

## **MECHANICS**

**9709**

(March, June, and November series 2020 – 2023 with Marking Scheme)

### **Momentum**

#### **Exercise - 1**

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1. Three small smooth spheres  $A$ ,  $B$  and  $C$  of equal radii and of masses 4 kg, 2 kg and 3 kg respectively, lie in that order in a straight line on a smooth horizontal plane. Initially,  $B$  and  $C$  are at rest and  $A$  is moving towards  $B$  with speed  $6 \text{ m s}^{-1}$ . After the collision with  $B$ , sphere  $A$  continues to move in the same direction but with speed  $2 \text{ m s}^{-1}$ .

(a) Find the speed of  $B$  after this collision. [2]

Sphere  $B$  collides with  $C$ . In this collision these two spheres coalesce to form an object  $D$ .

(b) Find the speed of  $D$  after this collision. [2]

(c) Show that the total loss of kinetic energy in the system due to the two collisions is 38.4 J. [2]

### Question 3: 9709\_y20\_sp\_4

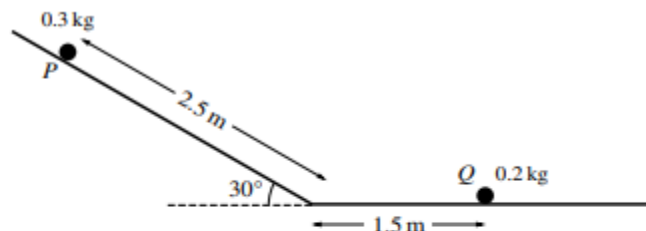
2. On a straight horizontal test track, driverless vehicles (with no passengers) are being tested. A car of mass 1600 kg is towing a trailer of mass 700 kg along the track. The brakes are applied, resulting in a deceleration of  $12 \text{ m s}^{-2}$ . The braking force acts on the car only. In addition to the braking force there are constant resistance forces of 600 N on the car and of 200 N on the trailer.

(d) After the collision, the van starts to move with speed  $5 \text{ m s}^{-1}$  and the car and trailer continue moving in the same direction with speed  $2 \text{ m s}^{-1}$ .

Find the mass of the van. [3]

### Question 6(d): 9709\_m20\_qp\_42

3.



A particle  $P$  of mass  $0.3 \text{ kg}$ , lying on a smooth plane inclined at  $30^\circ$  to the horizontal, is released from rest.  $P$  slides down the plane for a distance of  $2.5 \text{ m}$  and then reaches a horizontal plane. There is no change in speed when  $P$  reaches the horizontal plane. A particle  $Q$  of mass  $0.2 \text{ kg}$  lies at rest on the horizontal plane  $1.5 \text{ m}$  from the end of the inclined plane (see diagram).  $P$  collides directly with  $Q$ .

(a) It is given that the horizontal plane is smooth and that, after the collision,  $P$  continues moving in the same direction, with speed  $2 \text{ m s}^{-1}$ .

Find the speed of  $Q$  after the collision. [5]

(b) It is given instead that the horizontal plane is rough and that when  $P$  and  $Q$  collide, they coalesce and move with speed  $1.2 \text{ m s}^{-1}$ .

Find the coefficient of friction between  $P$  and the horizontal plane. [5]

### Question 7: 9709\_s20\_qp\_41

4. Small smooth spheres  $A$  and  $B$ , of equal radii and of masses  $4\text{ kg}$  and  $2\text{ kg}$  respectively, lie on a smooth horizontal plane. Initially  $B$  is at rest and  $A$  is moving towards  $B$  with speed  $10\text{ m s}^{-1}$ . After the spheres collide  $A$  continues to move in the same direction but with half the speed of  $B$ .

(a) Find the speed of  $B$  after the collision. [2]

A third small smooth sphere  $C$ , of mass  $1\text{ kg}$  and with the same radius as  $A$  and  $B$ , is at rest on the plane.  $B$  now collides directly with  $C$ . After this collision  $B$  continues to move in the same direction but with one third the speed of  $C$ .

(b) Show that there is another collision between  $A$  and  $B$ . [3]

(c)  $A$  and  $B$  coalesce during this collision.

Find the total loss of kinetic energy in the system due to the three collisions. [5]

**Question 4: 9709\_s20\_qp\_42**

5. Particles  $P$  of mass  $m\text{ kg}$  and  $Q$  of mass  $0.2\text{ kg}$  are free to move on a smooth horizontal plane.  $P$  is projected at a speed of  $2\text{ m s}^{-1}$  towards  $Q$  which is stationary. After the collision  $P$  and  $Q$  move in opposite directions with speeds of  $0.5\text{ m s}^{-1}$  and  $1\text{ m s}^{-1}$  respectively.

Find  $m$ . [3]

**Question 1: 9709\_s20\_qp\_43**

6. A particle  $B$  of mass  $5\text{ kg}$  is at rest on a smooth horizontal table. A particle  $A$  of mass  $2.5\text{ kg}$  moves on the table with a speed of  $6\text{ m s}^{-1}$  and collides directly with  $B$ . In the collision the two particles coalesce.

(a) Find the speed of the combined particle after the collision. [2]

(b) Find the loss of kinetic energy of the system due to the collision. [3]

**Question 1: 9709\_w20\_qp\_41**

7. Two particles  $P$  and  $Q$ , of masses  $0.2\text{ kg}$  and  $0.5\text{ kg}$  respectively, are at rest on a smooth horizontal plane.  $P$  is projected towards  $Q$  with speed  $2\text{ m s}^{-1}$ .

(a) Write down the momentum of  $P$ . [1]

(b) After the collision  $P$  continues to move in the same direction with speed  $0.3\text{ m s}^{-1}$ .

Find the speed of  $Q$  after the collision. [2]

**Question 1: 9709\_w20\_qp\_42**

8. Two small smooth spheres  $A$  and  $B$ , of equal radii and of masses  $4\text{ kg}$  and  $m\text{ kg}$  respectively, lie on a smooth horizontal plane. Initially, sphere  $B$  is at rest and  $A$  is moving towards  $B$  with speed  $6\text{ m s}^{-1}$ . After the collision  $A$  moves with speed  $1.5\text{ m s}^{-1}$  and  $B$  moves with speed  $3\text{ m s}^{-1}$ .

Find the two possible values of the loss of kinetic energy due to the collision. [6]

**Question 4: 9709\_w20\_qp\_43**

9. Two particles  $P$  and  $Q$  of masses  $0.2\text{ kg}$  and  $0.3\text{ kg}$  respectively are free to move in a horizontal straight line on a smooth horizontal plane.  $P$  is projected towards  $Q$  with speed  $0.5\text{ m s}^{-1}$ . At the same instant  $Q$  is projected towards  $P$  with speed  $1\text{ m s}^{-1}$ .  $Q$  comes to rest in the resulting collision.
- Find the speed of  $P$  after the collision. [3]

**Question 1: 9709\_m21\_qp\_42**

10. Three particles  $P$ ,  $Q$  and  $R$ , of masses  $0.1\text{ kg}$ ,  $0.2\text{ kg}$  and  $0.5\text{ kg}$  respectively, are at rest in a straight line on a smooth horizontal plane. Particle  $P$  is projected towards  $Q$  at a speed of  $5\text{ m s}^{-1}$ . After  $P$  and  $Q$  collide,  $P$  rebounds with speed  $1\text{ m s}^{-1}$ .
- (a) Find the speed of  $Q$  immediately after the collision with  $P$ . [3]
- $Q$  now collides with  $R$ . Immediately after the collision with  $Q$ ,  $R$  begins to move with speed  $V\text{ m s}^{-1}$ .
- (b) Given that there is no subsequent collision between  $P$  and  $Q$ , find the greatest possible value of  $V$ . [3]

**Question 3: 9709\_s21\_qp\_41**

11. A particle  $A$  is projected vertically upwards from level ground with an initial speed of  $30\text{ m s}^{-1}$ . At the same instant a particle  $B$  is released from rest  $15\text{ m}$  vertically above  $A$ . The mass of one of the particles is twice the mass of the other particle. During the subsequent motion  $A$  and  $B$  collide and coalesce to form particle  $C$ .
- Find the difference between the two possible times at which  $C$  hits the ground. [8]

