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Mechanics-1

Kinematics of Motion in a straight line.

Exercise-1 Solution (Revision)

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Suresh Goel
(Former Director)
Alliance World School
Noida-Delhi-N.C.R.
INDIA

(+91 9810444804)

(M-1) **Kinematics of motion in a straight line.**

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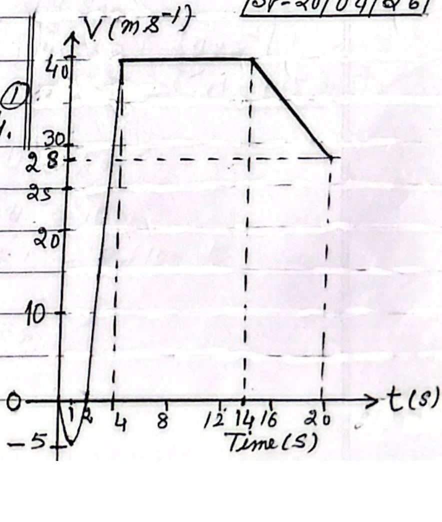
Ex-1 (Revision)

1. A particle P is projected vertically upwards with speed $20\,ms^{-1}$ from a point on the ground.
(a) Find the greatest height above the ground reached by P. ---[2]
(b) Find the total time from projection until P returns to the ground. [SP-20/04/Q1] --[2]

Solution (a) $u = 20\,ms^{-1}$, for greatest height h, final velocity $v = 0$
$v^2 = u^2 + 2gh \Rightarrow 0 = 20^2 + 2(-10)h \Rightarrow h = \underline{20m}$ ✓
(b) $v = u + at \Rightarrow 0 = 20 - 10t = t = 2s$ to reach the heighest point
it will be same time 2s to reach the ground.
$\therefore$ Total time to reach the ground $= 2 \times 2 = \underline{4s}$ ✓

2. A particle P moves in a straight line. The velocity $v\,ms^{-1}$ at time $t\,s$ is given by:
$v = 5t(t-2)$ for $0 < t \leq 4$ ---- ①
$v = k$ for $4 \leq t \leq 14$ ---- ②
$v = 68 - 2t$ for $14 < t \leq 20$ ---- ③
where k is a constant.
(a) Find k. ---[1]
(b) Sketch the velocity-time graph for $0 \leq t \leq 20$ --[3]
(c) Find the set of values of t for which the acceleration of P is positive. --[2]
(d) Find the total distance travelled by P in the interval $0 \leq t \leq 20$ --[5]
[SP-20/04/Q6]

Solution (a) $v = k$ from ② at $t = 4$ ✓
and $V = 5 \times 4 \times (4-2) = 40$ { from ① at $t = 4$.
$\therefore$ $K = 40$ ✓
(b) for Vel-time graph.
$t = 0,\ v = 0 \rightarrow (0,0)$
$t = 4 \rightarrow v = 40 \rightarrow (4,40)$
$t = 14 \rightarrow v = 40 \rightarrow (14,40)$
$t = 20 \rightarrow v = 28 \rightarrow (20,28)$
from ① Now $(2,0)$ and $(1,-5)$



(continued →)

(continued)

2(c) from ① $v = 5t^2 - 10t \qquad 1 \le t \le 4$

acc. $\frac{dv}{dt} = 10t - 10 > 0 \Rightarrow t > 1$

from ② $v =$ constant $\Rightarrow a = 0$
from ③ $a = -2 < 0$

∴ Acc. is positive for $1 < t < 4$ ✓

(d) $\text{Distance} = \left|\int_0^2 (5t^2-10t)\,dt\right| + \int_2^4 (5t^2-10t)\,dt + 40(14-4) + \frac{1}{2}(40+28)\times 6$

$= \left|\left[\frac{5}{3}t^3 - 5t^2\right]_0^2\right| + \left[\frac{5}{3}t^3 - 5t^2\right]_2^4 + 400 + 204$

$= \left|-\frac{20}{3}\right| + \left[\left(\frac{320}{3} - 80\right) - \left(\frac{40}{3} - 20\right)\right] + 604$

$= \frac{20}{3} + \frac{80}{3} + \frac{20}{3} + 604 = 40 + 604 = \underline{644}$ ✓

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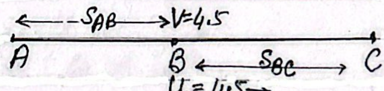
3. A cyclist travels along a straight road with constant acceleration. He passes through points A, B and C. The cyclist takes 2 seconds to travel along each of the sections AB and BC and passes through B with speed $4.5\,ms^{-1}$. The distance AB is $\frac{4}{5}$ of the distance BC.

(a) Find the acceleration of the cyclist. ---[5]

(b) Find AC [M-20/42/Q 4] ---[2]

Solution:



(a) $S_{AB} = Vt - \frac{1}{2}at^2$

$= 4.5\times 2 - \frac{1}{2}a\times 2^2$ ——— ①

$S_{BC} = ut + \frac{1}{2}at^2 = 4.5\times 2 + \frac{1}{2}a\times 2^2$ ——— ②

Given $S_{AB} = \frac{4}{5}S_{BC}$

$\Rightarrow \left[2\times 4.5 - \frac{1}{2}a\times 2^2\right] = \frac{4}{5}\left[4.5\times 2 + \frac{1}{2}a\times 2^2\right]$

$\Rightarrow 5[9-2a] = 4[9+2a] \Rightarrow 18a = 9 \Rightarrow \underline{a = 0.5\,ms^{-2}}$ ✓

(b) for ① $S_{AB} = 9 - \frac{1}{2}\times 0.5\times 4 = 8\,m$ ✓

$S_{BC} = 9 + \frac{1}{2}\times 0.5\times 4 = 10\,m$

$AC = AB + BC = 8 + 10 = 18$

∴ $AC = \underline{18\,m}$ ✓

4. A particle moves in a straight line through the point O. The displacement of the particle from O at time t s is S m, where,

$$S = t^2 - 3t + 2 \quad \text{for } 0 \le t \le 6$$
$$S = \frac{24}{t} - \frac{t^2}{4} + 25 \quad \text{for } t \ge 6.$$

(a) Find the value of t when the particle is instantaneously at rest during the first 6 seconds of its motion. ---[2]

At t=6, the particle hits a barrier at a point P and rebounds:

(b) Find the velocity with which the particle arrives at P and also the velocity with which the particle leaves P. ---[3]

(c) Find the total distance travelled by the particle in the first 10 seconds of its motion. $\boxed{\text{M-20/42/Q7}}$ --[5]

Solution (a) $S = t^2 - 3t + 2$ —① for $0 \le t \le 6$

$v = \frac{dS}{dt} = 2t - 3 = 0$ (for the particle to be at rest)

$\Rightarrow \underline{t = 1.5\,s}$ ✓

(b) $V = 2t - 3$ for $0 \le t \le 6$

∴ velocity at P (at 6 sec.) $v = 2\times6-3 = \underline{9\,ms^{-1}}$ ✓

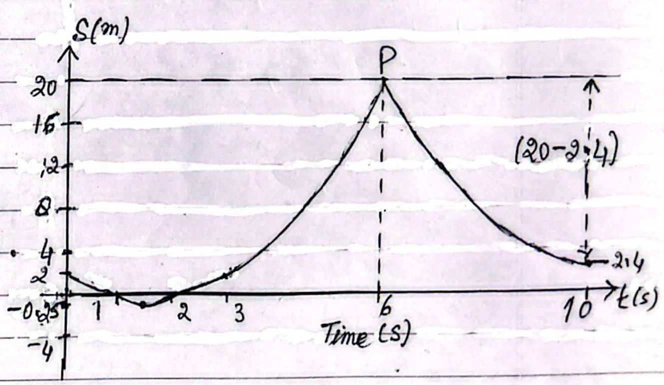
and after 6 sec. when leaving P, $S = \frac{24}{t} - \frac{t^2}{4} + 25$ —② $t \ge 6$

$$v = \frac{dS}{dt} = -\frac{24}{t^2} - \frac{1}{2}t$$

∴ Velocity when leaving P, $v = -\frac{24}{6^2} - \frac{1}{2}\times6 = \underline{-3.67\,ms^{-1}}$ ✓

(c) from ① $0 \le t \le 6$

$t=0 \to S=2$
$t=1 \to S=0$
$t=1.5 \to S=-0.25$
$t=2 \to S=0$
$t=3 \to S=2$
$t=6 \to S=20$
$t=10 \to S=2.4$



Required distance $0 \le t \le 10$.

$= 2 + 0.25 + 0.25 + 20 + (20 - 2.4)$

$= \underline{40.1\,m}$ ✓

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5. A particle P is projected vertically upwards with speed $5 ms^{-1}$ from a point A which is 2.8 m above the horizontal ground.

(a) Find the greatest height above the ground reached by P. ---[3]

(b) Find the length of time for which P is at a height more than 3.6 m above the ground.

[S-20/41/Q3] ---[4]

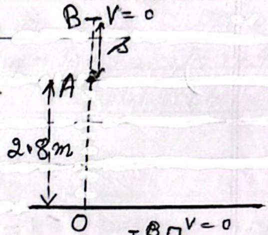
Solution (a) $v^2 = u^2 - 2gs$ (at the heighest point)

$0 = 5^2 - 2 \times 10 \times s$ ($v = 0$, $u = 5$)

$\Rightarrow AB = s = 1.25 m$

$\therefore$ Height above the ground

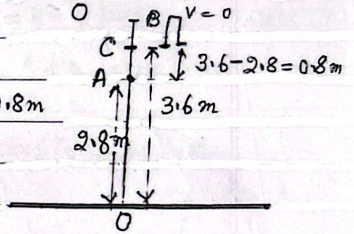
$= OA + AB = 2.8 + 1.25 = \underline{4.05 m}$



(b) At $h = 3.6 \Rightarrow OC = 3.6$

$\therefore$ distance above A = OC − OA

$= 3.6 - 2.8 = 0.8 m$



$s = ut - \frac{1}{2}gt^2$

$\Rightarrow 0.8 = 5t - \frac{1}{2} \times 10t^2$ ($u = 5 ms^{-1}$)

$\Rightarrow 5t^2 - 5t + 0.8 = 0$

$\Rightarrow 25t^2 - 25t + 4 = 0 \Rightarrow (5t-4)(5t-1) = 0 \Rightarrow t = 0.2 s$ & $0.8 s$

$\Rightarrow$ Particle takes 0.2 s to reach C from A and 0.8 s to come back at C again. hence total time above C = 0.8 − 0.2 = $\underline{0.6 s}$

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6. A particle moves in a straight line AB. The velocity v ms^{-1} of the particle t s after leaving A is given by, $v = k(t^2 - 10t + 21)$, where k is a constant. The displacement of the particle from A, in the direction towards B, is 2.85 m when $t = 3$ and is 2.4 m when $t = 6$.

(a) Find the value of k. Hence find an expression, in terms of t, for the displacement of the particle from A. ---[7]

(b) Find the displacement of the particle from A when its velocity is a minimum. S-20/41/Q6 --[4]

Solution (a) $v = k(t^2 - 10t + 21)$ —— ①

$\therefore$ displacement $S = \int k(t^2 - 10t + 21)\,dt = k\left(\frac{t^3}{3} - 5t^2 + 21t\right) + C$ —— ②

Given $S = 2.85$ at $t = 3 \Rightarrow 2.85 = k\left(\frac{3^3}{3} - 5\times3^2 + 21\times3\right) + C$ from ②

$\Rightarrow 27k + C = 2.85$ —— ③

and $S = 2.4$ at $t = 6 \Rightarrow 2.4 = k\left(\frac{6^3}{3} - 5\times6^2 + 21\times6\right) + C$

$\Rightarrow 18k + C = 2.4$ —— ④

solving ③ and ④ $\Rightarrow k = 0.05$ and $C = 1.5$

$\therefore$ displacement $S = 0.05\left(\frac{t^3}{3} - 5t^2 + 21t\right) + 1.5$ —— ⑤

(b) Now from ① $v = 0.05(t^2 - 10t + 21)$ —— ⑥ $(k = 0.05)$

$\frac{dv}{dt} = 0.05(2t - 10) = 0$ for Minimum v

$\Rightarrow t = 5s.$

$\therefore$ displacement at $t = 5$

$= 0.05\left(\frac{5^3}{3} - 5\times5^2 + 21\times5\right) + 1.5$ (from ⑤)

$= 0.05\times\frac{65}{3} + 1.5$

$= 1.083 + 1.5 = 2.583$ m

$\therefore S = \underline{2.58\text{ m}}$ ✓

7. A tram starts from rest and moves with uniform acceleration for 20 s. The tram then travels at a constant speed, $V\ ms^{-1}$, for 170 s before being brought to rest with a uniform deceleration of magnitude twice that of the acceleration. The total distance travelled by the tram is 2.775 km.

