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Mechanics 1

Momentum

Exercise 1 Solution (Revision)

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Momentum - Revision

1. Three small smooth spheres A, B and C of equal radii and of masses 4 kg, 2 kg and 3 kg respectively, lie in that order in a straight line on a smooth horizontal plane. Initially, B and C are at rest and A is moving towards B, with speed $6\,ms^{-1}$. After collision with B, sphere A continue to move in the same direction with speed $2\,ms^{-1}$.

(a) Find the speed of B after this collision. --- [2]

Sphere B collides with C. In this collision these two spheres coalesce to form an object D.

(b) Find the speed of D after this collision. --- [2]

(c) Show that the total loss of kinetic energy in the system due to two collisions is 38.4 J. [SP-20/04/Q3] --- [2]

Solution (a)

$u_A = 6\,ms^{-1}$ $\longrightarrow$, $m_A = 4\,kg$; $u_B = 0$, $m_B = 2\,kg$; $u_C = 0$, $m_C = 3\,kg$ $\longrightarrow$ After collision: $V_A = 2\,ms^{-1}$ (A) $\longrightarrow$ (B) $V_B = ?$

Momentum before collision before A & B collide $= m_A u_A + m_B u_B + m_C \cdot u_C$
$= 4\times6 + 2\times0 + 3\times0 = 24\,Ns$ ——— (1)

Momentum After A collide with B $= m_A \times V_A + m_B V_B$
$= 4\times2 + 2\times V_B$ ——— (2)

from (1) & (2) $8 + 2V_B = 24 \Rightarrow V_B = \underline{8\,ms^{-1}}$ ✓ (Principle of Conservation of momentum)

(b) when B & C collide:

$V_B = 8$ $\longrightarrow$, $m_B = 2\,kg$; $u_C = 0$, $m_C = 3\,kg$ After collision $\longrightarrow$ After collision (B & C coalesce) $\longrightarrow V_D = ?$

$m_D = m_B + m_C = 2 + 3 = 5\,kg$

$m_B \times V_B + m_C u_C = m_D \times V_D$ (Conservation of Momentum)

$\Rightarrow 2\times8 + 3\times0 = 5\times V_D \Rightarrow V_D = \frac{16}{5} = \underline{3.2\,ms^{-1}}$ ✓

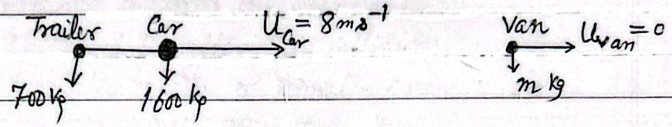
(c) Initial K.E $= \frac{1}{2} m_A \cdot u_A^2 = \frac{1}{2}\times4\times6^2 = 72\,J$ ——— (3)

Final K.E. $= \frac{1}{2} m_A \cdot V_A^2 + \frac{1}{2} m_D V_D^2 = \frac{1}{2}\times4\times2^2 + \frac{1}{2}\times5\times3.2^2$
$= 8 + 25.6 = 33.6\,J$ ——— (4)

$\therefore$ Loss in K.E $= 72 - 33.6$ (from (3) & (4))
$= \underline{38.4\,J}$ ✓

2. On a straight horizontal track, driverless vehicles are tested. A car of mass 1600 kg is towing a trailer of mass 700 kg along a track. A stationary Van is directly in front of the Car, Car hits the Van at a speed of $8\,ms^{-1}$.
After collision, the van starts to move with speed $5\,ms^{-1}$, and the Car and trailer continue moving in the same direction with speed $2\,ms^{-1}$.
Find the mass of the Van. [M-20/42/Q6(d)] ---[3]

Solution:



Momentum before the car hits the van,

$$m_{(Trailer + Car)} \times U_{Car} + m \times 0$$

$$= (2300) \times 8 + m \times 0 \quad \text{--- (1)}$$

Momentum After the Car hits the Van

$$m_{(Trailer + Car)} \times V_{Car} + m_{Van} \times V_{Van}$$

$$= 2300 \times 2 + m \times 5 \quad \text{--- (2)}$$

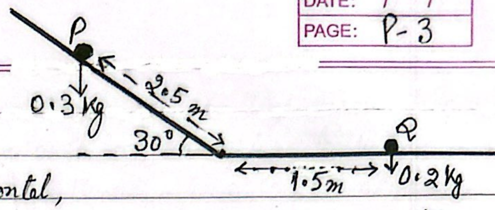
Using conservation of momentum,
from (1) and (2)

$$2300 \times 8 = 2300 \times 2 + 5m$$

$$\Rightarrow 5m = 13800$$

$$\Rightarrow m = \frac{13800}{5} = 2760$$

$\therefore$ mass of the van = 2760 kg.



3. A particle P of mass 0.3 kg, lying on a smooth plane inclined at 30° to the horizontal, is released from rest. P slides down the plane for a distance of 2.5 m and then reaches a horizontal plane. There is no change in speed when P reaches the horizontal plane. A particle Q of mass 0.2 kg lies at rest on the horizontal plane 1.5 m from the end of the inclined plane. P collides directly with Q.

(a) It is given that the horizontal plane is smooth and that, after collision, P continue to move in the same direction with speed $2 ms^{-1}$. Find the speed of Q after collision. --[5]

(b) It is given instead that the horizontal plane is rough and that when P and Q collide, they coalesce and move with speed $1.2 ms^{-1}$. Find the coefficient of friction between P and the horizontal plane. S-20/41/Q7 --[5]

Solution (a) Motion of particle P along the inclined plane;

$0.3g \sin 30° = 0.3a$ $(F = ma)$

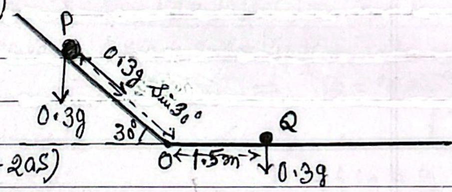
$\Rightarrow$ acc. $a = 5 ms^{-2}$

Let the speed at O, is $V ms^{-1}$

$V^2 = 0^2 + 2 \times 5 \times 2.5$ $(V^2 = u^2 + 2as)$

$= 25$

$V = 5 ms^{-1}$ ✓



now along the horizontal plane $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$\Rightarrow 0.3 \times 5 + 0.2 \times 0 = 0.3 \times 2 + 0.2 V_2$

$\therefore$ Velocity of Q after collision $V_2 = \underline{4.5 ms^{-1}}$ ✓

(b) Let the velocity of P before collision $= z ms^{-1}$

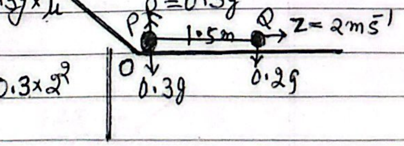
$0.3 \times z + 0 = (0.3 + 0.2) \times 1.2 \Rightarrow z = 2 ms^{-1}$

Now force of friction at P on reaching the horizontal plane $F = \mu R = 0.3g \times \mu$

$\therefore$ Work done by F on P = Change in K.E

$(0.3g \times \mu) \times 1.5 = \frac{1}{2} \times 0.3 \times 5^2 - \frac{1}{2} \times 0.3 \times 2^2$

$\Rightarrow \underline{\mu = 0.7}$ ✓



4. Small smooth spheres A and B of equal radii of masses 4 kg and 2 kg respectively lie on a smooth horizontal plane. Initially B is at rest and A is moving towards B with speed $10\,ms^{-1}$. After the spheres collide A continue to move in the same direction but with half the velocity of B.