**Question 6: 9709\_s21\_qp\_42**

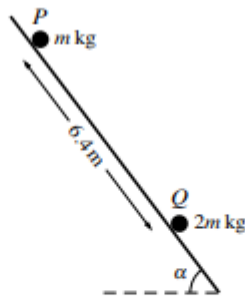
12. Particles  $P$  of mass  $0.4\text{ kg}$  and  $Q$  of mass  $0.5\text{ kg}$  are free to move on a smooth horizontal plane.  $P$  and  $Q$  are moving directly towards each other with speeds  $2.5\text{ m s}^{-1}$  and  $1.5\text{ m s}^{-1}$  respectively. After  $P$  and  $Q$  collide, the speed of  $Q$  is twice the speed of  $P$ .
- Find the two possible values of the speed of  $P$  after the collision. [4]

**Question 1: 9709\_s21\_qp\_43**

13. Two small smooth spheres  $A$  and  $B$ , of equal radii and of masses  $km\text{ kg}$  and  $m\text{ kg}$  respectively, where  $k > 1$ , are free to move on a smooth horizontal plane.  $A$  is moving towards  $B$  with speed  $6\text{ m s}^{-1}$  and  $B$  is moving towards  $A$  with speed  $2\text{ m s}^{-1}$ . After the collision  $A$  and  $B$  coalesce and move with speed  $4\text{ m s}^{-1}$ .
- (a) Find  $k$ . [3]
- (b) Find, in terms of  $m$ , the loss of kinetic energy due to the collision. [2]

**Question 2: 9709\_w21\_qp\_41**

14.



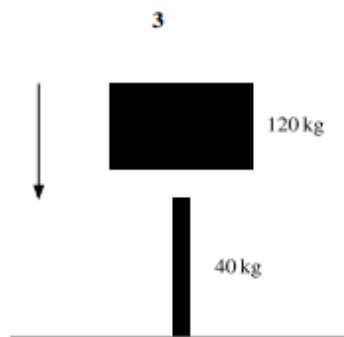
Particles  $P$  and  $Q$  have masses  $m$  kg and  $2m$  kg respectively. The particles are initially held at rest  $6.4$  m apart on the same line of greatest slope of a rough plane inclined at an angle  $\alpha$  to the horizontal, where  $\sin \alpha = 0.8$  (see diagram). Particle  $P$  is released from rest and slides down the line of greatest slope. Simultaneously, particle  $Q$  is projected up the same line of greatest slope at a speed of  $10 \text{ m s}^{-1}$ . The coefficient of friction between each particle and the plane is  $0.6$ .

- (a) Show that the acceleration of  $Q$  up the plane is  $-11.6 \text{ m s}^{-2}$ . [4]  
 (b) Find the time for which the particles are in motion before they collide. [5]  
 (c) The particles coalesce on impact.

Find the speed of the combined particle immediately after the impact. [4]

Question 7: 9709\_w21\_qp\_42

15.



A metal post is driven vertically into the ground by dropping a heavy object onto it from above. The mass of the object is  $120$  kg and the mass of the post is  $40$  kg (see diagram). The object hits the post with speed  $8 \text{ m s}^{-1}$  and remains in contact with it after the impact.

- (a) Calculate the speed with which the combined post and object moves immediately after the impact. [2]  
 (b) There is a constant force resisting the motion of magnitude  $4800 \text{ N}$ .

Calculate the distance the post is driven into the ground. [3]

Question 1: 9709\_w21\_qp\_43

16. A bead,  $A$ , of mass  $0.1\text{ kg}$  is threaded on a long straight rigid wire which is inclined at  $\sin^{-1}\left(\frac{7}{25}\right)$  to the horizontal.  $A$  is released from rest and moves down the wire. The coefficient of friction between  $A$  and the wire is  $\mu$ . When  $A$  has travelled  $0.45\text{ m}$  down the wire, its speed is  $0.6\text{ m s}^{-1}$ .

(a) Show that  $\mu = 0.25$ . [6]

Another bead,  $B$ , of mass  $0.5\text{ kg}$  is also threaded on the wire. At the point where  $A$  has travelled  $0.45\text{ m}$  down the wire, it hits  $B$  which is instantaneously at rest on the wire.  $A$  is brought to instantaneous rest in the collision. The coefficient of friction between  $B$  and the wire is  $0.275$ .

(b) Find the time from when the collision occurs until  $A$  collides with  $B$  again. [6]

#### Question 7: 9709\_m22\_qp\_42

17. Two particles  $A$  and  $B$ , of masses  $0.4\text{ kg}$  and  $0.2\text{ kg}$  respectively, are moving down the same line of greatest slope of a smooth plane. The plane is inclined at  $30^\circ$  to the horizontal, and  $A$  is higher up the plane than  $B$ . When the particles collide, the speeds of  $A$  and  $B$  are  $3\text{ m s}^{-1}$  and  $2\text{ m s}^{-1}$  respectively. In the collision between the particles, the speed of  $A$  is reduced to  $2.5\text{ m s}^{-1}$ .

(a) Find the speed of  $B$  immediately after the collision. [2]

After the collision, when  $B$  has moved  $1.6\text{ m}$  down the plane from the point of collision, it hits a barrier and returns back up the same line of greatest slope.  $B$  hits the barrier  $0.4\text{ s}$  after the collision, and when it hits the barrier, its speed is reduced by  $90\%$ . The two particles collide again  $0.44\text{ s}$  after their previous collision, and they then coalesce on impact.

(b) Show that the speed of  $B$  immediately after it hits the barrier is  $0.5\text{ m s}^{-1}$ . Hence find the speed of the combined particle immediately after the second collision between  $A$  and  $B$ . [7]

#### Question 7: 9709\_s22\_qp\_41

18. Small smooth spheres  $A$  and  $B$ , of equal radii and of masses  $5\text{ kg}$  and  $3\text{ kg}$  respectively, lie on a smooth horizontal plane. Initially  $B$  is at rest and  $A$  is moving towards  $B$  with speed  $8.5\text{ m s}^{-1}$ . The spheres collide and after the collision  $A$  continues to move in the same direction but with a quarter of the speed of  $B$ .

(a) Find the speed of  $B$  after the collision. [3]

(b) Find the loss of kinetic energy of the system due to the collision. [2]

#### Question 1: 9709\_s22\_qp\_42

19. Two particles  $P$  and  $Q$ , of masses  $0.3\text{ kg}$  and  $0.2\text{ kg}$  respectively, are at rest on a smooth horizontal plane.  $P$  is projected at a speed of  $4\text{ m s}^{-1}$  directly towards  $Q$ . After  $P$  and  $Q$  collide,  $Q$  begins to move with a speed of  $3\text{ m s}^{-1}$ .

(a) Find the speed of  $P$  after the collision. [2]

After the collision,  $Q$  moves directly towards a third particle  $R$ , of mass  $m\text{ kg}$ , which is at rest on the plane. The two particles  $Q$  and  $R$  coalesce on impact and move with a speed of  $2\text{ m s}^{-1}$ .

(b) Find  $m$ . [2]

Question 1: 9709\_s22\_qp\_43

20. Small smooth spheres  $A$  and  $B$ , of equal radii and of masses  $6\text{ kg}$  and  $2\text{ kg}$  respectively, lie on a smooth horizontal plane. Initially  $A$  is moving towards  $B$  with speed  $5\text{ m s}^{-1}$  and  $B$  is moving towards  $A$  with speed  $3\text{ m s}^{-1}$ . After the spheres collide, both  $A$  and  $B$  move in the same direction and the difference in the speeds of the spheres is  $2\text{ m s}^{-1}$ .

Find the loss of kinetic energy of the system due to the collision. [5]

Question 2: 9709\_w22\_qp\_41

21. Three particles  $A$ ,  $B$  and  $C$  of masses  $0.3\text{ kg}$ ,  $0.4\text{ kg}$  and  $m\text{ kg}$  respectively lie at rest in a straight line on a smooth horizontal plane. The distance between  $B$  and  $C$  is  $2.1\text{ m}$ .  $A$  is projected directly towards  $B$  with speed  $2\text{ m s}^{-1}$ . After  $A$  collides with  $B$  the speed of  $A$  is reduced to  $0.6\text{ m s}^{-1}$ , still moving in the same direction.