(a) Sketch a velocity-time graph for the motion, stating the total time for which the tram is moving. ---[2]

(b) Find V. --[2]

(c) Find the magnitude of the acceleration. --[2]

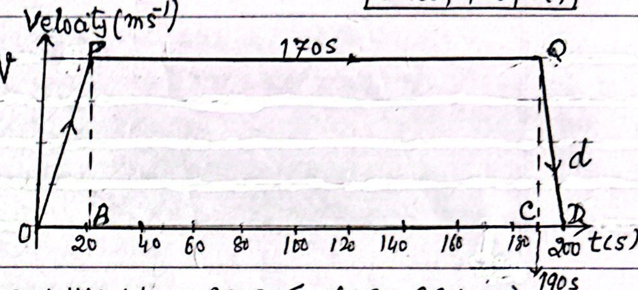
S-20/42/Q1

Solution (a) Trapezium.

acc. $a = \frac{V}{20}$

dec. $d = 2a = \frac{2V}{20} = \frac{V}{10}$

$\therefore$ Time $= 10$ s. ✓



Time $= 20 + 170 + 10 = 200$ s ✓ (OB + BC + CD)

(b) Distance = area of trapezium =

$= \frac{1}{2}(200 + 170) \times V = 2775$ m (2.775 km)

$\Rightarrow$ $185V = 2775$

$\Rightarrow$ $V = \underline{15\ ms^{-1}}$ ✓

(c) acc. $a = \frac{V}{t} = \frac{15}{20} = 0.75$

$\therefore$ $a = \underline{0.75}\ ms^{-2}$ ✓

8. A particle P moves in a straight line. The velocity $v\ ms^{-1}$ at time t s is given by:

$v = 2t+1$ for $0 \leq t \leq 5$

$v = 36 - t^2$ for $5 \leq t \leq 7$

$v = 2t - 27$ for $7 \leq t \leq 13.5$

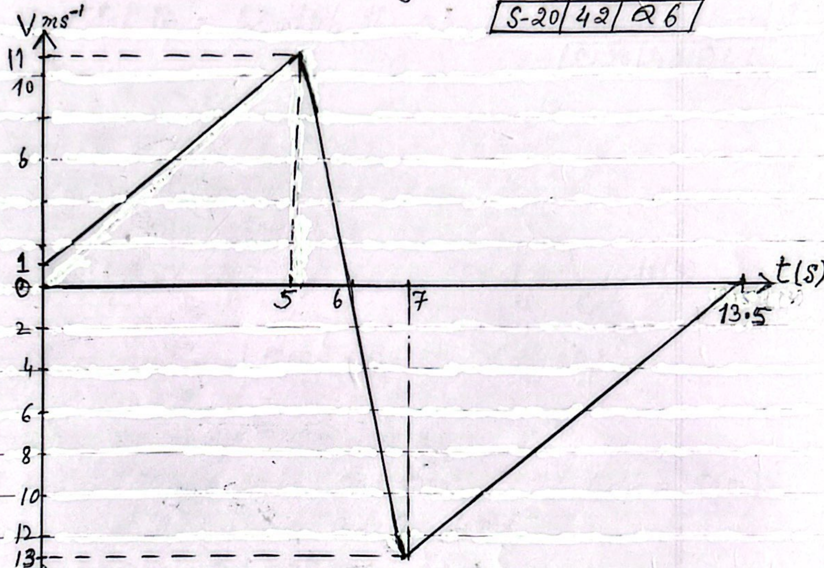
(a) Sketch the velocity-time graph for $0 \leq t \leq 13.5$ --- [3]

(b) Find the acceleration at the instant when $t=6$ --- [2]

(c) Find the total distance travelled by P in the interval $0 \leq t \leq 13.5$ --- [5]

S-20/42/Q6

Solution (a)



(b) $V = 36 - t^2$ $5 \leq t \leq 7$

acc. $\frac{dv}{dt} = -2t \Rightarrow$ acc at $t=6$ $\left(\frac{dv}{dt}\right)_{t=6} = -2\times 6 = \underline{-12}$ ✓

(c) Total distance $S = \int_0^5 (2t+1)\,dt + \int_5^6 (36-t^2)\,dt + \left| \int_6^7 (36-t^2)\,dt + \int_7^{13.5} (2t-27)\,dt \right|$

$= \left[t^2+t\right]_0^5 + \left[36t - \frac{t^3}{3}\right]_5^6 + \left| \left[36t - \frac{t^3}{3}\right]_6^7 + \left[t^2 - 27t\right]_7^{13.5} \right|$

$= 30 + (5.6) + |(137.7 - 144) + (-182.25 + 140)|$

$= 35.6 + |-48.55|$

$= 35.6 + 48.55$

$= \underline{84.15\ m}$ ✓

9. A car starts from rest and moves in a straight line with constant acceleration $a\ ms^{-2}$ for a distance of $50m$. The car then travels with constant velocity for $500m$ for a period of $25s$, before decelerating to rest. The magnitude of this deceleration is $2a\ ms^{-2}$.

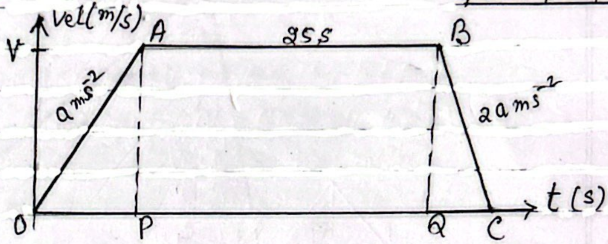
(a) Sketch the velocity-time graph for the motion of the car. ---[1]

(b) Find the value of a. ---[3]

(c) Find the total time for which the car is in motion. ---[3]

[S-20 | 43 | Q4]

Solution: (a)



(b) Vel. (constant for AB) $v = \frac{\text{distance}}{\text{time}} = \frac{500}{25} = 20\ ms^{-1}$ (for AB)

Now for OA → $20^2 = 0 + 2as$

$400 = 2a \times 50m$ (dis = 50m for OA)

$\Rightarrow$ Acc $\underline{a = 4\ ms^{-2}}$

(c) Time for OA, $v = 0 + at$

$\Rightarrow 20 = 4 \times t_1 = t_1 = 5$ sec. (OP = 5 sec)

for BC ⇒ $v = 2a \cdot t$

$20 = 2 \times 4 \times t_2$ (a = 4)

$t_2 = 2.5$ sec. (QC = 2.5s)

So total time $= 5 + 25 + 2.5 = \underline{32.5\ s}$ ✓

10. A particle travels in a straight line PQ. The velocity of the particle t s after leaving P is $v\ ms^{-1}$ where,

$$v = 4.5 + 4t - 0.5t^2$$

(a) Find the velocity of the particle at the instant when its acceleration is zero. ---[3]

The particle comes to instantaneous rest at Q.

(b) Find the distance PQ. ---[6]

S-20	43	Q6

<u>Solution:</u> (a) $v = 4.5 + 4t - 0.5t^2$ ——— (1)

acc. $\frac{dv}{dt} = 4 - t$ ——— (2)

for $a = 0 \Rightarrow 4 - t = 0 \Rightarrow t = 4$

from (1) at $t = 4 \Rightarrow v = 4.5 + 4 \times 4 - 0.5 \times 4^2$

$= \underline{12.5\ ms^{-1}}$ ✓

(b) At Q; $v = 0 \Rightarrow 4.5 + 4t - 0.5t^2 = 0$ (from (1))

$\Rightarrow t^2 - 8t - 9 = 0$

$(t-9)(t+1) = 0 \Rightarrow t = 9\,s$ or $t = -1$ ✗

$\therefore$ distance $PQ = \int v\,dt$

$= \int_0^9 (4.5 + 4t - 0.5t^2)\,dt$

$= \left[4.5t + 2t^2 - \frac{t^3}{6}\right]_0^9$

$= (202.5 - 121.5) - 0$

$= \underline{81\,m.}$ ✓

11. A particle P moves in a straight line. It starts from rest at a point O on the line and at time t s after leaving O it has acceleration a ms^{-2}, where $a = 6t - 18$.
Find the distance P moves before it comes to instantaneous rest. ---[6]
[W-20/41/Q4]

Solution: $a = 6t - 18$

$\therefore$ Velocity $v = \int (6t-18)dt$

$\Rightarrow v = 3t^2 - 18t + C$ ——— (1)

$0 = 0 + C$ $[\because$ at $t=0, v=0]$

from (1) $v = 3t^2 - 18t$ ——— (2)

$\therefore S = \int (3t^2 - 18t)dt$

$S = t^3 - 9t^2 + K$ $(\because t=0, S=0 \Rightarrow K=0$

$\therefore S = t^3 - 9t^2$ ——— (3)

Now at instantaneous rest $v = 0 \Rightarrow 3t^2 - 18t = 0$ from (2)

$3t(t-6) = 0$

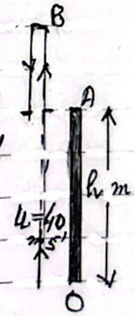
$\Rightarrow t = 0^*, t = 6$

$\therefore$ from (3) at $t = 6$; $S = 6^3 - 9 \times 6^2$

$= 216 - 324 = |-108|$

$\therefore$ distance travelled $= \underline{108\ m}$ ✓

11′ (Not in the Ex. 1) A particle is projected vertically upwards with speed $40 ms^{-1}$ alongside a building of height h m.



(a) Given that the particle is above the level of the top of building for 4 s, find h. ---[4]

(b) One second after the first particle is projected, a second particle is projected vertically upwards from the top of the building with speed $20 ms^{-1}$. Denoting the time after projection of the first particle by t s, find the value of t for which the two particles are at the same height above the ground. [W-20/42/Q5] --[4]

Solution (a) At the max height B, $v = 0 \rightarrow 0 = 40 - gt \Rightarrow t = 4s$

$\therefore$ time to the top of the building $\Rightarrow t = 4 - \frac{1}{2} \times 4 = 2s \Rightarrow h = 40 \times 2 - \frac{1}{2} g \times 2^2 = \underline{60\ m}$ ✓

(Continued →)

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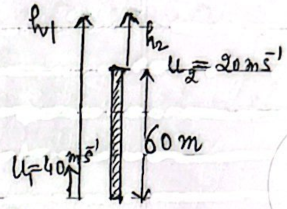
11′ (b) Height of First particle at time 't'

$$h_1 = 40t - \tfrac{1}{2}gt^2$$

$$\Rightarrow h_1 = 40t - 5t^2 \quad \text{—— (1)}$$



Second particle is projected of 1s ⇒ Time = (t−1)
height above the building at time t

$$h_2 = 20(t-1) - \tfrac{1}{2}g(t-1)^2$$

$$h_2 = 20(t-1) - 5(t-1)^2 \quad \text{—— (2)}$$

Now $h_1 = 60 + h_2$ [height of building = 60m]

$$\Rightarrow 40t - 5t^2 = 60 + 20(t-1) - 5(t-1)^2$$

$$\Rightarrow 40t - 5t^2 = 60 + 2t - 20 - 5t^2 + 10t - 5$$

$$\Rightarrow 5t = 35 \Rightarrow \underline{t = 3.5\,s}$$

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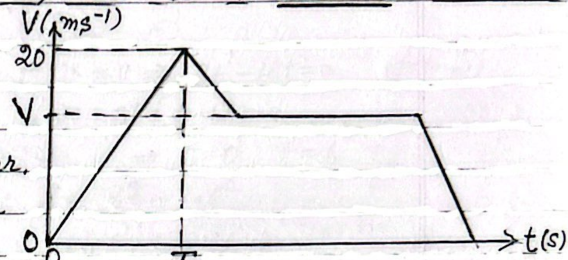
12. The diagram shows a velocity-time graph which models the the motion of a car. The graph consists of four straight line segments.



