07.01.24

M-1

Mechanics_1

Kinematics of Motion in Straight Line.

(i) 'Velocity - time' and 'Displacement-time' Graphs.

(ii) Constant Acceleration Formulae - SUVAT.

(iii) Application of Differentiation and Integration.

Notes

Includes questions from SP-2020 and W-19 Papers

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Kinematics of motion in a straight line
Notes

(i) Scalar quantities — distance and speed

(ii) Vector quantities — displacement, Velocity and acceleration.

(iii) The area under 'velocity-time' graph represents displacement.

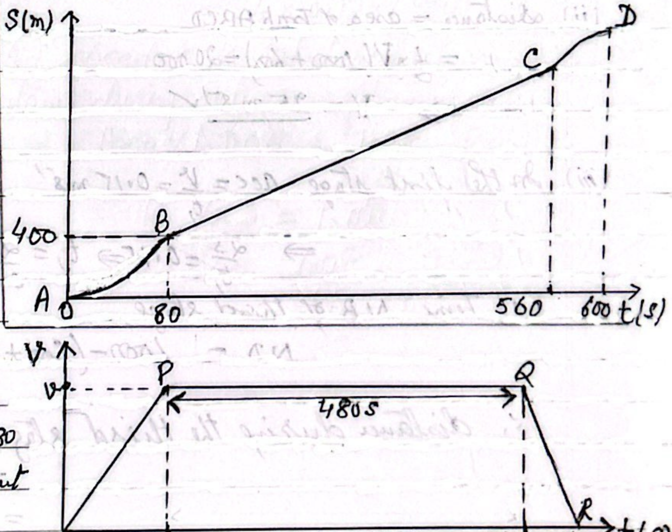
(iv) The gradient of a 'displacement-time' graph represents Velocity

(v) The gradient of 'velocity-time' graph represents Acceleration.

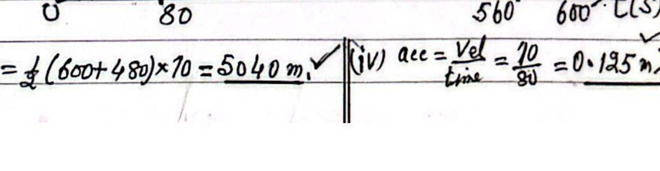
Example 1. The diagram shows the displacement-time graph for a car's journey. The graph consists of two curved parts AB and CD, and a straight line BC. The line BC is a tangent to the curve AB at B and a tangent to the curve CD at C. The gradient of the curves at $t=0$ and $t=600$ is zero, and the acceleration of the car is constant for $0<t<80$ and for $560<t<600$. The displacement of the car is 400 m when $t=80$.

(i) Sketch the velocity-time graph for the journey.
(ii) Find the velocity at $t=80$.
(iii) Find the total distance for the journey.
(iv) Find the acceleration of the car for $0<t<80$.

W-2005/04/Q5



Solution: (i) Vel-time graph.



(ii) Area of Triangle for $t=80$

$\frac{1}{2} \times 80 \times v = \text{displacement} = 400$

$\Rightarrow v = 10\,ms^{-1}$ ✓

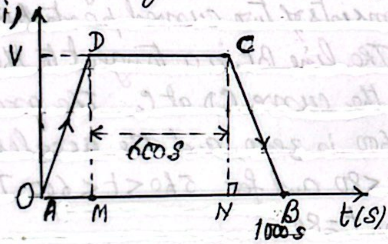
(iii) Total distance Trap OPQR $= \frac{1}{2}(600+480) \times 10 = 5040\,m$ ✓

(iv) $acc = \frac{Vel}{time} = \frac{10}{80} = 0.125\,ms^{-2}$

Example 2. A train travels from A to B, a distance of 20,000 m, taking 1000 s. The journey has three stages. In the first stage the train starts from rest at A and accelerates uniformly until its stage is $V m s^{-1}$. In the second stage the train travels at constant speed $V m s^{-1}$ for 600 s. During the third stage of the journey the train decelerates uniformly, coming to rest at B.

(i) Sketch the velocity-time graph for the train's journey.
(ii) Find the value of V.
(iii) Given that the acceleration of the train during the first stage of the journey is $0.15 \, m s^{-2}$, find the distance travelled by the train during the third stage of the journey. [W-08/04/Q6]

Solution: (i)



(ii) distance = area of Trap ABCD

$= \frac{1}{2} \times V(1000+600) = 20,000$

$\Rightarrow V = 25 \, m s^{-1}$

(iii) In the first stage $acc = \frac{V}{t_1} = 0.15 \, m s^{-1}$

$\Rightarrow \frac{25}{t_1} = 0.15 \Rightarrow t_1 = \frac{25}{0.15} = 166.66 \, s$

∴ Time NB of third stage,

$ND = 1000 - (600 + 166.66) = 233.33 \, s$

∴ distance during the third stage = area of Triangle CND

$= \frac{1}{2} \times 233.33 \times 25 \quad (V=25)$

$= 2917 \, m$

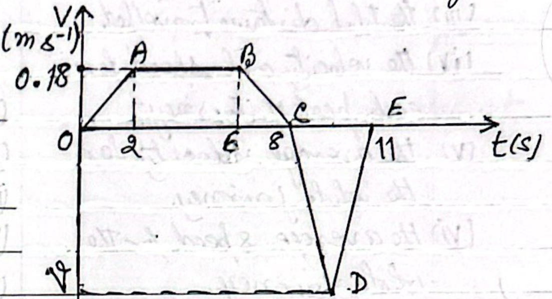
Example 3. The diagram shows the velocity-time graph for the motion of a machine's cutting tool. The graph consists of five straight line segments. The tool moves forward for 8s while cutting and then takes 3s to return to its starting position.

Find

(i) the acceleration of the tool during the first 2s of the motion.

(ii) the distance the tool moves forward while cutting.

(iii) the greatest speed of the tool during the return to its starting position.



S-10/41/Q2

Solution:

(i) acceleration during the first 2s $= \frac{v}{t} = \frac{0.18}{2} = 0.09\ ms^{-2}$

(ii) distance moved forward = area of trap OAB

$= \frac{1}{2} \times 0.18 \times (8+4) = \underline{1.08m}$ ✓ — ①

(iii) Let the greatest speed of the tool during return $= V\ ms^{-1}$

distance during return = forward distance

Area of $\Delta DCE = 1.08$ from ①

$\frac{1}{2} \times V \times 3 = 1.08$

$V = \frac{1.08}{1.5} = \underline{0.72\ ms^{-1}}$ ✓

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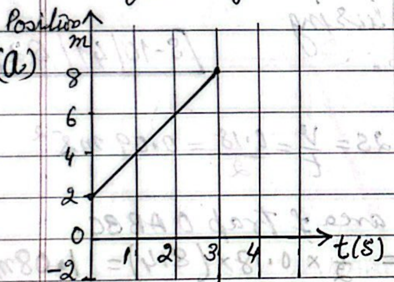
Example 4. For each of the following journeys find.

