MECHANICS

9709

(March, June and November series 2020 – 2023 With marking scheme)

Forces and Equilibrium

Exercise - 1

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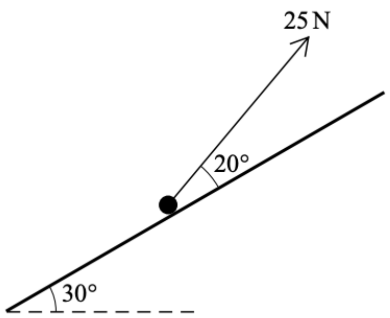
NOIDA

1. A particle of mass 20 kg is on a rough plane inclined at an angle of 30° to the horizontal. A force of magnitude 25 N, acting at an angle of 20° above a line of greatest slope of the plane, is used to prevent the particle from sliding down the plane. The coefficient of friction between the particle and the plane is μ.

(a) Complete the diagram below to show all the forces acting on the particle. [1]



(b) Find the least possible value of μ. [5]

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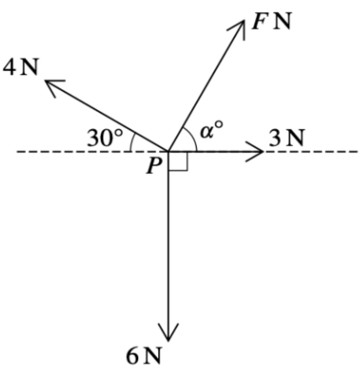
2.



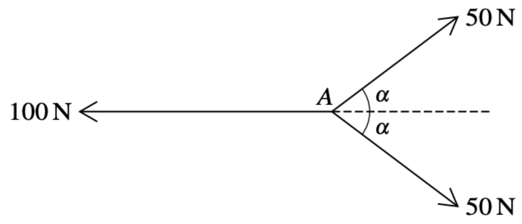
Coplanar forces, of magnitudes F N, 3 N, 6 N and 4 N, act at a point P, as shown in the diagram.

(a) Given that $\alpha = 60$, and that the resultant of the four forces is in the direction of the 3 N force, find F. [3]

(b) Given instead that the four forces are in equilibrium, find the values of F and α. [5]

Cambridge International AS & A Level Mathematics 9709 Paper 42 Q5 March 2020

3.



Three coplanar forces of magnitudes 100 N, 50 N and 50 N act at a point A, as shown in the diagram. The value of $\cos\alpha$ is $\frac{4}{5}$.

Find the magnitude of the resultant of the three forces and state its direction. [3]

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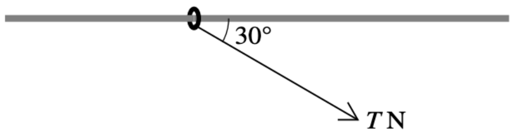
4.



The diagram shows a ring of mass 0.1 kg threaded on a fixed horizontal rod. The rod is rough and the coefficient of friction between the ring and the rod is 0.8. A force of magnitude T N acts on the ring in a direction at 30° to the rod, downwards in the vertical plane containing the rod. Initially the ring is at rest.

(a) Find the greatest value of T for which the ring remains at rest. [4]

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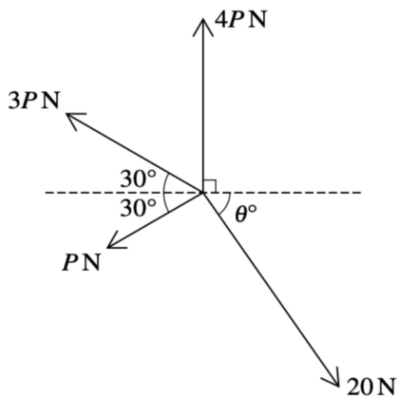
5.



Coplanar forces of magnitudes 20 N, P N, $3P$ N and $4P$ N act at a point in the directions shown in the diagram. The system is in equilibrium.

Find P and θ. [6]

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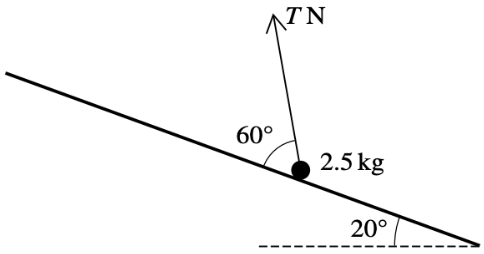
6.



A particle of mass 2.5 kg is held in equilibrium on a rough plane inclined at 20° to the horizontal by a force of magnitude T N making an angle of 60° with a line of greatest slope of the plane (see diagram). The coefficient of friction between the particle and the plane is 0.3.

Find the greatest and least possible values of T. [8]

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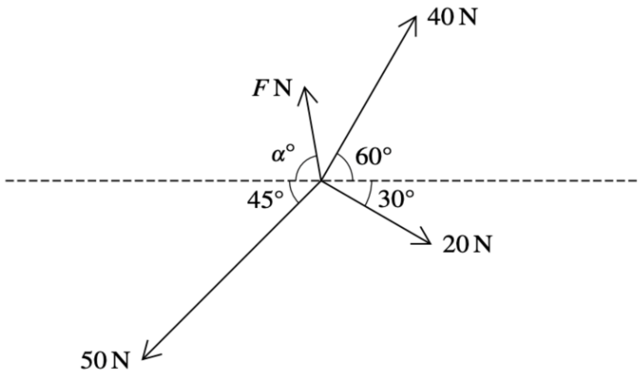
7.



Four coplanar forces of magnitudes 40 N, 20 N, 50 N and F N act at a point in the directions shown in the diagram. The four forces are in equilibrium.

Find F and α. [6]

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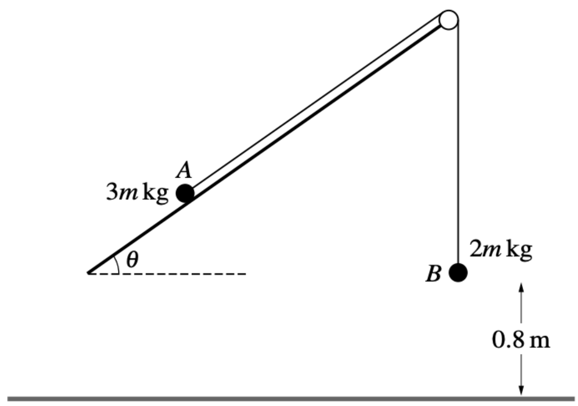
8.



Two particles A and B, of masses $3m$ kg and $2m$ kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley which is attached to the edge of a plane. The plane is inclined at an angle θ to the horizontal. A lies on the plane and B hangs vertically, 0.8 m above the floor, which is horizontal. The string between A and the pulley is parallel to a line of greatest slope of the plane (see diagram). Initially A and B are at rest.

(a) Given that the plane is smooth, find the value of θ for which A remains at rest. [3]

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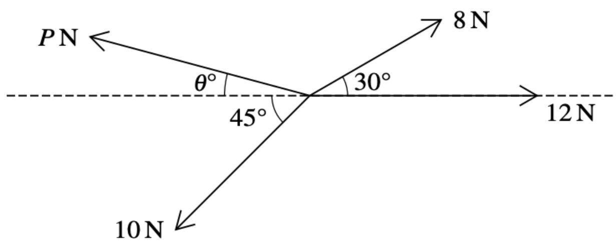
9.



Coplanar forces of magnitudes 8 N, 12 N, 10 N and P N act at a point in the directions shown in the diagram. The system is in equilibrium.

Find P and θ. [6]

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10. A block of mass 5 kg is placed on a plane inclined at 30° to the horizontal. The coefficient of friction between the block and the plane is μ.

(a)

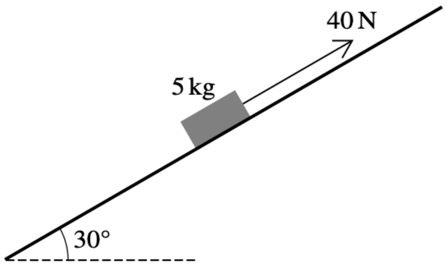

Fig. 6.1

When a force of magnitude 40 N is applied to the block, acting up the plane parallel to a line of greatest slope, the block begins to slide up the plane (see Fig. 6.1).

Show that $\mu < \frac{1}{5}\sqrt{3}$. [4]

(b)

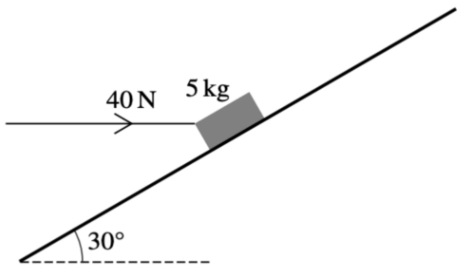

Fig. 6.2

When a force of magnitude 40 N is applied horizontally, in a vertical plane containing a line of greatest slope, the block does not move (see Fig. 6.2).

Show that, correct to 3 decimal places, the least possible value of μ is 0.152. [4]

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11. A string is attached to a block of mass 4 kg which rests in limiting equilibrium on a rough horizontal table. The string makes an angle of 24° above the horizontal and the tension in the string is 30 N.

(a) Draw a diagram showing all the forces acting on the block. [1]

(b) Find the coefficient of friction between the block and the table. [5]

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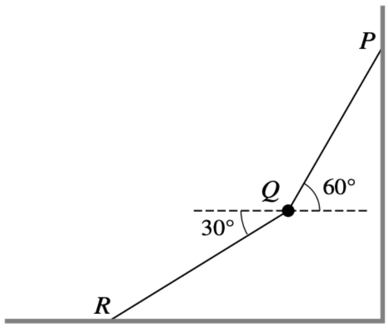
12.



A particle Q of mass 0.2 kg is held in equilibrium by two light inextensible strings PQ and QR. P is a fixed point on a vertical wall and R is a fixed point on a horizontal floor. The angles which strings PQ and QR make with the horizontal are 60° and 30° respectively (see diagram).

Find the tensions in the two strings. [5]

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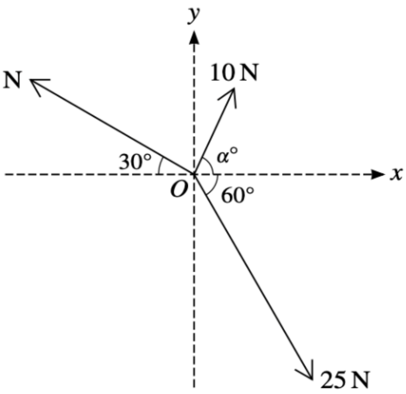
13.



Three coplanar forces of magnitudes 10 N, 25 N and 20 N act at a point O in the directions shown in the diagram.