The car accelerates at a constant rate of $2\,ms^{-2}$ from rest to a speed of $20\,ms^{-1}$ over a period of T s. It then decelerate at a constant rate for 5 seconds before travelling at a constant speed of $V\,ms^{-1}$ for 27.5 s. The car then decelerates to rest at a constant rate over a period of 5 s.

(a) Find T. --- [1]

(b) Given that the distance travelled up to the point at which the car begins to move with constant speed is one third of the total distance travelled, find V.

[W-20/42/Q4] --- [4]

Solution (a) acc. $a = \frac{V}{t} \Rightarrow \frac{20}{T} = 2 \Rightarrow T = 10\,s$

(b) Distance before constant speed

$$= \tfrac{1}{2} \times 10 \times 20 + \tfrac{1}{2}(20+V) \times 5$$

$$= 150 + 2.5V \quad \text{—— (1)}$$

Distance travelled after constant speed: $27.5V + \tfrac{1}{2} \times 5V = 30V$ —— (2)

from (1) and (2) and given,

$$(150 + 2.5V) = \tfrac{1}{3}(150 + 2.5V + 30V)$$

$$\Rightarrow V = \underline{12\,ms^{-1}}$$

13. A particle moves in a straight line from a point O with velocity $1.72\,ms^{-1}$. The acceleration $a\,ms^{-2}$ of the particle, t s after leaving O, is given by $a = 0.1t^{3/2}$.

(a) Find the value of t when the velocity of P is $3\,ms^{-1}$. ---[4]

(b) Find the displacement of P from O when $t=2$, giving your answer correct to 2 decimal places. --[3]

W-20/42/Q7

Solution (a) $v = \int a\,dt \Rightarrow v = \int 0.1t^{3/2}dt = 0.04t^{5/2} + C$ — ①

given $v = 1.72$ when $t=0 \Rightarrow C = 1.72$ from ①

$\therefore v = 0.04t^{5/2} + 1.72$ — ②

now given $v = 3 \Rightarrow 0.04t^{5/2} + 1.72 = 3$ (from ②)

$\Rightarrow 0.04t^{5/2} = 1.28$

$\Rightarrow t^{5/2} = 1.28/0.04 = 32 = 4^{5/2}$

$\Rightarrow \underline{t = 4}$ ✓

(b) Displacement $s = \int v\,dt = \int (0.04t^{5/2} + 1.72)dt$ from ②

$= \frac{2}{175}t^{7/2} + 1.72t + K$

$s=0$ for $t=0 \Rightarrow K=0$

$\therefore s = \frac{2}{175}t^{7/2} + 1.72t$ — ③

$\therefore$ displacement at $t=2$; $s = \frac{2}{175} \times 2^{7/2} + 1.72 \times 2$

$= \frac{16\sqrt{2}}{175} + 3.44$

$= 0.129 + 3.44$

$= \underline{3.57\,m}$ ✓

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14. A particle P is projected vertically upwards with speed $v\ ms^{-1}$ from a point on the ground. P reaches it greatest height after 3 s.

(a) Find v ---[1]

(b) Find the greatest height of P above the ground. ---[2]

W-20/43/Q1

Solution (a) $v = u + gt$ [at greatest height, $v = 0$]

$0 = v - 10 \times 3$

$\Rightarrow V = 30\ ms^{-1}$ ✓

(a) Now $V^2 = u^2 + 2gh \Rightarrow 0 = 30^2 - 2 \times 10 \times h \Rightarrow h = 45\ m$ ✓

15. A particle P moves in a straight line. It starts at a point O on the line and at time t s, after leaving O it has velocity $v\ ms^{-1}$, where $v = 4t^2 - 20t + 21$.

(a) Find the value of t for which P is at instantaneous rest. ---[2]

(b) Find the initial acceleration of P. ---[2]

(c) Find the minimum velocity of P. ---[2]

(d) Find the distance travelled by P during the time when its velocity is negative. W-20/43/Q5 --[4]

Solution (a) $v = 4t^2 - 20t + 21 = 0$ for instantaneous rest.

$(2t-3)(2t-7) = 0 \Rightarrow t = 1.5\ s$ and $t = 3.5\ s$ ✓

(b) acc. $a = \frac{dv}{dt} = \frac{d}{dt}(4t^2 - 20t + 21) = 8t - 20$

initial acceleration, ∴ acc. at $t = 0 \Rightarrow a = 8 \times 0 - 20 = -20\ ms^{-2}$ ✓

(c) for Min. v, $\frac{dv}{dt} = 0 \Rightarrow 8t - 20 = 0 \Rightarrow t = 2.5\ s$.

∴ Min $v = 4(2.5)^2 - 20 \times 2.5 + 21 = 25 - 50 + 21$

$V_{min} = -4\ ms^{-1}$ ✓

(d) distance $s = \int v\,dt = \int(4t^2 - 20t + 21)\,dt = \frac{4}{3}t^3 - 10t^2 + 21t + C$

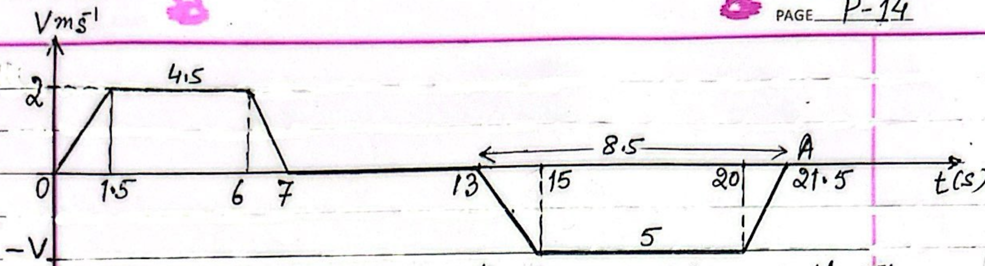
$s = 0$ for $t = 0 \Rightarrow C = 0$

∴ $s = \frac{4}{3}t^3 - 10t^2 + 21t$ —— ①

Velocity is negative $1.5 < t < 3.5$

∴ Distance $= \left[\frac{4}{3}t^3 - 10t^2 + 21t\right]_{1.5}^{3.5} = 8.16 - 13.5$

$= |-5.33| = 5.33\ m$ ✓



16. An elevator moves vertically, supported by a cable. The diagram shows a velocity-time graph which models the motion of the elevator. The graph consists of 7 straight line segments.

The elevator accelerates upwards from rest to a speed of $2\,ms^{-1}$ over a period 1.5s and then travels at this speed for 4.5s, before decelerating to rest over a period of 1s.
The elevator then remains at rest for 6s, before accelerating to a speed of $V\,ms^{-1}$ downwards over a period of 2s. The elevator travels at this speed for a period of 5s, before decelerating to rest over a period of 1.5s.

(a) Find the acceleration of the elevator during the first 1.5s. --- [1]

(b) Given that the elevator starts and finishes its journey on the ground floor, find V. --- [2]

(c) The combined weight of the elevator and passengers on its upward journey is 1500 Kg. Assuming that there is no resistance to motion, find the tension in the elevator cable on its upwards journey when the elevator is decelerating. --- [3]

[M-21/42/Q4]

Solution:

(a) Let the acceleration during first 1.5s is a.

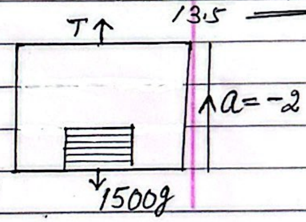
$v = u + at \Rightarrow 2 = 0 + a \times 1.5 \Rightarrow = \frac{2}{1.5} = \frac{4}{3}\,ms^{-2}$

(b) Upward distance and the downward distances are equal.

$\frac{1}{2}(7+4.5)\times 2 = \frac{1}{2}(8.5+5)\times V \Rightarrow 11.5 = \frac{1}{2}\times 13.5V \Rightarrow V = \frac{11.5\times 2}{13.5} = 1.7\,ms^{-1}$

(c) $W = 1500g$, for decleration $0 = 2 + a\times 1$

$\Rightarrow a = -2\,ms^{-2}$



Now $T - 1500g = 1500(-2)$

$T - 15000 = -3000 \Rightarrow T = \underline{12000\,N}$

17. A particle moves in a straight line. It starts from rest from a fixed point O, on a line. Its velocity at time t seconds after leaving O is $v\ ms^{-1}$, where $v = t^2 - 8t^{3/2} + 10t$

(a) Find the displacement of the particle from O when $t=1$ --- [4]

(b) Show that the minimum velocity of the particle is $-125\ ms^{-1}$ --- [7]

M-21/42/Q6

Solution: $u=0$, $t=0$ and $v = t^2 - 8t^{3/2} + 10t$ at time t.

(a) Displacement at $t=1$ $\nearrow$ $s = \int_0^1 v\,dt = \int_0^1 (t^2 - 8t^{3/2} + 10t)\,dt$

$$= \left[\frac{t^3}{3} - \frac{8t^{5/2}}{5/2} + \frac{10t^2}{2}\right]_0^1$$

$$s = \left(\frac{1}{3} - \frac{16}{5} + 5\right) - 0 = \frac{32}{15}\,m = \underline{2.13\,m}$$ ✓

(b) Now to find the minimum velocity.

$$V = t^2 - 8t^{3/2} + 10t \quad \text{---} (1)$$

$$\frac{dV}{dt} = 2t - 8\times\frac{3}{2}t^{1/2} + 10 = 0 \quad (2)$$ for any stationary point.

$\Rightarrow 2t - 12t^{1/2} + 10 = 0$ [Let $t^{1/2} = a \Rightarrow t = a^2$]

$\Rightarrow 2a^2 - 12a + 10 = 0$

$\Rightarrow a^2 - 6a + 5 = 0$

$(a-5)(a-1) = 0$

$\Rightarrow a = 5$ or $a = 1 \Rightarrow t^{1/2} = 5$; $t^{1/2} = 1$

$\Rightarrow t = 25$; $t = 1$

diff. (2) $\frac{d^2V}{dt} = 2 - 12\times\frac{1}{2}t^{-1/2} = 2 - \frac{6}{\sqrt{t}}$

$$\left(\frac{d^2V}{dt^2}\right)_{t=25} = 2 - \frac{6}{\sqrt{25}} = 2 - \frac{6}{5} = \frac{4}{5} > 0 \quad \text{Min} ✓$$

at $t=1$ $\left[\left(\frac{d^2V}{dt^2}\right)_{t=1} = 2-6 = -4 < 0\right.$ Max

$\therefore$ Velocity is min at $t = 25$

from (1) $V = 25^2 - 8\times 25^{3/2} + 10\times 25$

$= 625 - 1000 + 250$

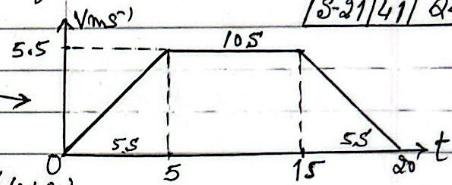
$\therefore$ Min. Velocity $= \underline{-125\ ms^{-1}}$ ✓

Q18. Two cyclists, Isabella and Maria, are having race. They both travel along a straight road with constant acceleration, starting from rest at point A. Isabella accelerates for 5s at a constant rate $a\,ms^{-2}$. She then travels at the constant speed she has reached for 10 s, before decelerating to rest at a constant rate over a period of 5s.
Maria accelerates at a constant rate, reaching a speed of $5\,ms^{-1}$ in a distance of 27.5 m. She then maintains this speed for a period of 10 s, before decelerating to rest at a constant rate over a period of 5 s.