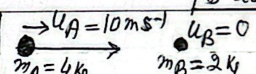
(a) Find the speed of B after collision. --[2]

A third small smooth sphere C, of mass 1 kg and with the same radius as A and B, is at rest on the plane. B now collides directly with C. After the collision B continues to move in the same direction but with one third the speed of C.

(b) Show that there is another collision between A and B. --[3]

(c) A and B coalesce during this collision. Find the total loss of kinetic energy in the system due to the three collisions. --[5]

S-20/42/Q4



after collision

$V_B = V$ (let) $\Rightarrow V_A = V/2$

Solution (a): Let speed of B after collision

$V_B = V$, $\therefore V_A = \frac{V}{2}$

Using conservation of momentum

$m_A \times u_A + m_B \cdot u_B = m_A V_A + m_B V_B \Rightarrow 4\times 10 + 2\times 0 = 4\times \frac{V}{2} + 2\times V$

$\Rightarrow V = 10 \Rightarrow V_B = 10\,ms^{-1}$ $V_A = \frac{10}{2} = 5\,ms^{-1}$

(b) for B & C

$2\times 10 + 1\times 0 \Rightarrow 2\cdot V_B + 1\cdot V_C$ $\left\{ \begin{array}{l} \text{Let } V_C = W \\ V_B = \frac{1}{3} V_C = \frac{W}{3} \end{array} \right.$

$\Rightarrow 20 = 2\times \frac{W}{3} + 1\times W$

$\left.\begin{array}{l} \Rightarrow V_C = W = 12\,ms^{-1} \\ \text{and } V_B = \frac{W}{3} = 4\,ms^{-1} \end{array}\right\}$

$\left.\begin{array}{l} \text{Now } V_B = 4\,ms^{-1} \\ \text{and } V_A = 5\,ms^{-1} \end{array}\right\} \Rightarrow V_A > V_B$

$\therefore$ There will be another collision between A & B.

(c) Now A and B coalesce.

$m_A \times V_A + m_B \times V_B = (m_A + m_B)\cdot z$ [Let the combined Vel = z of A & B]

$4\times 5 + 2\times 4 = (4+2) z \Rightarrow z = \frac{14}{3}\,ms^{-1}$ ✓

Initial K.E $= \frac{1}{2} m_A u_A^2 = \frac{1}{2}\times 4\times 10^2 = 200\,J$ — ①

Final K.E $= \frac{1}{2}(4+2)\times\left(\frac{14}{3}\right)^2 + \frac{1}{2}\times 1\times 12^2 = \frac{412}{3}\,J$ — ②

from ① & ②

$\therefore$ Loss in K.E. $= 200 - \frac{412}{3} = \frac{188}{3}\,J$ ✓

5. Particle P of mass m kg and Q of mass 0.2 kg are free to move on a smooth horizontal plane. P is projected at a speed of $2\,ms^{-1}$ towards Q which is stationary. After the collision P and Q move in opposite directions with speeds of $0.5\,ms^{-1}$ and $1\,ms^{-1}$ respectively. Find m. --[3]

S-20 / 43 / Q1

Solution: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ (Using conservation of momentum)

$m\times 2 + 0.2\times 0 = m(-0.5) + 0.2\times 1$

$\Rightarrow 2m = -0.5m + 0.2$

$\Rightarrow 2.5m = 0.2 \Rightarrow \quad m = \frac{0.2}{2.5} = \underline{0.08\ kg}$ ✓

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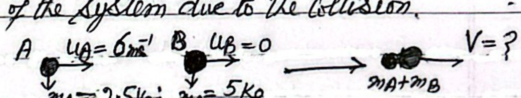
6. A particle B of mass 5 kg is at rest on a smooth horizontal table. A particle A of mass 2.5 kg moves on the table with a speed of $6\,ms^{-1}$ and collides directly with B. In the collision the two particles coalesce.

W-20 / 41 / Q1

(a) Find the speed of the combined particle after the collision. --[2]

(b) Find the loss of K.E of the system due to the collision. --[3]



Solution:

(a) Using conservation of momentum: Let the speed of combined particle is V.

$m_A \cdot u_A + m_B \cdot u_B = (m_A + m_B) \cdot V$

$2.5\times 6 + 5\times 0 = (2.5+5)V$

$\Rightarrow 15 = 7.5V \Rightarrow V = \frac{15}{7.5} = \underline{2\,ms^{-1}}$ ✓

(b) K.E. before collision $= \frac{1}{2}m_Au_A^2 + \frac{1}{2}m_B u_B^2 = \frac{1}{2}\times 2.5\times 6^2 + 0$

$= 45\,J$ —— (1)

K.E. after the collision $= \frac{1}{2}(m_A+m_B)\times V^2$

$= \frac{1}{2}(2.5+5)\times 2^2$

$= \frac{1}{2}\times 7.5\times 4 = 15\,J$ —— (2)

$\therefore$ Loss in K.E $= 45 - 15 = \underline{30\,J}$ ✓ (from (1) and (2))

7. Two particles P and Q, of masses 0.2 kg and 0.5 kg respectively, are at rest on a smooth horizontal plane. P is projected toward Q with speed $2 ms^{-1}$.

(a) Write down the momentum of P. --[1]

(b) After the collision P continues to move in the same direction with speed $0.3 ms^{-1}$.
Find the speed of Q after the collision. --[2]

W-20/42/Q1

Solution (a) Momentum of P $= m_P u_P = 0.2 \times 2 = \underline{0.4 \text{ kg ms}^{-1}}$ ✓

(b) Using conservation of momentum.

$$m_P \times u_P + m_Q \times u_Q = m_P \times V_P + m_Q \times V_Q$$

$$\Rightarrow 0.2 \times 2 + 0.5 \times 0 = 0.2 \times 0.3 + 0.5 \times V$$

$$\Rightarrow 0.4 = 0.06 + 0.5V$$

$$\Rightarrow 0.5V = 0.34$$

$$V = \frac{0.34}{0.5} = \underline{0.68 ms^{-1}}$$ ✓

8. Two small smooth spheres A and B, of equal radii and masses 4 kg and m kg respectively, lie on a smooth horizontal plane. Initially, sphere B is at rest and A is moving towards B with speed $6\,ms^{-1}$. After collision A moves with speed $1.5\,ms^{-1}$ and B with speed $3\,ms^{-1}$.
Find two possible values of the loss of kinetic energy due to the collision. ---[6]

[W-20 / 43 / Q4]

Solution: using conservation of momentum.