(a) Given that the component of the resultant force in the x-direction is zero, find α, and hence find the magnitude of the resultant force. [4]

(b) Given instead that $\alpha = 45$, find the magnitude and direction of the resultant of the three forces. [5]

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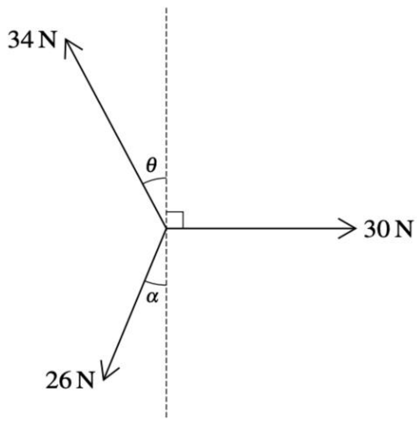
14.



Coplanar forces of magnitudes 34 N, 30 N and 26 N act at a point in the directions shown in the diagram.

Given that $\sin\alpha = \frac{5}{13}$ and $\sin\theta = \frac{8}{17}$, find the magnitude and direction of the resultant of the three forces. [6]

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15. A particle of mass 12 kg is stationary on a rough plane inclined at an angle of 25° to the horizontal. A pulling force of magnitude P N acts at an angle of 8° above a line of greatest slope of the plane. This force is used to keep the particle in equilibrium. The coefficient of friction between the particle and the plane is 0.3.

Find the greatest possible value of P. [6]

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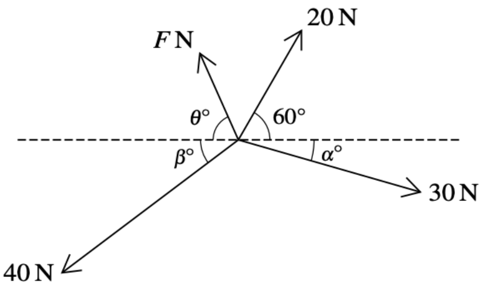
16.



Four coplanar forces act at a point. The magnitudes of the forces are 20 N, 30 N, 40 N and F N. The directions of the forces are as shown in the diagram, where $\sin \alpha° = 0.28$ and $\sin \beta° = 0.6$.

Given that the forces are in equilibrium, find F and θ. [6]

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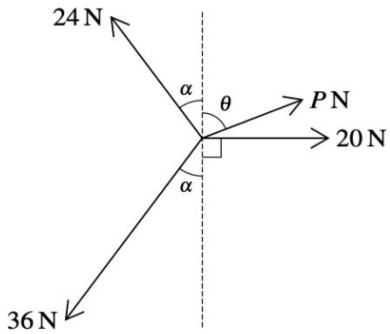
17.



Coplanar forces of magnitudes 24 N, P N, 20 N and 36 N act at a point in the directions shown in the diagram. The system is in equilibrium.

Given that $\sin \alpha = \frac{3}{5}$, find the values of P and θ. [6]

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18. A particle of mass 12 kg is stationary on a rough plane inclined at an angle of 25° to the horizontal. A force of magnitude P N acting parallel to a line of greatest slope of the plane is used to prevent the particle sliding down the plane. The coefficient of friction between the particle and the plane is 0.35.

(a) Draw a sketch showing the forces acting on the particle. [1]

(b) Find the least possible value of P. [5]

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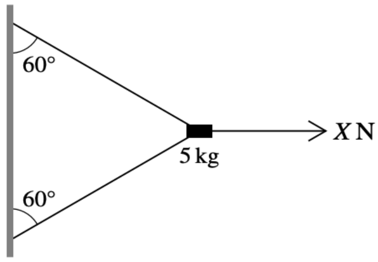
19.



A block of mass 5 kg is held in equilibrium near a vertical wall by two light strings and a horizontal force of magnitude X N, as shown in the diagram. The two strings are both inclined at 60° to the vertical.

(a) Given that $X = 100$, find the tension in the lower string. [4]

(b) Find the least value of X for which the block remains in equilibrium in the position shown. [4]

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20. A particle of mass 8 kg is suspended in equilibrium by two light inextensible strings which make angles of 60° and 45° above the horizontal.

(a) Draw a diagram showing the forces acting on the particle. [1]

(b) Find the tensions in the strings. [6]

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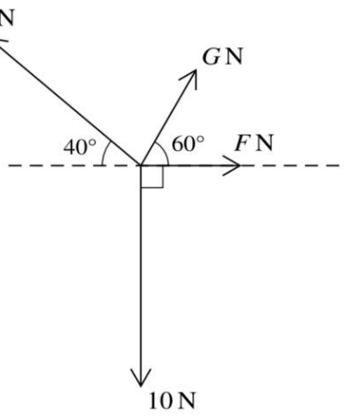
21.



Four coplanar forces act at a point. The magnitudes of the forces are 10 N, F N, G N and $2F$ N. The directions of the forces are as shown in the diagram.

(a) Given that the forces are in equilibrium, find the values of F and G. [5]

(b) Given instead that $F = 3$, find the value of G for which the resultant of the forces is perpendicular to the 10 N force. [2]

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22. A crate of mass 300 kg is at rest on rough horizontal ground. The coefficient of friction between the crate and the ground is 0.5. A force of magnitude X N, acting at an angle α above the horizontal, is applied to the crate, where $\sin\alpha = 0.28$.

Find the greatest value of X for which the crate remains at rest. [5]

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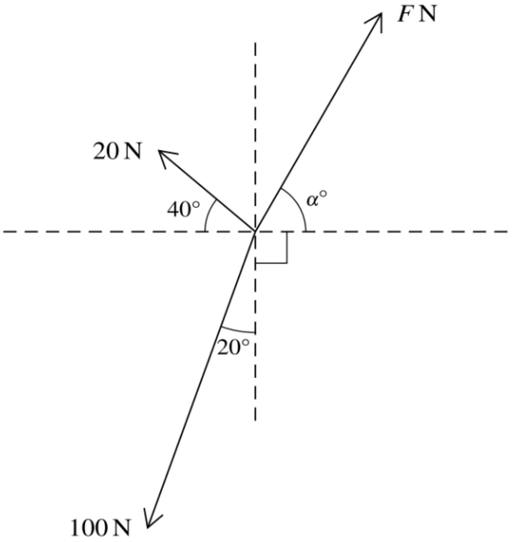
23.



Three coplanar forces of magnitudes 20 N, 100 N and F N act at a point. The directions of these forces are shown in the diagram.

Given that the three forces are in equilibrium, find F and α. [6]

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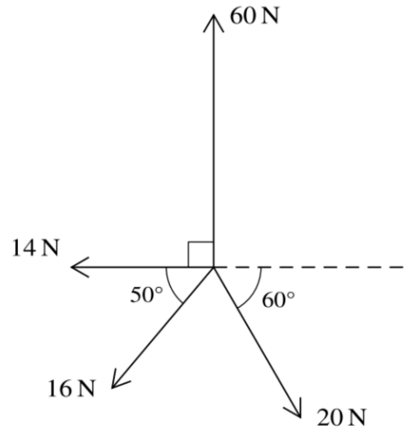
24.



Coplanar forces of magnitudes 60 N, 20 N, 16 N and 14 N act at a point in the directions shown in the diagram.

Find the magnitude and direction of the resultant force. [6]

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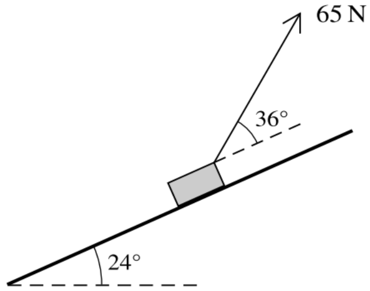
25.



A block of mass 12 kg is placed on a plane which is inclined at an angle of 24° to the horizontal. A light string, making an angle of 36° above a line of greatest slope, is attached to the block. The tension in the string is 65 N (see diagram). The coefficient of friction between the block and plane is μ. The block is in limiting equilibrium and is on the point of sliding up the plane.

Find μ. [6]

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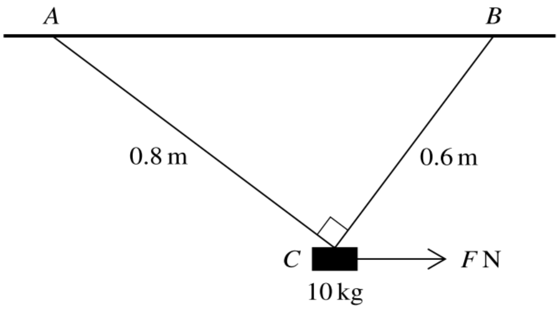
26.



The diagram shows a block of mass 10 kg suspended below a horizontal ceiling by two strings AC and BC, of lengths 0.8 m and 0.6 m respectively, attached to fixed points on the ceiling. Angle $ACB = 90°$. There is a horizontal force of magnitude F N acting on the block. The block is in equilibrium.

(a) In the case where $F = 20$, find the tensions in each of the strings. [5]

(b) Find the greatest value of F for which the block remains in equilibrium in the position shown. [3]

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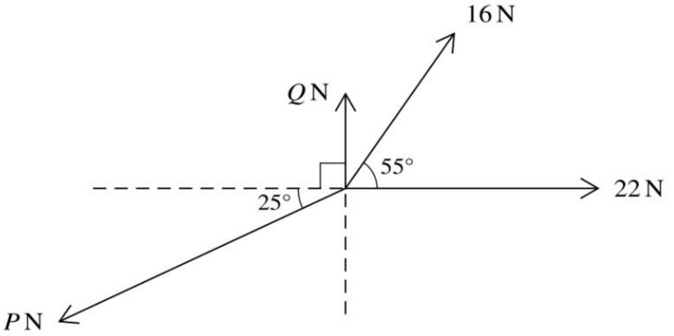
27.



Coplanar forces of magnitudes P N, Q N, 16 N and 22 N act at a point in the directions shown in the diagram. The forces are in equilibrium.

Find the values of P and Q. [5]

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28.

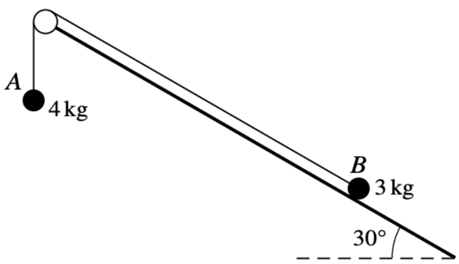

Fig. 6.1

Fig. 6.1 shows particles A and B, of masses 4 kg and 3 kg respectively, attached to the ends of a light inextensible string that passes over a small smooth pulley. The pulley is fixed at the top of a plane which is inclined at an angle of 30° to the horizontal. A hangs freely below the pulley and B is on the inclined plane. The string is taut and the section of the string between B and the pulley is parallel to a line of greatest slope of the plane.