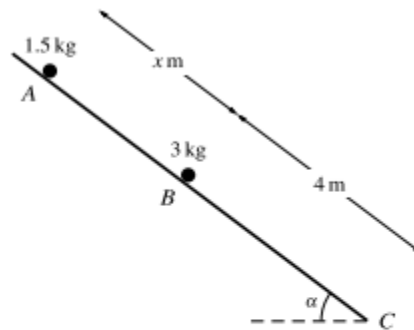
(a) Show that the speed of  $B$  after the collision is  $1.05\text{ m s}^{-1}$ . [2]

After the collision between  $A$  and  $B$ ,  $B$  moves directly towards  $C$ . Particle  $B$  now collides with  $C$ . After this collision, the two particles coalesce and have a combined speed of  $0.5\text{ m s}^{-1}$ .

(b) Find  $m$ . [2]

Question 6: 9709\_w22\_qp\_42

- 22.



Particles of masses  $1.5\text{ kg}$  and  $3\text{ kg}$  lie on a plane which is inclined at an angle of  $\alpha$  to the horizontal, where  $\tan \alpha = \frac{3}{4}$ . The section of the plane from  $A$  to  $B$  is smooth and the section of the plane from  $B$  to  $C$  is rough. The  $1.5\text{ kg}$  particle is held at rest at  $A$  and the  $3\text{ kg}$  particle is in limiting equilibrium at  $B$ . The distance  $AB$  is  $x\text{ m}$  and the distance  $BC$  is  $4\text{ m}$  (see diagram).

(a) Show that the coefficient of friction between the particle at  $B$  and the plane is  $0.75$ . [3]

The  $1.5\text{ kg}$  particle is released from rest. In the subsequent motion the two particles collide and coalesce. The time taken for the combined particle to travel from  $B$  to  $C$  is  $2\text{ s}$ . The coefficient of friction between the combined particle and the plane is still  $0.75$ .

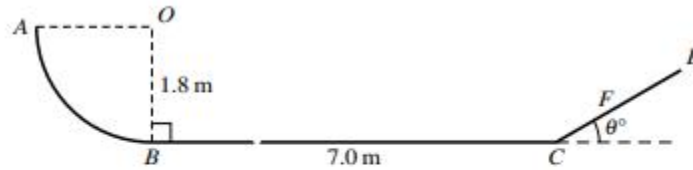
(b) Find  $x$ . [6]

(c) Find the total loss of energy of the particles from the time the  $1.5\text{ kg}$  particle is released until the combined particle reaches  $C$ . [3]



Question 7: 9709\_w22\_qp\_41

23.



The diagram shows a smooth track which lies in a vertical plane. The section  $AB$  is a quarter circle of radius  $1.8\text{ m}$  with centre  $O$ . The section  $BC$  is a horizontal straight line of length  $7.0\text{ m}$  and  $OB$  is perpendicular to  $BC$ . The section  $CFE$  is a straight line inclined at an angle of  $\theta^\circ$  above the horizontal.

A particle  $P$  of mass  $0.5\text{ kg}$  is released from rest at  $A$ . Particle  $P$  collides with a particle  $Q$  of mass  $0.1\text{ kg}$  which is at rest at  $B$ . Immediately after the collision, the speed of  $P$  is  $4\text{ m s}^{-1}$  in the direction  $BC$ . You should assume that  $P$  is moving horizontally when it collides with  $Q$ .

(a) Show that the speed of  $Q$  immediately after the collision is  $10\text{ m s}^{-1}$ . [4]

When  $Q$  reaches  $C$ , it collides with a particle  $R$  of mass  $0.4\text{ kg}$  which is at rest at  $C$ . The two particles coalesce. The combined particle comes instantaneously to rest at  $F$ . You should assume that there is no instantaneous change in speed as the combined particle leaves  $C$ , nor when it passes through  $C$  again as it returns down the slope.

(b) Given that the distance  $CF$  is  $0.4\text{ m}$ , find the value of  $\theta$ . [4]

(c) Find the distance from  $B$  at which  $P$  collides with the combined particle. [5]

Question 7: 9709\_m23\_qp\_42

24.

Two particles  $P$  and  $Q$ , of masses  $m\text{ kg}$  and  $0.3\text{ kg}$  respectively, are at rest on a smooth horizontal plane.  $P$  is projected at a speed of  $5\text{ m s}^{-1}$  directly towards  $Q$ . After  $P$  and  $Q$  collide,  $P$  moves with a speed of  $2\text{ m s}^{-1}$  in the same direction as it was originally moving.

(a) Find, in terms of  $m$ , the speed of  $Q$  after the collision. [2]

After this collision,  $Q$  moves directly towards a third particle  $R$ , of mass  $0.6\text{ kg}$ , which is at rest on the plane.  $Q$  is brought to rest in the collision with  $R$ , and  $R$  begins to move with a speed of  $1.5\text{ m s}^{-1}$ .

(b) Find the value of  $m$ . [2]

Question 1: 9709\_s23\_qp\_41

25.

Two particles  $A$  and  $B$ , of masses  $3.2\text{ kg}$  and  $2.4\text{ kg}$  respectively, lie on a smooth horizontal table.  $A$  moves towards  $B$  with a speed of  $v\text{ m s}^{-1}$  and collides with  $B$ , which is moving towards  $A$  with a speed of  $6\text{ m s}^{-1}$ . In the collision the two particles come to rest.

(a) Find the value of  $v$ . [2]

(b) Find the loss of kinetic energy of the system due to the collision. [2]



Question 2: 9709\_s23\_qp\_42

26. Two particles  $P$  and  $Q$ , of masses  $0.1\text{ kg}$  and  $0.4\text{ kg}$  respectively, are free to move on a smooth horizontal plane. Particle  $P$  is projected with speed  $4\text{ m s}^{-1}$  towards  $Q$  which is stationary. After  $P$  and  $Q$  collide, the speeds of  $P$  and  $Q$  are equal.

Find the two possible values of the speed of  $P$  after the collision.

[3]

Question 1: 9709\_s23\_qp\_43

## MARKING SCHEME

1.

Answer	Marks	Partial Marks	Guidance
Conservation of momentum $4 \times 6 [+ 0] = 4 \times 2 + 2v$	1	M1	For applying conservation of momentum
$v = 8\text{ [ms}^{-1}\text{]}$	1	A1	
	2		
$2 \times \text{their (8) [+ 0]} = 2v + 3v$	1	M1	For applying conservation of momentum
$v = 3.2\text{ [ms}^{-1}\text{]}$	1	A1	
	2		
Kinetic energy (KE) initial = $\frac{1}{2} \times 4 \times 6^2$ KE final = $\frac{1}{2} \times 4 \times 2^2 + \frac{1}{2} \times 5 \times 3.2^2$	1	M1	For use of $\frac{1}{2} \times m \times v^2$ , when either initial or final calculation correct using their value from (b)
Loss of KE = $72 - 33.6 = 38.4\text{ [J]}$	1	A1	AG For all correct
	2		

2.

$[2300 \times 8 + m \times 0 = 2300 \times 2 + m \times 5]$	M1	For applying the conservation of momentum equation to the system of car, trailer and van, where $m$ = mass of the van
	A1	Correct equation
$m = 2760\text{ kg}$	A1	
	3	

3.

$0.3g \sin 30 = 0.3a$ ( $a = 5$ ) (M1 for applying Newton's second law parallel to the plane)	M1
$v^2 = 0 + 2 \times 2.5 \times a$	M1
$v = 5$	A1
$0.3 \times 5 + 0 = 0.3 \times 2 + 0.2 w$	M1
Velocity of $Q = 4.5\text{ ms}^{-1}$	A1
	5

Answer	Marks
$0.3 \times z + 0 = 0.5 \times 1.2$	M1
Velocity of $P$ before collision $z = 2$	A1
Friction force on $P$ after reaches horizontal plane $F = \mu \times 0.3g$	B1
$\mu \times 0.3g \times 1.5 = \frac{1}{2} \times 0.3 \times 5^2 - \frac{1}{2} \times 0.3 \times 2^2$	M1
Coefficient $\mu = 0.7$	A1
<b>Alternative method for question 7(b)</b>	
$0.3 \times z + 0 = 0.5 \times 1.2$	M1
Velocity of $P$ before collision $z = 2$	A1
Friction force on $P$ after reaches horizontal plane $F = \mu \times 0.3g$	B1
$a = (5^2 - 2^2) / (2 \times 1.5) = 7, F = 0.3 \times 7$	M1
Coefficient $\mu = 0.7$	A1
	5

4.