Case I After collision, A continues to move in the same direction;

$$m_A \cdot u_A + m_B \cdot u_B = m_A \cdot V_A + m_B \cdot V_B$$

$$\Rightarrow 4\times 6 + m\times 0 = 4\times 1.5 + m\times 3$$

$$\Rightarrow 24 = 6 + 3m \Rightarrow m_B = 6\,kg. \quad \text{——— (1)}$$

Case II. After collision, A moves in the opposite direction.

$$m_A u_A + m_B \cdot u_B = m_A \times V_A + m_B \cdot V_B$$

$$\Rightarrow 4\times 6 + m\times 0 = 4\times(-1.5) + m\times 3$$

$$24 = -6 + 3m \Rightarrow 3m = 30$$

$$\Rightarrow m_B = 10\,kg \quad \text{——— (2)}$$

Now

$$\text{Initial } K.E = \frac{1}{2} m_A \cdot u_A^2 + \frac{1}{2} m_B \times u_B^2$$

$$= \frac{1}{2}\times 4\times 6^2 + \frac{1}{2}\times m\times 0 = 72\,J \quad \text{——— (3)}$$

$\boxed{\text{Case I}}$

$$K.E.\text{ after collision} = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2$$

$$= \frac{1}{2}\times 4\times 1.5^2 + \frac{1}{2}\times 6\times 3^2 \quad \left[\text{from (1) } m_B = 6\,kg\right]$$

$$= 4.5 + 27 = 31.5\,J \quad \text{——— (4)}$$

$$\therefore \text{Loss in } K.E = 72 - 31.5 = \underline{40.5\,J} \quad (\text{from (3) \& 3})$$

$\boxed{\text{Case II}}$

$$K.E\text{ after collision} = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2$$

$$= \frac{1}{2}\times 4\times 1.5^2 + \frac{1}{2}\times 10\times 3^2 \quad \left[\text{from (2) } m_B = 10\,kg\right]$$

$$= 4.5 + 45 = 49.5\,J \quad \text{——— (5)}$$

$$\therefore \text{Loss in } K.E = 72 - 49.5 = \underline{22.5\,J} \quad (\text{from (3) and (5)})$$

9. Two particles P and Q of masses 0.2 kg and 0.3 kg respectively are free to move in a horizontal straight line on a smooth horizontal plane. P is projected towards Q with a speed $0.5\,ms^{-1}$. At the same instant Q is projected towards P with speed $1\,ms^{-1}$. Q comes to rest in the resulting collision. Find the speed of P after collision. --- [3]

[M-21/42/Q1]

P $\rightarrow$ $V_1 = ?$ $\qquad\qquad$ $\leftarrow$ Q $V_2 = 0$

Solution: $m_1 = 0.2\,kg$, $\vec{u}_1 = 0.5\,ms^{-1}$ $\qquad$ $u_2 = -1\,ms^{-1}$, $m_2 = 0.3\,kg$

$$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$$

$$\Rightarrow 0.2 \times 0.5 + 0.3 \times (-1) = 0.2 \times V_1 + 0.3 \times 0$$

$$\Rightarrow \quad -0.2 = 0.2V_1 \Rightarrow V_1 = -1\,ms^{-1}$$

$\therefore$ speed of P after collision is $1\,ms^{-1}$ in the opposite direction.

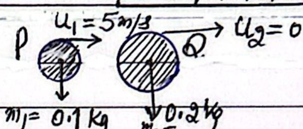
10. Three particles P, Q and R, of masses 0.1 Kg, 0.2 Kg and 0.5 Kg respectively, are at rest in a straight line on a smooth horizontal plane. Particle P projected towards Q at a speed of $5\,ms^{-1}$. After P and Q collide, P rebounds with speed $1\,ms^{-1}$.

(a) Find the speed of Q immediately after the collision with P. ---[3]

Q now collides with R. Immediately after the collision with Q, R begins to move with speed $V\,ms^{-1}$.

(b) Given that there is no subsequent collision between P and Q, find the greatest possible value of V. ---[3]

S-21/41/Q3



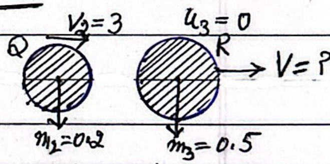
<u>Solution</u> (a) using conservation of momentum.

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$0.1 \times 5 + m_2 \times 0 = 0.1 \times (-1) + 0.2 \times v_2$$

$$0.5 + 0 = -0.1 + 0.2 v_2$$

$$\Rightarrow 0.2 v_2 = 0.6 \Rightarrow v_2 = \frac{0.6}{0.2} = \underline{3\,m/s}$$



(b) $m_2 \times v_2 + m_3 \times u_3 = m_2 v_3 + m_3 . V$

$$\Rightarrow 0.2 \times 3 + m_3 \times 0 = 0.2 \times (-1) + 0.5 \times V$$

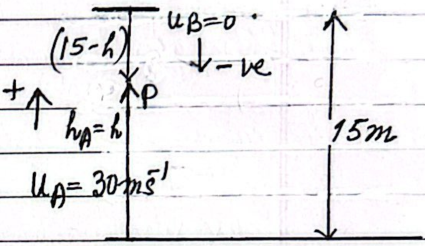
$$0.6 = -0.2 + 5V$$

$$\Rightarrow 5V = 0.8$$

$$V = \frac{0.8}{5} = \underline{1.6\,m/s}$$

[for the greatest value of V, $v_3 \geq -1$ take $v_3 = -1$]

11. A particle A is projected vertically upwards from level ground with an initial speed of $30\,ms^{-1}$. At the same instant a particle B is released from rest 15 m vertically above A. The mass of one of the particles is twice the mass of the other particle. During the subsequent motion A and B collide and coalesce to form particle C.
Find the difference between the two possible times at which C hits the ground. [S-21/42/Q6] ---[8]



Solution: $h_A = h = 30t - \frac{1}{2} \times 10 \times t^2$

$\Rightarrow h = 30t - 5t^2$ --- (1)

and $h_B = 15 - h = 0 + \frac{1}{2} \times 10t^2$

$\Rightarrow 15 - h = 5t^2$ --- (2)

add (1) and (2) $\quad 15 = 30t$

$\Rightarrow t = \frac{1}{2}\,s$ ✓

Let A and B meet at P, $t = \frac{1}{2}\,s$.