(a) It is given that the plane is rough and the particles are in limiting equilibrium.

Find the coefficient of friction between B and the plane. [6]

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29. A particle P of mass 0.4 kg is in limiting equilibrium on a plane inclined at 30° to the horizontal.

(a) Show that the coefficient of friction between the particle and the plane is $\frac{1}{3}\sqrt{3}$. [3]

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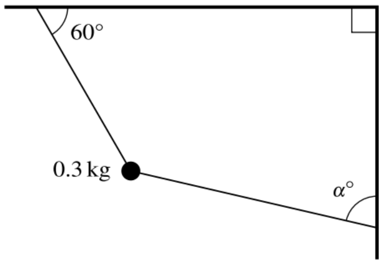
30.



A particle of mass 0.3 kg is held at rest by two light inextensible strings. One string is attached at an angle of 60° to a horizontal ceiling. The other string is attached at an angle $\alpha°$ to a vertical wall (see diagram). The tension in the string attached to the ceiling is 4 N.

Find the tension in the string which is attached to the wall and find the value of α. [6]

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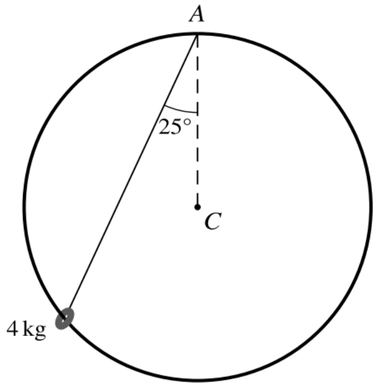
31.



A ring of mass 4 kg is threaded on a smooth circular rigid wire with centre C. The wire is fixed in a vertical plane and the ring is kept at rest by a light string connected to A, the highest point of the circle. The string makes an angle of 25° to the vertical (see diagram).

Find the tension in the string and the magnitude of the normal reaction of the wire on the ring. [6]

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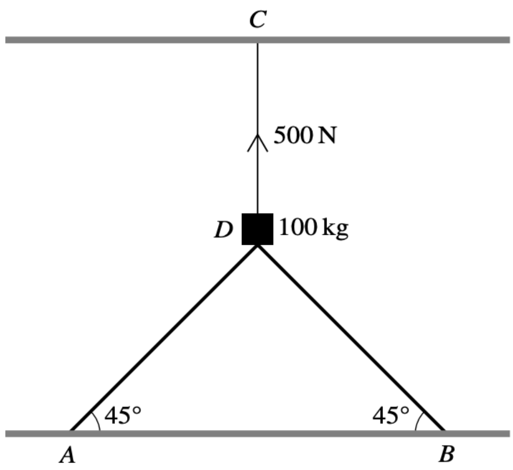
32.



The diagram shows a block D of mass 100 kg supported by two sloping struts AD and BD, each attached at an angle of 45° to fixed points A and B respectively on a horizontal floor. The block is also held in place by a vertical rope CD attached to a fixed point C on a horizontal ceiling. The tension in the rope CD is 500 N and the block rests in equilibrium.

(a) Find the magnitude of the force in each of the struts AD and BD. [3]

A horizontal force of magnitude F N is applied to the block in a direction parallel to AB.

(b) Find the value of F for which the magnitude of the force in the strut AD is zero. [3]

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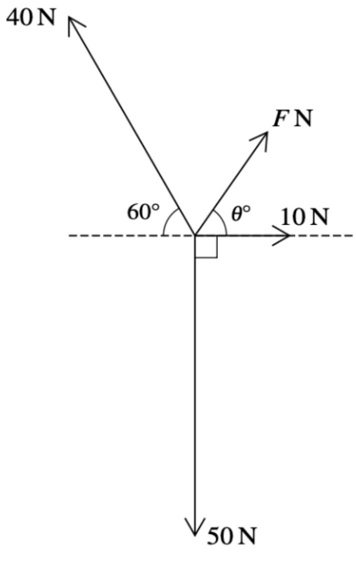
33.



Four coplanar forces act at a point. The magnitudes of the forces are F N, 10 N, 50 N and 40 N. The directions of the forces are as shown in the diagram.

(a) Given that the forces are in equilibrium, find the value of F and the value of θ. [6]

(b) Given instead that $F = 10\sqrt{2}$ and $\theta = 45$, find the direction and the exact magnitude the resultant force. [3]

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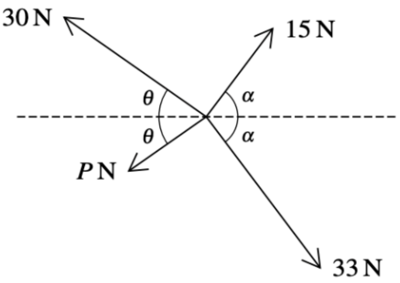
34.



Coplanar forces of magnitudes 30 N, 15 N, 33 N and P N act at a point in the directions shown in the diagram, where $\tan\alpha = \frac{4}{3}$. The system is in equilibrium.

(a) Show that $\left(\frac{14.4}{30 - P}\right)^2 + \left(\frac{28.8}{P + 30}\right)^2 = 1$. [4]

(b) Verify that $P = 6$ satisfies this equation and find the value of θ. [2]

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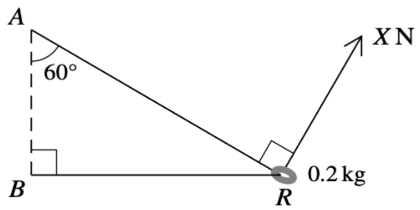
35.



A smooth ring R of mass 0.2 kg is threaded on a light string ARB. The ends of the string are attached to fixed points A and B with A vertically above B. The string is taut and angle $ABR = 90°$. The angle between the part AR of the string and the vertical is 60°. The ring is held in equilibrium by a force of magnitude X N, acting on the ring in a direction perpendicular to AR (see diagram).

Calculate the tension in the string and the value of X. [5]

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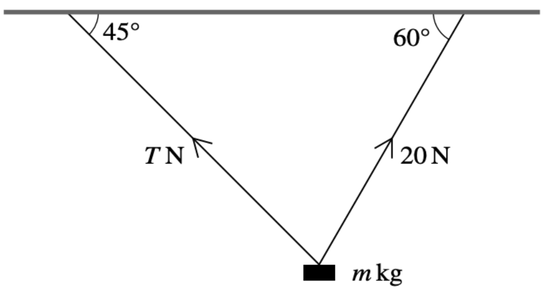
36.



A block of mass m kg is held in equilibrium below a horizontal ceiling by two strings, as shown in the diagram. One of the strings is inclined at 45° to the horizontal and the tension in this string is T N. The other string is inclined at 60° to the horizontal and the tension in this string is 20 N.

Find T and m. [5]

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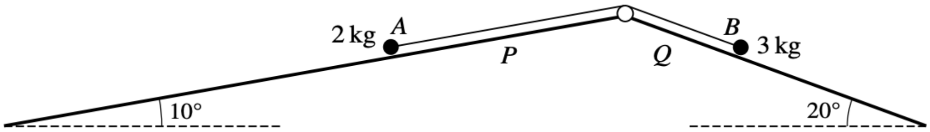
37.



As shown in the diagram, particles A and B of masses 2 kg and 3 kg respectively are attached to the ends of a light inextensible string. The string passes over a small fixed smooth pulley which is attached to the top of two inclined planes. Particle A is on plane P, which is inclined at an angle of 10° to the horizontal. Particle B is on plane Q, which is inclined at an angle of 20° to the horizontal. The string is taut, and the two parts of the string are parallel to lines of greatest slope of their respective planes.

(a) It is given that plane P is smooth, plane Q is rough, and the particles are in limiting equilibrium.

Find the coefficient of friction between particle B and plane Q. [5]

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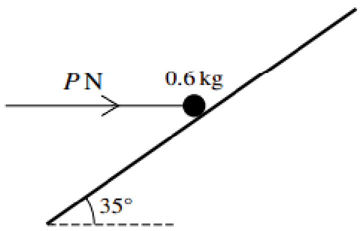
38.

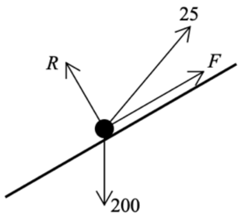


A particle of mass 0.6 kg is placed on a rough plane which is inclined at an angle of 35° to the horizontal. The particle is kept in equilibrium by a horizontal force of magnitude P N acting in a vertical plane containing a line of greatest slope (see diagram). The coefficient of friction between the particle and plane is 0.4.

Find the least possible value of P. [6]

Mark Scheme

1. **(b)** Find the time that it takes for the blocks to reach a speed of $1.2\,\mathrm{m\,s^{-1}}$ from rest. [2]

Answer	Marks	Partial Marks	Guidance
Correct force diagram with 3 extra forces shown 	**1**	**B1**	Accept 200 or 20g for weight
For resolving forces in the direction parallel to and/or perpendicular to the plane	1	**M1**	
$F + 25\cos 20° = 20 \times g \times \sin 30°$	1	**A1**	
$R + 25\sin 20° = 20 \times g \times \cos 30°$	1	**A1**	
$[F = 76.5...]$ $[R = 164.65...]$ $\mu = \frac{\text{their } (76.5...)}{\text{their } (164.65...)}$	1	**M1**	Use $F = \mu R$ to evaluate μ
$= 0.465$	1	**A1**	
	5		

2.

Answer	Mark	Guidance
$[4 \sin 30 + F \sin 60 - 6 = 0]$	**M1**	Resolve forces vertically and equate to zero
Correct equation	**A1**	
$F = 4.62$	**A1**	Allow $F = \frac{8}{\sqrt{3}}$ or $F = \frac{8}{3}\sqrt{3}$
	3	

Answer	Marks	Guidance
Resolve forces either vertically or horizontally	**M1**	
$F \sin \alpha + 4 \sin 30 - 6 = 0$ and $F \cos \alpha + 3 - 4 \cos 30 = 0$	**A1**	Both equations correct $[F \sin \alpha = 4]$ $[F \cos \alpha = 0.464102...]$
$[F^2 = 4^2 + 0.464^2]$ or $\left[F = \frac{4}{\sin 83.4} = \frac{0.464}{\cos 83.4}\right]$	**M1**	Attempt to solve for F using Pythagoras or from a value found for α
$\left[\alpha = \tan^{-1}\left(\frac{4}{0.464}\right)\right]$ or $\left[\alpha = \sin^{-1}\left(\frac{4}{4.03}\right) = \cos^{-1}\left(\frac{0.464}{4.03}\right)\right]$	**M1**	Attempt to solve for α using trigonometry or from a value found for F
$F = 4.03$ and $\alpha = 83.4$	**A1**	Both correct as shown $[F = 4.0268…, \alpha = 83.382…]$
	5	

3.