(a) Given $a = 1.1$, find which cyclist travels further. ---[5]

(b) Find the value of a for which the two cyclists travel the same distance. --[2]

[S-21/41/ Q4]



Solution (a) $a = 1.1$, $t = 5s$ (for Isabella)

$v = 0 + at = 1.1 \times 5 = 5.5\,ms^{-1}$

Distance travelled by Isabella:

$= \text{Area of Trapezium} = \frac{1}{2} \times 5.5(10+20)$

$S_I = \underline{82.5}$ ✓

For Maria: $S_M = 27.5 + 5 \times 10 + \frac{1}{2} \times 5 \times 5 = \underline{90}$ ✓

Distance travelled by Isabella is 82.5 m and by Maria is 90 m.

Hence Maria goes further.

(b) $\frac{1}{2} a \times 5^2 + 10 \times 5a + \frac{1}{2} a \times 5^2 = 90$

$\Rightarrow 75a = 90$

$\Rightarrow \underline{a = 1.2\,ms^{-2}}$ ✓

19. A particle moving in a straight line starts from rest at a point A and comes instantaneously to rest at point B. The acceleration of the particle at t s after leaving A is $a\ ms^{-1}$, where, $a = 6t^{1/2} - 2t$

(a) Find the value of t at point B: --[3]

(b) Find the distance travelled from A to the point at which the acceleration of the particle is again zero. --[5]

S-21/41/Q5

Solution:
(a) $a = \frac{dv}{dt} = 6t^{1/2} - 2t$ (Given)

$v = \int (6t^{1/2} - 2t)\,dt$

$v = 4t^{3/2} - t^2 + c$ —— ①

$v = 0$ for $t = 0 \Rightarrow c = 0$

from ① $v = 4t^{3/2} - t^2$ —— ②

Now at B, $v = 0 \Rightarrow 4t^{3/2} - t^2 = 0$

$t^{3/2}[4 - t^{1/2}] = 0$

$\Rightarrow t^{1/2} = 4$ or $t = 0^{x}$

$\Rightarrow t = 16\ s$ ✓

(b) acc. $a = 6t^{1/2} - 2t = 0$

$2t^{1/2}[3 - t^{1/2}] = 0 \Rightarrow t^{1/2} = 3,\ t = 0^{x}$

$t^{1/2} = 3 \Rightarrow t = 9\,s$

$v = \frac{ds}{dt} = 4t^{3/2} - t^2$ from ②

$s = \int_0^9 (4t^{3/2} - t^2)\,dt$

$= \left[\frac{8}{5}t^{5/2} - \frac{t^3}{3}\right]_0^9$

$s = \frac{8}{5}\times(9)^{5/2} - \frac{1}{3}\times 9^3$

$= 145.8\ m$ ✓

20. A particle P moving in a straight line starts from rest at a point O and comes to rest 16 s later. At time t s after leaving O, the acceleration $a\ ms^{-2}$ of P is given by:

$a = 6 + 4t \qquad 0 \le t < 2,$

$a = 14 \qquad 2 \le t < 4,$

$a = 16 - 2t \qquad 4 \le t \le 16$

There is no sudden change in velocity at any instant.

(a) Find the value of t when the velocity of P is $55\ ms^{-1}$ --[5]

(b) Complete the sketch of the velocity-time graph. S-21/42/Q7 - [2]

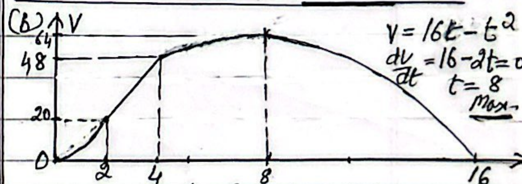
(c) Find the distance travelled by P when it is decelerating. --[3]

Solution: (a) $V = \begin{cases} 6t + 2t^2 + c & 0 \le t < 2 \\ \text{or } 6t + 2t^2 \text{ — } & 0 \le t < 2 \checkmark \quad c = 0, (V = 0, t = 0) \\ 14t + c & 2 \le t < 4 \quad V = 20, t = 2 \\ \Rightarrow c = -8 & \\ 14t - 8 & 2 \le t < 4 \checkmark \\ 16t - t^2 + c & [t = 4, V = 48] \\ 16t - t^2; & 4 \le t < 16 \quad c = 0 \end{cases}$

for $V = 55 = 16t - t^2 \quad 4 \le t \le 16$

$\Rightarrow t^2 - 16t + 55 = 0 \Rightarrow t = 5,\ t = 11$ ✓

(b)



(c) It decelerates from $t = 8$ to $t = 16$

$S = \int_8^{16} (16t - t^2)\,dt = \left[8t^2 - \frac{1}{3}t^3\right]_8^{16} = 341\frac{1}{3}$ ✓

Scanned with CamScanner

21. A particle is projected vertically upwards with speed $u\ ms^{-1}$ from a point on horizontal ground. After 2 seconds, the height of the particle above the ground is 24 m.

(a) Show that $u = 22$. ---[2]

(b) The height of the particle above the ground is more than h m for a period of 3.6 s. Find h. ---[4]

S-21/43/Q4

Solution (a) $h = 24\,m$, $t = 2\,s$, upwards

$h = ut - \frac{1}{2}gt^2$

$24 = u\times 2 - \frac{1}{2}\times 10\times 2^2 \Rightarrow u = 22\ ms^{-1}$

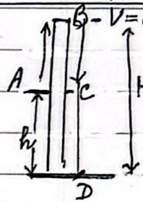
(b) B is the heighest point.

$BD = H$

$v^2 = u^2 - 2gH$

$0 = 22^2 - 2\times 10\times H$

$H = 24.2\,m$



(Time above h) Time to travel $AB + BC = 3.6\,s$

Time to fall $BC = \frac{3.6}{2} = 1.8\,s$

$BC = 0 + \frac{1}{2}gt^2 = \frac{1}{2}\times 10\times 1.8^2 = 16.2\,m$

$\therefore$ Required $h = BD - BC = 24.2 - 16.2 = 8\,m$

$\Rightarrow h = 8\,m$

22. A particle moves in a straight line and passes through the point A at time $t=0$. The velocity of the particle at time t s after leaving A is $v\ ms^{-1}$, where $v = 2t^2 - 5t + 3$

(a) Find the times at which the particle is instantaneously at rest. Hence find the minimum velocity of the particle. --[4]

(b) Sketch the velocity-time graph for the first 3 seconds of motion. --[3]

(c) Find the distance travelled between the two times when the particle is instantaneously at rest. --[3]

[S-21/43/Q6]

Solution: (a) $v = 2t^2 - 5t + 3$ ① for instantly at rest

$= (2t-3)(t-1) = 0$

$\Rightarrow t = 1$ or $t = 1.5$ ✓

for minimum velocity $\frac{dv}{dt} = 0$

$\frac{dv}{dt} = 4t - 5 = 0 \Rightarrow t = \frac{5}{4} = 1.25$

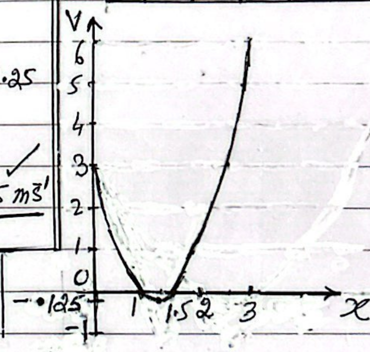
∴ for ① minimum vel.

$V_{min} = (2\times\frac{5}{4} - 3)(\frac{5}{4} - 1)$ ✓

$= -\frac{2}{4} \times \frac{1}{4} = -0.125\ ms^{-1}$

(b)

t	0	1	1.25	1.5	3
V	3	0	$\bar{0}.125$	0	6



(c) $V = \frac{ds}{dt} = 2t^2 - 5t + 3$

$s = \int_1^{1.5} (2t^2 - 5t + 3)\,dt$

Distance $= \left[\frac{2}{3}t^3 - \frac{5}{2}t^2 + 3t\right]_1^{1.5} = 0.0417$ ✓

23. A bus moves from rest with constant acceleration for 12s, It then moves with constant speed for 30s before decelerating uniformly to rest in a further 6s, The total distance travelled is 585m,

(a) Find the constant speed of the bus, ---[2]

(b) Find the magnitude of the deceleration. --[1]

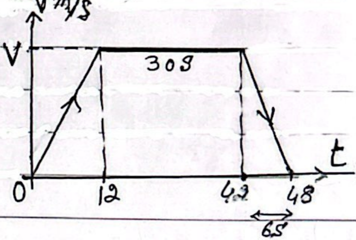
W-21/41/Q1

Solution (a) Let the constant speed = $v\ ms^{-1}$

Total distance travelled;

$\frac{1}{2}(48+30)v = 585$ (Given)

$v = 585/39 = \underline{15\,ms^{-1}}$ ✓



(b) Deceleration (magnitude) $= \frac{v}{t} = \frac{15}{6} = 2.5$

Dec. $= \underline{2.5\,ms^{-2}}$

24. A particle P moves in a straight line starting from a point O and comes to rest 14 s later. At t s after leaving O, the velocity v ms^{-1} of P is given by:

where p and q are constants. $\begin{cases} v = pt^2 - qt & 0 \le t \le 6 \\ v = 63 - 4.5t & 6 \le t \le 14 \end{cases}$

The acceleration of P is zero when $t = 2$

(a) Given that there is no instantaneous changes in velocity, find p and q. --[3]

(b) Sketch the velocity-time graph. --[3]

(c) Find the total distance travelled by P during the 14 s. --[5]

[W-21|41|Q6]

Solution: $v = pt^2 - qt$ ① $0 \le t \le 6$

acc. $\frac{dv}{dt} = 2pt - q$

(a) at $t = 2$, acc $= 4p - q = 0$ ② (Given)

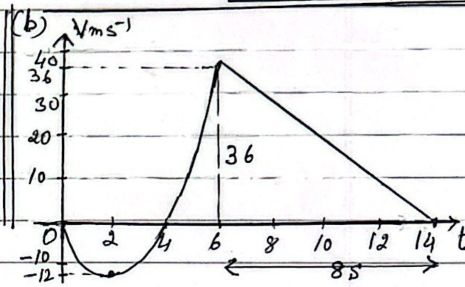
at $t = 6$ $\begin{Bmatrix} pt^2 - qt \\ 63 - 4.5t \end{Bmatrix} \Rightarrow 36p - 6q = 36$

$\Rightarrow 6p - q = 6$ --③

Solving ② & ③ $\underline{p = 3}$ ✓ and $\underline{q = 12}$ ✓

Now $\begin{cases} v = 3t^2 - 12t & 0 \le t \le 6 \\ v = 63 - 4.5t & 6 \le t \le 14 \end{cases}$

(b)



(c) Total distance travelled in 14 s = |Area between the curve and x-axis t = 0 to 4|
+ (Area under the curve and x-axis t = 4, to 6)
+ (area of triangle t = 6 to 14)

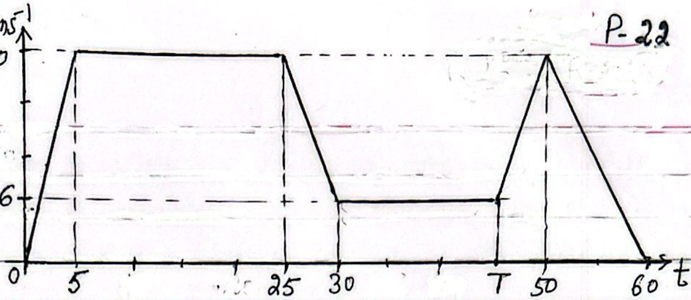
$= \left| \int_0^4 (3t^2 - 12t)dt \right| + \int_4^6 (3t^2 - 12t)dt + \frac{1}{2} \times (14 - 6) \times 36$

$= \left| \left[t^3 - 6t^2 \right]_0^4 \right| + \left[t^3 - 6t^2 \right]_4^6 + \frac{1}{2} \times 8 \times 36$

$= |(64 - 96)| + [(216 - 216) - (64 - 96)] + 144$

$= 32 + 32 + 144 = \underline{208\,m}$ ✓

25.