$4 \times 10 [+0] = 4 \times 0.5v + 2v$	M1
$v_A = 5$ and $v_B = 10$	A1
	2
Conservation of momentum $B, C$ $2 \times 10 [+0] = 2 \times v + 3v$	M1
$v = 4$	A1
$v_A > v_B$ , hence another collision	A1
	3
Conservation of momentum $A, B$	M1
$4 \times 5 + 2 \times 4 = 4v + 2v \quad v = \frac{14}{3} \text{ (ms}^{-1}\text{)}$	A1
KE initial = $\frac{1}{2} \times 4 \times 10^2$	M1
KE final = $\frac{1}{2} \times 6 \times \left(\frac{14}{3}\right)^2 + \frac{1}{2} \times 1 \times 12^2$	A1
Loss of KE = $200 - \frac{412}{3} = \frac{188}{3}$	A1
	5

5.

Use of conservation of momentum	<b>M1</b>
$m \times 2 + 0 = m \times (-0.5) + 0.2 \times 1$	<b>A1</b>
$m = 0.08$	<b>A1</b>
	<b>3</b>

6.

$6 \times 2.5 = 2.5v + 5v$	<b>M1</b>	Apply conservation of momentum, 3 terms implied
$v = 2 \text{ ms}^{-1}$	<b>A1</b>	
	<b>2</b>	
Use KE = $\frac{1}{2}mv^2$ either before or after collision	<b>M1</b>	Allow this for either particle
KE(before) = $0.5 \times 2.5 \times 6^2$ KE(after) = $0.5 \times 7.5 \times 2^2$	<b>A1 FT</b>	Both correct FT on v
Loss of KE = 30 J	<b>A1</b>	
	<b>3</b>	

7.

Momentum = $0.2 \times 2 = 0.4 \text{ kg ms}^{-1}$	<b>B1</b>	
	<b>1</b>	
$0.4 = 0.2 \times 0.3 + 0.5v$	<b>M1</b>	Apply conservation of momentum, 3 terms
$v = 0.68 \text{ ms}^{-1}$	<b>A1 FT</b>	FT on answer in 1(a)
	<b>2</b>	

8.

For using conservation of momentum (either case)	<b>M1</b>	
$6 \times 4 = 3m + 4 \times 1.5$ or $6 \times 4 = 3m - 4 \times 1.5$	<b>A1</b>	
$m = 6$ and $m = 10$	<b>A1</b>	
KE <sub>A</sub> initial = $\frac{1}{2} \times 4 \times 6^2$ (72 J) or KE <sub>A</sub> after = $\frac{1}{2} \times 4 \times 1.5^2$ (4.5 J) or KE <sub>B</sub> after = $\frac{1}{2} \times 6 \times 3^2$ (27 J) or KE <sub>B</sub> after = $\frac{1}{2} \times 10 \times 3^2$ (45 J)	<b>B1 FT</b>	KE = $\frac{1}{2} \times m \times v^2$ FT 4.5m for KE <sub>B</sub>
KE loss = $[\frac{1}{2} \times 4 \times 6^2 - \frac{1}{2} \times 4 \times 1.5^2 - \frac{1}{2} \times 6 \times 3^2]$ or $[\frac{1}{2} \times 4 \times 6^2 - \frac{1}{2} \times 4 \times 1.5^2 - \frac{1}{2} \times 10 \times 3^2]$	<b>M1</b>	Uses KE loss = KE before – KE after
Loss of KE = 40.5 J or 22.5 J	<b>A1</b>	
	<b>6</b>	

9.

$\pm 0.2 \times 0.5$ or $\pm 0.3 \times 1$	<b>B1</b>	For initial momentum for either particle. Allow kg or g.
$0.2 \times 0.5 + 0.3 \times (-1) = 0.2 \times v + 0$	<b>M1</b>	For conservation of momentum. Dimensions correct. Allow if 3 relevant momentum terms are seen regardless of sign.
Speed = $1 \text{ m s}^{-1}$	<b>A1</b>	Allow if final answer given as $v = 1$ or speed = 1 from an equation whose solution is $v = -1$
	<b>3</b>	

### 10.

Use of conservation of momentum, 3 terms	<b>M1</b>	Correct dimensions
$0.1 \times 5 + 0 = 0.1 \times (-1) + 0.2 \times (\pm v)$	<b>A1</b>	
$v = 3 \text{ m s}^{-1}$	<b>A1</b>	A0 for $v = -3$
	<b>3</b>	
$0.2 \times \textit{their 3} + 0 = 0.2 \times u + 0.5V$	<b>M1</b>	Use of conservation of momentum, 3 terms, correct dimensions. Allow $u = 0$ used or if $Q$ and $R$ coalesce
$u \geq -1$	<b>B1</b>	Allow $u = -1$ . Allow equality for finding greatest value of $V$ . Condition for no collision with $P$ , may be a statement.
Greatest $V = 1.6$	<b>A1 FT</b>	FT on <i>their 3</i> from <b>3(a)</b> if $u = -1$ used.
	<b>3</b>	

### 11.

Answer	Marks	Guidance
$s_A = \pm(30t - 5t^2)$ or $s_B = \pm 5t^2$	<b>B1</b>	Use of constant acceleration equations to find expressions for displacements of <i>A</i> or <i>B</i> .
$s_A + s_B = 15$ leading to $15 = 30t$ leading to $t = 0.5$	<b>B1</b>	Use $s_A + s_B = 15$ to find time at which particles collide.
$t = 0.5$ leading to $v_A = \pm 25$ and $v_B = \pm 5$	<b>B1</b>	Find speed of particles at $t = 0.5$ before collision.
$t = 0.5$ leading to $h_A = \pm\left(30 \times 0.5 - \frac{1}{2}g \times 0.5^2\right) = \pm 13.75$	<b>B1</b>	Find position of <i>A</i> or <i>B</i> at which collision occurs at $t = 0.5$ Alternatively allow $h_B = \pm 1.25$ as displacement of <i>B</i>
$25 \times (2m) - 5(m) = (3m)v \rightarrow v_1 = 15$ $25(m) - 5 \times (2m) = (3m)v \rightarrow v_2 = 5$	<b>M1</b>	Use of conservation of momentum, either case, using <i>their</i> $v_A$ and $v_B \neq 0$ or $30$ , with 3 terms.
	<b>A1</b>	Both values of $v$ correct

Answer	Marks	Guidance
Particle $C_1$ $-13.75 = 15t - 5t^2$ Particle $C_2$ $-13.75 = 5t - 5t^2$	<b>M1</b>	Use of $s = ut + \frac{1}{2}at^2$ OE to find $t$ , using either <i>their</i> numerical $v_1$ or numerical $v_2$ from a relevant conservation of momentum equation.
$t_{C_1}, t_{C_2} = 3.74, 2.23$ leading to $T = 1 + \sqrt{5} - \sqrt{3} = 1.50$	<b>A1</b>	Find $T = t_{C_1} - t_{C_2}$ from $t_{C_1} = 3.736$ and $t_{C_2} = 2.232$
	<b>8</b>	Subscripts 1 and 2 refer to the two cases.
<b>Alternative method for the final two marks</b>		
$0 = 15 - gt_1$ , $0 = 5 - gt_2 \rightarrow t_1 = 1.5$ , $t_2 = 0.5$ Total heights $h_1 = 13.75 + 11.25 = 25$ Or $h_2 = 13.75 + 1.25 = 15$ $25 = 5T_1^2$ and $15 = 5T_2^2 \rightarrow T_1 = \sqrt{5}$ , $T_2 = \sqrt{3}$	<b>M1</b>	Use of $v = u - gt$ to find time to highest point for either case and use of $v^2 = u^2 - 2gs$ to find total height reached for either case, using either <i>their</i> numerical $v_1$ or numerical $v_2$ from a relevant conservation of momentum equation. Use $s = 0 + \frac{1}{2}gT^2$ to find time to reach ground (either case).
$T = 1.5 + \sqrt{5} - (0.5 + \sqrt{3}) = 1 + \sqrt{5} - \sqrt{3} = 1.50$	<b>A1</b>	Find difference in total times $T = (t_1 + T_1) - (t_2 + T_2)$

12.