Answer	Marks
Resultant = $100 - 2 \times 50\cos \alpha$	**M1**
20 N	**A1**
Direction is to the left (or equivalent)	**B1**
	3

4.

Answer	Marks
Resolving forces in either direction	M1
$R = T\sin 30 + 0.1g,\ F = T\cos 30$	A1
$T\cos 30 = 0.8\,(T\sin 30 + 0.1g)$	M1
$T = 1.72\ (1.7166...)$	A1
	4

5.

Answer	Marks
Resolving forces in either direction	M1
$20\cos\theta = 4P\cos 30$	A1
$4P + 2P\sin 30 = 20\sin\theta$	A1
$\cos\theta = \frac{\sqrt{3}}{10}P$ $\sin\theta = \frac{P}{4}$ $\frac{3}{100}P^2 + \frac{1}{16}P^2 = 1$	M1
$P = 3.29$	A1
$\theta = 55.3$	A1
	6

6.

Answer	Marks
$T\sin 60 + R = 25\cos 20$	B1
Attempt at resolving in any direction	M1
$T\cos 60 = F + 25\sin 20$	A1
$T\cos 60 + F = 25\sin 20$	A1
Use of $F = \mu R$	M1
$T\cos 60 = 25\sin 20 \pm 0.3(25\cos 20 - T\sin 60)$ $T = \frac{25\sin 20 \pm 0.3\times 25\cos 20}{\cos 60 \pm 0.3\sin 60}$	M1
$T = 6.26$	A1
$T = 20.5$	A1
	8

7.

Answer	Marks
Attempt to resolve, either direction with correct number of terms	M1
$F\cos\alpha = 40\sin 30 + 20\sin 60 - 50\sin 45\ (= 1.965...)$	A1
$F\sin\alpha = 50\cos 45 + 20\cos 60 - 40\cos 30\ (= 10.714...)$	A1
Method for either F or α	M1
$F = \sqrt{\left((1.965...)^2 + (10.714...)^2\right)} = 10.9(10.893)$	A1
$\alpha = \tan^{-1}(10.714... / 1.965...) = 79.6\ (79.606...)$	A1
	6

8.

Answer	Marks
$T - 2mg = 0$	**B1**
$3mg \sin\theta - T = 0$ (**M1** for resolving forces parallel to the plane and solving for θ)	**M1**
$\theta = 41.8$ (41.810...)	**A1**
	3

9.

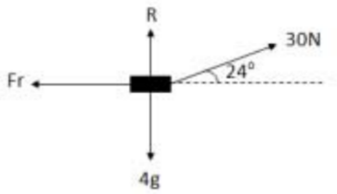
Answer	Marks	Guidance
Resolve forces either horizontally or vertically	**M1**	Correct number of relevant terms
$P\cos\theta = 12 + 8\cos 30 - 10\cos 45$ [= 11.857]	**A1**	
$P\sin\theta = 10\sin 45 - 8\sin 30$ [= 3.071]	**A1**	
$P = \sqrt{(11.857^2 + 3.071^2)}$	**M1**	OE. Use of correct method for finding P
$\theta = \tan^{-1}\left(\frac{3.071}{11.857}\right)$	**M1**	OE. Use of correct method for finding θ
$P = 12.2$ and $\theta = 14.5$	**A1**	Both correct
	6	

10.

Answer	Mark	Guidance
$R = 5g\cos 30$ [$= 25\sqrt{3}$]	**B1**	
$40 - 5g\sin 30 - F > 0$	**M1**	State that the net force up the plane is positive, 3 terms
$F = \mu \times 5g\cos 30$	**M1**	For using $F = \mu R$ with R as a component of $5g$ to obtain an equality/inequality in μ only with 3 terms
$\mu < \frac{1}{5}\sqrt{3}$	**A1**	AG
Alternative scheme for question 6(a)		
$R = 5g\cos 30$ [$= 25\sqrt{3}$]	**B1**	
$40 - 5g\sin 30 - F = 5a$	**M1**	Acceleration $a > 0$
$F = \mu \times 5g\cos 30$ [$40 - 5g\sin 30 - \mu \times 5g\cos 30 = 5a$]	**M1**	For using $F = \mu R$ with R as a component of $5g$ to obtain an equality in μ and a
$\mu < \frac{1}{5}\sqrt{3}$	**A1**	AG. From $\mu = \frac{1}{5}\sqrt{3} = \frac{a}{g}\cos 30$ with $a > 0$
	4	

Answer	Mark	Guidance
Attempt to resolve forces parallel to or perpendicular to the inclined plane, 3 relevant terms in either direction	**M1**	
$R = 5g\cos 30 + 40\sin 30$ [$= 20 + 25\sqrt{3} = 63.3$]	**A1**	
$F = 40\cos 30 - 5g\sin 30$ [$= 20\sqrt{3} - 25 = 9.64$]	**A1**	
$\mu \geqslant 0.152$	**B1**	AG. Using $F \leqslant \mu R$
Alternative method for question 6(b)		
Attempt to resolve forces horizontally or vertically with 3 relevant terms in either direction	**M1**	
$40 = R\sin 30 + F\cos 30$ [$40 = \frac{1}{2}R + \sqrt{3}/2F$]	**A1**	
$5g = R\cos 30 - F\sin 30$ [$5g = \sqrt{3}/2R - \frac{1}{2}F$]	**A1**	
$\mu \geqslant 0.152$	**B1**	AG. Solve for R and F and use $F \leqslant \mu R$

11.

Answer	Marks	Guidance
	**B1**	4 forces, labelled
	1	
For resolving horizontally or vertically	**M1**	
$30 \cos 24 = F$ $(F = 27.406\ldots)$	**A1**	
$R + 30 \cos 24 = 40$ $(R = 27.797\ldots)$	**A1**	
$\mu = \dfrac{30\cos 24}{40 - 30\sin 24}$	**M1**	Using $\mu = F/R$
$\mu = 0.986$ $(0.9859\ldots)$	**A1**	
	5	

12.

Answer	Marks	Guidance
For attempting to resolve forces in either direction.	**M1**	Correct number of relevant terms.
$T_P \cos 60 = T_R \cos 30$	**A1**	
$T_P \sin 60 = T_R \sin 30 + 0.2g$	**A1**	
Attempt to solve simultaneously for either tension.	**M1**	From 2 equations, with correct number of relevant terms.
$T_P = 3.46$ N and $T_R = 2$ N	**A1**	Both correct. Allow $T_P = 2\sqrt{3}$ N.
Alternative method for question 3		
$\dfrac{T_P}{\sin 60} = \dfrac{T_R}{\sin 150} = \dfrac{0.2g}{\sin 150}$	**M1**	Attempt one pair of Lami's equations. Correct angles.
One pair correct	**A1**	
Equations all correct	**A1**	
Solve for T_P or T_R	**M1**	From equations of the correct form.
$T_P = 3.46$ N and $T_R = 2$ N	**A1**	Both correct. Allow $T_P = 2\sqrt{3}$ N
	5	

13.

Answer	Marks	Guidance
$20\cos 30 = 25\cos 60 + 10\cos\alpha$ $[17.32 = 12.5 + 10\cos\alpha, \ \to \cos\alpha = 0.4821]$	M1	For resolving forces horizontally, all relevant terms included
$\alpha = 61.2$	A1	From $\alpha = 61.18$
Resultant $= 20\sin 30 + 10\sin 61.2 - 25\sin 60$ $[= 10 + 8.761 - 21.651]$	M1	For resolving forces vertically, all relevant terms included
Magnitude of resultant force = 2.89 N	A1	A0 for –2.89 N or for ±2.89 N. Allow 2.89 N downwards
	4	
$X = 25\cos 60 + 10\cos 45 - 20\cos 30$ $= 12.5 + 7.07107 - 17.32051 = 2.25056$ $Y = 20\sin 30 + 10\sin 45 - 25\sin 60$ $= 10 + 7.07107 - 21.65064 = -4.57957$	M1	For either horizontal or vertical component, correct number of relevant terms. Allow $\pm X$ and/or $\pm Y$
	A1	For both correct, allow unsimplified
$R = \sqrt{X^2 + Y^2}$	M1	OE. Using a method to find the resultant force, using expressions for *X* and *Y* with at least 5 relevant terms.
$\alpha = \tan^{-1}\dfrac{Y}{X}$	M1	OE. A method to find the direction, using expressions for *X* and *Y* with at least 5 relevant terms.
Resultant = 5.10 N, Direction = 63.8° below positive *x*-axis	A1	For both correct, angle clearly explained. May use a diagram with a correct arrow and arc for angle. Allow angle 296° (measured anticlockwise from +ve *x*-axis)
	5	

14.

Answer	Marks	Guidance
Resolve either horizontally or vertically with correct number of terms.	M1	Allow θ and α as in the question for this mark
$[X =] 30 - 34 \times \frac{8}{17} - 26 \times \frac{5}{13} [= 4]$	A1	Allow $\pm X$ as they may resolve forces left or right Allow $[X =] 30 - 34\sin 28 - 26\sin 23$ angle 2s.f. or better
$[Y =] 34 \times \frac{15}{17} - 26 \times \frac{12}{13} [= 6]$	A1	Allow $\pm Y$ as they may resolve forces up or down Allow $[Y =]$ $34\cos 28 - 26\cos 23$ angle 2s.f. or better
$[R =] \sqrt{X^2 + Y^2}$	M1	Attempt to solve for the magnitude of the force
$[\beta =] \tan^{-1}\left(\frac{Y}{X}\right)$ or $[\beta =] \tan^{-1}\left(\frac{X}{Y}\right)$	M1	Attempt to solve for the direction of the resultant force

Answer	Marks	Guidance
$R = \sqrt{52} = 2\sqrt{13} = 7.21$ N **and** $\beta = 56.3$ above 30N force or anticlockwise from 30N force	A1	Both correct with correct explanation of the direction. Must be a correct and clear explanation.
	6	

15.