The diagram shows a velocity-time graph which models the motion of a car. The graph consists of six straight line segment. The car accelerates from rest to a speed of $20\,ms^{-1}$ over a period of 5s, and then travels at this speed for a further 20s. The car then decelerates to a speed of $6\,ms^{-1}$ over a period of 5s. The speed is maintained for a further $(T-30)$s. The car then accelerates again to a speed of $20\,ms^{-1}$ over a period of $(50-T)$s, before decelerating to rest over a period of 10s.

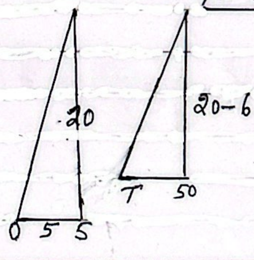
(a) Given that during the two stages of the motion when the car is accelerating, the accelerations are equal, find the value of T. ---[2]

(b) Find the total distance travelled by the car during the motion. ---[2]

[W-21/42/Q1]

<u>Solution</u> (a) using similarity of triangles

$$\frac{20-6}{50-T} = \frac{20}{5} \quad \left(\frac{V_1}{t_1} = \frac{V_2}{t_2}\right) \text{(accelerations are equal)}$$



$$\Rightarrow 50-T = \frac{14}{4} \Rightarrow T = 50 - \frac{14}{4}$$

$$T = 46.5\,s$$

(b) Distance (for $T = 46.5$) $= \frac{1}{2}\times5\times20 + 20\times20 + \frac{1}{2}\times5(20+6) + 6(T-30) + \frac{1}{2}(50-T)(20+6) + \frac{1}{2}10\times20$

$= 50 + 400 + 65 + 6\times16.5 + \frac{1}{2}\times3.5\times26 + 100$

$= 50 + 400 + 65 + 99 + 45.5 + 100$

$= 759.5\,m$

26. A cyclist starts from rest at a point A and travels along a straight road AB, coming to rest at B. The displacement of the cyclist from A at time t s after the start is s m, where

$$s = 0.004(75t^2 - t^3)$$

(a) Show that the distance AB is 250 m. ---[4]

(b) Find the maximum velocity of the cyclist. ---[3]

[W-21/42/Q4]

Solution (a) $s = 0.004(75t^2 - t^3)$ — ①

Vel. $v = \frac{ds}{dt} = 0.004(150t - 3t^2)$

$v = 0.6t - 0.012t^2$ — ②

Now $v = 0 \Rightarrow 0.6t - 0.012t^2 = 0$

$0.06t[1 - 0.02t] = 0$

$\Rightarrow t = 0$ (at A); $t = 50$ (at B)

∴ distance AB, Put $t = 50$ in ①

$s = 0.004(75 \times 50^2 - 50^3)$

$= 50^2(0.3 - 0.2)$

$\underline{s} = 2500 \times 0.1 = \underline{250\,m}$ ✓

To find max. velocity $\frac{dv}{dt} = 0$

$\frac{dv}{dt} = 0.6 - 0.024t$ — ③

for stationary value of v

$0.6 - 0.024t = 0$

$t = \frac{0.6}{0.024} = 25$ ✓

diff ③ $\frac{d^2v}{dt^2} = -0.24 < 0$ hence Max at $t = 25$ ✓

Max. Vel at $= 25$ $= 0.004(150 \times 25 - 3 \times 25^2)$

$= 0.004 \times 25(150 - 75)$

Max Vel $= 0.004 \times 25 \times 75 = \underline{7.5}$ ✓

27. A particle P moves in a straight line, starting for rest at a point O on the line. A time t s after leaving O the acceleration of P is $k(16-t^2)\,ms^{-2}$, where k is a positive constant, and the displacement from O is s m. The velocity of P is $8\,ms^{-1}$ when $t=4$.

(a) Show that $s = \frac{1}{64}t^2(96-t^2)$ ---[5]

(b) Find the speed of P at the instant that it returns to O. ---[3]

(c) Find the maximum displacement of the particle from O. ---[3]

W-21 | 43 | Q5

Solution (a) $a = \frac{dv}{dt} = (16k - kt^2)$ --- ①

$v = \int (16k - kt^2)\,dt$

$v = 16kt - \frac{1}{3}kt^3 + \cancel{c}$ — ② $[c=0 \text{ as } v=0, t=0]$

$v=8$ for $t=4$ in ②

$8 = 16k\times 4 - \frac{1}{3}k\times 4^3 \Rightarrow k = \frac{3}{16}$ ✓

from ② $v = 3t - \frac{t^3}{16}$ — ③ $\left(= \frac{ds}{dt}\right)$

$S = \int \left(3t - \frac{t^3}{16}\right) dt$

$s = \frac{3}{2}t^2 - \frac{t^4}{64} = \frac{t^2}{64}(96-t^2)$ — ④ ✓

(b) $s=0$, from ④

$\Rightarrow \frac{t^2}{64}(96-t^2) = 0$

$t^2 = 96 \Rightarrow t = 4\sqrt{6}$ ✓

from ③. at $t = 4\sqrt{6}$

$V = 3\sqrt{96} - \frac{(\sqrt{96})^3}{16}$

$= \sqrt{96}\left[3 - \frac{96}{16}\right]$

$= -3\sqrt{96}$

$= -29.4\,ms^{-1}$ ✓

(c) for max displacement $v=0$

from ③ $3t - \frac{t^3}{16} = 0$

$t\left[3 - \frac{t^2}{16}\right] = 0$

$\Rightarrow t^2 = 48 \Rightarrow, t = 4\sqrt{3}$ ✓

from ④ for $t = 4\sqrt{3}$

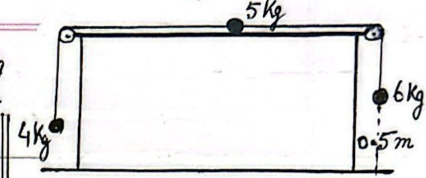
Max distance.

$s = \frac{1}{64}\times(4\sqrt{3})^2\left[96 - (4\sqrt{3})^2\right]$

$= \frac{48}{64}(96-48) = \frac{48\times 48}{64} = 36$

$\therefore$ Max distance $= \underline{36\,m}$ ✓

28. The diagram shows a particle of mass 5 kg on a rough horizontal table, and two light inextensible strings attached to it passing over smooth pulleys fixed at the edges of the table. Particles of masses 4 kg and 6 kg hang freely at the ends of the strings. The particle of mass 6 kg is 0.5 m above the ground. The system is in limiting equilibrium.



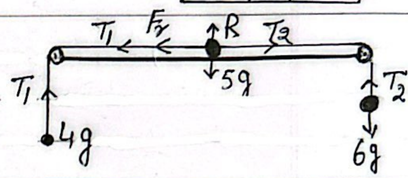
Show that the coefficient of friction between the 5 kg particle and the table is 0.4. --[2]

[W-21/43/Q6(a)]

Solution: Normal reaction $R = 5g$

System is in equilibrium,

$T_1 = 4g$, $T_2 = 6g$, force of friction $= F_r$



$\therefore \quad F_r + T_1 = T_2 \Rightarrow F_r + 4g = 6g \Rightarrow F_r = 6g - 4g = 2g.$

$\therefore$ Coefficient of friction $\mu = \frac{F}{R} = 2g/5g = \underline{0.4}$ ✓

29. A particle P is projected vertically upwards from horizontal ground with speed u m s^{-1}. P reaches a maximum height of 20 m above the ground.

(a) Find the value of u. ---[2]

(b) Find the total time for which P is atleast 15 m above the ground. --[3]

[M-22/22/Q2]

Solution (a) for max height $V = 0$

$0 = u^2 - 2g \times 20 \quad [v^2 = u^2 - 2gh]$

$u^2 = 2 \times 10 \times 20 = 400 \Rightarrow u = \underline{20 \text{ m/s}}$

(b) $S = ut - \frac{1}{2}gt^2$

$15 = 20t - \frac{1}{2} \times 10\, t^2 \quad [S = 15\text{ m}]$

$\Rightarrow 5t^2 - 20t + 15 = 0$

$t^2 - 4t + 3 = 0$

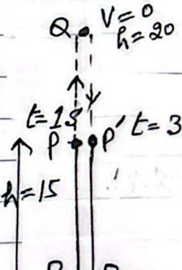
$(t-3)(t-1) = 0 \Rightarrow t = 1, 3$ s.

$h \geq 15$

from PQ and back to P'

Time $= 3 - 1$

$= \underline{2 \text{ seconds}}$ ✓



30. A cyclist starts from rest at a fixed point O and moves a straight line, before coming to rest k seconds later. The acceleration of the cyclist at time t s after leaving O is a ms^{-2}, where $a = 2t^{-1/2} - \frac{3}{5}t^{1/2}$ for $0 < t \le k$.

(a) Find the value of k. --- [4]

(b) Find the maximum speed of the cyclist. --- [3]

(c) Find an expression for the displacement from O in terms of t. Hence find the total distance travelled by the cyclist from the time at which she reaches her maximum speed until she comes to rest. --- [4]

[M-22/22/Q6]

Solution (a) acceleration $a = 2t^{-\frac{1}{2}} - \frac{3}{5}t^{1/2}$ for $0 < t \le k$. --- ①

velocity $v = \int (2t^{-1/2} - \frac{3}{5}t^{1/2})dt = 4t^{1/2} - \frac{2}{5}t^{3/2} + c$; $\{v = 0, t = 0 \Rightarrow c = 0$

$\therefore v = 4t^{1/2} - \frac{2}{5}t^{3/2}$ --- ②

for particle at rest $v = 0$, $4t^{1/2} - \frac{2}{5}t^{3/2} = 0 \Rightarrow t^{1/2}[4 - \frac{2}{5}t] = 0$

$\Rightarrow t = 10 \Rightarrow k = 10s$ --- ③

(b) Max speed when acceleration $a = 2t^{-1/2} - \frac{3}{5}t^{1/2} = 0$

$\Rightarrow t^{-1/2}[2 - \frac{3}{5}t] = 0 \Rightarrow t = \frac{10}{3}s$

from at $t = \frac{10}{3}$, from ② $t^{1/2}[4 - \frac{2}{5}t]$

$= \sqrt{\frac{10}{3}}\left[4 - \frac{2}{5} \times \frac{10}{3}\right] = 4.87\, ms^{-1}$ (4.868)

(c) from ② displacement $s = \int (4t^{1/2} - \frac{2}{5}t^{3/2})dt$

$s = \frac{8}{3}t^{3/2} - \frac{4}{25}t^{5/2} + c$ [$s = 0, t = 0 \Rightarrow c = 0$

Now Max speed at $t = 10/3$, at rest $t = 10s$ $(t \ne 0)$

$\therefore$ Required distance $= \left[\frac{8}{3}t^{3/2} - \frac{4}{25}t^{5/2}\right]_{10/3}^{10}$

$= \left[t^{3/2}\left(\frac{8}{3} - \frac{4}{25}t\right)\right]_{10/3}^{10}$

$= \left[10^{3/2}\left(\frac{8}{3} - \frac{4}{25} \times 10\right) - \left(\frac{10}{3}\right)^{3/2}\left(\frac{8}{3} - \frac{4}{25} \times \frac{10}{3}\right)\right]$

$= 33.73 - 12.98 = 20.7m$ (20.74)

31. A car starts from rest and moves in a straight line with constant acceleration for a distance of 200m, reaching a speed of $25 ms^{-1}$. The car then travels at the speed for 400 m, before decelerating uniformly to rest over a period of 5s.

(a) Find the time for which the car is accelerating. --- [2]

(b) Sketch the velocity-time graph for the motion of the car, showing the key points. --- [2]

(c) Find the average speed of the car during the motion. --- [3]

S-22/41/Q1

Solution (a) $200 = \frac{(0+25)}{2} t$ $\quad \left(S = \frac{(u+v)}{2} t\right) \Rightarrow t = \frac{200 \times 2}{25} = 16\,s$ ✓

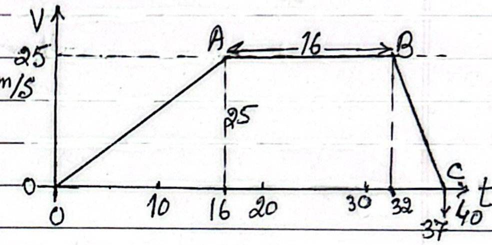
(b) Velocity time graph:

Trapezium;

(0,0), (16,25), (32,25) (37,0),

distance 400m, speed = $25 ms^{-1}$

for AB → $t = \frac{S}{t} = \frac{400}{25} = 16\,s.$



(C)

Total distance = area of Trapezium OABC $= \frac{1}{2}(37+16) \times 25 = 662.5\,m$

Time = 37s.