$0.4 \times 2.5 - 0.5 \times 1.5$	<b>M1</b>	Attempt momentum before impact.
$0.4 \times 2.5 - 0.5 \times 1.5 = 0.4v + 0.5 \times 2v$	<b>M1</b>	Use of conservation of momentum, either case.
$0.4 \times 2.5 - 0.5 \times 1.5 = 0.4v + 0.5 \times 2v$ or $0.4 \times 2.5 - 0.5 \times 1.5 = -0.4v + 0.5 \times 2v$	<b>A1</b>	One correct equation
Speed is $0.179 \text{ m s}^{-1}$ or $0.417 \text{ m s}^{-1}$	<b>A1</b>	Both values
	<b>4</b>	

13.

Attempt at use of conservation of momentum	<b>M1</b>	4 terms implied, i.e. $m$ and $km$ included before and after collision. Velocity after collision is the same for $m$ and $km$ .
$km \times 6 - m \times 2 = (km + m) \times 4$	<b>A1</b>	
$k = 3$	<b>A1</b>	
	<b>3</b>	
KE initial = $\frac{1}{2} \times km \times 6^2 + \frac{1}{2} \times m \times (-2)^2$ KE after = $\frac{1}{2} \times (km + m) \times 4^2$	<b>M1</b>	Attempt at any of the three possible KE terms, unsimplified. $k$ need not be substituted here.
Loss of KE = $24m$ J	<b>A1 FT</b>	KE loss = $56m - 32m$ FT on <i>their</i> $k$ , KE loss = $(10k - 6)m$ , $k > 0.6$ .
	<b>2</b>	

#### 14.

For $Q$ : $-2mg \sin \alpha - F = 2ma$ $[-16m - 7.2m = 2ma]$ $R = 2mg \cos \alpha$ $[= 12m]$	<b>M1</b>	Apply Newton's 2nd law along or perpendicular to the plane to particle $Q$ . Must use values for $\alpha$ or $\sin \alpha$ or $\cos \alpha$ .
	<b>A1</b>	Both correct.
$F = 0.6 \times 2mg \cos \alpha = 0.6 \times 0.6 \times 20m$ $[= 7.2m]$ $[2(m)a = -2(m)g(0.8) - 0.6 \times 2(m)g(0.6)]$	<b>M1</b>	Using $F = 0.6R$ where $R$ is a component of $2mg$ only
Acceleration of $Q$ up the plane while moving up the plane is $a = -11.6 \text{ ms}^{-2}$	<b>A1</b>	<b>AG</b>
	<b>4</b>	
For $P$ : $mg \sin \alpha - 0.6R = ma$ , leading to $8m - 3.6m = ma$ $[R = mg \cos \alpha = 6m, a = 4.4 \text{ ms}^{-2}]$	<b>M1</b>	Apply Newton's 2nd law to attempt to find the acceleration of particle $P$ . Must use values for $\alpha$ or $\sin \alpha$ .
$Q$ comes to rest when $10 - 11.6T_1 = 0$ , $\left[ T_1 = \frac{25}{29} = 0.862 \right]$	<b>M1</b>	For using constant acceleration equations to attempt to determine when $v_Q = 0$ .
For $P$ $s_{P(\text{down})} = \frac{1}{2} \times 4.4 \times T_1^2$ $[= 1.635]$ For $Q$ $s_{Q(\text{up})} = 10T_1 + \frac{1}{2} \times (-11.6) \times T_1^2$ $[= 4.31]$	<b>M1</b>	Use constant acceleration equations to attempt to find either $s_{P(\text{down})}$ or $s_{Q(\text{up})}$ at time $T_1$ .
$d = 6.4 - s_{P(\text{down})} - s_{Q(\text{up})}$ $[= 0.455]$ and to find $T_2$ $[= 0.12]$ by using $d = s_{P2} - s_{Q2} = (4.4T_1) \times T_2$ $[s_{P2}$ and $s_{Q2}$ are distances travelled by $P$ and $Q$ in time $T_2]$	<b>M1</b>	For attempting to find the extra distance $d$ $[= 0.455]$ needed to reach 6.4 m and using $u_P = 4.4T_1$ at $T_1$ to find $T_2$ as $d = (4.4T_1)T_2 + \frac{1}{2} \times 4.4T_2^2 - \frac{1}{2} \times 4.4T_2^2$ .
Time before collision = $[t = T_1 + T_2 = 0.862 + 0.12 =] 0.982$	<b>A1</b>	$t = 0.98194357 \dots$

Alternative method for Question 7(b)		
For $P$ : $mg \sin \alpha - 0.6R = ma$ , leading to $8m - 3.6m = ma$ [ $R = mg \cos \alpha = 6m, a = 4.4 \text{ ms}^{-2}$ ]	M1	Apply Newton's 2nd law to attempt to find the acceleration of particle $P$ . Must use values for $\alpha$ or $\sin \alpha$
$Q$ comes to rest when $10 - 11.6T_1 = 0$ , $\left[ T_1 = \frac{25}{29} = 0.862 \right]$	M1	For using constant acceleration equations to attempt to determine when $v_Q = 0$
For $P$ $s_{P(\text{down})} = \frac{1}{2} \times 4.4 \times t^2$ For $Q$ $s_{Q(\text{up})} = 10T_1 + \frac{1}{2} \times (-11.6)T_1^2 - \frac{1}{2} \times 4.4(t - T_1)^2$	M1	Use constant acceleration equations to attempt to find either $s_{P(\text{down})}$ or $s_{Q(\text{up})}$ at time $t$ where $t$ is the total time before collision.
$\frac{1}{2} \times 4.4t^2 + 10T_1 + \frac{1}{2} \times (-11.6)T_1^2 - \frac{1}{2} \times 4.4(t - T_1)^2 = 6.4$	M1	For using $s_{P(\text{down})} + s_{Q(\text{up})} = 6.4$ and solving for $t$
Time before collision is $t = 0.982 \text{ s}$	A1	$t = 0.98194357\dots$
	5	
Special case for those who do not take into account the fact that $Q$ comes to rest and then changes its direction		
For $P$ : $mg \sin \alpha - 0.6R = ma$ , leading to $8m - 3.6m = ma$ [ $R = mg \cos \alpha = 6m, a = 4.4 \text{ ms}^{-2}$ ]	M1	Apply Newton's 2nd law to attempt to find the acceleration of particle $P$ . Must use values for $\alpha$ or $\sin \alpha$ .
For $P$ $s_{P(\text{down})} = (\pm) \frac{1}{2} \times 4.4t^2$ For $Q$ $s_{Q(\text{up})} = (\pm) 10t + \frac{1}{2} \times (-11.6)t^2$	M1	For using constant acceleration equations to attempt to find either $s_{P(\text{down})}$ or $s_{Q(\text{up})}$ .
$s_p + s_q = 6.4$ leading to $\frac{1}{2} \times 4.4t^2 + 10t + \frac{1}{2} \times (-11.6)t^2 = 6.4$	M1	For applying $(\pm) s_p + (\pm) s_q = 6.4$ using their expressions for $s_p$ and $s_q$ to set up and solve a 3-term quadratic equation in $t$ to obtain at least 1 solution.