Answer	Marks	Guidance
For resolving either parallel to or perpendicular to the plane	M1	Three relevant terms in either equation.
$P\cos 8 = F + 12g\sin 25$	A1	
$12g\cos 25 = R + P\sin 8$	A1	
$F = 0.3R$	M1	Use $F = 0.3R$, where R must involve components of both $12g$ and P.
$P\cos 8 = 0.3(12g\cos 25 - P\sin 8) + 12g\sin 25$	M1	For attempting to solve for P, using equations with the correct number of relevant terms in both.
$P = 80.8$	A1	From $P = 80.755\ldots$ Allow $P \leqslant 80.8$ If more than one case is considered for direction of friction then a choice must be made for final answer.
Alternative mark scheme for Question 4		
For resolving forces either vertically or horizontally	M1	Correct number of terms in either equation.
$R\cos 25 + P\sin 33 = 12g + F\sin 25$	A1	

Answer	Marks	Guidance
$P\cos 33 = F\cos 25 + R\sin 25$	A1	
$F = 0.3R$	M1	Use $F = 0.3R$
Solve a pair of simultaneous equations in P and R May see $R = 97.5$	M1	For attempting to solve for P, using equations with the correct number of relevant terms.
$P = 80.8$	A1	From $P = 80.755\ldots$ Allow $P \leqslant 80.8$ If more than one case is considered for direction of friction then a choice must be made for final answer.
	6	

Answer	Marks	Guidance
$P\cos 33 = F\cos 25 + R\sin 25$	A1	
$F = 0.3R$	M1	Use $F = 0.3R$
Solve a pair of simultaneous equations in P and R May see $R = 97.5$	M1	For attempting to solve for P, using equations with the correct number of relevant terms.
$P = 80.8$	A1	From $P = 80.755\ldots$ Allow $P \leqslant 80.8$ If more than one case is considered for direction of friction then a choice must be made for final answer.
	6	

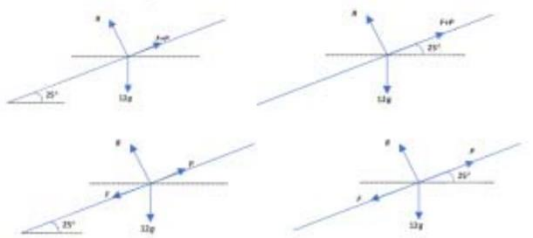
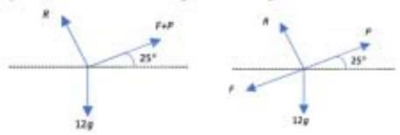
16.

Answer	Marks	Guidance
$F\sin\theta + 20\sin 60 - 30\sin\alpha - 40\sin\beta = 0$	M1	For resolving in either direction
Vertical: $F\sin\theta + 20\sin 60 - 30 \times 0.28 - 40 \times 0.6 = 0$ [$F\sin\theta = 15.07949\ldots$]	A1	
Horizontal: $F\cos\theta + 40 \times 0.8 - 30 \times 0.96 - 20\cos 60 = 0$ [$F\cos\theta = 6.8$]	A1	
$\theta = \tan^{-1}\dfrac{15.0794\ldots}{6.8}$	M1	For method for finding θ
$F = \sqrt{15.07949\ldots^2 + 6.8^2}$	M1	For method for finding F
$\theta = 65.7$, $F = 16.5$	A1	
	6	

17.

Answer	Marks	Guidance
Attempt at resolving in any direction	**M1**	Correct number of terms. No substitution for α required.
$P\cos\theta=(36-24)\cos 36.9$ or $P\cos\theta=(36-24)\times 0.8$	**A1**	
$P\sin\theta+20=(24+36)\sin 36.9=14.4+21.6$ or $P\sin\theta+20=60\times 0.6=36$	**A1**	
$P\cos\theta=9.6, P\sin\theta=16 \quad P=\sqrt{16^2+9.6^2}$	**M1**	Correct method for solving equations for *P*. OE
$\theta=\tan^{-1}\left(\frac{16}{9.6}\right)$	**M1**	Correct method for solving equations for θ. OE e.g. $\theta=\tan^{-1}\left(\frac{5}{3}\right)$
$P=18.7$ $\theta=59[.0]$	**A1**	Allow $P=\frac{16\sqrt{34}}{5}$. Allow $P=18.6$.
	6	

18.

Answer	Marks	Guidance
Correct 4 force diagram 	**B1**	Angles shown. *F* either up/down slope. 
	1	
Attempt to resolve forces parallel to the plane	**M1**	Three terms, allow sign errors.
$P+F=12g\sin 25 \quad [=50.7]$	**A1**	Must have correct direction of *F* here.
$R=12g\cos 25 \quad [=108.8]$	**B1**	
$F=120\cos 25\times 0.35 \quad [=38.1]$ $P+38.1=120\sin 25$	**M1**	Attempt to solve for *P* using equations with the correct number of terms. *R* must be a single-term component of 12*g*.
$P=12.6$	**A1**	$P=12.64926\ldots$ Allow $P=12.7$
	5	

19.

Answer	Marks	Guidance
Horizontal: $100 - T_U \sin 60 - T_L \sin 60 = 0$ Vertical: $T_U \cos 60 - T_L \cos 60 - 5g = 0$ Perp to T_U $T_L \cos 30 + 5g \cos 30 = 100 \cos 60$	M1	Resolve horizontally or vertically or perpendicular to the upper string to reach an equation. Correct number of terms, Allow X for 100 in horizontal equation.
	A1	Either horizontal and vertical equations correct or perpendicular correct. Must see $X = 100$ used for A1.
Solve for either T_L or T_U using equation(s) with no missing term.	M1	May see $T_U = 107.74$
$T_L = 7.74$ N	A1	Allow 7.73
	4	
Horizontal: $X - T_{up} \sin 60 = 0$ Vertical: $T_{up} \cos 60 - 5g = 0$ Perp to T_{up} $5g \cos 30 = X \cos 60$	M1	Resolve either horizontally or vertically or perpendicular to the upper string. Must be using the tension $T_{low} = 0$. Equivalent to Lami as: $\frac{5g}{\sin 150} = \frac{X}{\sin 120}\left(= \frac{T_{up}}{\sin 90}\right)$
	A1	Either horizontal and vertical equations correct or perpendicular correct.
Eliminate T_{up} and/or solve for X	M1	$T_{up} = 100$
Least value of $X = 86.6$	A1	Allow $X = 50\sqrt{3}$
	4	

20.

Answer	Marks	Guidance
Correct 3 force diagram, including angles shown	B1	
	1	
$T_1 \cos 60 = T_2 \cos 45$	M1	Resolving forces horizontally.
$T_1 \sin 60 + T_2 \sin 45 = 8g$	M1	Resolving forces vertically.
$T_1 \cos 60 = T_2 \cos 45$ and $T_1 \sin 60 + T_2 \sin 45 = 8g$	A1	
Attempting to solve for either T_1 or T_2	M1	
$T_1 = 58.6$ N	A1	
$T_2 = 41.4$ N	A1	
Alternative method for question 2(b)		
$\frac{T_1}{\sin 135} = \frac{T_2}{\sin 150} = \frac{80}{\sin 75}$	M1	Applies Lami's Theorem – at least two terms correct.
	A1	
$T_1 = \frac{80 \sin 135}{\sin 75}$	M1	Solves for T_1.
$T_1 = 58.6$ N	A1	
$T_2 = \frac{80 \sin 150}{\sin 75}$	M1	Solves for T_2.
$T_2 = 41.4$ N	A1	
	6	

Answer	Marks	Guidance
Correct 3 force diagram, including angles shown	**B1**	
	1	
$T_1 \cos 60 = T_2 \cos 45$	**M1**	Resolving forces horizontally.
$T_1 \sin 60 + T_2 \sin 45 = 8g$	**M1**	Resolving forces vertically.
$T_1 \cos 60 = T_2 \cos 45$ and $T_1 \sin 60 + T_2 \sin 45 = 8g$	**A1**	
Attempting to solve for either T_1 or T_2	**M1**	
$T_1 = 58.6$ N	**A1**	
$T_2 = 41.4$ N	**A1**	
Alternative method for question 2(b)		
$\frac{T_1}{\sin 135} = \frac{T_2}{\sin 150} = \frac{80}{\sin 75}$	**M1**	Applies Lami's Theorem – at least two terms correct.
	A1	
$T_1 = \frac{80 \sin 135}{\sin 75}$	**M1**	Solves for T_1.
$T_1 = 58.6$ N	**A1**	
$T_2 = \frac{80 \sin 150}{\sin 75}$	**M1**	Solves for T_2.
$T_2 = 41.4$ N	**A1**	
	6	

21.

Answer	Marks	Guidance
Attempt to resolve vertically or horizontally	**M1**	Correct number of terms.
$G \sin 60° + 2F \sin 40° - 10 = 0$	**A1**	Correct resolution vertically.
$F + G \cos 60° - 2F \cos 40° = 0$	**A1**	Correct resolution horizontally.
Attempt to solve simultaneously for F or G	**M1**	From equations with 3 relevant terms in each
$F = 4.53$, $G = 4.82$	**A1**	For both correct.
	5	
$G \sin 60° + 2 \times 3 \sin 40° - 10 = 0$	**M1**	Resolve forces parallel to the 10 N force and equate this expression to zero, 3 terms.
$G = 7.09$ to 3 sf	**A1**	
	2	

22.

Answer	Marks	Guidance
For attempt at resolving horizontally or vertically	M1	Allow sin/cos mix. Allow sign error. Allow *g* missing. Correct number of terms.
$R = 300g - 0.28X$ or $R = 300g - X\sin 16.3$	A1	$\alpha = 16.26\ldots$
$0.96X - F = 0$ or $0.96X - 0.5(300g - X\sin\alpha) = 0$ Or $X\cos 16.3 - F = 0$ or $X\cos 16.3 - 0.5(300g - X\sin\alpha) = 0$	A1	Or using *their F*
Use of $F = 0.5R$	M1	Use to get an equation in X only. Allow sin/cos mix. Allow sign error. Allow *g* missing. Must be from 2 term R, which is a linear combination of 300(g) and a component of X
$X = 1360$ [1363.63…]	A1	
	5	

23.