(i) the initial and final position.
(ii) the total displacement
(iii) the total distance travelled
(iv) the velocity and speed for each part of Journey.
(v) the average velocity for the whole journey.
(vi) the average speed for the whole journey.

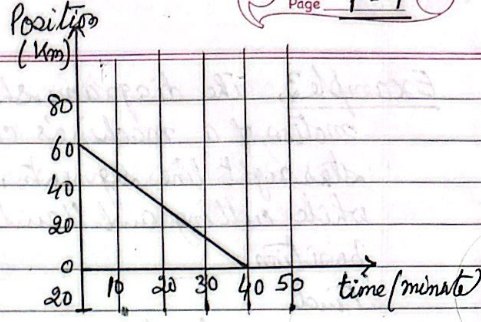
(a)



(i) 2 m; 8 m
(ii) 6 m
(iii) 6 m
(iv) $2\,ms^{-1}$; $2\,ms^{-1}$
(v) $2\,ms^{-1}$
(vi) $2\,ms^{-1}$.

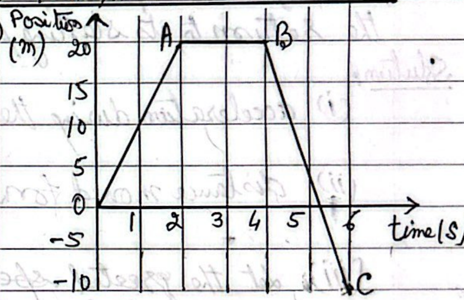
(d) (i) 0 Km, 25 Km (ii) 25 Km
(iii) 65 Km
(iv) AB: $-10\,Kmh^{-1}$, $10\,Kmh^{-1}$
BC: $11.25\,Kmh^{-1}$, $11.25\,Kmh^{-1}$,
(v) $4.167\,Kmh^{-1}$
(vi) $10.83\,Kmh^{-1}$

(b)



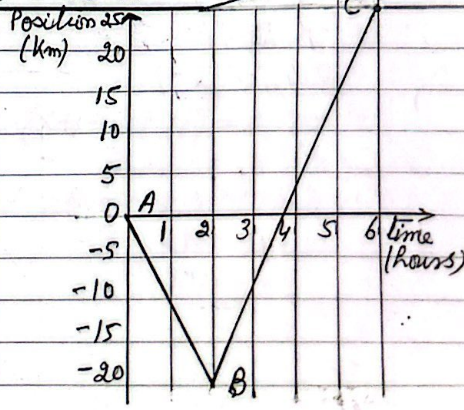
(i) 60 Km, 0 Km
(ii) −60 Km
(iii) 60 Km
(iv) −90 Km/h; 90 Km/h
(v) −90 Km/h
(vi) 90 Km/h.

(c)



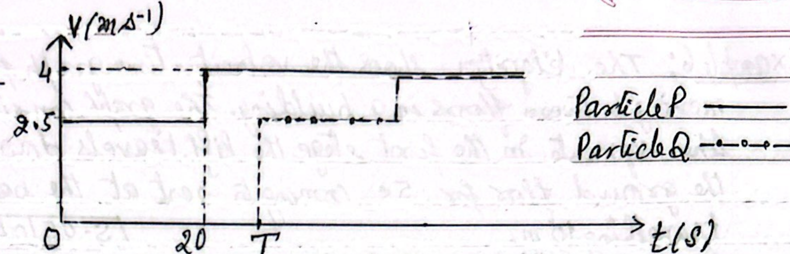
(i) 0 m, −10 m (ii) −10 m (iii) 50 m
(iv) OA: $10\,ms^{-1}$, $10\,ms^{-1}$; AB: $0\,ms^{-1}$, $0\,ms^{-1}$
BC: $-15\,ms^{-1}$, $15\,ms^{-1}$, (v) $-1.67\,ms^{-1}$
(vi) $8.33\,ms^{-1}$

(d)



Kinematics of motion in a straight line.

Example 5:



The diagram shows the velocity-time graph for the motion of two particles P and Q, which travel in the same direction along a straight line. P and Q both start at the same point X on the line, but Q starts to move T s later than P. Each particle moves with speed $2.5\ ms^{-1}$ for the first 20 s of its motion. The speed of each particle changes instantaneously to $4\ ms^{-1}$ after it has been moving for 20 s and the particle continues at this speed.

(i) make rough copy of the diagram and shade the region whose area represents the displacement of P from X at the instant when Q starts. ---[1]

It is given that P has travelled 70 km at the instant when Q starts.

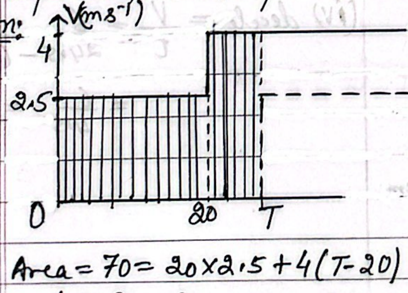
(ii) Find the value of T. --[2]

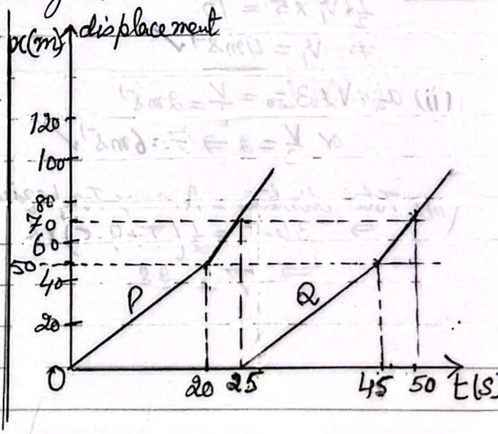
(iii) Find the distance between P and Q when the Q's speed reaches $4\ ms^{-1}$ ..[2]

(iv) Sketch a single diagram showing the displacement-time graph for both P and Q, with values shown on the t-axis at which the speed of either particle changes. [S-11/43/Q4] --[2]

Solution:

(i)





(ii) Area $= 70 = 20 \times 2.5 + 4(T-20)$

$\Rightarrow 4T - 80 = 20 \Rightarrow T = 25\,s$

(iii) Distance $= 70 + (4-2.5) \times 20$

$= 100\ m$ ✓

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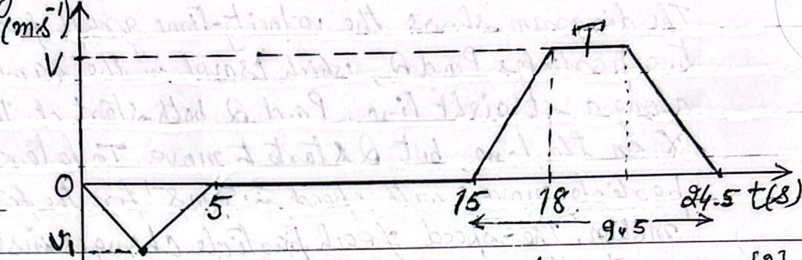
Example 6: The diagram shows the velocity-time graph for a lift moving between floors in a building. The graph consists of straight line segments. In the first stage the lift travels downwards from the ground floor for 5s, coming to rest at the basement after travelling 10 m. S-05/04/Q6



(i) Find the greatest speed reached during this stage. ---[2]

The second stage consists of a 10s wait at the basement. In the third stage, the lift travels upwards until it comes to rest at a floor 34.5 m above the basement, arriving 24.5 s after the start of the first stage. The lift accelerates at $2ms^{-2}$ for the first 3s of the third stage, reaching a speed of $V ms^{-1}$.