Time that particles are in motion before collision = $t = 1 \text{ s}$	A1	Must reject $t = 16/9$ <b>Maximum mark 4 out of 5</b>
	4	
$u_{P(\text{down})} = 0 + 4.4 \times 0.982 [= 4.3208]$	B1 FT	Allow $\pm 4.4$ . FT on <i>their</i> 4.4 and <i>their</i> 0.982
$u_{Q(\text{down})} = 4.4 \times 0.12 [= 0.528]$	B1 FT	Allow $\pm 4.4$ . FT on <i>their</i> 4.4 and <i>their</i> 0.12
$\pm m \times 4.3208 \pm 2m \times 0.528 = \pm (m + 2m)v$ [Correct equation is $m \times 4.3208 + 2m \times 0.528 = \pm (m + 2m)v$ ]	M1	Apply conservation of momentum, 4 terms, using <i>their</i> $u_p$ and $u_q$ values with $m$ and $2m$ respectively. Velocity of $P$ and $Q$ after impact must be equal.
Speed of combined particle immediately after impact = $v = 1.79 \text{ ms}^{-1}$	A1	Must be positive
Special case for those who do not take into account the fact that $Q$ comes to rest and then changes its direction		
$u_{P(\text{down})} = 0 + 4.4 \times 1 [= 4.4]$	B1 FT	Allow $\pm 4.4$ , FT on <i>their</i> 1 and <i>their</i> 4.4
$u_{Q(\text{up})} = 10 - 11.6 \times 1 [= -1.6]$ so $u_{Q(\text{down})} = 1.6$	B1 FT	Allow $\pm (10 - 11.6 \times 1)$ , FT on <i>their</i> 1
$\pm m \times 4.4 \pm 2m \times 1.6 = \pm (m + 2m)v$	M1	Apply conservation of momentum, 4 terms, using <i>their</i> $u_p$ and $u_q$ values with $m$ and $2m$ respectively. Velocity of $P$ and $Q$ after impact must be equal.
Speed of combined particle immediately after impact = $v = 2.53 \text{ ms}^{-1}$	A1	Allow $v = \frac{38}{15}$ . Must be positive.
	4	

15.



$120 \times 8 = 120v + 40v$	<b>M1</b>	Applying conservation of momentum.
$v = 6 \text{ ms}^{-1}$	<b>A1</b>	
	<b>2</b>	
$1600 - 4800 = 160a$ leading to $a = -20$	<b>M1</b>	Applying Newton's 2nd law to the system.
$0 = 6^2 + 2 \times (-20) \times s$	<b>M1</b>	Use of constant acceleration equations such as $v^2 = u^2 + 2as$ .
Distance travelled by post = 0.9 m	<b>A1</b>	
<b>Alternative method for question 1(b)</b>		
Initial KE = $\frac{1}{2} \times 160 \times 6^2$	<b>M1</b>	Use of KE = $\frac{1}{2} mv^2$ for combined mass.
$\frac{1}{2} \times 160 \times 6^2 + 160 \times 10 \times s = 4800s$	<b>M1</b>	Forms work/energy equation.
Distance travelled by post = 0.9 m	<b>A1</b>	
	<b>3</b>	

## 16.

a)

$0.6^2 = 0 + 2a \times 0.45$	<b>M1</b>	Use of constant acceleration equations to find $a$ .
$a = 0.4$	<b>A1</b>	
$R = 0.1g \times \cos \alpha = 0.1g \times \frac{24}{25} = 0.1g \times \cos 16.3^\circ \left[ R = \frac{24}{25} = 0.96 \right]$	<b>B1</b>	Must use a value for $\cos \alpha$ .
$0.1g \times \frac{7}{25} - F = 0.1 \times 0.4 \left[ 0.28 - F = 0.04 \rightarrow F = 0.24 \right]$	<b>M1</b>	Newton's second law, 3 terms.
$F = \mu \times 0.1g \times \frac{24}{25} \left[ F = \frac{24\mu}{25} = 0.96\mu \right]$	<b>M1</b>	Use of $F = \mu R$ , where $R$ is a component of $0.1g$
$\mu = 0.25$	<b>A1</b>	<b>AG</b> Must be from exact working $\mu = 0.25$ only

Alternative scheme for question 7(a)		
Attempt PE loss or KE gain	<b>M1</b>	Use of either $PE = mgh$ or $KE = \frac{1}{2}mv^2$
$PE \text{ loss} = 0.1 \times g \times 0.45 \sin 16.3 = 0.1 \times g \times 0.45 \times \frac{7}{25} \left[ = \frac{63}{500} = 0.126 \right]$ $KE \text{ gain} = \frac{1}{2} \times 0.1 \times 0.6^2 \left[ = \frac{9}{500} = 0.018 \right]$	<b>A1</b>	Both correct.
$R = 0.1g \times \cos \alpha = 0.1g \times \frac{24}{25} = 0.1g \times \cos 16.3^\circ \left[ R = \frac{24}{25} = 0.96 \right]$	<b>B1</b>	Must use a value for $\cos \alpha$ .
$0.1 \times g \times 0.45 \times \frac{7}{25} = \frac{1}{2} \times 0.1 \times 0.6^2 + F \times 0.45$ $\left[ \frac{63}{500} = \frac{9}{500} + \mu \times \frac{54}{125} \right]$ or $[0.126 = 0.018 + \mu \times 0.432]$	<b>M1</b>	Use of work-energy equation as PE loss = KE gain + WD against friction
$F = \mu \times 0.1g \times \frac{24}{25} \left[ F = \frac{24\mu}{25} = 0.96\mu \right]$	<b>M1</b>	Use of $F = \mu R$ , where $R$ is a component of $0.1g$
$\mu = 0.25$	<b>A1</b>	<b>AG</b> Must be from exact working $\mu = 0.25$ only
	<b>6</b>	

b)

$0.1 \times 0.6 = 0.5v$	<b>M1</b>	Use of conservation of momentum, 2 terms.
$v = 0.12$	<b>A1</b>	
For $B$ $0.5g \times \frac{7}{25} - 0.275 \times 0.5g \times \frac{24}{25} = 0.5a$ [leading to $a = 0.16$ ]	<b>B1</b>	Apply Newton's second law for particle $B$ , 3 terms. Allow correct unsimplified expression in $a$ only.
$s_A = 0 + \frac{1}{2} \times 0.4t^2$ $s_B = 0.12t + \frac{1}{2} \times 0.16t^2$	<b>*M1</b>	Attempt an expression for either $s_A$ or $s_B$ . Must see $u_A = 0$ and $u_B \neq 0$ but $u_B$ must have been found from a momentum equation.
For both $s_A$ and $s_B$ and attempt to solve $s_A = s_B$ to find $t$	<b>DM1</b>	Must be from 3 terms leading to a 2-term quadratic. If energy used in 7(a) then must find $a = 0.4$ for $A$ . Their working must be leading to a positive $t$ value.
Required time is $t = 1$ s	<b>A1</b>	
	<b>6</b>	

17.

$0.4 \times 3 + 0.2 \times 2 = 0.4 \times 2.5 + 0.2v$	<b>M1</b>	Use of conservation of momentum with 4 terms. Allow sign errors.
$v = 3 \text{ ms}^{-1}$	<b>A1</b>	Allow M1A0 if $g$ included with the masses.
	<b>2</b>	
For $A \pm 0.4g \sin 30^\circ = 0.4a$ or for $B \pm 0.2g \sin 30^\circ = 0.2a$ or $\pm mg \sin 30^\circ = ma$	<b>M1</b>	For either. Allow sin/cos mix.
$a = \pm 5$ or $\pm g \sin 30^\circ$	<b>A1</b>	Allow $g \sin 30^\circ$ without working for M1A1
For $B$ when hits barrier $v^2 = 3^2 + 2 \times 5 \times 1.6 \Rightarrow v = 5$ OR $v = u + at \Rightarrow v = 3 + 5 \times 0.4 \Rightarrow v = 5$	<b>M1</b>	Using <i>their</i> $a \neq \pm g$ and <i>their</i> $v$ from part (a) OR: use of $s = \frac{u+v}{2}t$ $1.6 = \frac{3+v}{2} \times 0.4 \Rightarrow v = 5$ OR $\frac{1}{2} \times 0.2 \times v^2 - \frac{1}{2} \times 0.2 \times 3^2 = 0.2 \times 1.6 \times g \sin 30$
Speed after hitting barrier $= 0.1 \times 5 = 0.5$	<b>A1</b>	<b>AG</b>
$v_A = 2.5 + 5 \times 0.44 [= 4.7]$ $v_B = -0.5 + 5 \times 0.04 [= -0.3]$ or $v_B = 0.5 + (-5) \times 0.04 [= 0.3]$	<b>*M1</b>	Use of $v = u + at$ for either with correct $t$ -value, with initial speeds $\pm 2.5$ or $\pm 0.5$ <i>their</i> $\pm a \neq \pm g$
$0.4 \times 4.7 + 0.2 \times (-0.3) = 0.6 v_{\text{comb}}$	<b>DM1</b>	Use of $v = u + at$ for BOTH with correct $t$ -values, initial speeds $\pm 2.5, \pm 0.5$ and $\pm$ <i>their</i> acceleration (same for both) and use of conservation of momentum with correct number of terms. Allow sign errors.
$v_{\text{comb}} = 3.03 \text{ ms}^{-1}$	<b>A1</b>	Allow $v = \frac{91}{30} = 3 \frac{1}{30}$ Allow DM1A0 if $g$ included with the masses.
	<b>7</b>	