∴ Average speed $= S/t = 662.5/37 = 17.9\,ms^{-1}$ ✓

32. A particle starts from a point O and moves in a straight line. The velocity $v\ ms^{-1}$ of the particle at time t s after leaving O is given by: $v = k(3t^2 - 2t^3)$; where k is a constant.

(a) Verify that the particle returns to O when t=2. --- [4]

(b) It is given that the acceleration of the particle is $-13.5\ ms^{-2}$ for the positive value of t at which v=0. Find k and hence find the total distance travelled in the first two seconds of motion. --- [6]

S-22/41/Q6

Solution (a) $V = k(3t^2 - 2t^3)$ —① ⇒ distance $S = \int k(3t^2 - 2t^3)\,dt = k\left[t^3 - \frac{1}{2}t^4\right] + C$

∴ $S = k(t^3 - \frac{1}{2}t^4)$ ---② $\quad$ (at $t=0, S=0 \Rightarrow C=0$)

Now at $t=2$ → $S = k\left[2^3 - \frac{1}{2} \times 2^4\right] = 0$ ✓

∴ The particle returns at O at $t = 2s$

(continued →)

(Continued →)

32(b). from ① $v = k(3t^2 - 2t^3) = 0 \Rightarrow k \times t^2(3-2t) = 0 \Rightarrow t = 0, t = 1.5$

Now diff ① acceleration:

$$a = \frac{dv}{dt} = k(6t - 6t^2) \quad \text{③}$$

Given that $a = -13.5$, when $v = 0$ ($t = 1.5$)

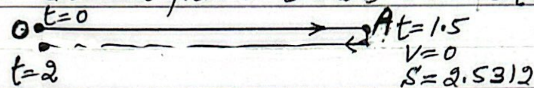
from ③ $-13.5 = k(6 \times 1.5 - 6 \times 1.5^2) \Rightarrow -4.5k = -13.5$

$\Rightarrow \underline{k = 3}$ ✓

Now from ② $S = k\left(t^3 - \frac{1}{2}t^4\right) = 3\left(t^3 - \frac{1}{2}t^4\right)$ — ④ $[k = 3]$

∴ Distance from $t = 0$ to $t = 1.5 = 3\left[t^3 - \frac{1}{2}t^4\right]_0^{1.5} = 3\left(1.5^3 - \frac{1}{2} \times 1.5^4\right) - 0$

$= 2.5312$ m.



∴ Total distance travelled in first two seconds $= 2 \times 2.531$
$= \underline{5.06\text{ m}}$ ✓

33. A particle A, moving along a straight horizontal track with constant speed 8 ms^{-1}, passes a fixed point O. Four seconds later, another particle B passes O, moving along a parallel track in the same direction as A. Particle B has speed 20 ms^{-1} when it passes O and has a constant deceleration of 2 ms^{-1}. B comes to rest when it returns to O.

(a) Find expression, in terms of t, for the displacement from O of each particle t seconds after B passes O. --[3]

(b) Find the value of t when the particles are at the same distance from O; [3]

(c) Sketch the displacement-time graph for both particles; $0 \le t \le 20$. --[3]

[S-22/42/Q4]

Solution

(a) for B passes through O, time $= t$

$u = 20\text{ ms}^{-1}$, $a = -2$

$S_B = 20t - \frac{1}{2} \times 2 \times t^2$ $\left(s = ut + \frac{1}{2}at^2\right)$

$S_B = \underline{20t - t^2}$ ✓ --①

for A, time $= (t+4)$, speed $= 8\text{ ms}^{-1}$

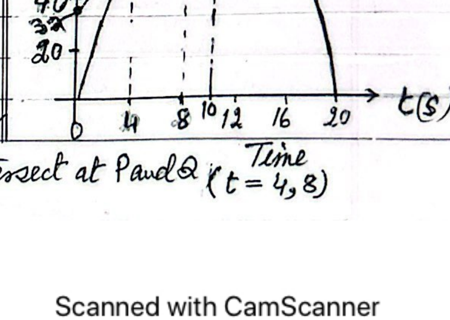
$\Rightarrow S_A = ut_A = 8(t+4) =$

$S_A = \underline{32 + 8t}$ ✓ ②

(b) $S_A = S_B \Rightarrow 32 + 8t = 20t - t^2$

$\Rightarrow t^2 - 12t + 32 = 0 \Rightarrow \underline{t = 4; 8}$ ✓

(c) $S_A = 32 + 8t$; $S_B = 100 - (t-10)^2$



Intersect at P and Q ($t = 4, 8$)

34. A particle P moves in a straight line. The velocity $v\ ms^{-1}$ at time t seconds is given by: $V = 0.5t$ for $0 \le t \le 10$
$V = 0.25t^2 - 8t + 60$ for $10 \le t \le 20$

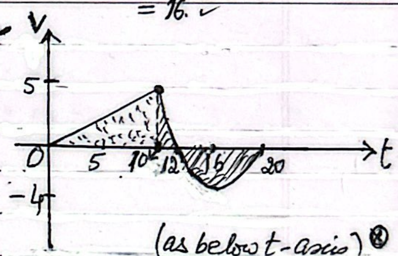
(a) Show that there is an instantaneous change in the acceleration of the particle at $t = 10$. ---[3]

(b) Find the total distance covered by P in the interval $0 \le t \le 20$ --[6]

S-22/42/Q7

Solution: $V = 0.5t$ for $0 \le t \le 10$ -- ①
$V = 0.25t^2 - 8t + 60$ for $10 \le t \le 20$ -- ②
Diff ① $acc = \frac{dV}{dt} = 0.5$ -- ③ [for t=10]
Diff ② $acc.\ \frac{dV}{dt} = 2 \times 0.25t - 8$
$= 0.5t - 8$ -- ④
from $\left(\frac{dv}{dt}\right)_{t=10} = 0.5 \times 10 - 8 = -3$ -- ⑤
from ③ & ⑤ we find change in acc, at $t = 10s$,

ⓑ distance travelled $0 \le t \le 10$
from ① $S = \int_0^{10} 0.5t\,dt = \left[\frac{t^2}{4}\right]_0^{10}$
$= 25m$ ✓ --- ⑥
$V = 0.25t^2 - 8t + 60$; $10 \le t \le 20$
$V = \frac{1}{4}[t^2 - 32t + 240] = 0$
for $t = 12, 20$ ✓ $(V=0)$
Vertex of graph of V,
$t = -\frac{(-32)}{2\times1}$ $\left[t = -\frac{b}{2a}\right]$
$= 16.$ ✓



(b) continued →
now for distance travelled $10 \le t \le 12$, and $12 \le t \le 20$, from ②
$= \int (0.25t^2 - 8t + 60)\,dt$
$S = \frac{0.25t^3}{3} - \frac{8}{2}t^2 + 60t$
Now find for $10 \le t \le 12$ & $12 \le t \le 20$

(as below t-axis) ⊗ this value will come negative

$$\text{Total distance} = 25 + \left[\tfrac{1}{12}t^3 - 4t^2 + 60t\right]_{10}^{12} - \left[\tfrac{1}{12}t^3 - 4t^2 + 60t\right]_{12}^{20}$$
$$= 25 + \left(288 - \frac{850}{3}\right) - \left(\frac{800}{3} - 288\right)$$
$$= 25 + \frac{14}{3} - \left(-\frac{64}{3}\right) ⊗$$
$$= 51m ✓$$

35. A particle P is projected vertically upwards from horizontal ground. P reaches a maximum height of 45 m. After reaching the ground, P comes to rest without rebounding.

(a) Find the speed at which P was projected. ---[2]

(b) Find the total time for which the speed of P is at least $10\,ms^{-1}$. ---[3]

[S-22/43/Q2]

Solution (a) Max height = 45 m, Max height V = 0

$0 = u^2 - 2g \times 45 \quad [V^2 = u^2 - 2gh]$

$\Rightarrow u^2 = 2 \times 10 \times 45 = 900 \Rightarrow u = \sqrt{900}$

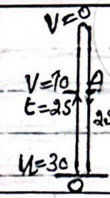
$u = 30\,ms^{-1}$.

for $V = 10\,ms^{-1} \rightarrow V = u - gt$

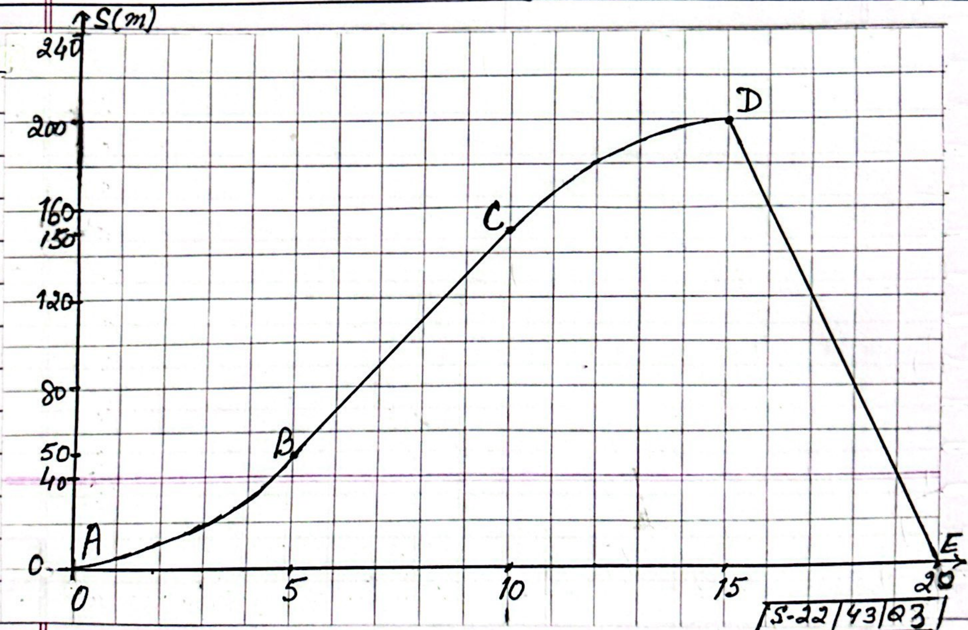
$\Rightarrow 10 = 30 - 10t \Rightarrow t = 2$ sec.

Total Time $V \geq 10$ for OA + AO

$T = 2 \times 2 = 4\,s$.



36.



[S-22/43/Q3]

The displacement of a particle moving in a straight line is s metres, at time t seconds after leaving a fixed point O. The particle starts from rest and passes through points P, Q and R, at times $t = 5$, $t = 10$ and $t = 15$ respectively, and returns to O at time $t = 20$. The distances OP, OQ and OR are 50 m, 150 m and 200 m respectively.

The diagram shows a displacement-time graph which models the motion of the particle from $t = 0$ to $t = 20$. The graph consists of two curved segments AB and CD and two straight line segments BC and DE.

(Continued →)

(→ Continued)

36 (a) Find the speed of the particle between $t=5$ and $t=10$ ---[1]

(b) Find the acceleration of the particle between $t=0$ and $t=5$, given that it is constant. ---[2]

(c) Find the average speed of the particle during its motion. ---[2]

[S-22/43/Q3]

Solution (a) Speed between $t=5$ and $t=10$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{(150-50)}{10-5} = 20\,ms^{-1}$$

(b) acceleration between $t=0$ and $t=5$.

$V = 20\,ms^{-1}$, $u=0$,

$20 = 0 + a\times 5$ $[V = u+at]$

$\Rightarrow a = 4\,ms^{-2}$

(c) $$\text{Average speed} = \frac{\text{Total distance}}{\text{Total Time}}$$

$$= \frac{50+100+50+200}{20}$$

$$= 20\,ms^{-1}$$

3.7 A particle P moves in a straight line through a point O. The velocity $v\ ms^{-1}$ of P, at t s after passing O, is given by: $v = \frac{9}{4} + \frac{b}{(t+1)^2} - ct^2$ where b and c are positive constants.