Find (ii) Find the value of V. ---[2]

(iii) the time during the third stage for which the lift moving at constant speed. ---[3]

(iv) the deceleration of the lift in the final part of the third stage. ---[2]

Solution: (i) Area of Triangle

$\frac{1}{2} \times V_1 \times 5 = 10$

$\Rightarrow V_1 = 4ms^{-1}$ ✓

(ii) acceleration $= \frac{V}{t} = 2ms^{-1}$

or $\frac{V}{3} = 2 \Rightarrow V = 6ms^{-1}$ ✓

(iii) Total distance = Area of Trapezium

$\Rightarrow 34.5 = \frac{1}{2}(T + 9.5) \times 6$

$\Rightarrow T = 2S$

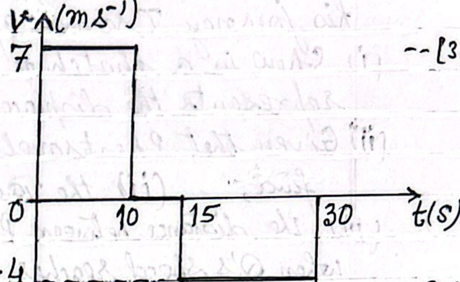
(iv) decele. $= \frac{V}{t} = \frac{6}{24.5 - (18+2)}$

$= \frac{6}{4.5} = \frac{4}{3} m s^{-2}$ ✓

Example 7: A boy runs from a point A to a point C. He pauses at C and then walks back towards A until reaching the point B, where he stops. The diagram shows the graph of v against t, where $v\ ms^{-1}$ is the boy's velocity at time t seconds after leaving A. The boy runs and walks in the same straight line throughout.

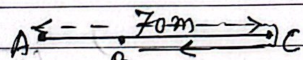
(i) Find the distances AC and AB. --[3]

(ii) Sketch the graph of x against t, where x metres is the boy's displacement from A. Show clearly the values of t and x when the boy arrives at C, and when he arrives at B. [3]



[S-04/04/Q3]

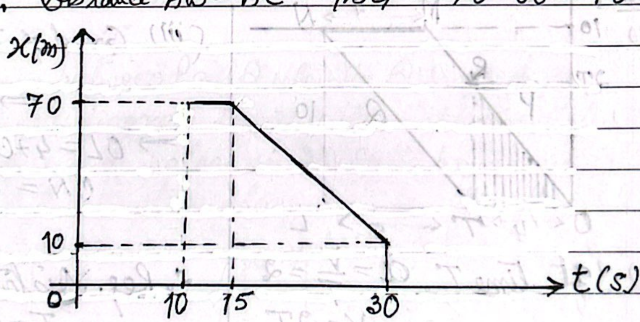
Solution: (i) Distance $AC = V \times t = 7 \times 10 = 70\,m$



and Distance $CB = V \times t = -4 \times 15 = -60\,m$

$\therefore$ Distance $AB = AC - |BC| = 70 - 60 = 10\,m$

(ii)



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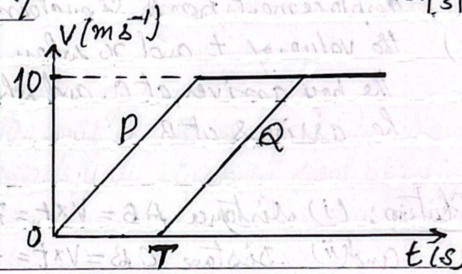
Example 8: The diagram shows the velocity-time graph for the motion of two cyclists P and Q, who travel in the same direction along a straight line. Both cyclists start from rest at the same point O and both accelerate at $2ms^{-2}$ up to a speed of $10 ms^{-1}$. Both then continue at a constant speed of $10ms^{-1}$. Q starts his journey T seconds after P.

(i) Show in a sketch of the diagram the region whose area represents the displacement of P, from O, at the instant when Q starts. --[1]

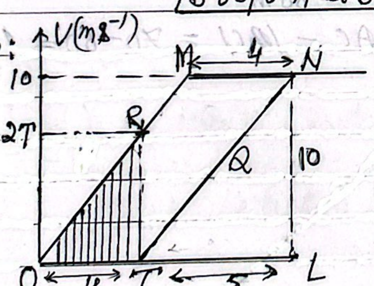
Given that P has travelled 16m at the instant when Q starts, find, (ii) the value of T --[3]

(iii) the distance between P and Q when Q's speed reaches $10 ms^{-1}$. ---[2]



[S-03/04/Q3]

Solution:



(i) at time T, $a = \frac{V}{T} = 2$

$\Rightarrow V = 2T$

area under $V = 2T$ from $t=0$ to $t=2T$,

(ii) Area ORT = distance

$\frac{1}{2} \times 2T \times T = 16 \Rightarrow T = 4s$

(iii) Grad of $TN = \frac{10}{TL} = 2$ (acc)

$\Rightarrow TL = 5$

$\Rightarrow OL = 4+5 = 9m$

$MN = OT = 4m$ (opp. sides of ||gm)

$\therefore$ Req. Distance

$=$ ar Trap OMNL $-$ ar ΔNTL

$= \frac{1}{2} \times 10(9+4) - \frac{1}{2} \times 10 \times 5$

$= 5 \times 13 - 25$

$= 40m$

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Kinematics of motion in a straight line

Constant acceleration formula

SUVAT formulae.

u = Initial velocity Unit: ms^{-1}
v = Final velocity : ms^{-1}
t = Time : s
a = Acceleration (constant) : ms^{-2}
s = displacement : m

(i) $v = u + at$ (ii) $s = \left(\frac{v+u}{2}\right) \times t$ $\left[a = \frac{v-u}{2}\right]$
(iii) $s = ut + \frac{1}{2}at^2$
(iv) $v^2 = u^2 + 2as$
(v) $s = vt - \frac{1}{2}at^2$

Note: If $s = s_0$ when $t = 0$
Then replace s by $(s - s_0)$

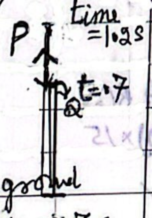
Example 9: Two particles P and Q are projected vertically upwards from horizontal ground at the same instant. The speeds of projection of P and Q are $12\,ms^{-1}$ and $7\,ms^{-1}$ respectively and the heights of P and Q above ground, t seconds after projection, are h_P m and h_Q m, respectively. Each particle comes to rest on returning to ground. [S-11 | 42 | Q5]

(i) Find the set of values of t for which the particles are travelling in opposite directions. --[3]

(ii) At a certain instant, P and Q are above the ground and $3h_P = 8h_Q$. Find the velocities of P and Q at this instant. --[5]

Solution:
(i) for P. $(v = u + at)$
Max height $0 = 12 - 9t$
$\Rightarrow t = 1.2s$,

for Q.
Max height $0 = 7 - 9t$
$\Rightarrow t = 0.7s$.