### 18.

Conservation of momentum	<b>M1</b>	3 terms; allow M1 if speed of $A$ after collision is $\frac{1}{4} \times 8.5$ . Allow $5 \times 8.5 = 5X + 3Y$ where $ X $ and $ Y $ are different which may be seen by later work. If $ X $ and $ Y $ are subsequently used as being equal then M0.
$5 \times 8.5 = 5 \times 0.25v + 3v$	<b>A1</b>	OE e.g. $5 \times 8.5 = 5V + 3 \times 4V$
Speed of $B = 10 \text{ ms}^{-1}$	<b>A1</b>	Do not award if 10 from using $mgv$ , maximum 2/3 -10 is A0 as speed required not velocity
	<b>3</b>	
KE before $= \frac{1}{2} \times 5 \times 8.5^2 [= 180.625]$  KE after $= \frac{1}{2} \times 5 \times 2.5^2 + \frac{1}{2} \times 3 \times 10^2 [= 15.625 + 150 = 165.625]$	<b>1</b>	Attempt at any of the 3 terms for KE, using their $10 \text{ ms}^{-1}$ Not $\frac{1}{2} \times (5+3) \times 8.5^2$ , not $\frac{1}{2} \times (5+3) \times 2.5^2$ not $\frac{1}{2} \times (5+3) \times 10^2$ unless $ X  =  Y $ seen
KE loss $[= 180.625 - 165.625] = 15 \text{ J}$	<b>A1</b>	Accept -15, AWR T $\pm 15.0$
	<b>2</b>	

### 19.

$0.3 \times 4 + 0 = 0.3v + 0.2 \times 3$	<b>M1</b>	For attempt at use of conservation of momentum
Speed = $2 \text{ ms}^{-1}$	<b>A1</b>	
	<b>2</b>	
$0.2 \times 3 + 0 = (0.2 + m) \times 2$	<b>M1</b>	For attempt at use of conservation of momentum
$m = 0.1$	<b>A1</b>	
	<b>2</b>	

**20.**

Use conservation of momentum $6 \times 5 + 2 \times (-3) = 6v_A + 2v_B$	<b>*M1</b>	4 dimensionally correct terms. Allow sign errors, $v_A$ and $v_B$ must be different.
Use $v_B = v_A + 2$ or $v_A = v_B - 2$ with their momentum equation and solve for $v_A$ or $v_B$	<b>DM1</b>	Allow $v_B = v_A \pm 2$ or $v_A = v_B \pm 2$ .
$v_A = 2.5$ or $v_B = 4.5$	<b>A1</b>	
Attempt at initial KE, or final KE, or change in KE for $A$ , or change in KE for $B$ Initial KE = $\frac{1}{2} \times 6 \times 5^2 + \frac{1}{2} \times 2 \times (-3)^2 [= 84]$ Final KE = $\frac{1}{2} \times 6 \times (\text{their } 2.5)^2 + \frac{1}{2} \times 2 \times (\text{their } 4.5)^2$ Change in KE for $A = \pm \left( \frac{1}{2} \times 6 \times 5^2 - \frac{1}{2} \times 6 \times (\text{their } 2.5)^2 \right)$ Change in KE for $B = \pm \left( \frac{1}{2} \times 2 \times (-3)^2 - \frac{1}{2} \times 2 \times (\text{their } 4.5)^2 \right)$	<b>M1</b>	Allow use of their $v_A$ and/or $v_B$ . Allow if 2 KE equations seen.
Loss of KE = 45 J	<b>A1</b>	Allow -45 J. Allow if $mgv$ used in momentum equation.
	<b>5</b>	

**21.**

$0.3 \times 2 [+0] = 0.3 \times 0.6 + 0.4 \times v$	<b>M1</b>	For use of conservation of momentum. Must be 3 terms. Allow sign errors.
Speed of $B = 1.05 \text{ ms}^{-1}$	<b>A1</b>	AG Allow M1 A0 if $g$ included with the masses.
	<b>2</b>	
$0.4 \times 1.05 [+0] = (0.4 + m) \times 0.5$	<b>M1</b>	For use of conservation of momentum. Must be 3 terms. Allow sign errors.
$m = 0.44$ or $\frac{11}{25}$	<b>A1</b>	Allow M1 A0 if $g$ included with the masses.
	<b>2</b>	

**22.**

$R = 3g \cos \alpha = 3 \times 10 \times 0.8$	<b>B1</b>	
$F = 3g \sin \alpha = 3 \times 10 \times 0.6$	<b>M1</b>	Resolving parallel to plane.
$\mu = \frac{18}{24} = 0.75$ or $\mu = \frac{3g \sin \alpha}{3g \cos \alpha} = \tan \alpha = 0.75$	<b>A1</b>	Uses $\mu = \frac{F}{R}$ AG.
	<b>3</b>	
$a = g \sin \alpha$ or PE loss = $1.5gx \sin \alpha$ for <i>AB</i> and $a = 0$ for <i>BC</i> $4.5g \times \sin \alpha - 0.75 \times 4.5g \cos \alpha = 4.5a$ leading to $a = 0$	<b>B1</b>	Accelerations for <i>AB</i> and <i>BC</i> .
$v_1^2 = 2 \times g \sin \alpha \times x$ or $[1.5g \times x \sin \alpha = 0.5 \times 1.5 \times v_1^2]$	<b>M1</b>	Uses 'suvat' or PE loss = KE gain for <i>AB</i> .
$v_1^2 = 20x \sin \alpha = 12x$ leading to $v_1 = \sqrt{12x}$	<b>A1</b>	
$1.5 \times \sqrt{12x} + 0 = 4.5 \times v_2$ leading to $v_2 = \frac{1}{3} \sqrt{12x}$	<b>M1</b>	Conservation of momentum.
$4 = \frac{2}{3} \times \sqrt{12x}$	<b>M1</b>	Use of $s = vt$ on <i>BC</i> since $a = 0$ .
$x = 3$	<b>A1</b>	

<b>Alternative Method for 7(b)</b>		
$a = g \sin \alpha$ or PE loss = $1.5gx \sin \alpha$ for <i>AB</i> and $a = 0$ for <i>BC</i> $4.5g \times \sin \alpha - 0.75 \times 4.5g \cos \alpha = 4.5a$ leading to $a = 0$	<b>B1</b>	Accelerations for <i>AB</i> and <i>BC</i> .
$4 = 2v_2$ leading to $v_2 = 2$	<b>M1</b>	Uses $s = vt$ on <i>BC</i> since $a = 0$ .
$1.5 \times v_1 + 0 = 4.5 \times 2$	<b>M1</b>	Conservation of momentum.
$v_1 = 6$	<b>A1</b>	Velocity before collision.
$6^2 = 2 \times g \sin \alpha \times x$ or $1.5g \times x \sin \alpha = 0.5 \times 1.5 \times 6^2$	<b>M1</b>	Uses <i>suvat</i> or PE loss = KE gain for <i>AB</i> .
$x = 3$	<b>A1</b>	
	<b>6</b>	
KE = $0.5 \times 4.5 \times 2^2 = 9\text{J}$	<b>B1</b>	KE gain for <i>AC</i> .
PE loss = $15 \times (4+3) \times \frac{3}{5} + 30 \times 4 \times \frac{3}{5} = 135\text{J}$	<b>M1</b>	Evaluates PE loss for <i>AC</i> .
Loss of energy = 126J	<b>A1</b>	
	<b>3</b>	

23.