Answer	Marks	Guidance
Alternative mark scheme for question 4: For candidates who resolve in other directions		
Attempt to resolve (e.g. parallel or perpendicular to 100 N)	M1	For resolving. Allow sin/cos mix. Allow sign error. Correct number of terms.
$F\sin(\alpha+20)+20\sin 20-100=0$ $[F\sin(\alpha+20)=93.159\ldots]$	A1	
$F\cos(\alpha+20)-20\cos 20=0$ $[F\cos(\alpha+20)=18.793\ldots]$	A1	
$F=\sqrt{93.159\ldots^2+18.793^2}$	M1	OE; Attempt to solve for F; one term missing in total
$\alpha=\tan^{-1}\left(\frac{93.159\ldots}{18.793\ldots}\right)-20$	M1	OE; Attempt to solve for α; one term missing in total
$F = 95[.0]$, $\alpha = 58.6$	A1	$F = 95.0364\ldots$ and $\alpha = 58.5943\ldots$
	6	

Answer	Marks	Guidance
Attempt to resolve in any direction	M1	For resolving. Allow sin/cos mix. Allow sign error. Correct number of terms.
$F\cos\alpha - 20\cos 40 - 100\sin 20 = 0$ $[F\cos\alpha = 15.320\ldots + 34.202\ldots = 49.5229\ldots]$	A1	
$F\sin\alpha + 20\sin 40 - 100\cos 20 = 0$ $[F\sin\alpha = 93.969\ldots - 12.855\ldots = 81.1135\ldots]$	A1	
$F=\sqrt{(49.5229\ldots)^2+(81.1135\ldots)^2}$	M1	OE; Attempt to solve for F; one term missing in total
$\alpha=\tan^{-1}\left(\frac{81.1135\ldots}{49.5229\ldots}\right)$	M1	OE; Attempt to solve for α; one term missing in total
$F = 95(.0)$, $\alpha = 58.6$	A1	$F = 95.0364\ldots$ and $\alpha = 58.5943\ldots$
Alternative mark scheme for question 4: For candidates who use cosine and/or sine rule		
Attempt at cosine rule from triangle of forces	M1	Must use lengths 100 and 20 with a suitable angle
$F^2 = 100^2 + 20^2 - 2\times100\times20\cos 70$	A1	Correct
$F = 95[.0]$	A1	
$\frac{95.0364}{\sin 70}=\frac{20}{\sin\beta}$ OR $\frac{95.0364}{\sin 70}=\frac{100}{\sin\gamma}$	M1	Attempt at sin rule
	A1	where $\beta=(70-\alpha)$ where $\gamma=(40+\alpha)$
$\alpha = 58.6$	A1	$\alpha = 58.5943\ldots$

Answer	Marks	Guidance
Alternative mark scheme for question 4: For candidates who resolve in other directions		
Attempt to resolve (e.g. parallel or perpendicular to 100 N)	M1	For resolving. Allow sin/cos mix. Allow sign error. Correct number of terms.
$F\sin(\alpha+20)+20\sin 20-100=0$ $[F\sin(\alpha+20)=93.159\ldots]$	A1	
$F\cos(\alpha+20)-20\cos 20=0$ $[F\cos(\alpha+20)=18.793\ldots]$	A1	
$F=\sqrt{93.159\ldots^2+18.793^2}$	M1	OE; Attempt to solve for F; one term missing in total
$\alpha=\tan^{-1}\left(\frac{93.159\ldots}{18.793\ldots}\right)-20$	M1	OE; Attempt to solve for α; one term missing in total
$F=95[.0]$, $\alpha=58.6$	A1	$F=95.0364\ldots$ and $\alpha=58.5943\ldots$
	6	

Answer	Marks	Guidance
Alternative mark scheme for question 4: For candidates who resolve in other directions		
Attempt to resolve (e.g. parallel or perpendicular to 100 N)	M1	For resolving. Allow sin/cos mix. Allow sign error. Correct number of terms.
$F\sin(\alpha+20)+20\sin 20-100=0$ $[F\sin(\alpha+20)=93.159\ldots]$	A1	
$F\cos(\alpha+20)-20\cos 20=0$ $[F\cos(\alpha+20)=18.793\ldots]$	A1	
$F=\sqrt{93.159\ldots^2+18.793^2}$	M1	OE; Attempt to solve for F; one term missing in total
$\alpha=\tan^{-1}\left(\frac{93.159\ldots}{18.793\ldots}\right)-20$	M1	OE; Attempt to solve for α; one term missing in total
$F=95[.0]$, $\alpha=58.6$	A1	$F=95.0364\ldots$ and $\alpha=58.5943\ldots$
	6	

24.

Answer	Marks	Guidance
Resolving either direction	M1	3 terms; allow sign errors and allow sin/cos mix
$(X=)\pm(20\cos 60-14-16\cos 50)$ $[=\mp 14.2846\ldots\ldots\ldots]$	A1	
$(Y=)\pm(60-20\sin 60-16\sin 50)$ $[=\pm 30.42278\ldots\ldots\ldots]$	A1	
$R=\sqrt{(14.2846\ldots)^2+(30.42278\ldots)^2}$	M1	Attempt to solve for R; one missing term in total
$\theta=\tan^{-1}\left(\frac{30.42278\ldots}{14.2846\ldots}\right)\left[=\tan^{-1}(2.1297\ldots\ldots)\right]$ OR $\alpha=\tan^{-1}\left(\frac{14.2846\ldots}{30.42278\ldots}\right)\left[=\tan^{-1}(0.4596\ldots\ldots)\right]$	M1	Attempt to solve for θ or α; one missing term in total
$R=33.6$ N Direction is 64.8° above the 14 N force or 25.2° above the negative x-axis or 25.2° left of the 60 N force or bearing 335° or 115° anticlockwise from the positive x-axis	A1	Both correct. OE; allow 64.9, 25.1 Giving an angle only is insufficient. Direction may be seen on a diagram, with minimum of arrow on resultant. Arrows on both components only is A0 as it doesn't show the direction of the resultant. However the direction is stated, it must be able to be drawn uniquely.
	6	

25.

Answer	Marks	Guidance
Attempt at resolving parallel to the plane	*M1	3 terms. Allow sign errors, sin/cos mix. Allow g missing, otherwise dimensionally correct.
$65\cos 36 = 12g \times \sin 24 + F$	A1	$F = 3.777707\ldots\ldots$
Attempt at resolving perpendicular to the plane	*M1	3 terms. Allow sign errors, sin/cos mix. Allow g missing, otherwise dimensionally correct.
$12g \times \cos 24 = R + 65\sin 36$	A1	$R = 71.419\ldots\ldots$
Use $F = \mu R$ $\left[\mu = \frac{65\cos 36 - 12g \times \sin 24}{12g \times \cos 24 - 65\sin 36} = \frac{52.586 - 48.808}{109.625 - 38.206} = \frac{3.777\ldots}{71.419\ldots}\right]$	DM1	To get an equation in μ only. Dependent on two previous M marks. Allow g missing
$\mu = 0.0529$	A1	Allow AWRT 0.053 Do not accept fractional equivalent.
	6	

Answer	Marks	Guidance
Attempt at resolving parallel to the plane	*M1	3 terms. Allow sign errors, sin/cos mix. Allow g missing, otherwise dimensionally correct.
$65\cos 36 = 12g \times \sin 24 + F$	A1	$F = 3.777707\ldots\ldots$
Attempt at resolving perpendicular to the plane	*M1	3 terms. Allow sign errors, sin/cos mix. Allow g missing, otherwise dimensionally correct.
$12g \times \cos 24 = R + 65\sin 36$	A1	$R = 71.419\ldots\ldots$
Use $F = \mu R$ $\left[\mu = \frac{65\cos 36 - 12g \times \sin 24}{12g \times \cos 24 - 65\sin 36} = \frac{52.586 - 48.808}{109.625 - 38.206} = \frac{3.777\ldots}{71.419\ldots}\right]$	DM1	To get an equation in μ only. Dependent on two previous M marks. Allow g missing
$\mu = 0.0529$	A1	Allow AWRT 0.053 Do not accept fractional equivalent.
	6	

26.

Answer	Marks	Guidance
$T_A \times 0.8 - T_B \times 0.6 - 20 = 0$ or $T_A \times 0.6 + T_B \times 0.8 - 10g = 0$	M1	Resolving horizontally or vertically
$T_A \times 0.8 - T_B \times 0.6 - 20 = 0$	A1	
$T_A \times 0.6 + T_B \times 0.8 - 10g = 0$	A1	
$0.8T_A - \frac{0.6(10g - 0.6T_A)}{0.8} = 20 \quad \rightarrow \quad T_A = \ldots$	M1	Attempt to solve simultaneously
$T_A = 76$ N, $T_B = 68$ N	A1	
	5	

Answer	Marks	Guidance
$T_A \times 0.6 - 10g = 0 \Rightarrow T_A = \frac{500}{3}$	B1	From using $T_B = 0$
$T_A \times 0.8 - F = 0$	M1	
$F = \frac{400}{3}$	A1	Allow $F = 133$ to 3 s.f.
	3	

27.

Answer	Marks	Guidance
Attempt at resolving horizontally or vertically	**M1**	Allow sign errors, allow sin/cos mix. 3 terms.
$P\cos 25 = 22 + 16\cos 55$	**A1**	
$Q + 16\sin 55 = P\sin 25$	**A1**	Allow *their P*.
Attempt to solve for P or Q	**M1**	No missing/extra terms.
$P = 34.4 \quad Q = 1.43$	**A1**	$P = 34.40025941,\ Q = 1.431745128$.
	5	

28.

Answer	Marks	Guidance
$T = 4g$	**B1**	soi
$R = 3g\cos 30$	**B1**	
Attempt to resolve parallel to the plane	**M1**	3 terms, allow g missing. Allow sign errors, sin/cos mix.
$F = T - 3g\sin 30$	***A1**	May see $F = 25$.
Eliminate T and use $F = \mu R$ to get an equation in μ only	**DM1**	Where R is a component of *their* weight.
Coefficient of friction = 0.962	**A1**	allow $\frac{5\sqrt{3}}{9}$. allow 0.96. If F negative must say why using positive for this mark.
	6	

29.

Answer	Marks	Guidance
$R = 0.4g\cos 30\ \left[=2\sqrt{3}\right]$ or F or $\mu R = 0.4g\sin 30\ \left[=2\right]$	**B1**	Use of m instead of 0.4 condoned.
$0.4g\sin 30 - \mu\, 0.4g\cos 30 = 0$	**M1**	For using $F = \mu R$. Allow sin/cos mix. Both must be different components of their weight only, not a 2 term R. Allow sign errors. Allow g omitted.
$\mu\left[=\frac{4\sin 30}{4\cos 30}\right] = \frac{1}{3}\sqrt{3}$ or $\frac{\sqrt{3}}{3}$.	**A1**	**AG** (exact answer only) If zero scored then **SC B1** for [Angle of friction = 30° so] $\mu = \tan 30 = \frac{1}{3}\sqrt{3}$. Allow full marks if using m in place of 0.4 or W in place of mg or $0.4g$ A0 for $\mu = 0.577\ldots = \frac{1}{3}\sqrt{3}$, but A1(ISW) for $\mu = \frac{1}{3}\sqrt{3} = 0.577\ldots$
	3	

30.