Q is coming downwards after 0.7s
Whereas P is moving upwards $t = 1.2s$
when $.7 < t < 1.2s$, P and Q are in opp. direction

(ii) $3h_P = 8h_Q$
$\Rightarrow 3\left[12t - \frac{1}{2}g t^2\right] = 8\left[7t - \frac{1}{2}gt^2\right]$
$\Rightarrow t = 8/g$
$\therefore V_P = 12 - g \times \frac{8}{g} = 4\,ms^{-1}$ ✓
$V_Q = 7 - g \times \frac{8}{g} = -1\,ms^{-1}$ ✓

Example 10: A train starts from rest at a station A and travels in a straight line to station B, where it comes to rest. The train moves with constant acceleration $0.025\,ms^{-2}$ for the first 600 s, with constant speed for the next 2600 s, and finally with constant deceleration $0.0375\,ms^{-2}$.

(i) Find the total time for the train to travel from A to B. --[ ]

(ii) Sketch the velocity-time graph for the journey and find the distance AB. --[3]

(iii) The speed of the train t seconds after leaving A is $7.5\,ms^{-1}$. State the possible values of t. [S-11/41/Q5] [1]

Solution:

(i) Velocity after accelerat $v = 0 + .025 \times 600$

$v = 15\,ms^{-1}$

Finally when dec. $= 0.0375$

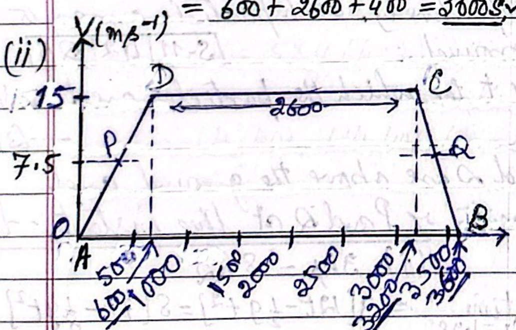
time t_3; $0 = 15 - 0.0375 \times t_3$

$\Rightarrow t_3 = 400\,s$

Total time $= t_1 + t_2 + t_3$

$= 600 + 2600 + 400 = \underline{3600\,s}$ ✓

(ii)



Distance = Area of Trap ABCD

$= \frac{1}{2}(3600 + 2600) \times 15$

$= \underline{46500\,m}$ ✓

(iii) In the first stage

when $a = 0.025\,ms^{-2}$

$v = 7.5\,ms^{-1}$, $t = ?$

$v = u + at$ at P

$7.5 = 0 + 0.025t$

$\Rightarrow \underline{t = 300\,s}$ ✓

Again in the third stage,

at Q, $u = 7.5\,ms^{-1}$

$v = 0$

$a = 0.0375$

$0 = 7.5 - 0.0375 \times t$

$\Rightarrow t = 200$

∴ time after leaving A

$= 3600 - 200$

$= \underline{3400\,s}$ ✓

Example 11. A balls moves on the horizontal surface of a billiards table with deceleration of constant magnitude $d\ ms^{-2}$. The ball starts at A with speed $1.4\ ms^{-1}$ and reaches the edge of the table at B, 1.2 s later with speed $1.1\ ms^{-1}$.

(i) Find the distance AB and the value of d. --[3]

AB is at right angles to the edge of the table containing B. The table has a low wall along each of its edges and the ball rebounds from the wall at B and move directly towards A. The ball comes to rest at C where the distance BC is 2 m.

(ii) Find the speed with which the ball starts to move towards A and the time taken for the ball to travel from B to C. --[3]

(iii) Sketch a velocity-time graph for the motion of the ball, from the time the ball leaves A until it comes to rest at C, showing on the axes the values of the velocity and time when the ball is at A, at B and at C. $\boxed{S\text{-}10/43/Q5}$ --[2]

Solution: (i) $S = \left(\frac{V+U}{2}\right)\times t \Rightarrow AB = \left(\frac{1.1+1.4}{2}\right)\times 1.2$

$\Rightarrow AB = 1.5\ m$ ✓

Also $V = U + dt \Rightarrow d = \frac{V-U}{t} = \frac{1.1-1.4}{1.2}$

$\Rightarrow d = -0.25$ ✓

(ii) Motion $B \to C$, $BC = 2\ m$, $V = 0$, $U = ?$, $d = -0.25$

$V^2 = U^2 + 2ds \Rightarrow 0 = U^2 - 2\times(-0.25)\times 2$

$\Rightarrow U = 1\ ms^{-1}$ ✓

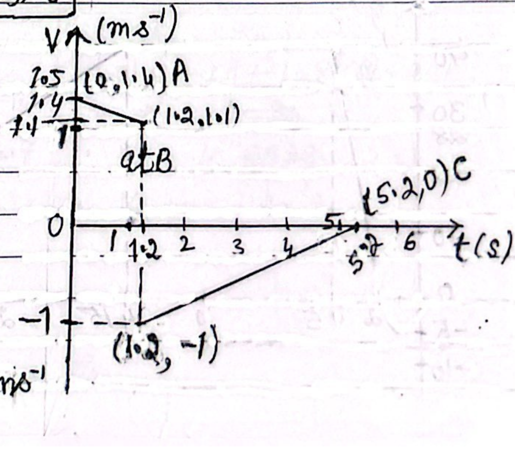
Also $S = Ut - \frac{1}{2}at^2$

$\Rightarrow 2 = 0 - \frac{1}{2}(-0.25)t^2$

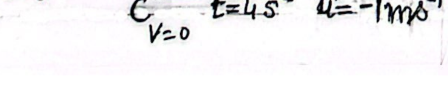
$\Rightarrow t^2 = 2\times 8 = 16$

$\Rightarrow t = 4\ s$ ✓

(iii) Velocity-time graph



Drawing on table.



Example 12: A particle P is projected vertically with speed $20\,ms^{-1}$ from a point on the ground.

(a) Find the greatest height above the ground reached by P. --[2]

(b) Find the total time from projection until P returns to the ground. --[2]

$\boxed{SP-20/04/Q1}$

Solution: $v^2 = u^2 + 2gh$

(a) $0 = 20^2 + 2(-10)h \Rightarrow h = \underline{20m}$ ✓

(b) $v = u + at \Rightarrow 0 = 20 - 10 \times t \Rightarrow t = 2s$ to reach the highest pt.