Now at P: $V_A = 30 - 10 \times \frac{1}{2}$

$V_A = 25\,m\,s^{-1}$ --- (3)

$V_B = 0 - 10 \times \frac{1}{2} = -5\,m\,s^{-1}$ --- (4)

Case I:

Let $m_A = 2m$, $m_B = m$

using conservation of momentum;

$m_A V_A + m_B V_B = (m_A + m_B)V$

$\Rightarrow 2m \times 25 + m \times (-5) = (2m + m)V$

$\Rightarrow 45m = 3mV$

$\Rightarrow V = 15\,m/s$ --- (5)

from (1) $h = 30 \times \frac{1}{2} - 5 \times \left(\frac{1}{2}\right)^2 \left[\text{at } t = \frac{1}{2}\right]$

$h = 13.75\,m$ ✓ --- (6)

Now C to reach ground:

$h = Vt + \frac{1}{2} g t^2$

$-13.75 = 15t - 5t^2$

$5t^2 - 15t - 13.75 = 0$

$\Rightarrow t^2 - 3t - 2.75 = 0$

Solving $t_1 = \frac{3 \pm \sqrt{20}}{2} = 3.74$ ✓ --- (7)

Again Case II: $m_A = m$, $m_B = 2m$

$\therefore m \times 25 - 5 \times 2m = 3mV$

$\Rightarrow 15m = 3mV \Rightarrow V = 5$ --- (8)

for (1) $-13.75 = 5t - 5t^2$

$\Rightarrow t^2 - t - 2.75 = 0$

$t_2 = \frac{1 \pm \sqrt{12}}{2} = 2.23$ ✓ --- (9)

for (7) and (9)

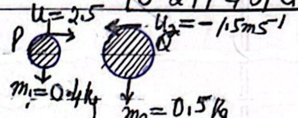
Difference between two times

$T = t_1 - t_2 = 3.74 - 2.23$

$= 1.51$ seconds.

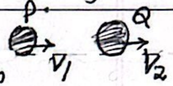
12. Particles P of mass 0.4 Kg and Q of mass 0.5 Kg are free to move on a horizontal plane. P and Q are moving directly towards each other with speeds $2.5\,ms^{-1}$ and $1.5\,ms^{-1}$ respectively. After P and Q collide, the speed of Q is twice the speed of P. --[4]
Find the two possible values of the speed of P after the collision.

[S-21/43/Q1]



Solution: P: $m_1 = 0.4$ Kg, $u_1 = 2.5\,ms^{-1}$
Q: $m_2 = 0.5$ Kg, $u_2 = -1.5\,ms^{-1}$

After collision: Let $V_1 = V$



Case I: $V_2 = 2V$ {when Q moves in the direction of P.

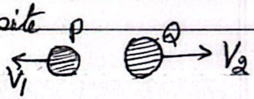
$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$

$0.4\times2.5 - 0.5\times1.5 = 0.4V + 0.5\times2V$

$\Rightarrow 1 - 0.75 = 1.4V \Rightarrow V = \frac{0.25}{1.4} = 0.179\,ms^{-1}$

Speed of $P = 0.179\,ms^{-1}$ ✓

Case II when P and Q move in the opposite directions and $V_2 = 2V$



$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$

$0.4\times2.5 - 0.5\times1.5 = -0.4V + 0.5\times2V$

$\Rightarrow 1 - 0.75 = -0.4V + V \Rightarrow 0.25 = 0.6V$

$\Rightarrow V = \frac{0.25}{0.6} = 0.417\,ms^{-1}$

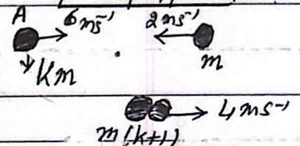
Speed of $P = 0.417\,ms^{-1}$

13. Two small smooth spheres A and B, of equal radii and of masses km kg and m kg respectively, whose $K > 1$, are free to move on a smooth horizontal plane. A is moving towards B with speed $6ms^{-1}$ and B is moving towards A with speed $2ms^{-1}$. After the collision A and B coelesce and move move with speed $4ms^{-1}$.

(a) Find k. ---[3]

(b) Find in terms of m, the loss of kinetic energy due to the collision. --[2]

W-21/41/Q2



Solution: Using conservation of momentum:

(a) $m_1u_1 + m_2u_2 = (m_1 + m_2)v$

$\Rightarrow km \times 6 - m \times 2 = (km + m) \times 4$

$\Rightarrow 6k - 2 = 4k + 4$

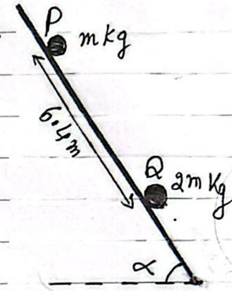
$\Rightarrow 2k = 6 \Rightarrow \underline{k = 3}$ ✓

(b) Initial K.E $= \frac{1}{2}km \cdot 6^2 + \frac{1}{2}m(-2)^2$

Final K.E $= \frac{1}{2}(km + m) \times 4^2$

(for $k=3$) $\rightarrow$ ∴ Loss in K.E $= (54m + 2m) - 32m = \underline{24m\ J}$

14. Particles P and Q have masses m kg and $2m$ kg respectively. The particles are initially held at rest 6.4 m apart on the same line of greatest slope of a rough plane inclined at an angle α to the horizontal, where $\sin\alpha = 0.8$. Particle P is released from rest and slides down the line of greatest slope. Simultaneously, particle Q is projected up the same line of greatest slope at a speed of $10\ ms^{-1}$. The coefficient of friction between each particle and the plane is 0.6.



(a) Show that the acceleration of Q up the plane is $-11.6\ ms^{-2}$ --- [4]

(b) Find the time for which the particles are in motion before they collide. --- [5]

(c) The particles coalesce on impact. Find the speed of the combined particle immediately after the impact. --- [4]

W-21/42/Q7

Solution (a) For Q. $[\sin\alpha = 0.8,\ \cos\alpha = 0.6]$

along the plane: $-2mg\sin\alpha - F = 2m.a$

$\Rightarrow -2m \times 10 \times 0.8 - F = 2ma$

$\Rightarrow -16m - F = 2ma$ —— ①

perp. to Plane: $R = 2mg\cos\alpha = 2m \times 10 \times .6 = 12m$

$\therefore F = \mu R = 0.6 \times 12m = 7.2m$ —— ②

from ① & ② $2ma = -16m - 7.2m$

$a = -11.6\ ms^{-2}$ ✓

(b) For P: $mg\sin\alpha - 0.6R = ma$ —— ③

$R = mg\cos\alpha = 6m$ —— ④

frm ③ & ④ $m \times 10 \times 0.8 - 0.6 \times (6m) = ma$

for P $\Rightarrow 8m - 3.6m = ma \Rightarrow a = 4.4\ ms^{-2}$ ✓

Now Q comes to rest when: $V = u + at_1$

$\Rightarrow 0 = 10 - 11.6t_1 \Rightarrow t_1 = 0.862$ ✓

for Q → $S_Q(up) = ut_1 + \frac{1}{2}at_1^2 = 10 \times .862 + \frac{1}{2}(-11.6)(0.862)^2$
$= 4.31$ ✓

for P → $S_P(down) = 0 + \frac{1}{2} \times 4.4 \times (0.862)^2 = 1.635$ ✓

$\therefore$ After Time t_1; the remaining dis $d = 6.4 - (4.31 + 1.635) = 0.455$ ✓