Answer	Marks	Guidance
Attempt to resolve either direction	M1	Correct number of terms. Allow sin/cos mix. Allow sign errors. Allow g missing.
$0.3g + T\cos\alpha° - 4\sin 60° = 0$ ($T\cos\alpha° = 0.464\ldots$)	A1	OE
$T\sin\alpha° - 4\cos 60° = 0$ ($T\sin\alpha° = 2$)	A1	OE If the two Ts are different, award maximum A1A0 unless subsequently stated that the two Ts are the same.
$\alpha = \tan^{-1}\left(\frac{4\cos 60°}{4\sin 60° - 0.3g}\right) = \tan^{-1}\left(\frac{2}{0.464\ldots}\right)$	M1	Attempt to solve for α. No missing/extra terms. Allow g missing. Must get to '$\alpha =$'.
$T = \frac{4\cos 60}{\sin(their\,\alpha)} = \sqrt{(4\cos 60°)^2 + (4\sin 60° - 0.3g)^2} = \sqrt{2^2 + (0.464\ldots)^2}$	M1	OE Attempt to solve for T. No missing/extra terms. Allow g missing. Must get to '$T =$'.
Tension = 2.05 N $\alpha = 76.9$	A1	For both AWRT 2.05, 76.9 (Tension = 2.05314… N $\alpha = 76.9356\ldots$)
Alternative method for Q3 using triangle of forces		
Attempt at cosine rule from triangle of forces	M1	Must use lengths 4 and $0.3g$ with a suitable angle. Allow g missing.
$T^2 = 4^2 + (0.3g)^2 - 2\times4\times(0.3g)\times\cos 30$	A1	
Tension = 2.05	A1	Tension = 2.05314… AWRT 2.05
Attempt at sin rule	M1	Must have angle 30° and another angle in terms of α with correct numerators, but allow g missing.
$\frac{Their\,T}{\sin 30} = \frac{4}{\sin(180-\alpha)}$ or $\frac{Their\,T}{\sin 30} = \frac{0.3g}{\sin(\alpha - 30)}$	A1	Correct. Allow $\sin\alpha$ instead of $\sin(180-\alpha)$.
$\alpha = 76.9$	A1	$\alpha = 76.9356\ldots$ AWRT 76.9

Answer	Marks	Guidance
Alternative method for Q3 using Lami's theorem		
Attempt at Lami's theorem	M1	Must have numerators correct and at least one angle correct. Allow g missing.
$\frac{4}{\sin\alpha} = \frac{0.3g}{\sin(210-\alpha)} = \frac{T}{\sin(150)}$	A1 A1	A1 for two parts second A1 for all three.
$\alpha = \tan^{-1}\left(\frac{4\sin 210}{0.3g + 4\cos 210}\right)$	M1	For solving for α using compound angle formula. Must be correct for their angles. Allow g missing.
$T = \frac{4\sin(150)}{\sin\alpha}$ or $T = \frac{0.3g\sin(150)}{\sin(210-\alpha)}$	M1	For solving for T using their α. Allow g missing.
Tension = 2.05 N $\alpha = 76.9$	A1	For both AWRT 2.05, 76.9
	6	
SC: Tension and the 4N force considered in the wrong directions		
Attempt to resolve either direction	M1	Correct number of terms. Allow sin/cos mix. Allow sign errors. Allow g missing.
$T\cos 60° - 4\sin\alpha° = 0$ And: $T\sin 60° - 4\cos\alpha° - 0.3g = 0$	A1	For both OE If the two Ts are different, they get **SC A0** unless they subsequently state that the two Ts are the same.
$\left(\frac{T\cos 60°}{4}\right)^2 + \left(\frac{T\sin 60° - 0.3g}{4}\right)^2 = 1 \Rightarrow \frac{1}{4}T^2 + \frac{3}{4}T^2 - 3\sqrt{3}T + 9 = 16$ $\Rightarrow T^2 - 3\sqrt{3}T - 7 = 0 \Rightarrow T = 6.31 (\text{or } -1.11)$ OR: $4\sqrt{3}\sin\alpha - 4\cos\alpha = 3 \Rightarrow 8\sin(\alpha - 30) = 3 \Rightarrow \alpha = \sin^{-1}\frac{3}{8} + 30$	M1	OE Attempt to solve for T *or* α. No missing/extra terms. Allow g missing. Must get to '$T =$' or 'α='.

Answer	Marks	Guidance
$T = 6.31\text{N}$ $\alpha = 52.0$	A1	(T = 6.30617…, α=52.0243…)
	6	

31.

Answer	Marks	Guidance
$T \cos 25 = 40 + R \cos 50$	M1	Resolving in any direction e.g. horizontal, vertical, along radius or tangent.
$R \sin 50 = T \sin 25$	M1	Resolving in a second direction.
Radially: $T \cos 25 = R + 40 \cos 50$ Tangentially: $T \sin 25 = 40 \sin 50$ Parallel to T: $T = R \cos 25 + 40 \cos 25$ Perpendicular to T: $R \sin 25 = 40 \sin 25$ Vertically: $T \cos 25 = 40 + R \cos 50$ Horizontally: $R \sin 50 = T \sin 25$	A1	Two correct equations.
Solving equation(s) to find either T or R	M1	
$T = 72.5$ N	A1	From 72.504….
$R = 40$ N	A1	
	6	

32.

Answer	Marks	Guidance
Attempt to resolve vertically	M1	4 terms; allow with T_A and T_B; allow sign errors; allow g missing.
$500 + T\cos 45 + T\cos 45 - 100g = 0$ OR $500 + T_A\cos 45 + T_B\cos 45 - 100g = 0$ AND $T_A(\sin 45) = T_B(\sin 45)$	A1	Must have $T_A = T_B = T$. Allow if $500 - 2T\cos 45 - 100g = 0$. Allow $500 - T_A\cos 45 - T_B\cos 45 - 100g = 0$ AND $T_A(\sin 45) = T_B(\sin 45)$.
$T = 354\text{N}$	A1	Allow $250\sqrt{2}$, $\frac{500}{\sqrt{2}}$. Allow if $500 - 2T\cos 45 - 100g = 0$ to obtain $T = -354$ and then state magnitude is 354. If T_A and T_B are different values then A0.
Alternative Method 1 for Question 5(a): Resolving perpendicular to a strut		
Resolve perpendicular to T_A or T_B	M1	3 terms; allow sign errors; allow g missing.
$T_A(\text{or } T_B) + 500\cos 45 = 100g\cos 45$	A1	Allow $T_A(\text{or } T_B) + 100g\cos 45 = 500\cos 45$.
$T_A = T_B = 354$	A1	Allow $250\sqrt{2}$, $\frac{500}{\sqrt{2}}$.

Answer	Marks	Guidance
Attempt to resolve vertically	M1	4 terms; allow with T_A and T_B; allow sign errors; allow g missing.
$500+T\cos 45+T\cos 45-100g=0$ OR $500+T_A\cos 45+T_B\cos 45-100g=0$ AND $T_A(\sin 45)=T_B(\sin 45)$	A1	Must have $T_A=T_B=T$. Allow if $500-2T\cos 45-100g=0$. Allow $500-T_A\cos 45-T_B\cos 45-100g=0$ AND $T_A(\sin 45)=T_B(\sin 45)$.
$T=354\text{N}$	A1	Allow $250\sqrt{2}$, $\frac{500}{\sqrt{2}}$. Allow if $500-2T\cos 45-100g=0$ to obtain $T=-354$ and then state magnitude is 354. If T_A and T_B are different values then A0.
Alternative Method 1 for Question 5(a): Resolving perpendicular to a strut		
Resolve perpendicular to T_A or T_B	M1	3 terms; allow sign errors; allow g missing.
$T_A(\text{or } T_B)+500\cos 45=100g\cos 45$	A1	Allow $T_A(\text{or } T_B)+100g\cos 45=500\cos 45$.
$T_A=T_B=354$	A1	Allow $250\sqrt{2}$, $\frac{500}{\sqrt{2}}$.

Answer	Marks	Guidance
Alternative Method 2 for Question 5(a): Using triangle of forces		
Attempt Pythagoras on a right-angled triangle of forces or use of trigonometry $\left[\begin{array}{c} T_A^2+T_B^2=(100g-500)^2 \\ \text{OR } \sin 45 \text{ or } \cos 45=\frac{100g-500}{T_A} \text{ or } \frac{100g-500}{T_B}\end{array}\right]$	M1	4 terms; allow with T_A and T_B; allow sign errors; allow g missing.
$T^2+T^2=(100g-500)^2$ OR $T_A^2+T_B^2=(100g-500)^2$ AND $T_A(\sin 45)=T_B(\sin 45)$ OR $\sin 45=\frac{T_A(\text{or } T_B)}{100g-500}$ OR $\cos 45=\frac{T_A(\text{or } T_B)}{100g-500}$	A1	Allow $\sin 45=\frac{T_A(\text{or } T_B)}{500-100g}$ OR $\cos 45=\frac{T_A(\text{or } T_B)}{500-100g}$.
$T_A=T_B=354$	A1	Allow $250\sqrt{2}$, $\frac{500}{\sqrt{2}}$.
Alternative Method 3 for Question 5(a): Using Lami's Theorem		
Attempt at Lami	M1	Allow with T_A and T_B; allow sign errors; allow g missing.
$\frac{100g-500}{\sin 90}=\frac{T_A(\text{or } T_B)}{\sin 135}$	A1	Allow $\frac{500-100g}{\sin 90}=\frac{T_A(\text{or } T_B)}{\sin 135}$. Allow $\frac{100g-500}{\sin 270}=\frac{T_A(\text{or } T_B)}{\sin 45}$.
$T_A=T_B=354$	A1	Allow $250\sqrt{2}$, $\frac{500}{\sqrt{2}}$.
	3	

Answer	Marks	Guidance
Attempt to resolve vertically **and** horizontally	**M1**	3 terms vertically and 2 terms horizontally; allow sign errors; allow g missing. Must have $T_A = 0$.
$T_B \cos 45 + 500 - 100g = 0$ and $F - T_B \sin 45 = 0$	**A1**	Allow $-T_B \cos 45 + 500 - 100g = 0$ and $F + T_B \sin 45 = 0$ OR $T_B \cos 45 + 500 - 100g = 0$ and $F + T_B \sin 45 = 0$ OR $-T_B \cos 45 + 500 - 100g = 0$ and $F - T_B \sin 45 = 0$. For both equations correct.
$F = 500$	**A1**	awrt 500 to 3sf.
Alternative Method 1 for Question 5(b): Resolving perpendicular to T_B		
Attempt to resolve perpendicular to T_B	**M1**	3 terms; allow sign errors; allow g missing. Must have $T_A = 0$.
$F \cos 45 + 500 \cos 45 = 100g \cos 45$	**A1**	Allow $-F \cos 45 + 500 \cos 45 = 100g \cos 45$.
$F = 500$	**A1**	awrt 500 to 3sf.