$\therefore$ time for P to return the ground $= 2 \times 2 = \underline{4s}$ ✓

Example 13: A particle P moves in a straight line. The velocity $v\,ms^{-1}$ at time $t\,s$ is given by:

$v = 5t(t-2)$ for $0 \le t \le 4$, —— ①

$v = k$ for $4 \le t \le 14$ —— ②

$v = 68 - 2t$ for $14 \le t \le 20$. —— ③

where k is a constant.

(a) Find k. --[1] (b) Sketch the velocity-time graph for $0 \le t \le 20$. --[3]

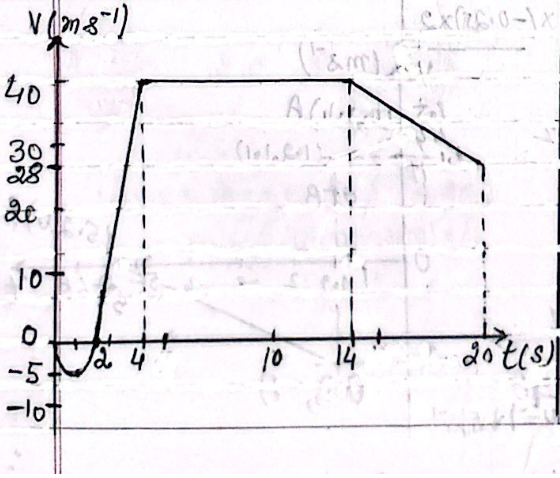
(c) Find the set of values of t for which the acceleration of P is positive. --[2]

(d) Find the total distance travelled by P in the interval $0 \le t \le 20$ --[5]

$\boxed{SP-20/04/Q6}$

Solution: (a) $\begin{cases} V = k ✓ \text{ for } t = 4 \text{ from } ② \\ V = 5 \times 4(4-2) \text{ for } t = 4 \text{ frm } ① \\ \quad = 40 ✓ \end{cases}$ and $\Rightarrow \underline{k = 40}$ ✓

(b) $(0,0), (4,40), (14,40), (20,28)$
$(2,0), (1,-5)$



(c) from ① $v = 5t^2 - 10t$

acc. $= \frac{dv}{dt} = 10t - 10 > 0$

$\Rightarrow t > 1$

for ② V is const, acc = 0

$\therefore \underline{1 < t < 4}$ for acc > 0

(d) distance

$= \left|\int_0^2 (5t^2 - 10t)dt\right| + \int_2^4 (5t^2 - 10t)dt$
$+ 40 \times 10 + \frac{1}{2}(40 \times 28) \times 6$

$= \left|\left[5\frac{t^3}{3} - 5t^2\right]_0^2\right| + \left[5\frac{t^3}{3} - 5t^2\right]_2^4$
$+ 400 + 204$

$= \underline{644\,m}$ ✓

Example 14: A cyclist travels along a straight road with constant acceleration. He passes through points A, B and C. The cyclist takes 2 seconds to travel along each of the sections AB and BC, and passes through B with speed $4.5\ m s^{-1}$. The distance AB is $\frac{4}{5}$ of the distance BC.

(a) Find the acceleration of the cyclist. --[5]

(b) Find AC. [M-20/42/Q 4] --[2]

Solution (a) $S_{AB} = Vt - \frac{1}{2}at^2 = 4.5\times2 - \frac{1}{2}a\times2^2$ —①

and $S_{BC} = ut + \frac{1}{2}at^2 = 4.5\times2 + \frac{1}{2}a\times2^2$ —②

Given $S_{AB} = \frac{4}{5}BC$

∴ fr ① & ② $\left[2\times4.5 - \frac{1}{2}a\times2^2\right] = \frac{4}{5}\left[4.5\times2 + \frac{1}{2}a\times2^2\right]$

$\Rightarrow \underline{a = 0.5\ ms^{-2}}$ ✓

(b) from ① $S_{AB} = 4.5\times2 - \frac{1}{2}\times0.5\times2^2 = 8\ m$

fr ② $S_{BC} = 4.5\times2 + \frac{1}{2}\times0.5\times2^2 = 10\ m.$

∴ $AC = AB + BC = 8 + 10 = \underline{18\ m}$ ✓

Example 15: Two particles A and B move in the same vertical line. Partical A is projected vertically upwards from the ground with speed $20ms^{-1}$. One second later particle B is dropped from rest from a height of 40 m.

(i) Find the height above the ground at which the two particle collide. [4]

(ii) Find the difference in the speeds of the two particles at the instant when collision occurs. [W-19/42/Q5] -- [3]

Solution: $h_A = 20t - \frac{1}{2}\times10t^2$ —①

(i) $h_B = \pm\frac{1}{2}\times10\times(t-1)^2$ —②

when they meet,

$20t - \frac{1}{2}\times10t^2 + \pm\frac{1}{2}\times10\times(t-1)^2 = 40$

$\Rightarrow 10t - 35 = 0 \Rightarrow t = 3.5\ s$

from ①∴ height at collision $= \underline{8.75\ m}$ ✓

(ii) $V_A = u - gt = 20 - 10\times3.5 = -15\ ms^{-1}$ ✓

$V_B = -g(t-1) = -10\times2.5 = -25\ ms^{-1}$

difference $= -15 - (-25)$

$= \underline{10\ ms^{-1}}$ ✓

Example 16: A car travels along a straight road with constant acceleration. It passes through points P, Q, R and S. The times taken for the car to travel from P to Q, Q to R and R to S are each equal to 10 s. The distance QR is 1.5 times the distance PQ. At point Q the speed of the car is $20\,ms^{-1}$.

(i) Show that the acceleration of the car is $0.8\,ms^{-2}$. --- [3]

(ii) Find the distance QS and hence find the average speed of the car between Q and S. [W-19/43/Q4] --- [3]

Solution: $S_{PQ} = Vt - \frac{1}{2}at^2 = 20\times10 - 0.5a\times10^2 = 200 - 50a$ —— ①

(i) and $S_{QR} = ut + \frac{1}{2}at^2 = 20\times10 + 0.5a\times10^2 = 200 + 50a$ —— ②

Given $QR = 1.5 \times PQ \Rightarrow 200 + 50a = 1.5(200 - 50a)$

$\Rightarrow 100 = 125a \Rightarrow a = 0.8\,ms^{-1}$ ✓

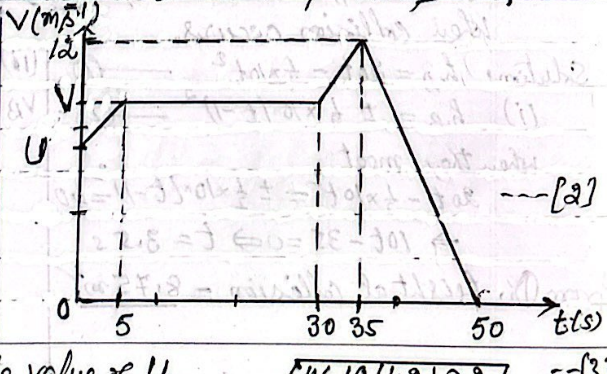
(ii) $QS = 20\times20 + \frac{1}{2}\times0.8\times20^2 = 560\,m$,

Average speed between Q and S $= \frac{560}{20} = 28\,ms^{-1}$ ✓

Example 17: The diagram shows a velocity-time graph which models the motion of a tractor. The graph consists of four straight line segments. The tractor passes a point O at time $t=0$ with speed $U\,ms^{-1}$. The tractor accelerates at a speed of $V\,ms^{-1}$ over a period of 5 s, and then travels at this speed for a further 25 s. The tractor then accelerates to a speed of $12\,ms^{-1}$ over a period of 5 s. The tractor then decelerates to rest over a period of 15 s.