Part (b) Continued:

Now let time 't_2' is required to meet P & Q $d = 0.455$

for P = $S_P' = (4.4t_1)t_2 + \frac{1}{2} \times 4.4 \times t_2^2$ — ⑤

for Q = $S_Q' = 0 + \frac{1}{2} \times 4.4 t_2^2$ — ⑥

$S_P' - S_Q' = d \Rightarrow 4.4 \times t_1 \times t_2 = d$

$\Rightarrow 4.4 \times 0.862 \times t_2 = 0.455$

$t_2 = 0.12$

$\therefore$ Total Time before P & Q collide
$= t_1 + t_2 = 0.862 + 0.12 = 0.982$ ✓

(c) $u_P(down) = 0 + 4.4 \times 0.982 = 4.3208$ ✓

$u_Q(down) = 0 + 4.4 \times 0.12 = 0.528$ ✓

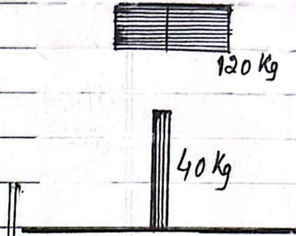
using conservation of momentum when the collide and Coalesce:

$m \times 4.3208 + 2m \times 0.528 = (m + 2m) \times V$

$\Rightarrow 3V = 5.3768$

$\Rightarrow V = 1.79\ ms^{-1}$ ✓

15. A metal post is driven vertically into the ground by dropping a heavy object onto it from above. The mass of the object is 120 kg and the mass of the post is 40 kg. The object hits the post with speed $8 ms^{-1}$, and remains in contact with it after the impact.



(a) Calculate the speed with which the combined post and object moves immediately after the impact. ---[2]

(b) There is a constant force resisting the motion of magnitude 4800 N. Calculate the distance the post is driven into the ground. --[3]

W-21/43/Q1

Solution: using conservation of momentum;

(a) $120 \times 8 + 40 \times 0 = (120+40)v$

$960 = 160v \Rightarrow \underline{v = 6 ms^{-1}}$ ✓

(b) $(120+40)g - 4800 = 160a$

$\Rightarrow a = -\frac{3200}{160} = -20 ms^{-2}$

$(v^2 = u^2 + 2as)$

$0 = 6^2 + 2 \times (-20)s$ [from part (a) $u = 6ms^{-1}$]

$\Rightarrow \underline{s = 0.9 m}$ ✓

Scanned with CamScanner

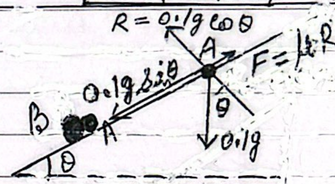
16 A bead, A, of mass 0.1 Kg is threaded on a long straight rigid wire which is inclined at $\sin^{-1}(7/25)$ to the horizontal. A is released from rest and moves down the wire. The coefficient of friction between A and the wire is μ. When A has travelled 0.45 m down the wire, its speed is $0.6\ ms^{-1}$.

(a) Show that $\mu = 0.25$ ---[6]

Another bead B, of mass 0.5 Kg is also threaded on the wire. At the point where A has travelled 0.45 m down the wire, it hits B which is instantaneouly at rest on the wire. A is brought to instantaneous on rest in the collision. The coefficient of friction between B and the wire is 0.275.

(b) Find the time from when the collision occurs until A collides with B again. ---[6]

[M-22/42/Q7]



Solution (a) $u = 0,\ V = 0.6,\ S = 0.45$

$V^2 = u^2 + 2aS \Rightarrow 0.6^2 = 0 + 2a \times 0.45$

$\Rightarrow a = .36/.9 = 0.4$ ✓

$\sin\theta = \frac{7}{25},\ \cos\theta = \frac{24}{25}$

$R = 0.1g\cos\alpha = \frac{24}{25}$ --- ①

along the thread downwards

$0.1g\sin\theta - F = ma$

$\frac{7}{25} - F = 0.1 \times 0.4$

$\Rightarrow F = \frac{7}{25} - 0.04$ —— ②

also $F = \mu R = \frac{24}{25}\mu$ ③ from ①

from ② & ③ $\frac{24}{25}\mu = \frac{7}{25} - 0.04$

$\Rightarrow \mu = \frac{25}{24} \times \frac{6}{25} = \frac{1}{4}$

$\mu = 0.25$

(b) when A and B collide

$m_A \cdot u_A + m_B \cdot u_B = m_A \cdot V_A + m_B \cdot V_B$

$0.1 \times 0.6 + 0 = 0 + 0.5 \times V_B$

$\Rightarrow V_B = \frac{.1 \times 0.6}{0.5} = 0.12$ ✓ $= V_B$

for B after Collision

$0.5g\sin\theta - \mu R = 0.5a$

$\Rightarrow 5 \times \frac{7}{25} - 0.275 \times 0.5g \times \frac{24}{25} = 0.5a$

$\Rightarrow a = 0.16$ for B ✓

$S_A = 0 + \frac{1}{2} \times 0.4t^2,$ —— ③

$S_B = 0.12t + \frac{1}{2} \times 0.16t^2$ —— ④

for A and B collide again $S_A = S_B$

$\Rightarrow 0.2t^2 = 0.12t + 0.08t^2$

$\Rightarrow 0.12t^2 - 0.12t = 0$

$\Rightarrow 0.12t[t-1] = 0 \Rightarrow t = 1s$ ✓ $(t = 0$ ✗$)$

17 Two particles A and B, of masses 0.4 kg and 0.2 kg respectively, are moving down the same line of greatest slope of a smooth plane. The plane is inclined at 30° to the horizontal, and A is higher up the plane than B. When the particles collide, the speeds of A and B are $3\,ms^{-1}$ and $2\,ms^{-1}$ respectively. In the collision between the particles, the speed of A is reduced to $2.5\,ms^{-1}$.

