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Mechanics - M_1

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Momentum

Notes

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Momentum

§ Momentum:

The momentum of an object is defined as its mass x velocity.

Given mass $= m$ kg

velocity $= v \; ms^{-1}$

Then the momentum of the object $p = m \cdot v$ $kg\,ms^{-1}$ (or Ns)

Note: Momentum is a vector quantity and its direction is same as the direction of the velocity.

Example 1: Show that a ball of mass 3.0 kg moving at a velocity of $15\,ms^{-1}$ and a person of mass 50 kg moving at a velocity of $0.9\,ms^{-1}$, have the same momentum.

Solution: momentum of the ball $= m_1 v_1 = 3 \times 15 = 45\,kg\,ms^{-1}$ —(1)

and the momentum of the person $= m_2 v_2 = 50 \times 0.9 = 45\,kg\,ms^{-1}$ —(2)

from (1) & (2). we get the required result.

§ Newtons Second Law of Motion:

The rate of change of momentum of an object is proportional to the force applied.

$$F \propto \frac{mv - mu}{t} = m\frac{(v-u)}{t} = ma \quad \text{[here } a \text{ is acceleration]}$$

or $F = k\,ma$ (where k is a constant)

for $m = 1\,kg$, $a = 1\,ms^{-2}$ and $F = 1N \Rightarrow k = 1$.

$$\Rightarrow F = ma = m\left(\frac{v-u}{t}\right)$$

If initial velocity $u = 0 \Rightarrow$

$$F = \frac{v \times m}{t} \Rightarrow v \times m = F \times t$$

hence Unit momentum is $kg\,ms^{-1}$ or Ns

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Example 2. A force of 15 N acts for 20 s on an object of mass 100 kg, which is initially at rest.
Calculate:
(i) the change of momentum of the object.
(ii) the velocity of the object at 20 s.

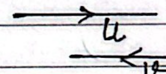
Solution:

(i) Change of momentum $= f \times t = 15 \times 20 = 300$ Ns ✓
(ii) Momentum at 20 s $= 300$ Ns as the object was initially at rest.

$$\therefore \text{Velocity} = \frac{\text{momentum}}{\text{mass}} \quad [\because \text{momentum} = m \times v]$$

$$= \frac{300}{100} = 3.0\ ms^{-1}$$ ✓

§ To find the momentum when direction of the velocity changes to opposite direction:

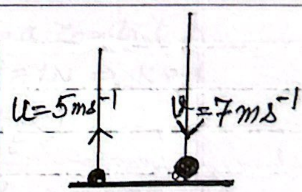


Change of momentum $= m(u - (-v)) = m(u+v)$ ✓

Example 3: A ball of mass 2 kg is thrown upwards with an initial velocity of $5\ ms^{-1}$. It is travelling at $7\ ms^{-1}$ just before it lands. Find the change in its momentum.

Solution:

Change of momentum
$= m(v - u)$
$= 2[7 - (-5)]$
$= 2 \times 12$
$= 24$ Ns ✓



Momentum

§ Collisions and conservation of Momentum:

When objects interact, they exert equal and opposite forces on each other.

§ Principle of conversation of momentum:

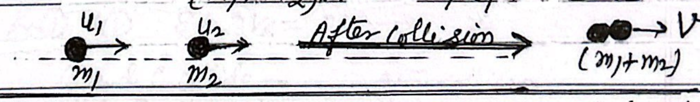
If two bodies (moving along the same straight line) collide with each other then the total final momentum remains same as the total initial momentum.

Then $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

§ Note:

If the colliding objects stick together as a result of collision (Coalesce), they have the same velocity V, then,

$$(m_1+m_2) \cdot V = m_1u_1 + m_2u_2 \checkmark$$



Example 4: A rail wagon of mass 3000 kg moving along a level track at a speed of $2 ms^{-1}$ collides with and couple to a second rail wagon of mass 2000 kg which is initially stationary. Calculate the speed of the two wagons after collision.

Solution: Total initial momentum $= m_1u_1 + m_2u_2 = 3000 \times 2 + 2000 \times 0$
$= 6000$ NS — ①

Total final momentum $= (m_1+m_2)V = (3000+2000)V$
$= 5000V$ Ns — ②

from ① and ②
Using the principle of conservation of momentum.