(i) Given that the acceleration of the tractor between $t=30$ and $t=35$ is $0.8\,ms^{-2}$, find the value of V. --- [2]

(ii) Given also that the total distance covered by the tractor in the 50 s of motion is 375 m, find the value of U. [W-19/42/Q2] --- [3]



(Continued →)

M-1

(Continued→)

(Example 17:) **Solution** (i) $\frac{V_1 - U_1}{t_1} = a \Rightarrow \frac{(12-V)}{35-30} = 0.8 \Rightarrow V = 8\, ms^{-1}$ ✓

(ii) Total distance $= [\frac{1}{2}(V+8)\times 5 + 25\times 8 + \frac{1}{2}\times(8+12)\times 5 + \frac{1}{2}\times 15\times 12 =$

$\Rightarrow V = 6\, ms^{-1}$ ✓

(M-1) Kinematics of Motion in Straight Line

Application of Differentiation & Integration

- Differentiation: Given $y = f(x)$

Instantaneous rate of change of y w.r.t x is defined as differentiation of y w.r.t x, denoted by $\frac{dy}{dx}$ or $f'(x)$

Formulae:

(i) $y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$

(ii) $y = (ax+b)^n \Rightarrow \frac{dy}{dx} = a(ax+b)^{n-1}$

(iii) $y = c$ (constant) $\Rightarrow \frac{dc}{dx} = 0$

(iv) $y = af(x) + c \Rightarrow \frac{dy}{dx} = a \cdot f'(x)$

Example 1 (i) $y = x^7 \Rightarrow \frac{dy}{dx} = 7x^6$ ✓ or $\frac{d}{dx}x^7 = 7x^6$ ✓

or $f(x) = x^7 \Rightarrow f'(x) = 7x^6$. ✓

(ii) $y = 4x^5 - 3x^2 + 6$

$\frac{dy}{dx} = 4 \times 5x^4 - 3 \times 2x + 0 = 20x^4 - 6x$ ✓

Example 2 (i) $\frac{d}{dx}(3x+5)^9 = 3 \times 9(3x+5)^8 = 27(3x+5)^8$ ✓

(ii) $\frac{d}{dx}\sqrt{x} = \frac{d}{dx}x^{\frac{1}{2}} = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$ ✓

(iii) $\frac{d}{dx}\frac{1}{x} = \frac{d}{dx}x^{-1} = -1 \cdot x^{-2} = -\frac{1}{x^2}$ ✓

- Application of Diff in Motion of a straight line:

1. Given distance $s = f(t)$; t is time

The velocity $= \frac{ds}{dt}$ or $f'(t)$ ✓

and Acceleration $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}$ ✓

Example: $v = 6t - 2t^2$

$\Rightarrow$ Acceleration $a = \frac{dv}{dt} = 6 - 4t$ ✓

- **Integration:**

(i) $\int x^n \, dx = \dfrac{x^{n+1}}{(n+1)} + c$

(ii) $\int (ax+b)^n \, dx = \dfrac{(ax+b)^{n+1}}{a(n+1)} + c$

(iii) $\int c \, dx = cx + d$

Example 1

(i) $\int x^7 dx = \dfrac{x^8}{8} + c$

(ii) $\int (5x+8)^6 dx = \dfrac{(5x+8)^7}{5 \times 7} + c$

(iii) $\int \sqrt{x} \, dx = \int x^{1/2} dx = \dfrac{x^{3/2}}{3/2} = \dfrac{2}{3} x^{3/2} + c$

(iv) $\int (x^3 + 4x^2 - 8x + 9) \, dx$
$= \left(\dfrac{x^4}{4} - \dfrac{4x^3}{3} - \dfrac{8x^2}{2} + 9x + c \right)$

- **Application of Integration to the motion of a particle in Motion:**

(i) Given $v = f(t) \Rightarrow$ distance $s = \int v \, dt = \int f(t) dt$

(ii) Given acc. $a = f(t) \Rightarrow$ Velocity $v = \int a \, dt = \int f(t) dt$

Example (i) $v = 6t^2 - 30t + 24$

distance $s = \int v \, dt = \int (6t^2 - 30t + 24) dt$
$= 6 \times \dfrac{t^3}{3} - 30 \dfrac{t^2}{2} + 24t + c$
$= (2t^3 - 15t^2 + 24t + c)$

and distance travelled from $t = 1s$ to $4s$

$= \int_1^4 f(t) dt = \left[2t^3 - 15t^2 + 24t \right]_1^4$

$= 27 m$

Example 18: A particle moves in a straight line through the point O. The displacement of the particle from O at time t s is s m, where

$$s = t^2 - 3t + 2 \quad \text{for } 0 \le t \le 6$$
$$s = \frac{24}{t} - \frac{t^2}{4} + 25 \quad \text{for } t \ge 6$$

(a) Find the value of t when the particle is instantaneously at rest during the first 6 seconds of its motion. --[2]

At $t = 6$, the particle hits a barrier at a point P and rebounds.

(b) Find the velocity with which the particle arrives at P and also the velocity with which the particle leaves P. --[3]

(c) Find the total distance travelled by the particle in the first 10 seconds of its motion. [M-20/42/Q7] --[5]

Solution: $s = t^2 - 3t + 2$ —① for $0 \le t \le 6$

(a) $V = \frac{ds}{dt} = 2t - 3 = 0$ for the particle to be on rest,

$\Longrightarrow t = 1.5$ s. ✓

(b) for first 6 seconds $V = 2t - 3$

Vel at P (at 6 sec.) $= 2\times6 - 3 = 9\ ms^{-1}$ ✓

and after 6 sec. when leaving P, $s = \frac{24}{t} - \frac{t^2}{4} + 25$ —②

$$V = \frac{ds}{dt} = -\frac{24}{t^2} - \frac{1}{2}t$$

∴ Velocity when leaving P $= -\frac{24}{6^2} - \frac{1}{2}\times6 = -3.67\ ms^{-1}$ ✓

