrow T = \frac{500}{\sqrt{2}}$ --- ①

$= \underline{354\,N}$

(b)

Resolving vertically:



$T_B \cos 45 + 500 - 100g = 0$ --- ②

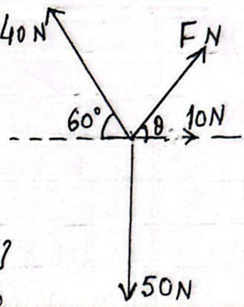
Resolving horizontally: $F - T_B \sin 45° = 0$ --- ③

$\Rightarrow F - \frac{T_B}{\sqrt{2}} = 0$ { from ② $\frac{T_B}{\sqrt{2}} = 500$

$\Rightarrow F - 500 = 0$

$\Rightarrow \underline{F = 500\,N}$

33. Four coplanar forces act at a point. The magnitude of the forces are F N, 10 N, 50 N and 40 N. The direction of the forces are as shown in the diagram.



(a) Given that the forces are in equilibrium, find the value of F and the value of θ. ---[6]

(b) Given instead that $F = 10\sqrt{2}$ and $\theta = 45$, find the direction and the exact magnitude of the resultant force. ---[3]

[S-23/41/Q5]

Solution:
(a) Resolving vertically: $F\sin\theta + 40\sin 60 - 50 = 0$ --- ①

Resolving Horizontally: $F\cos\theta + 10 - 40\cos 60 = 0$ --- ②

from ① $F\sin\theta = 50 - 40\cdot\frac{\sqrt{3}}{2} = 50 - 20\sqrt{3}$ --- ③

fr ② $F\cos\theta = 40\times\frac{1}{2} - 10 = 10$ --- ④

fr ③ & ④ $\tan\theta = \frac{50 - 20\sqrt{3}}{10} = 5 - 2\sqrt{3} \Rightarrow \theta = \tan^{-1}(5 - 2\sqrt{3}) = \underline{56.9^\circ}$ ✓

Square and add ③ & ④ $F = \sqrt{(5-2\sqrt{3})^2 + 10^2} = \sqrt{(15.358)^2 + 10^2}$

$F = \underline{18.3\ N}$ ✓

(b) Now $F = 10\sqrt{2}$ and $\theta = 45$

fm ① Vertical component $R_y = 10\sqrt{2}\sin 45^\circ + 40\sin 60 - 50 = (20\sqrt{3} - 40) = -(40 - 20\sqrt{3})$ --- ⑤

Horizontal Component $R_x = 10\sqrt{2}\cos 45 + 10 - 40\cos 60^\circ = 0$ --- ⑥

sq. and add ⑤ & ⑥ $R = \sqrt{(20\sqrt{3} - 40)^2 + 0^2} = \underline{(40 - 20\sqrt{3})}$ ✓

and the <u>same direction as 50 N</u>. (as X-component is zero)

$R = -(40 - 20\sqrt{3})$ ↓

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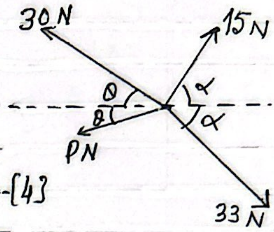
34. Coplanar forces of magnitudes 30 N, 15 N, 33 N and P N act at a point in the directions shown in the diagram, where $\tan\alpha = \frac{4}{3}$. The system is in equilibrium.

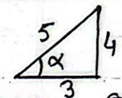


(a) Show that $\left(\frac{14.4}{30-P}\right)^2 + \left(\frac{28.8}{P+30}\right)^2 = 1$ ---[4]

(b) Verify that $P = 6$, satisfies this equation and find the value of θ. ---[2]

S-23/42/Q3

Solution: $\tan\alpha = \frac{4}{3} \Rightarrow \cos\alpha = 3/5$ and $\sin\alpha = \frac{4}{5}$



(a) Resolving along horizontal: $33\cos\alpha + 15\cos\alpha = P\cos\theta + 30\cos\theta$

$\Rightarrow (33+15)\times\frac{3}{5} = (P+30)\cos\theta \Rightarrow \cos\theta = \left(\frac{28.8}{P+30}\right)$ --- ①