$5000V = 6000$

$\Rightarrow V = 1.2 ms^{-1} \checkmark$

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Example 5. Three small smooth spheres A, B and C of equal radii and of masses 4 kg, 2 kg and 3 kg respectively, lie in that order in a straight line on a smooth horizontal plane. Initially, B and C are at rest and A is moving towards B, with speed 6 m s⁻¹. After the collision with B, sphere A continues to move in the same direction with speed 2 m s⁻¹.

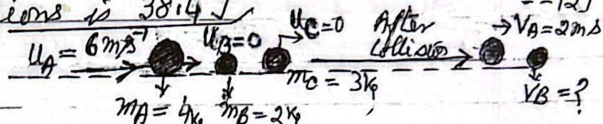
(a) Find the speed of B after this collision. ---[2]

Sphere B collides with C. In this collision these two spheres coalesce to form an object D. [SP-20/04/Q3]

(b) Find the speed of D after this collision. ---[2]

(c) Show that the total loss of kinetic energy in the system due to the two collisions is 38.4 J. ---[2]

Solution (a)



Momentum of the system before A collides B $= m_A \times u_A + m_B \times u_B + m_C \times v_C$

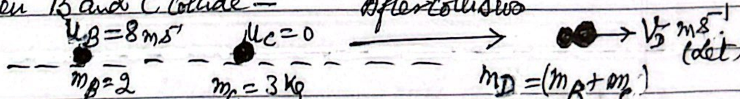
$= 4\times 6 + 2\times 0 + 3\times 0 = 24\,Ns + 0 + 0 = 24\,Ns$ —(1)

After A collides with B,

$= 4\times 2 + V_B \times 2$ —(2)

fr (1) & (2) $8 + 2V_B = 24 \rightarrow V_B = \underline{8\,ms^{-1}}$ ✓

(b) Again when B and C collide —



$m_D = (m_B + m_C) = (2+3) = 5\,kg$

Total Moment of B and C before collision

$m_B \times u_B + m_C \times u_C = 2\times 8 + 3\times 0 = 16\,Ns$ —(3)

Momentum after B & C collide

$M_D \times V_D = 5 \cdot V_D$ —(4)

fr (3) & (4) → $5V_D = 16 \rightarrow \underline{V_D = 3.2\,ms^{-1}}$ ✓

(c) Initial K.E $= \frac{1}{2} m_A u_A^2 = \frac{1}{2}\times 4 \times 6^2 = 72\,J$ —(5)

Final K.E $= \frac{1}{2} m_A \cdot V_A^2 + \frac{1}{2} m_D \cdot V_D^2 = \frac{1}{2}\times 4\times 2^2 + \frac{1}{2}\times 5\times 3.2^2 = 33.6\,J$ —(6)

∴ Loss in K.E $= 72 - 33.6 = \underline{38.4\,J}$ ✓

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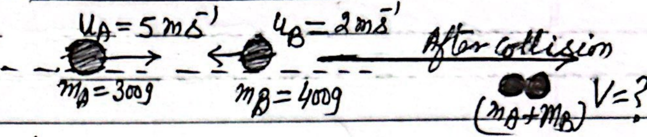
Example 6. A ball of mass 300g moving at $5 ms^{-1}$, makes a direct collision with another ball of mass 400g moving at $2 ms^{-1}$. On the impact the ball coalesce.
Find the speed of balls after collision if the balls were initially moving towards one-another.

Solution:



Total initial momentum $= m_A \times u_A + m_B \times u_B$
$= 0.300 \times 5 + 0.400 \times (-2)$
$= 1.5 - 0.8$
$= 0.7 \ Ns$ ——— (1)

Total final momentum $= (m_A + m_B) \times V_{A+B}$
$= (0.3 + 0.4) \times V_{AB}$
$= 0.7 V$ ——— (2)

∴ using the conservation of momentum (principle) for (1) & (2)

$0.7 V = 0.7 \ NS$
$\therefore V = 1 ms^{-1}$

∴ Speeds of balls after collision is $1 ms^{-1}$ in the initial direction of ball A.