(c) for ① $0 \le t \le 6$

$t = 0,\ s = 2$ m

$t = 1.5,\ s = -0.25$ m

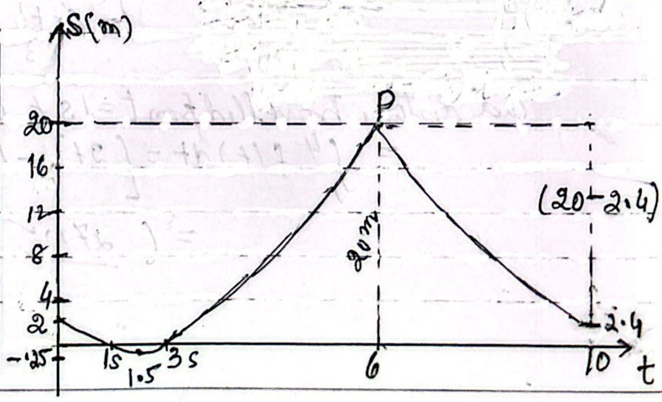
$t = 6,\ s = 20$ m

$t = 10,\ s = 2.4$ m

Required distance

$= 2 + 0.25 + 0.25 + 20 + (20 - 2.4)$

$= \underline{40.1\ m}$ ✓



Example 19: A particle moves in a straight line, starting from rest at a point O, and comes to instantaneous rest at a point P. The velocity of the particle at time t s after leaving O is v ms^{-1}, where $v = 0.6t^2 - 0.12t^3$

(i) Show that the distance OP is 6.25 m. --[5]

On another occasion, the particle also moves in the same straight line. On this occasion, the displacement of the particle at time t s after leaving O is s m, where

$$s = kt^3 + ct^5$$

It is given that the particle passes point P with velocity 1.25 ms^{-1} at time $t=5$

(ii) Find the values of the constants k and c. --[5]

(iii) Find the acceleration of the particle at time $t=5$ --[2]

W-19/41/Q7

Solution: for instantaneous rest $v=0 \Rightarrow 0.6t^2 - 0.12t^3 = 0$

$\Rightarrow t^2(0.6 - 0.12t) \Rightarrow t=0 ; t=5$ ✓

(i) $OP = \int_0^5 v\,dt = \int_0^5 (0.6t^2 - 0.12t^3)\,dt = [0.2t^3 - 0.03t^4]_0^5$

$\Rightarrow OP = 6.25m$ ✓

(ii) Now $s = kt^3 + ct^5$

$\Rightarrow v = \frac{ds}{dt} = 3kt^2 + 5ct^4$ ①

$\Rightarrow$ at $t=5$; $s = k\times5^3 + c\times5^5 = 6.25$

and $v = 3k\times5^2 + 5c\times5^4 = 1.25$

$\Rightarrow \begin{cases} 125k + 3125c = 6.25 \\ 75k + 3125c = 1.25 \end{cases}$

$\Rightarrow k = 0.1$ and $c = -0.002$ ✓

(iii) from ① $v = .3t^2 - 0.01t^4$

Acceleration $a = \frac{dv}{dt} = 0.6t - 0.04t^3$ ②

At $t=5$, $a = 0.6\times5 - 0.04\times5^3$

$= 3 - 5$

$= -2 ms^{-2}$ ✓

Example 20: A particle moves in a straight line. The displacement of the particle at time t s is s m, where $s = t^3 - 6t^2 + 4t$. Find the velocity of the particle at the instant when its acceleration is zero. [W-19/42/Q1] ---[4]

Solution: $s = t^3 - 6t^2 + 4t$

$v = \frac{ds}{dt} = 3t^2 - 12t + 4$ —— ①

and $a = \frac{dv}{dt} = 6t - 12$ —— ②

Now from ② $a = 0 \Rightarrow 6t - 12 = 0 \Rightarrow t = 2$

$\therefore$ from ① at $t = 2$, $v = 3\times2^2 - 12\times2 + 4 = \underline{-8\,ms^{-1}}$ ✓

Example 21: Particle P travels in a straight line from A to B. The velocity of P at time t s after leaving A is denoted by $v\,ms^{-1}$, where $v = 0.04t^3 + ct^2 + kt$; P takes 5 s to travel from A to B and it reaches B with speed $10\,ms^{-1}$. The distance AB is 25 m.

(i) Find the values of constants c and k. [W-19/43/Q6] ---[6]

(ii) Show that the acceleration of P is minimum when $t = 2.5$ --[3]

Solution: $v = 0.04t^3 + ct^2 + kt$ —— ①

from ① at $t = 5$, $v = 10 \Rightarrow 10 = 0.04\times5^3 + c\times5^2 + k\times5$

$\Rightarrow 25c + 5k = 5$ —— ②

Integrating ① $s = \int(0.04t^3 + ct^2 + kt)\,dt$

or $s = 0.04\frac{t^4}{4} + c\frac{t^3}{3} + k\frac{t^2}{2} + d$

Now $s = 25$, at $t = 5$, $[t = 0, s = 0 \Rightarrow d = 0]$

$\Rightarrow 25 = 0.01\times5^4 + \frac{5^3}{3}c + \frac{5^2}{2}k$

$\Rightarrow \frac{125}{3}c + \frac{25}{2}k = 18.75$ —— ③

solving ② & ③ $\Rightarrow c = -0.3$ and $k = 2.5$ ✓

from ①

$v = 0.04t^3 - 0.3t^2 + 2.5t$ —— ④

diff ④ $a = 0.12t^2 - 0.6t + 2.5$ —— ⑤

$\Rightarrow \frac{da}{dt} = 0.24t - 0.6 = 0$ for 'a' to be minimum.

$\Rightarrow t = \frac{0.6}{0.24} = \underline{2.5\,s}$ ✓

Example 22: A particle P start at the point O and travels in a straight line. At time t seconds after leaving O the velocity of P is $v\ ms^{-1}$, where $v = 0.75t^2 - 0.0625t^3$. Find

(i) the positive value of t for which the acceleration is zero. ---[3]

(ii) the distance travelled by P before it changes its direction of motion. --[5]

[S-12/41/Q4]

Solution: $v = 0.75t^2 - 0.0625t^3$ —— ①

(i) acc. $a = \frac{dv}{dt} = 1.5t - 0.1875t^2 = 0 \Rightarrow t(1.5 - 0.1875t) = 0$

$\Rightarrow \underline{t = 8s}$ ✓

(ii) $v = t^2 \times 0.0625[12 - t] = 0$

at $t = 12$, the direction of motion changes.