(a) Find the speed of B immediately after the collision. ...[2]

After the collision, when B has moved 1.6 m down the plane from the point of collision, its hits a barrier and returns back up the same line of greatest slope. B hits the barrier 0.4 s after the collision, and when it hits the barrier, its speed reduced by 90%. The two particles collide again 0.44 s after their previous collision, and they then coalesce on impact.

(b) Show that the speed of B immediately after it hits the barrier is $0.5\,ms^{-1}$. Hence find the speed of the combined particle immediately after the second collision between A and B. ---[7]

S-22/41/Q7

Law of conservation of momentum.

Solution (a) $m_A \cdot u_A + m_B u_B = m_A \cdot V_A + m_B \cdot V_B$

$0.4 \times 3 + 0.2 \times 2 = 0.4 \times 2.5 + 0.2 \times V_B$

$\Rightarrow 0.2 V_B = 0.6 \Rightarrow V_B = 3\,ms^{-1}$

(b) For A: $0.4g \sin 30^\circ = 0.4 a_1$

$\Rightarrow \qquad \Rightarrow a_1 = 5\,ms^{-2}$

for B: $0.2g \sin 30^\circ = 0.2 a_2$

$\Rightarrow a_2 = 5$

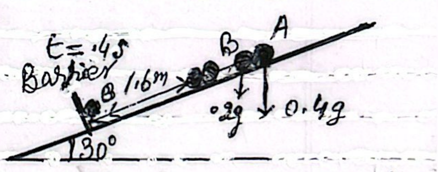
Now for B when hits the barrier

$V^2 = u^2 + 2g \Rightarrow V^2 = 3^2 + 2 \times 5 \times 1.6$

$\Rightarrow V_{B_1} = 5$ ✓

Now speed of B after hitting the barrier $V_{B_2} = 0.1 \times 5 = 0.5$ ✓

(90% less)



Now after $t = 0.44$ second of collision

$V_{A_1} = u + at = 2.5 + 5 \times 0.44 = 4.7\,ms^{-1}$

and $V_{B_3} = u + at = 0.5 - 5 \times 0.04$

$= -0.3\,ms^{-1}$

Now after coalesce; (after 4.4 s)

$m_A \cdot u_A + m_B u_B = (m_A + m_B) V$

$0.4 \times 4.7 + 0.2 \times (-0.3) = (.4 + .2) V$

$\Rightarrow 0.6 V = 1.82$

$V = 1.82/0.6$

$V = 3.03\,ms^{-1}$

18. Small smooth spheres A and B, of equal radii and masses of 5 kg and 3 kg respectively, lie on a smooth horizonal plane. Initially B is at rest and A is moving towards B with speed $8.5\,ms^{-1}$. The spheres collide and after the collision A continues to move in the same direction but with a quarter of the speed of B.

(a) Find the speed of B after the collision. ---[3]

(b) Find the loss of kinetic energy of the system due to the collision. --[2]

[S-22/42/Q1]

Solution (a) Conservation of momentum:

$m_A \cdot u_A + m_B \cdot u_B = m_A V_A + m_B \cdot V_B$

$\Rightarrow 5 \times 8.5 + 3 \times 0 = 5 \times (\frac{1}{4}V) + 3 \times V$

$\frac{17}{4} V = 42.5 \Rightarrow V_B = 10\,ms^{-1}$ ✓

(b) $K.E \text{ before} = \frac{1}{2} \times 5 \times 8.5^2 = 180.625$ ✓

$K.E \text{ After} = \frac{1}{2} \times 5 \times 2.5^2 + \frac{1}{2} \times 3 \times 10^2$

$= 165.625$ ✓

$K.E \text{ loss} = 180.625 - 165.625 = 15\,J$ ✓

19. Two particles P and Q, of masses 0.3 kg and 0.2 kg respectively, are at rest on a smooth horizontal plane. P is projected at a speed of $4\,ms^{-1}$ directly towards Q. After P and Q collide, Q begins to move with a speed of $3\,ms^{-1}$.

(a) Find the speed of P after collision. ---[2]

After the collision, Q moves directly towards a third particle R, of mass m kg, which is at rest on the plane. The two particles Q and R coalesce on impact and move with a speed of $2\,ms^{-1}$.

(b) Find m. ---[2]

[S-22/43/Q1]

Solution (a) Conservation of momentum:

$m_P \times u_p + m_q \cdot u_q = m_P \cdot V_p + m_p \cdot V_q$

$0.3 \times 4 + 0.2 \times 0 = 0.3 \times V_p + 0.2 \times 3$

$0.3\,V_p = 1.2 - 0.6 = 0.6$

$V_p = 0.6/0.3 = 2\,ms^{-1}$

(b) $m_q \cdot u_q + m_R \cdot u_r = (m_q + m_r) \times V_{comb.}$

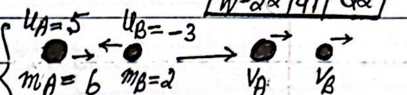
$0.2 \times 3 + 0 = (0.2 + m) \times 2$

$0.6 = 0.4 + 2m$

$\Rightarrow 2m = 0.2$

$m = 0.1\,kg$ ✓

20. Small smooth spheres A and B, of equal radii and of masses 6 kg and 2 kg respectively, lie on a smooth horizontal plane. Initially A is moving towards B with speed $5\,ms^{-1}$ and B is moving towards A with speed $3\,ms^{-1}$. After the spheres collides, both A and B move in the same direction and the difference in the speeds of the spheres is $2\,ms^{-1}$.
Find the loss of kinetic energy of the system due to the collision. --[5]
[W-22/41/Q2]



$\therefore V_B = V_A+2$

Solution: Using conservation of momentum:

$m_A \cdot u_A + m_B u_B = m_A V_A + m_B \cdot V_B$

$\Rightarrow 6\times 5 + 2\times(-3) = 6V_A + 2V_B$

$\Rightarrow 6V_A + 2V_B = 24$ --- ①

and $V_B = V_A + 2$ --- ②

Solving ① and ② $V_A = 2.5$ and $V_B = 4.5$

Now Initial $K.E = \frac{1}{2}\times 6\times 5^2 + \frac{1}{2}\times 2\times(-3)^2 = 84\,J$ ✓ $[K.E = \frac{1}{2}mv^2]$

Final $K.E = \frac{1}{2}\times 6\times(2.5)^2 + \frac{1}{2}\times 2\times(4.5)^2 = 39\,J$

Change in $K.E = 84 - 39 = \underline{45\,J}$ ✓

21. Three particles A, B and C of masses 0.3 Kg, 0.4 Kg and m kg respectively lie at rest in a straight line on a smooth horizontal plane. The distance between B and C is 2.1 m. 'A' is projected directly towards B with speed $2\,ms^{-1}$. After 'A' collides with B the speed of A is reduced to $0.6\,ms^{-1}$, still moving in the same direction.