Attempt to use conservation of energy $\left[ \frac{1}{2} \times 0.5v^2 = 0.5g \times 1.8 \right]$ or $\left[ \frac{1}{2} \times mv^2 = mg \times 1.8 \right]$	<b>M1</b>	2 terms, dimensionally correct. Do not allow from use of constant acceleration.
$v = 6$	<b>A1</b>	Do not allow from use of constant acceleration.
Attempt at conservation of momentum $\left[ 0.5 \times 6(+0) = 0.5 \times 4 + 0.1w \right]$	<b>M1</b>	3 terms; allow sign errors; allow <i>their</i> $v = 6$ or just $v$ ; allow if using $mgv$ (consistently in all terms).
Speed of $Q$ ( $=w$ ) = $10 \text{ ms}^{-1}$	<b>A1</b>	AG Do not allow from use of constant acceleration. Do not allow if using $mgv$ . Use of constant acceleration gets M0 A0 M1 A0 maximum.
	<b>4</b>	<b>SC Assuming elastic collision</b> <b>M1A1</b> $0.5g \times 1.8 = \frac{1}{2} \times 0.1w^2 + \frac{1}{2} \times 0.5 \times 4^2$ <b>M1</b> For attempt at conservation of energy, 3 terms; allow sign errors. <b>B1</b> Speed of $Q$ ( $=w$ ) = $10 \text{ ms}^{-1}$

Attempt at conservation of momentum $\left[ 0.1 \times 10 = (0.1 + 0.4) \times z \Rightarrow z = 2 \right]$	<b>*M1</b>	3 terms, allow sign errors, allow if using $mgv$ .
Attempt to use conservation of energy $\left[ \frac{1}{2} \times (0.1 + 0.4) \times (\text{their } 2)^2 = (0.1 + 0.4)gh \Rightarrow h = 0.2 \right]$	<b>*DM1</b>	Dependent on previous M mark. 4 terms, dimensionally correct. Do not allow from use of constant acceleration. <i>their</i> 2 $\neq 10$ .
Use trigonometry to get an equation in $\theta$ and solve for $\theta$ $\left[ \theta = \sin^{-1} \left( \frac{\text{their } 0.2}{0.4} \right) \right]$	<b>DM1</b>	Dependent on previous 2 M marks. Using <i>their</i> $h$ and $0.4$ . Allow sin/cos mix.
$\theta = 30$	<b>A1</b>	Do not allow if using $mgv$ .
<b>Alternative method for Question 7(b): Using constant acceleration</b>		
Attempt at conservation of momentum $\left[ 0.1 \times 10 = 0.5 \times z \Rightarrow z = 2 \right]$	<b>*M1</b>	2 terms, allow sign errors, allow if using $mgv$ .
Attempt at use of constant acceleration $\left[ 0^2 = (\text{their } 2)^2 \pm 2 \times a \times 0.4 \Rightarrow a = \mp 5 \right]$	<b>*DM1</b>	Dependent on previous M mark. Uses constant acceleration with $u = \text{their } 2$ and $s = 0.4$ to get an equation in $a$ ; <i>their</i> 2 $\neq 10$ .
Use N2L to get an equation in $\theta$ leading to a positive value of $\theta$ and solve for $\theta$ $\left[ (0.5) \text{their } a  = (0.5)g \sin \theta \right]$	<b>DM1</b>	Dependent on previous 2 M marks. Using <i>their</i> $a$ ; May have $m$ for $0.5$ . Allow sin/cos mix.
$\theta = 30$	<b>A1</b>	Do not allow if using $mgv$ .
	<b>4</b>	

$Q$ takes 0.7s to travel from $B$ to $C$	<b>B1</b>	
$0.4 = \frac{(their\ 2) + 0}{2} t \Rightarrow t = 0.4$	<b>B1FT</b>	SOI FT $their\ 2$ from (b), $t = \frac{0.8}{their\ 2}$ . For use of $s = \left(\frac{u+v}{2}\right)t$ to get a time up the slope. Allow for total time on slope from $0 = (their\ 2)t - \frac{1}{2}(their\ a)t^2 \Rightarrow t = 0.8$ .
Distance between $P$ moved is $(0.7 + 0.8) \times 4 (= 6)$	<b>B1</b>	Allow 1 m from point $C$ .
Set up equation in $t$ using $4t$ , $(their\ 2)t$ and $their\ 6$ and solve for $t$ [ $4t + (their\ 2)t = (their\ 1)$ OR $(their\ 6) + 4t + (their\ 2)t = 7$ ]	<b>M1</b>	Must have considered all parts of motion to find times from relevant equations.
Distance from $B = 6\frac{2}{3}$ m	<b>A1</b>	

<b>Alternative method for last 3 marks of Question 7(c)</b>		
[Time for $P = \frac{b}{4}$ and [Time for $QR = \frac{7-b}{2}$ OR [Time for $P = \frac{7-c}{4}$ and [Time for $QR = \frac{c}{2}$	<b>B1</b>	Where $b$ is distance from $B$ OR Where $c$ is distance from $C$ .
Attempt to form an equation from use of total time and solve for $b$ (or $c$ ) [ $\frac{7-b}{2} + 0.7 + 0.4 + 0.4 = \frac{b}{4} \Rightarrow b = 6\frac{2}{3}$ ] OR $\frac{c}{2} + 0.7 + 0.4 + 0.4 = \frac{7-c}{4} \Rightarrow c = \frac{1}{3}$	<b>M1</b>	Where $b$ is distance from $B$ OR Where $c$ is distance from $C$ . Must have considered all parts of motion to find times from relevant equations.
Distance from $B = 6\frac{2}{3}$ m	<b>A1</b>	
	<b>5</b>	

24.



$m \times 5 + 0 = m \times 2 + 0.3v$	<b>M1</b>	Attempt at conservation of momentum; 3 non-zero terms (with $m$ appearing in two terms); allow sign errors.
Speed = $10m$ ( $\text{m s}^{-1}$ )	<b>A1</b>	M1A0 if using $g$ in momentum terms. $v = -10m$ is A0.
	<b>2</b>	
$0.3 \times 10m + 0 = 0 + 0.6 \times 1.5$ [ $3m = 0.9$ ]	<b>M1</b>	Attempt at conservation of momentum between $Q$ and $R$ (so must be using correct masses of 0.3 and 0.6) to form a linear equation in $m$ using their answer from (a); 2 non-zero terms; allow sign errors.
$m = 0.3$	<b>A1FT</b>	FT $\frac{3}{\text{their +ve coefficient of } m \text{ from (a)}}$ Condone including kg in answer.
	<b>2</b>	

25.

$\pm[3.2v + 2.4 \times (-6)] = 0$	<b>M1</b>	Attempt at conservation of momentum; 2 non-zero terms; allow sign errors.
$v = 4.5$	<b>A1</b>	M1A0 for use of $mgv$ . $v = -4.5$ is A0.
	<b>2</b>	

$\text{KE} = \pm \frac{1}{2} \times 3.2 \times (\text{their } 4.5)^2$ OR $\pm \frac{1}{2} \times 2.4 \times 6^2$	<b>M1</b>	Attempt at either KE term, using <i>their</i> $v$ . Do not allow $\frac{1}{2} \times 3.2 \times (\text{their } 4.5 \pm 6)^2$ , or $\frac{1}{2} \times 2.4 \times (\text{their } 4.5 \pm 6)^2$ , or $\frac{1}{2} \times (3.2 + 2.4) \times (\text{their } 4.5 \pm 6)^2$ , or $\frac{1}{2} \times 3.2 \times (\text{their } 4.5 - 0)^2$ , or $\frac{1}{2} \times 2.4 \times (6 - 0)^2$ .
$\text{KE}_{\text{loss}} = 75.6 \text{ J}$	<b>A1</b>	Allow $-75.6$ . Note $\frac{1}{2} \times (3.2 + 2.4) \times 6^2$ or $\frac{1}{2} \times (3.2 + 2.4) \times (\text{their } 4.5)^2$ is M1A0.
	<b>2</b>	

26.

For attempt at use of conservation of momentum in one case	<b>M1</b>	$0.1 \times 4 + 0 = 0.4v + 0.1v$ or $0.1 \times 4 + 0 = 0.4v + 0.1(-v)$ OE. Must have correct number of terms. Allow sign errors.
Speed = $0.8 \text{ [ms}^{-1}\text{]}$ or $\frac{4}{5}$	<b>A1</b>	Must be positive. Allow Max M1A1A0 if $g$ included with the masses.
Speed = $\frac{4}{3} \text{ [ms}^{-1}\text{]}$ Allow 1.33	<b>A1</b>	Must be positive.
	<b>3</b>	