Now.

distance $s = \int v\,dt = \int (0.75t^2 - 0.0625t^3)dt$

$\Rightarrow s = 0.25t^3 - 0.0625\frac{t^4}{4} + C$ but $t = 0,\ s = 0$

$\Rightarrow C = 0$

$\therefore$ distance at $t = 12$; $s = 0.025 \times 12^3 - \frac{0.0625}{4} \times 12^4 = \underline{108\ m}$ ✓

Example 23: A particle P moves in a straight line, starting from point O with velocity $2ms^{-1}$. The acceleration of P at time t s after leaving O is $2t^{2/3}\ ms^{-2}$;

(i) Show that $t^{5/3} = 5/6$ when the velocity of P is $3ms^{-1}$. ---[4]

(ii) Find the distance of P from O when the velocity of P is $3ms^{-1}$. ---[3]

[S-12/42/Q3]

Solution: Given acc. $a = 2t^{2/3}$

(i) Integrating $v = \int 2t^{2/3}dt$

$v = 2 \times \frac{t^{5/3}}{5/3} + C$ —— ①

Given $v = 2$ when $t = 0 \Rightarrow C = 2$

from ① $v = 1.2t^{5/3} + 2$ —— ②

when $v = 3 \Rightarrow 1.2t^{5/3} + 2 = 3$

$\Rightarrow \underline{t^{5/3} = 5/6}$ ✓ —— ③

(ii) Integrating ②

$s = \int v\,dt = \int (1.2t^{5/3} + 2)\,dt$

$\Rightarrow s = 0.45t^{8/3} + 2t + C$

$[s = 0, t = 0$

$\Rightarrow C = 0$

$\therefore s = 0.45t^{8/3} + 2t$

$s = t[0.45t^{5/3} + 2]$

$= 0.738[0.45 \times \frac{5}{6} + 2]$

$= \underline{2.13}$ ✓

[from ③ $t = \left(\frac{5}{6}\right)^{3/5} = 0.738$]

Example 23: A particle travels in a straight line from a point P to a point Q. If velocity t seconds after leaving P is $v\,ms^{-1}$, where $v = 4t - \frac{1}{16}t^3$. The distance PQ is 64 m.

(i) Find the time taken for the particle to travel from P to Q. --[5]

(ii) Find the set of values of t for which the acceleration of the particle is positive. [S-11/41/Q6] --[4]

Solution; $v = 4t - \frac{1}{16}t^3$ —— ①

(i) Integrating $S = \int(4t - \frac{1}{16}t^3)\,dt$

$\Rightarrow S = 2t^2 - \frac{t^4}{64} + C$

Now $t=0, S=0 \Rightarrow C=0$

$\therefore S = 2t^2 - \frac{t^4}{64} = PQ = 64$ given

$\Rightarrow t^4 - 128t^2 + 64^2 = 0$

$\Rightarrow (t^2-64)^2 = 0 \Rightarrow \underline{t = 8s}$ ✓

(ii) Diff ①

$a = \frac{dv}{dt} = 4 - \frac{3}{16}t^2 > 0$ Positive

$\Rightarrow t^2 < \frac{64}{3}$

$\Rightarrow 0 < t < \frac{8}{\sqrt{3}}$

or $\underline{0 < t < 4.62}$ ✓

Example 24. A walker travels along a straight road passing through the points A and B on the road with speeds $0.9\,ms^{-1}$ and $1.3\,ms^{-1}$ respectively. The walker's acceleration between A and B is constant and equal to $0.004\,ms^{-2}$.

(i) Find the time taken by the walker to travel from A to B, and find the distance AB. --[3]

A cyclist leaves A at the same instant as the walker. She starts from rest and travels along the straight road, passing through B at the same instant as the walker. At time t s after leaving A the cyclist's speed is $kt^3\,ms^{-1}$, where k is a constant.

(ii) Show that when $t = 64.05$ the speed of the walker and the speed of the cyclist are same, correct to 3 significant figures. --[5]

(iii) Find the cyclist's acceleration at the instant she passes through B. --[2]

[S-11/42/Q7]

Solution: $V = u + at$

$\Rightarrow 1.3 = 0.9 + 0.004T$

$\Rightarrow \underline{T = 100s}$ ✓

Now $v^2 = u^2 + 2aS$

$1.3^2 = 0.9^2 + 2\times0.004S \Rightarrow S = 110m$

$\therefore \underline{AB = 110m}$ ✓

(Continued→)

(Continued →)

24(ii) speed of Cyclist $= kt^3$

distance $S_c = \int kt^3\,dt = k\frac{t^4}{4} + C$

$t=0, S_c=0 \Rightarrow C=0$

$\therefore$ dis for Cyclist at $t=100$ s

$S_c = k\cdot\frac{100^4}{4} = 110 \Rightarrow k = 4.4\times10^{-6}$ ✓

at $t = 64.05$ s $\quad | V_c = 4.4\times10^{-6}\times t^3$

$V_w = 0.9 + 0.004\times 64.05$

and $V_c = 0 + 4.4\times10^{-6}\times 64.05$

both are equal to 1.16 correct to 3 s.f.

(iii) for cyclist

$V = kt^3$

$\therefore$ acc. $\frac{dV}{dt} = 3kt^2$

$\therefore$ acc of Cyclist at B, $t=100$ s

$= 3\times4.4\times10^{-6}\times(100)^2$

$= 3\times4.4\times\frac{1}{100}$

$= 0.132\ ms^{-2}$ ✓

Example 25: A partical travels in a straight line from A to B in 20 s. Its accelesation t seconds after leaving A is a ms^{-2}, where $a = \frac{3}{160}t^2 - \frac{1}{800}t^3$. It is given that the particle comes to rest at B.

(i) Show that the initial speed of the particle is zero. --[4]

(ii) Find the maximum speed of the particle. --[2]

(iii) Find the distance AB. [S-11/43/Q7] --[4]

Solution: $a = \frac{3}{160}t^2 - \frac{1}{800}t^3$ —①

(i) Integrating $v = \frac{1}{160}t^3 - \frac{1}{3200}t^4 + c$ —②

$[v=0$ at $t=20 \Rightarrow 0 = \frac{8000}{160} - \frac{160000}{3200} + c_1$

$\Rightarrow c_1 = 0]$

$\therefore$ from ② $v = \frac{1}{160}t^3 - \frac{1}{3200}t^4$ —③

Proves $v=0$ at $t=0$ ✓

or initial speed of the particle is zero.

(ii) for Max value of v, $\frac{dv}{dt} = a = 0$

from ① $\frac{t^2}{800}(15-t) = 0 \Rightarrow t = 15$

for V_{max} put $t=15$ in ③

$V_{max} = \frac{1}{160}\times15^3 - \frac{1}{3200}\times15^4 = 5.27\,ms^{-1}$

(iii) for distance integrating ③

$s = \int\left(\frac{1}{160}t^3 - \frac{1}{3200}t^4\right)dt$

$s = \frac{1}{640}t^4 - \frac{1}{16000}t^5 + C_2$

$[s=0$ when $t=0 \Rightarrow C_2 = 0]$

$\therefore s = \frac{1}{640}t^4 - \frac{1}{16000}t^5$ —④

$\therefore$ for $t = 20$ s

$AB = \frac{1}{640}\times20^4 - \frac{1}{16000}\times20^5$

$= 20^4\left[\frac{1}{640} - \frac{\cancel{20}^{1}}{\cancel{16000}_{800}}\right]$

$= 20^4\left[\frac{10-8}{6400}\right]$

$= 2\times\frac{160000}{6400}$

$\therefore AB = 2\times25 = 50$m ✓