(a) Show that the speed of B after collision is $1.05\,ms^{-1}$. --- [2]

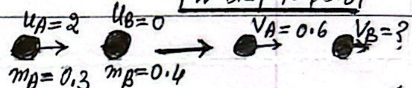
After the collision between A and B, B moves directly towards C. Particle B now collide with C. After this collision, the two particles coalesce and have a combined speed of $0.5\,ms^{-1}$. --- [2]

(b) Find m.

(c) Find the time that it takes, from the instant when B and C collide, until A collides with the combined particle. --- [5]

W-22/42/Q6

Solution (a) Using conservation of momentum.

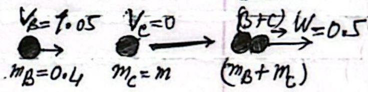


$$m_A \times u_A + m_B \times u_B = m_A V_A + m_B \cdot V_B$$

$$\Rightarrow 0.3 \times 2 + 0.4 \times 0 = 0.3 \times 0.6 + 0.4 \times V_B \Rightarrow 0.4 V_B = 0.42 \Rightarrow V_B = 1.05\,ms^{-1}$$

Now $m_c = m$

(b) $m_B \times V_B + m_C \times V_C = (m_B + m_C) \times W_{(B+C)}$ (Coalesce)



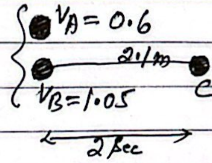
$$0.4 \times 1.05 + m \times 0 = (0.4 + m) \times 0.5$$

$$\Rightarrow 0.42 = 0.2 + 0.5m \Rightarrow m = \frac{0.22}{0.5} = 0.44$$

$$\Rightarrow m_c = m = 0.44\,kg$$

(c) Time B takes to reach C $= \frac{2.1\,m}{V_B} = \frac{2.1}{1.05}$

$= 2$ Sec.



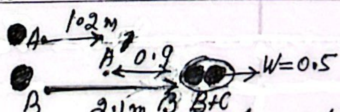
A travels in 2 sec $= 0.6 \times 2 = 1.2\,m$

When B collides C,

distance between A' & (BC) combined

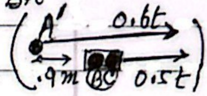
$= 2.1\,m - 1.2\,m = 0.9\,m$

Now

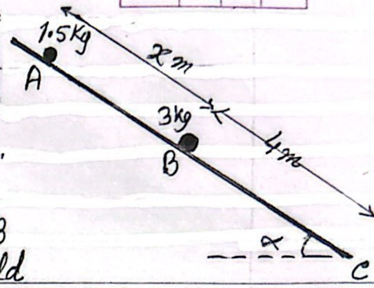


Now if the required time $= t$ s

distances $\Rightarrow \underset{A'}{0.6 \times t} - \underset{BC}{0.5 \times t} = 0.9\,m \Rightarrow t = 9\,s$



22. Particles of masses 1.5 kg and 3 kg lie on a plane which is inclined at an angle of α to the horizontal, where $\tan\alpha = 3/4$. The section of the plane from A to B is smooth and the section of the plane from B to C is rough. The 1.5 kg particle is held at rest at A and the 3 kg particle is in limiting equilibrium at B. The distance AB is x m and the distance BC is 4 m.



(a) Show that the coefficient of friction between the particle B and plane is 0.75. --- [3]

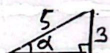
The 1.5 kg is released from rest. In the subsequent motion the two particles collide and coalesce. The time taken for the combined particle to travel from B to C is 2 s. The coefficient of friction between the combined particle and the plane is still 0.75.

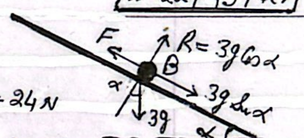
(b) Find x. [6]

(c) Find the total loss of kinetic energy of the particles from the time the 1.5 kg particle is released until the combined particle reaches C. --- [3]

W-22/43/Q7

Solution. $\tan\alpha = 3/4 \Rightarrow \sin\alpha = 3/5$; $\cos\alpha = \frac{4}{5}$

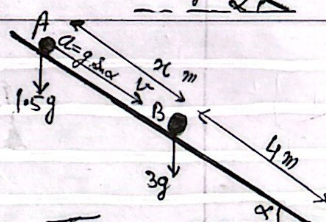




(a) Resolving at B; perp to the plane: $R = 3g\cos\alpha = 3\times10\times0.8 = 24\,N$

Along the plane $F = 3g\sin\alpha = 3\times10\times0.6 = 18\,N$

$F = \mu R \Rightarrow \mu = \frac{F}{R} = \frac{18}{24} = 0.75$ ✓



(b) At A, acc $a = g\sin\alpha$, dis $AB = x$ m

∴ velocity of A, when reaches at $B = v_1$

$v_1^2 = 0 + 2g\sin\alpha \cdot x \Rightarrow v_1^2 = 2\times10\times0.6x = 12x \Rightarrow v_1 = \sqrt{12x}$

Now when A collides with B, (using conservation of momentum)

$m_A\cdot v_1 + m_B\times0 = (m_A+m_B)\cdot v_2 \Rightarrow 1.5\times\sqrt{12x}+0 = 4.5\times v_2 \Rightarrow v_2 = \frac{1}{3}\sqrt{12x}$ ✓

for combined motion of (A+B) at B; $4.5g\sin\alpha - 0.75\times4.5g\times\cos\alpha = 4.5a$

for motion BC → dis = 4m, time = 2s, a = 0 ⟸ ⟹ $0 = 4.5a \Rightarrow a = 0$ ✓

and $v_2 = \frac{1}{3}\sqrt{12x}$

$4 = \frac{1}{3}\sqrt{12x}\times2 + 0 \Rightarrow 12x = 36$ ($S = ut + \frac{1}{2}at^2$)

$\Rightarrow x = 3\,m$ ✓

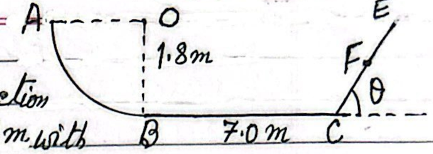
(c) $v_2 = \frac{1}{3}\sqrt{12x} = \frac{1}{3}\sqrt{12\times3} = \frac{1}{3}\times6 = 2\,m/s$

For motion B→C

$K.E = \frac{1}{2}\times4.5\times2^2 = 9\,J.$

P.E. Loss $= mgh = (15\times(4+3)\frac{3}{5} + 30\times4\times3/5) = 135\,J.$

∴ Loss of Energy $= 135 - 9 = 126\,J$ ✓



23. The diagram shows a smooth track which lies in a vertical plane. The section AB is a quarter circle of radius 1.8 m with centre O. The section BC is a horizontal straigh line of length 7.0m and OB is perpendicular to BC. The section CFE is a straight line inclined at an angle of $\theta°$ above the horizontal.