At $t=5$, the velocity of P is zero and its acceleration is $-\frac{13}{12}\,ms^{-2}$.

(a) Show that $b=9$ and find the value of c. ---[5]

(b) Given that the velocity of P is zero only at $t=5$, find the distance travelled in the first 10 seconds of motion. ---[5]

[S-22/43/Q7]

Solution: (a) $v = \frac{9}{4} + \frac{b}{(t+1)^2} - ct^2$ --- (1)

at $t=5$; $v = \frac{9}{4} + \frac{b}{(5+1)^2} - c\times 5^2 = 0$ (given)

$\Rightarrow \frac{b}{36} - 25c = -\frac{9}{4}$ --- (2)

Diff (1)

$a = -\frac{2b}{(t+1)^3} - 2ct$ --- (3)

at $t=5$, $a = -\frac{2b}{(5+1)^3} - 2c\times 5 = -\frac{13}{12}$ given

$\Rightarrow -\frac{2b}{216} - 10c = -\frac{13}{12}$

$\Rightarrow -\frac{b}{108} - 10c = -\frac{13}{12}$ --- (4)

Solving (2) & (4) $\Rightarrow$ $b = 9$ and $c = 0.1$

(b) Integrating (1)

distance $s = \int \left(\frac{9}{4} + \frac{9}{(t+1)^2} - 0.1t^2\right) dt$

$s = \frac{9}{4}t - \frac{9}{(t+1)} - \frac{1}{30}t^3 (+k)$

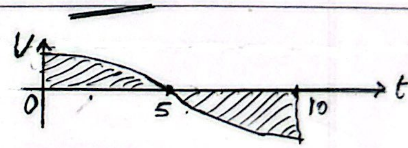
Req dis from $t=0$ to $t=10$

$$\left[\frac{9}{4}t - \frac{9}{t+1} - \frac{t^3}{30}\right]_0^5 - \left[\frac{9}{4} - \frac{9}{t+1} - \frac{t^3}{30}\right]_5^{10}$$

$= (5.583+9) - (-11.651 - 5.583)$

$= 5.583 + 9 + 11.651 + 5.583$

$= 31.8\,m$



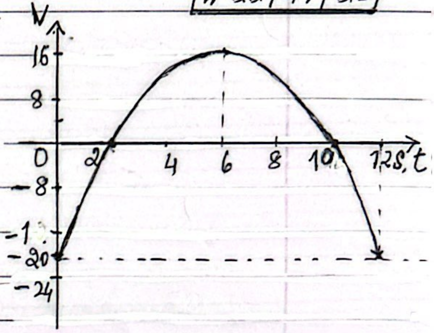
38. A particle P moves on the x-axis from origin O with an initial velocity of $-20\,ms^{-1}$. The acceleration $a\,ms^{-2}$ at time t s after leaving O is given by $a = 12 - 2t$.

(a) Sketch a velocity-time graph for $0 \le t \le 12$, indicating the times when P is at rest. ...[5]

(b) Find the total distance travelled by P in the interval $0 \le t \le 12$. ...[5]

W-22/41/Q5

Solution: (a) Acceleration $a = \frac{dv}{dt} = 12 - 2t$



$\Rightarrow$ Velocity $v = \int (12-2t)dt$

$\Longrightarrow v = 12t - t^2 + C$ --(1)

given at $t=0$, $V=-20$

from (1) $-20 = C$

Hence $V = 12t - t^2 - 20$ ---(2)

Now the particle P is at rest for $V=0$

from (2) $12t - t^2 - 20 = 0$

$-(t-10)(t-2)=0 \Longrightarrow t = 2s$ and $t = 10s$

(b) from (1) $v = \frac{ds}{dt} = 12t - t^2 - 20$

distance:

$\Rightarrow S = \int (12t - t^2 - 20)dt = (6t^2 - \frac{1}{3}t^3 - 20t)$

Required distance travelled by P in the interval $0 \le t \le 12$

$S = -S_1(0 \le t \le 2) + S_2(2 \le t \le 10) - S_3(10 \le t \le 12)$
(below x-axis) (below x-axis)

$$S = -\left[6t^2 - \tfrac{1}{3}t^3 - 20t\right]_0^2 + \left[6t^2 - \tfrac{1}{3}t^3 - 20t\right]_2^{10} - \left[6t^2 - \tfrac{1}{3}t^3 - 20t\right]_{10}^{12}$$

$$= -\left(-\frac{56}{3} - 0\right) + \left(\frac{200}{3} - \left(-\frac{56}{3}\right)\right) - \left(48 - \frac{200}{3}\right)$$

$$= \frac{56}{3} + \frac{256}{3} + \frac{56}{3} = \frac{368}{3} = 122.666$$

$S = \underline{123\,m}$

39. A particle P travels in a straight line, starting at rest from a point O. The acceleration of P at time t s after leaving O is denoted by a ms^{-2};

where: $\begin{cases} a = 0.3t^{1/2} & \text{for } 0 \le t \le 4, \\ a = -kt^{-3/2} & \text{for } 4 < t \le T \end{cases}$

where k and T are constant

(a) Find the velocity of P at t = 4. ---[2]

(b) It is given that there is no change in the velocity of P at t = 4, and that the velocity of P at t = 16 is $0.3 ms^{-1}$.
Show that k = 2.6 and find an expression, in terms of t, for the velocity of P for $4 \le t \le T$ --[4]

(c) Given that P comes to instentaneous at t = T, find the exact value of T. --[2]

(d) Find the total distance travelled between t = 0 and t = T. [W-22/42/Q7] --[4]

Solution:

(a) acc. $a = \frac{dv}{dt} = 0.3t^{1/2}$; $0 \le t \le 4$

$v = \int 0.3t^{1/2} dt = 0.3\frac{t^{3/2}}{3/2} + c$

$v = 0.2t^{3/2} + c$

given $v = 0, t = 0 \Rightarrow c = 0$

$\therefore v = 0.2t^{3/2}$ --- (1) $0 \le t \le 4$

$\therefore$ at $t = 4$, $v = 0.2 \times 4^{3/2} = 0.2 \times 8 = \underline{1.6 ms^{-1}}$

(b) Now when $4 < t \le T$

$v = \int -kt^{-3/2} dt = \frac{-k}{-1/2} t^{-1/2} + d \Rightarrow v = 2kt^{-1/2} + d$ --- (2)

for $t = 4$, $v = 1.6$

$\Rightarrow 1.6 = 2k \times 4^{-1/2} + d$

$\Rightarrow k + d = 1.6$ --- (3)

Again at $t = 16$, $v = 0.3 ms^{-1}$ in (2)

$0.3 = 2k \times 16^{-1/2} + d \Rightarrow \frac{k}{2} + d = 0.3$ --- (4)

Solving (4) and (3) $\underline{k = 2.6}$ ✓

and $d = -1$

Hence from (1)

$v = 2 \times 2.6\, t^{-1/2} - 1$

or $\underline{v = 5.2t^{-1/2} - 1}$ ✓ --- (5)

(c) $t = T$, $v = 0$ given

from (5) $0 = 5.2T^{-1/2} - 1$

$\Rightarrow 5.2T^{-1/2} = 1 \Rightarrow T^{1/2} = 5.2$

$T = (5.2)^2 = \underline{27.04}$ ✓

(d) from (1)

$S_1 = \int_0^4 0.2t^{3/2} dt$

$= \left[0.2\frac{t^{5/2}}{5/2}\right]_0^4$

$= \left[0.08t^{5/2}\right]_0^4 = 0.08 \times 32$

$= 2.56$ --- (6)

from (5)

$S_2 = \int_4^{27.04} (5.2t^{-1/2} - 1) dt$

$= \left[5.2 \cdot \frac{t^{1/2}}{1/2} - t\right]_4^{27.4}$

$= \left[10.4t^{1/2} - t\right]_4^{27.4}$

$= 10.24$ --- (7)

Hence Required distance

$S = S_1 + S_2$ from (6) & (7)

$= 2.56 + 10.24$

$= \underline{12.8 m}$ ✓

40. A particle P is projected vertically upwards with speed $u\,ms^{-1}$ from a point on the ground. P reaches its greatest height after 3 s.
(a) Find u ---[1]
(b) Find the greatest height of P above the ground. --[2]
W-22/43/Q1

Solution: $v = u + gt$ --- ①
(a) Now for greatest height $v = 0$ and $t = 3$ gives
from ① $0 = u - 10\times 3 \Rightarrow \underline{u = 30\,ms^{-1}}$

(b) $v^2 = u^2 - 2gh$
for greatest height, $v = 0$
$0 = 30^2 - 2\times 10\,h$ $(u = 30)$
$\Rightarrow h = 30^2/20 = \underline{45\,m}$

41. A particle P travels in the positive direction along a straight line with constant acceleration. P travels a distance of 52 m during the 2nd second of its motion and a distance of 64 m during the 4th second of its motion.
(a) Find the initial speed and the acceleration of P. ...[5]
(b) Find the distance travelled by P during the first 10 seconds of its motion.
W-22/43/Q4 [2]

Solution: Distance in 2nd second $(S = ut + \frac{1}{2}at^2)$
(a) = distance in 2 seconds − dis in 1 second.
$\Rightarrow 52 = [u\times 2 + \frac{1}{2}a(2)^2] - [u\times 1 + \frac{1}{2}a\cdot(1)^2]$
$\Rightarrow 52 = (2u + 2a) - (u + a/2) \Rightarrow u + \frac{3}{2}a = 52$ --- ①
distance in 4th second = dis in 4 sec − dis. in 3 sec.
$\Rightarrow$ $64 = (4u + \frac{1}{2}a(4)^2) - (3u + \frac{1}{2}a(3)^2)$
$64 = (4u + 8a) - (3u + \frac{9}{2}a) \Rightarrow u + \frac{7}{2}a = 64$ -- ②
Solving ① and ②
$\underline{a = 6\,ms^{-2}}$ and $\underline{u = 43\,ms^{-1}}$. ✓

(b) distance in 10 seconds, $a = 6$, $u = 43$
$S = 43\times 10 + \frac{1}{2}\times 6\times 10^2$ $(S = ut + \frac{1}{2}at^2)$
$= 430 + 300$
distance = $\underline{730\,m}$ ✓

42. Particles X and Y move in a straight line through points A and B. Particle X start from rest at A and moves towards B. At the same instant, Y starts from rest at B.

At t seconds after the particles start moving:

- the acceleration of X in the direction AB is given by $(12t+12)\,ms^{-2}$.
- the acceleration of Y in the direction AB is given by $(24t-8)\,ms^{-2}$.

(a) It is given that the velocities of X and Y are equal when they collide. Calculate the distance AB. ---[6]

(b) It is given instead that AB = 36 m; Verify that X and Y collide after 3s. ---[2]

[W-22/43/Q5]

Solution:

(a) $a_x = \frac{d}{dt}V_x = 12t+12 \Rightarrow V_x = \int(12t+12)dt \Rightarrow V_x = 6t^2+12t$ --- ①

$a_y = \frac{d}{dt}V_y = 24t-8 \Rightarrow V_y = \int(24t-8)\,dt \Rightarrow V_y = 12t^2-8t$ --- ②

from ① & ②

Given $V_x = V_y \Rightarrow \quad 6t^2+12t = 12t^2-8t \Rightarrow 6t^2+20t=0 \Rightarrow t(6t-20t)=0$

$\Rightarrow t = 10/3\ s$ ✓ --- ③

from ①

Now $V_x = \frac{d}{dt}S_x = 6t^2+12t \Rightarrow \underset{(\text{distance})}{S_x} = \int(6t^2+12t)dt = 2t^3+6t^2$ ---- ④

from ②

and $V_y = \frac{d}{dx}S_y = 12t^2-8t \Rightarrow \underset{(\text{distance})}{S_y} = \int(12t^2-8t)dt = 4t^3-4t^2$ --- ⑤

distance $AB = (2t^3+6t^2)-(4t^3-4t^2) = -2t^3+10t^2$ --- ⑥

$\therefore$ from ③ $t=\frac{10}{3} \Rightarrow AB = -2\left(\frac{10}{3}\right)^3 + 10\left(\frac{10}{3}\right)^2 = \frac{-2000}{27}+\frac{1000}{9}$

$\Rightarrow AB = \frac{1000}{27} = 37$ ✓

(b) From ⑥ $AB = -2t^3+10t^2$

at $t=3 \Rightarrow AB = -2\times(3)^3+10\times 3^2$

$= -54+90 = 36m$

Hence verified that at $t=3$, $AB = 36m$. ✓

Scanned with CamScanner

43. A particle P is projected vertically upwards from horizontal ground with speed $15\,ms^{-1}$.

(a) Find the speed of P when it is 10m above the ground. ---[2]

At the same instant that P is projected, a second particle Q is dropped from a height of 18m above the ground in the same vertical line as P.