Resolving vertically: $15\sin\alpha + 30\sin\theta = 33\sin\alpha + P\sin\theta$

$\Rightarrow (30-P)\sin\theta = (33-15)\times\frac{4}{5} \Rightarrow \sin\theta = \frac{14.4}{(30-P)}$ --- ②

we know $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \left(\frac{14.4}{30-P}\right)^2 + \left(\frac{28.8}{P+30}\right)^2 = 1$ --- ③ (from ① & ②)

(b) $P = 6$, put in ③

$\left(\frac{14.4}{30-6}\right)^2 + \left(\frac{28.8}{6+30}\right)^2 = 1 \Rightarrow \left(\frac{14.4}{24}\right)^2 + \left(\frac{28.8}{36}\right)^2 = 1 \Rightarrow$

$\Rightarrow \left(\frac{144}{240}\right)^2 + \left(\frac{288}{360}\right)^2 = 1 \Rightarrow \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = 1$

$\Rightarrow \frac{9}{25} + \frac{16}{25} = 1$

is True.

from ① $\cos\theta = \frac{28.8}{P+30} = \frac{28.8}{6+30}$ (from $P = 6$)

$= \frac{28.8}{36} = 0.8$

$\Rightarrow \theta = \cos^{-1}(0.8)$

$= 36.9°$

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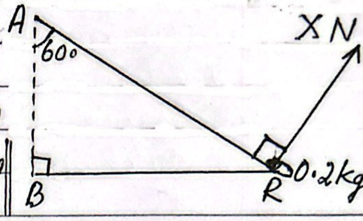
35 A smooth ring R of mass 0.2 kg is threaded on a light string ARB. The ends of the strings are attached to fixed points A and B with A vertically above B. The string is taut and angle ABR = 90°. The angle between the part AR of the string and the vertical is 60°. The ring is held in equilibrium by a force of magnitude X N, acting on the ring in a direction perpendicular to AR.
Calculate the tension in the string and the value of X. ---[5]



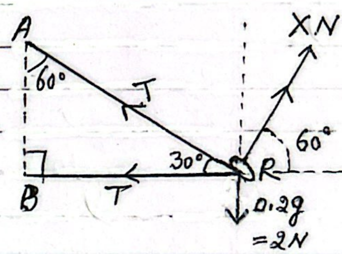
S-23/43/Q3

Solution: Resolving the forces vertically:

$X \sin 60° + T \sin 30° - 0.2g = 0$

$\Rightarrow \frac{\sqrt{3}}{2} X + \frac{1}{2} T = 2$

$\Rightarrow \sqrt{3}X + T = 4$ --- (1)



Resolving horizonally:

$X \cos 60 - T - T \cos 30 = 0$

$\frac{1}{2}X - \left(1 + \frac{\sqrt{3}}{2}\right)T = 0$ --- (2)

$\Rightarrow \frac{1}{2}X - \left(1 + \frac{\sqrt{3}}{2}\right)(4 - \sqrt{3}X) = 0$ (fm (1) $T = (4 - \sqrt{3}X)$)

$\Rightarrow \frac{1}{2}X - 4 - 2\sqrt{3} + \sqrt{3}X + \frac{3}{2}X = 0$

$\Rightarrow 2X + \sqrt{3}X = 4 + 2\sqrt{3} \Rightarrow X \cdot (2 + \sqrt{3}) = 2(2 + \sqrt{3})$

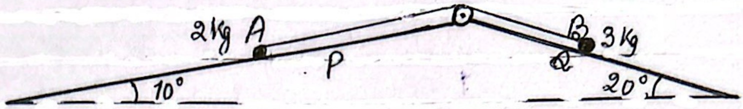
$\Rightarrow X = 2$

Put $X = 2$ in (1) $\sqrt{3} \times 2 + T = 4$

$\Rightarrow T = 4 - 2\sqrt{3}$

$T = 0.536\,N$

37

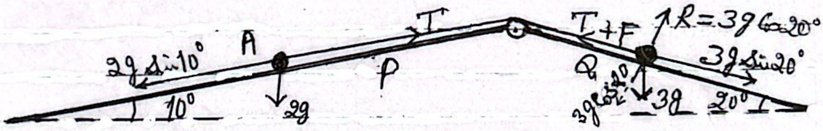


As shown in the diagram, particles A and B of masses 2 kg and 3 kg respectively are attached to the ends of a light inextensible string. The string passes over a small fixed smooth pully which is attached to the top of two inclined planes. Particle A is on plane P, which is inclined at an angle of 10° to the horizontal. Particle Q is on plane Q, which is inclined at an angle of 20° to the horizontal. The string is taut, and the two parts of the string are parallel to lines of greatest slope of their respective planes.