The partical P of mass 0.5 Kg is released from rest at A. Particle P collides with a particle Q of mass 0.1Kg which at rest at B. Immediately after the collision, the speed of P is $4\,ms^{-1}$ in the direction of BC. P is moving horizontally when it collides with Q.

(a) Show that the speed of Q immediately after the collision is $10\,ms^{-1}$. ---[4]

When Q reaches C, it collides with a particle R of mass 0.4kg which is at rest at C. The two particles coalesce. The combind particle comes instantaneously to rest at F. There is no instantaneous change in speed as the combined particle leaves leaves C, nor when it passes through C again as it returns down the slope.

(b) Given that the distance CF is 0.4m, find the value of θ. ---[4]

(C) Find the distance from B at which P collides with the combined particle. --[5]

[M-23/42/Q7]

Solution (a) Motion from A→B; (using conservation of energy) $\frac{1}{2}mV^2 = mgh$

$\Rightarrow \frac{1}{2}\times 0.5\times V^2 = 0.5\times g\times 1.8$ [V = Velocity of P at B) [h = 1.8;

$\Rightarrow V = 6.$

Now using conservation of momentum.

$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$

$0.5\times 6 + 0.1\times 0 = 0.5\times 4 + 0.1\times V_2$

$0.1V_2 = 1 \Rightarrow V_2 = 10$

∴ Speed of Q = $10\,ms^{-1}$ (V)

(b) Conservation of momentum

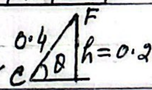
$m_Q\times V_B = m(\text{combined})\times V_Z$

$0.1\times 10 = (0.1+0.4)\cdot V_C \Rightarrow V_C = 2\,ms^{-1}$

using conservation of energy for C→F

$\frac{1}{2}(0.1+0.4)\times 2^2 = (0.1+0.4)gh \Rightarrow h = 0.2\,m.$

$\Rightarrow$ $\sin\theta = \frac{0.2}{0.4} = \frac{1}{2}$



$\theta = \sin^{-1}\frac{1}{2} = 30°$

(C)

Partical Q takes time B to C = $\frac{BC}{Speed} = \frac{7}{10}$ = 0.7 seconds.

Time of combined partical for C→F = $\frac{Dis}{Speed}$

$= \frac{0.4}{\text{Average Speed}} = \frac{0.4}{\left(\frac{2+0}{2}\right)} = 0.4s$

Total time from C to F and back = $0.4\times 2 = 0.8s$

Speed of P after collision with Q = $4\,ms^{-1}$

∴ distance travelled by P = $(0.7+0.8)\times 4$

$= 6\,m$

Now let the time when comb. particle reaches → back at C, and approaches to P

$4t + 2t = 1 \Rightarrow t = \frac{1}{6}$

distance = $4\times\frac{1}{6} = \frac{2}{3}\,m \Rightarrow$ Total Distance = $6\frac{2}{3}\,m$

24. Two particles P and Q, of masses m kg and 0.3 kg respectively, are at rest on a smooth horizontal plane. P is projected at a speed of $5\ ms^{-1}$ directly towards Q. After P and Q collide, P moves with a speed of $2\ ms^{-1}$ in the same direction as it was moving originally.

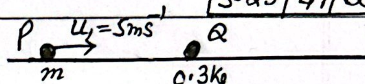
(a) Find, in terms of m, the speed of Q after the collision. ---[2]

After this collision, Q moves directly towards a third particle R, of mass 0.6 kg, which is at rest on the plane. Q is brought to rest in the collision with R, and R begins to move with a speed of $1.5\ ms^{-1}$.

(b) Find the value of m. ---[2]

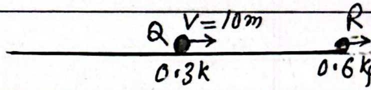
S-23/41/Q1

Solution: Conservation of momentum:



(a)
$$m_1 u_1 + m_2 u_2 = m_1 V_1 + m_2 V_Q$$
$$m \times 5 + 0.3 \times 0 = m \times 2 + 0.3 \times V_Q$$
$$\Rightarrow \quad 0.3\, V_Q = 3m \Rightarrow V_Q = \underline{10m}\ (ms^{-1})$$

(b) Conservation of momentum:



$$m_Q \cdot u_Q + m_R \cdot u_R = m_Q \times V_Q + m_R \times V_R$$
$$0.3 \times (10m) + 0.6 \times 0 = m_Q \times 0 + 0.6 \times 1.5$$
$$\Rightarrow 3m = 0.9$$
$$\Rightarrow m = \underline{0.3}\ Kg$$

25. Two particles A and B, of masses 3.2 Kg and 2.4 Kg respectively, lie on a smooth horizontal table. A moves towards B with a speed of $v\,ms^{-1}$, and collides with B, which is moving towards A with a speed of $6\,ms^{-1}$. In the collision the two particles come to rest.

(a) Find the value of v. ---[2]

(b) Find the loss of kinetic energy of the system due to the collision. --[2]

[S-23/42/Q2]

Solution (a) Conservation of momentum

$A \bullet \rightarrow v \qquad 6 \leftarrow \bullet B$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \Rightarrow 3.2\times v + 2.4\times(-6) = 0+0$$

$$\Rightarrow v = \frac{6\times 2.4}{3.2} = 4.5\,ms^{-1}$$

(b) Initial K.E $= \frac{1}{2}\times 3.2\times(4.5)^2 + \frac{1}{2}\times 2.4\times(-6)^2 = 75.6$

Final K.E = 0 (both the particles are at rest)

$\therefore$ Loss in K.E = $\underline{75.6}$

26. Two particles P and Q, of masses 0.1 Kg and 0.4 Kg respectively, are free to move on a smooth horizontal plane. Particle P is is projected with speed $4\,ms^{-1}$ towards Q which is stationary. After P and Q collide, the speeds of P and Q are equal.

Find the two possible values of the speeds of P after collision. --[3]

[S-23/43/Q1]

Solution: Using Conservation of momentum, $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Case I $\quad 0.1\times 4 + 0.4\times 0 = 0.1v + 0.4v$

$\Rightarrow 0.5v = 0.4 \Rightarrow \underline{v = 0.8\,ms^{-1}}$ } both move in the same direction

Case II Both move in opposite directions after collision.

$0.1\times 4 + 0.4\times 0 = 0.1(-v) + 0.4(v)$ $\quad$ P $\bullet\rightarrow\bullet$ Q $\Rightarrow$ $\leftarrow$P $\bullet\bullet$ Q$\rightarrow$ after collision

$\Rightarrow 0.4 = 0.3v \Rightarrow v = \frac{4}{3}$

$= 1.33\,ms^{-1}$

P move with $v = \underline{1.33\,ms^{-1}}$ in reverse direction.