Answer	Marks	Guidance
Alternative Method 2 for Question 5(b): Using Lami's Theorem		
Attempt at Lami	**M1**	Allow sign errors.
$\frac{100g - 500}{\sin 135} = \frac{F}{\sin 135} \left(= \frac{T_B}{\sin 90} \right)$	**A1**	Allow $\frac{500 - 100g}{\sin 135} = \frac{F}{\sin 135}$ or $\frac{100g - 500}{\sin 45} = \frac{F}{\sin 45}$ or $\frac{100g - 500}{\sin 45} = \frac{F}{\sin 225}$ or $\frac{100g - 500}{\sin 225} = \frac{F}{\sin 45}$.
$F = 500$	**A1**	awrt 500 to 3sf.
Alternative Method 3 for Question 5(b): Using triangle of forces		
Attempt use of trigonometry on right angled triangle	**M1**	Allow sign errors; allow g missing.
$\tan 45 = \frac{F}{100g - 500}$	**A1**	Allow $\tan 45 = \frac{F}{500 - 100g}$
$F = 500$	**A1**	awrt 500 to 3sf.
	3	

33.

Answer	Marks	Guidance
Resolving either direction.	M1	3 terms; allow sign errors and allow sin/cos mix. Must be an equation with either = 0 or with an attempt to balance forces.
Vertical: $F\sin\theta + 40\sin 60 - 50 = 0$	A1	$\left[F\sin\theta = 50 - 20\sqrt{3} = 15.358...\right]$
Horizontal: $F\cos\theta + 10 - 40\cos 60 = 0$	A1	$\left[F\cos\theta = 10\right]$
$\theta = \tan^{-1}(5-2\sqrt{3})$	M1	Attempt to solve for θ; one missing term in total $\theta = \tan^{-1}1.535898...$
$F = \sqrt{15.358...^2 + 10^2}$	M1	Attempt to solve for F: one missing term in total.
$\theta = 56.9$, $F = 18.3$	A1	Both correct (18.327530…, 56.932462…).
	6	
$(Y =)\pm(10\sqrt{2}\sin 45 + 40\sin 60 - 50)\,[=\pm(20\sqrt{3}-40)]$	B1	Allow non-exact values for $\sqrt{2}$ etc. in correct expression.
$(X =)\pm(10\sqrt{2}\cos 45 + 10 - 40\cos 60)\,[=0]$	B1	Allow non-exact values for $\sqrt{2}$ etc. in correct expression. Could be implied by correct answer.
Resultant force is $40-20\sqrt{3}$ (N) in the same direction as the 50(N) force.	B1	Allow vertically downwards, south, 180°, negative y-direction. Resultant force must be exact and positive (so $20\sqrt{3}-40$ is B0).
	3	

34.

Answer	Marks	Guidance
Resolving either direction.	M1	Correct number of terms, allow sign errors, allow sin/cos mix. Do not allow with just $\sin\alpha$ and $\cos\alpha$.
$(33+15)\times\frac{3}{5} = P\cos\theta + 30\cos\theta$ OR $(33+15)\cos\left(\tan^{-1}\frac{4}{3}\right) = P\cos\theta + 30\cos\theta$ OR $19.8 + 9 = P\cos\theta + 30\cos\theta$	A1	OE, but see note for final A1. Allow: $28.8 = (P+30)\cos\theta$ $(33+15)\cos 53(.1) = P\cos\theta + 30\cos\theta$ $19.81 + 9.01 = P\cos\theta + 30\cos\theta$ $19.86 + 9.03 = P\cos\theta + 30\cos\theta$.
$15\times\frac{4}{5} + 30\sin\theta = 33\times\frac{4}{5} + P\sin\theta$ OR $15\sin\left(\tan^{-1}\frac{4}{3}\right) + 30\sin\theta = 33\sin\left(\tan^{-1}\frac{4}{3}\right) + P\sin\theta$ OR $12 + 30\sin\theta = 26.4 + P\sin\theta$	A1	OE, but see note for final A1. Allow: $14.4 = (30-P)\sin\theta$ $15\sin 53(.1) + 30\sin\theta = 33\sin 53(.1) + P\sin\theta$ $12.00 + 30\sin\theta = 26.39 + P\sin\theta$ $11.98 + 30\sin\theta = 26.35 + P\sin\theta$.
[Use $\cos^2\theta + \sin^2\theta = 1$ with] $\cos\theta = \frac{28.8}{P+30}$ and $\sin\theta = \frac{14.4}{30-P}$ to get $\left(\frac{14.4}{30-P}\right)^2 + \left(\frac{28.8}{P+30}\right)^2 = 1$	A1	AG. Must have evidence of where 28.8 and 14.4 come from. A0 for any error seen. A0 if use of inexact angles seen. Any inexact decimals seen for force components, i.e. if 14.4 and/or 28.8 have come from rounding to 3sf, scores M1A1A1A0 max 3/4. If exact values of $\sin\alpha$ and $\cos\alpha$ not shown (e.g. $28.8 = (P+30)\cos\theta$ or $14.4 = (30-P)\sin\theta$ from no working), this scores M1A1A1A0 max 3/4 marks.
	4	

Answer	Marks	Guidance
Sub $P=6$ into $\left(\frac{14.4}{30-P}\right)^2+\left(\frac{28.8}{P+30}\right)^2$ to get $\left[\left(\frac{14.4}{24}\right)^2+\left(\frac{28.8}{36}\right)^2\right]=\left(\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2=0.36+0.64=1$	B1	Must see either $\left(\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2=1$ or $0.36+0.64=1$ as minimum working.
$\theta=36.9$	B1	AWRT 36.9 .
	2	

35.

Answer	Marks	Guidance
For attempt to resolve in one direction	M1	Must use 0.2 substituted for m if just awarding M1 for vertical equation. Must have correct number of relevant terms (forces must have components as required). Allow sin/cos mix. Allow sign errors. Allow g missing.
$X\sin 60+T\sin 30-0.2g=0$	A1	OE. Correct vertical.
$X\cos 60-T-T\cos 30=0$	A1	OE. Correct horizontal. If the two Ts are different, they can get max M1A1A0M0A0, unless they subsequently state that the two Ts are equal.
For attempt to solve for tension or X	M1	Must have correct number of relevant terms in both equations. Must get to '$T=$' or '$X=$'. Allow g missing. Can be implied by correct answers. If no working shown their values must follow from their equations.
$X=2$, tension in string = 0.536[N]	A1	Allow exact value of tension = $4-2\sqrt{3}$. Allow awrt 2.00 for X.

36.

Answer	Mark	Guidance
20 cos 60 = T cos 45	M1	Resolve forces horizontally, 2 terms
$T=10\sqrt{2}$ or $T=14.1$	A1	
20 sin 60 + T sin 45 = mg or W	M1	Resolve forces vertically, 3 terms
20 sin 60 + T sin 45 = mg	A1	
$m=2.73$ [$=\sqrt{3}+1$]	A1	
Alternative method for question 3		
$\left[\frac{T}{\sin 150}=\frac{mg \text{ or } W}{\sin 75}=\frac{20}{\sin 135}\right]$	M1	Attempt at one pair of terms using Lami's Method
$\frac{T}{\sin 150}=\frac{mg}{\sin 75}=\frac{20}{\sin 135}$	A1	All terms correct in Lami's Method
Attempt to solve for either T or m or W	M1	
$T=10\sqrt{2}$ or $T=14.1$	A1	
$m=2.73$ [$=\sqrt{3}+1$]	A1	
	5	

Answer	Mark	Guidance
Alternative method for question 3		
$\left[\frac{T}{\sin 30} = \frac{mg \text{ or } W}{\sin 105} = \frac{20}{\sin 45}\right]$	**M1**	Attempt the triangle of forces method and state one equation which involves any two of the forces *T*, *m* and 20.
$\frac{T}{\sin 30} = \frac{mg}{\sin 105} = \frac{20}{\sin 45}$	**A1**	All correct
Attempt to solve for either *T* or *m* or *W*	**M1**	
$T = 10\sqrt{2}$ or $T = 14.1$	**A1**	
$m = 2.73$ [$= \sqrt{3} + 1$]	**A1**	
	5	

37.

Answer	Marks	Guidance
[$T = 2g \sin 10$] or [$3g \sin 20 = F + T$]	**M1**	Resolve forces parallel to plane *P* for particle *A* or parallel to plane *Q* for Particle *B*
$T = 2g \sin 10$ **and** $3g \sin 20 = F + T$	**A1**	
$R = 30 \cos 20$ (= 28.19...)	**B1**	Resolving forces perpendicular to plane *Q* for particle *B*
$\mu = \frac{3g \sin 20 - 2g \sin 10}{30 \cos 20}$	**M1**	Using $\mu = F/R$
$\mu = 0.241$ (=0.2407…)	**A1**	
	5	

38.

Answer	Marks	Guidance
Attempt at resolving parallel or perpendicular to the plane.	***M1**	3 terms, allow sign errors, allow sin/cos mix, allow *g* missing. Forces that need resolving should be resolved, forces that do not need resolving should not be resolved.
$R = P\sin 35 + 0.6g\cos 35$ $[R = (0.573\ldots)P + 4.914\ldots]$	**A1**	
$F + P\cos 35 = 0.6g\sin 35$ $[F + (0.819\ldots)P = 3.441\ldots]$	**A1**	Their *F*.
Use of $F = 0.4R$	***M1**	Where *R* is initially a linear combination of a *P* component and a weight component (or a mass component).
Solve for *P*.	**DM1**	From equations with the correct number of relevant resolved terms. $R = \frac{0.6g}{\cos 35 + 0.4\sin 35} = 5.7222$. Must get to $P = \ldots$, e.g. $P = \frac{0.6g\sin 35 - 0.4 \times 0.6g\cos 35}{\cos 35 + 0.4\sin 35}$ If no working seen, allow this mark if correct solution for their equations. If $F \leqslant 0.4R$ used, it should be used correctly. e.g. $0.6g \sin 35 - P \cos 35 \leqslant 0.4(P \sin 35 + 0.6g \cos 35)$.
$P = 1.41$	**A1**	AWRT 1.41 . If $P \geqslant 1.41$ seen, must then state the least value explicitly for A1.

Alternative for Question 5: Resolving vertically and horizontally		
Attempt at resolving vertically or horizontally.	*M1	3 terms, allow sign errors, allow sin/cos mix, allow g missing. Forces that need resolving should be resolved, forces that do not need resolving should not be resolved.
$R\cos 35 + F\sin 35 = 0.6g$	A1	*Their* F or R.
$P + F\cos 35 = R\sin 35$	A1	*Their* F or R.
Use of $F = 0.4R$	*M1	To get 2 equations, one in R (or F) and the other in P and R (or P and F) from resolved equations with correct number of relevant terms. Allow g missing.
Solve for P	DM1	From equations with the correct number of relevant resolved terms. May see $R = \frac{0.6g}{\cos 35 + 0.4\sin 35} = 5.7222$; Must get to $P =$ …, e.g. $P = \frac{0.6g\sin 35 - 0.4 \times 0.6g\cos 35}{\cos 35 + 0.4\sin 35}$. If no working seen, allow this mark if correct solution for *their* equations.
$P = 1.41$	A1	AWRT 1.41 .
	6	