It is given that plane P is smooth, plane Q is rough, and the particles are in limiting equilibrium. Find the coefficient of friction between particle B and plane Q. --[5]

[W-20/43/Q7(a)]

Solution:



for particle A, along the the plane P, $T = 2g\sin 10°$ —— ①

for particle B, along the plane Q, $T+F = 3g\sin 20°$ —— ② (F is the force of friction)

for particle B, perp. to the plane Q $R = 30\cos 20 = 28.19$ —— ③

from ① and ② $2g\sin 10° + F = 3g\sin 20°$

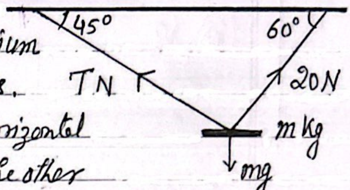
$F = 3g\sin 20 - 2g\sin 10° = 6.787$ —— ④

Now $F = \mu R$

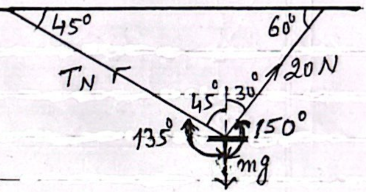
$\Rightarrow \mu = \dfrac{F}{R} = \dfrac{6.787}{28.19} = 0.2407$ (from ③ & ④)

$\therefore \mu = \underline{0.241}$ ✓

36. A block of mass m kg is held in equilibrium below a horizontal ceiling by two strings. One of the strings is inclined at 45° to the horizontal and the tension in this string is T N. The other string is inclined at 60° to the horizontal and the tension in this string is 20 N. Find T and m. [W-20/42/Q3] --[5]



Solution: Using Lami's Theorem:

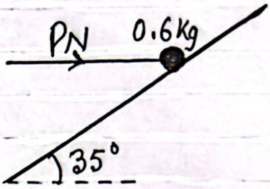


$$\frac{T}{\sin 150^\circ} = \frac{mg}{\sin 75^\circ} = \frac{20}{\sin 135^\circ}$$

$$\Rightarrow \frac{T}{\frac{1}{2}} = \frac{10m}{0.966} = \frac{20}{\frac{1}{\sqrt{2}}}$$

$\Rightarrow T = 10\sqrt{2} = \underline{14.1\ N}$ ✓ and $\underline{m = 2.73}$ ✓

38 A particle of mass 0.6 Kg is placed on a rough plane which is inclined at an angle of 35° to the horizontal. The particle is kept in equilibrium by a horizontal force of magnitude P N acting in a vertical plane containing line of greatest slope. The coefficient of friction between the particle and plane is 0.4.

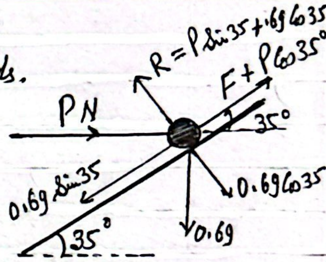


Find the least possible value of P. ---[6]

S-23/42/Q5

Solution: Force of friction $F = \mu R = 0.4R$ ---①

Particle is trying to move downward → F is upwards.



Forces, perpendicular to the Plane,

$$R = P\sin 35° + 0.6g\cos 35°$$

$$\Rightarrow R = (0.573)P + 4.914$$

$$\therefore F = \mu R = 0.4[(0.573)P + 4.914]$$

$$F = 0.229P + 1.966 \quad ---②$$

Resolving along the plane;

$$F + P\cos 35° = 0.6g\sin 35°$$

$$[(0.229)P + 1.966] + (0.819)P = 3.44 \quad \text{(from ②)}$$

$$\Rightarrow (1.048)P = 1.48$$

$$\Rightarrow P = \frac{1.48}{1.048} = 1.41$$

$$\therefore P = \underline{1.41}$$