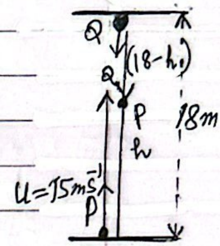
(b) Find the height above the ground at which the two particles collide. ---[3]

[M-23/42/Q2]

Solution (a) $u = 15\,ms^{-1}$, $h = 10m$, $v = ?$

$v^2 = 15^2 - 2g \times 10 \qquad (v^2 = u^2 + 2gh)$

$v^2 = 225 - 200 = 25 \Rightarrow v = 5\,ms^{-1}$ ✓



(b) Let P and Q meet at a height 'h' ($s = ut + \frac{1}{2}at^2$)

for P: $h = 15t - \frac{1}{2}gt^2$ --- ①

for Q: $18 - h = 0 + \frac{1}{2}gt^2$ --- ②

add ① and ② $\Rightarrow 18 = 15t \Rightarrow t = \frac{18}{15} = \frac{6}{5}$ second

for ① $h = 15 \times \frac{6}{5} - \frac{1}{2} \times 10 \times \left(\frac{6}{5}\right)^2 \Rightarrow h = 18 - 7.2 = 10.8m$

$\therefore h = 10.8m$ ✓

---

44. A particle moves in a straight line starting from rest from a point O. The acceleration of the particle at time t s after leaving O is $a\,ms^{-2}$, where $a = 4t^{1/2}$

(a) Find the speed of the particle when $t = 9$. ---[2]

(b) Find the time after leaving O at which the speed (in ms^{-1}) and the distance travelled (in metres) are numerically equal. ---[3]

[M-23/42/Q3]

Solution (a) acc. $a = 4t^{1/2}$, $u = 0$, $t = 9$

$v = \int 4t^{1/2}\,dt$

$v = 4 \times \frac{t^{3/2}}{3/2} + c$

$\Rightarrow v = 8/3\, t^{3/2} + c$

for $t = 0, v = 0 \Rightarrow c = 0$

$\therefore v = \frac{8}{3}t^{3/2}$ --- ①

at $t = 9s \Rightarrow v = 72\,ms^{-1}$.

(b) distance $s = \int \frac{8}{3}t^{3/2}\,dt$

$s = 8/3 \cdot \frac{t^{5/2}}{5/2} + k$

$s = \frac{16}{15}t^{5/2} + k$

Now $s = 0, t = 0 \Rightarrow k = 0$

$\therefore s = \frac{16}{15}t^{5/2}$ --- ②

Now given $v = s$ numerically

for ① & ② $8/3\, t^{3/2} = \frac{16}{15}t^{5/2} \Rightarrow t = \frac{8}{3} \times \frac{15}{16}$

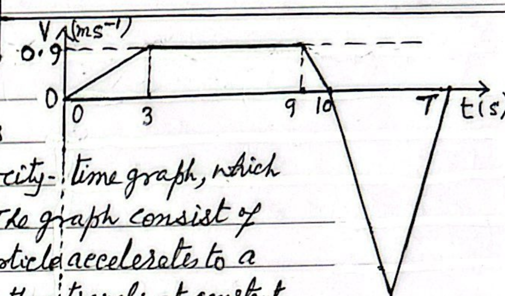
$\therefore t = \frac{5}{2}$ s ✓

45. A particle moves in a straight line starting from rest. The displacement s m of the particle from a fixed point O on the line at t s is given by $s = t^{5/2} - \frac{15}{4}t^{3/2} + 6$
Find the value of s when the particle is again at rest. ---[4]
[S-23/41/Q3]

Solution: $s = t^{5/2} - \frac{15}{4}t^{3/2} + 6$ --- ①
Speed $V = \frac{ds}{dt} = \frac{5}{2}t^{3/2} - \frac{45}{8}t^{1/2}$
for particle at rest $V = 0$
$\Rightarrow \frac{1}{8}t^{1/2}[20t - 45] = 0$
$\Rightarrow t = \frac{45}{20} = 9/4$ ✓

from ① at $t = 9/4$
$s = \left(\frac{9}{4}\right)^{5/2} - \frac{15}{4}\times\left(\frac{9}{4}\right)^{3/2} + 6$
$s = 0.9375$ m (or $s = \frac{15}{16}$)

46. The velocity of a particle at time t s after leaving a fixed point O is v ms^{-1}. The diagram shows a velocity-time graph, which models the motion of the particle. The graph consist of 5 straight line segments. The particle accelerates to a speed of 0.9 ms^{-1} in period of 3s, then travels at constant speed for 6 s, and then comes to instantaneously to rest 1s later. The particle then moves back and returns to rest at O at time T s.



(a) Find the distance travelled by the particle in first 10s of its motion. ---[2]
(b) Given that $T = 12$, find the minimum velocity of the particle. ---[2]
(c) Given instead that the greatest speed of the particle is 3 ms^{-1}, find the value of T and hence find the average speed of the particle for the whole of motion. [4]
[S-23/41/Q4]

Solution (a) Distance in 10s = area Trapezium
$= \frac{1}{2}(6+10)\times 0.9 = \underline{7.2 \text{ m}}$ ✓

(b) Area of Triangle $= \frac{1}{2}(12-10)\times V_{min} = -7.2$
$\Rightarrow V_{min} = \underline{-7.2 \text{ m}s^{-1}}$ ✓

(c) $\frac{1}{2}(T-10)\times 3 = 7.2$
$\Rightarrow T = 14.8$ s
Total distance $= 2\times 7.2 = 14.4$ m
hence average speed for the whole of motion $= \frac{\text{Distance}}{\text{Time}} = \frac{14.4}{14.8}$
$= \frac{36}{37}$ s (or 0.973 s)

47\. A particle P starts at rest and moves in a straight line from a point O. At time t s after leaving O, the velocity P, v ms^{-1}, is given by, $v = bt + ct^{3/2}$, where b and c are constants. P has velocity 8 ms^{-1} when t=4 and has velocity 13.5 ms^{-1} when t=9.

(a) Show that b=3 and c= −0.5. ---[1]

(b) Find the acceleration of P when t=1. --[2]

(c) Find the positive value of t when P is at instantaneous rest and find the distance of P from O at this instant. ---[5]

(d) Find the speed of P at the instant it returns to O. ---[3] [S-23/42/Q6]

Solution: $v = bt + ct^{3/2}$ ---(1)

(a) at t=4, v=8 ⇒ $8 = 4b + 8c$ --(2)

and at t=9, v=13.5 ⇒ $13.5 = 9b + 27c$ -(3)

Solving (2)&(3) $\underline{b=3}$ ✓ and $\underline{c=-0.5}$ ✓

(b) acc. $a = \frac{dv}{dt} = b + \frac{3}{2}ct^{1/2}$

$= 3 + \frac{3}{2}(-0.5)\times 1$ [t=1, b=3, c=−0.5]

$= \underline{2.25\ ms^{-2}}$ ✓

(c) for P at rest v=0

from (1) $3t + (-0.5)t^{3/2} = 0$,

⇒ $t[3 - 0.5t^{1/2}] = 0 \Rightarrow t^{1/2} = \frac{3}{0.5} = 6$

$\underline{t = 36\ s}$ ✓

distance $s = \int(3t - 0.5t^{3/2})\,dt$

$s = \frac{3}{2}t^2 - 0.5\times\frac{2}{5}t^{5/2} + D$

$s = \frac{3}{2}t^2 - \frac{1}{5}t^{5/2}$ [D=0 as s=0 at t=0] → (4)

hence at t=36

$s = \frac{3}{2}\times 36^2 - \frac{1}{5}\cdot 36^{5/2}$

$= \underline{388.8\ m}$ ✓

(d) P returns to O

distance s=0

from (4) $\frac{3}{2}t^2 - \frac{1}{5}t^{5/2} = 0$

$\Rightarrow t^2\left(\frac{3}{2} - \frac{1}{5}t^{1/2}\right) = 0$

$\Rightarrow \frac{3}{2} - \frac{1}{5}\sqrt{t} = 0$ or $t = 0$

$\sqrt{t} = \frac{15}{2}$ (7.5)

$t = 56.25$ s ✓ $\left(\frac{225}{4}\right)$

speed from (1)

$v = 3t - 0.5t^{3/2}$

at t = 56.25

$v = 3\times 56.25 - 0.5\cdot(56.25)^{3/2}$

$= \underline{42.2\ ms^{-1}}$ ✓ (42.1875)

48. A particle starts from rest from a point O and moves in a straight line. The acceleration of the particle at time t s after leaving O is a ms^{-2}, where $a = kt^{1/2}$ for $0 \le t \le 9$ and k is a constant. The velocity of the particle at $t=9$ is $1.8\,ms^{-1}$.

(a) Show that $k = 0.1$ --[3]

for $t>9$, the velocity $v\,ms^{-1}$ of the particle is $v = 0.2(t-9)^2 + 1.8$

(b) Show that the distance travelled in the first 9 seconds is one tenth of the distance travelled between $t=9$ to $t=18$. --[4]

(c) Find the greatest acceleration of the particle during the first 10 seconds of its motion. --[3]

[S-23/43/Q5]

Solution

acc. $a = kt^{1/2}$ -- ① ; $0 \le t \le 9$

(a) $V = \int k t^{1/2} dt$

$V = \frac{2}{3}k\, t^{3/2} + C$ -- ②

Given $t=0,\ V=0 \Rightarrow C=0$

for $t=9,\ V=1.8$

② $\Rightarrow 1.8 = \frac{2}{3}k \times 27 \Rightarrow k = 0.1$ ✓

from ②

(b) $V = \frac{2}{3} \times 0.1\, t^{3/2}$ $\quad 0 \le t \le 9$

$S = \int_0^9 \frac{2}{3} \times 0.1\, t^{3/2}\, dt$

$S = \left[\frac{2}{3} \times 0.1 \times \frac{t^{5/2}}{5/2}\right]_0^9$

$= \frac{2}{3} \times 0.1 \times \frac{2}{5}[243] = \frac{4}{150} \times 243$

$S = 6.48$ for $0 \le t \le 9$ ——③

and Now for $9 \le t \le 18$; $V = 0.2(t-9)^2 + 1.8$ --④

$S = \int_9^{18} [0.2(t-9)^2 + 1.8]\, dt$

$= \left[0.2\frac{(t-9)^3}{3} + 1.8t\right]_9^{18}$

$= (0.2 \times 243 + 32.4) - (0 + 16.2) = 64.8$ --⑤

from ③ & ⑤ $64.8 = 10 \times 6.48$ ✓

Hence the required result.

(c) Now

$acc = 0.1 \cdot t^{1/2}$ $\quad 0 \le t \le 9$

max at $t = 9 = 0.1 \times \sqrt{9} = 0.3\,ms^{-2}$ ---⑥

for $9 \le t \le 10$

$V = 0.2(t-9)^2 + 1.8$

$acc = \frac{dV}{dt} = 0.2 \times 2(t-9)$

at $t = 10$

$acc = 0.4 \times 1 = 0.4$ -⑦

from ⑥ & ⑦

greatest acceleration in first 10 sec. $= 0.4\,ms^{-2}$ ✓