07.01.2024

M-1

Mechanics-1

Newton's Laws of Motion Revision.

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M.1

Newtons Laws of Motion - Revision P-1

Car towing a trailer by mean of a light rigid tow-bar

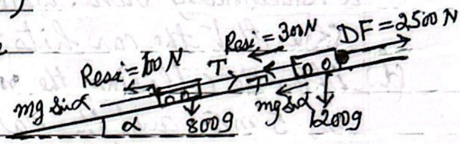
1. A car of mass 1200 kg is pulling a trailer of mass 800 kg up a hill inclined at an angle of $\sin^{-1}(0.1)$ to the horizontal. The car and the trailer are connected by a light tow-bar which is parallel to the road. The driving force of the car's engine is 2500 N and the resistances to the car and the trailer are 300 N and 100 N respectively.

(a) Find the acceleration of the system and the tension in the tow-bar. [4]

(b) When the car and the trailer are travelling at a speed of $30\,ms^{-1}$, the driving force becomes zero.
Find the time in seconds, before the system comes to rest and the force in the tow-bar during this time. [SP-20/04/Q5] [5]

(Given $\alpha = \sin^{-1} 0.1 \Rightarrow \sin\alpha = 0.1$)

Solution: Components of Forces along the plane



(a) for car moving upwards

$2500 - (1200g + 800g)\sin\alpha - 400 = 2000a$ —— ①

$\Rightarrow 2500 - 2000 \times 10 \times 0.1 - 400 = 2000a$

$\Rightarrow 100 = 2000a \Rightarrow a = \frac{100}{2000} = .05$

$\therefore a = \underline{0.05\,ms^{-2}}$ ✓

Let the tension in the tow-bar = T

$\Rightarrow 2500 - T - 300 - 1200g\sin\alpha = 1200 \times 0.05$ (for Car's motion)

$\Rightarrow 2500 - 300 - 1200 - 60 = T$

$\Rightarrow T = \underline{940 N}$ ✓

(b) when the Driving force becomes zero.

for ① $-2000g \times 0.1 - 400 = 2000a$

$\Rightarrow a = -1.2$ ✓

also $v = u + at$, when system is at rest $\Rightarrow 0 = 30 - 1.2t \Rightarrow \underline{t = 25s}$ ✓

for trailer.

$T - 100 - 800g \times 0.1 = 800 \times -1.2$

$\Rightarrow \underline{T = -60N}$ ✓

M.1

Car towing a trailer and then→ Braking the Car (Thrust in Tow Bar) P-2

2\. On a straight horizontal test track, driverless vehicles (with no passengers) are being tested. A car of mass 1600 kg is towing a trailer of mass 700 kg along the track. The brakes are applied, resulting in a deceleration of 12 m s^{-2}. The braking force acts on the car only. In addition to the braking force there are constant resistances forces of 600 N on the car and 200 N on the trailer.

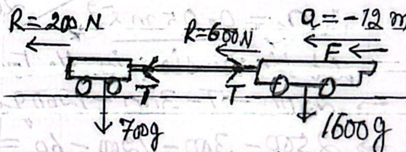
(a) Find the magnitude of force in the tow-bar. --[2]

(b) Find the braking force. --[2]

(c) At the instant when the brakes are applied, the car has speed 22 m s^{-1}. At this moment the car is 17.5 m away from a stationary van, which is direcly in front of the car. Show that the car hits the van at a speed of 8 m s^{-1}. --[2]

(d) After the collision, the van starts to move with a speed of 5 m s^{-1} and the car and trailer continue moving in the same direction with speed 2 m s^{-1}. Find the mass of Van. [M-20/42/Q6] --[3]

Solution: Force in the tow-bar = T



(a) for Trailer:

$-T-200 = 700\times -12$

$\Rightarrow T = +8200 N$

Magnitude of force in tow-bar = 8200 N ✓

⊗ Thrust in Tow-bar = T

(b) for car

$T-F-600 = 1600\times -12$

$\Rightarrow 8200-F-600 = -19200$

$-F = -19200-7600$

$\therefore F = -26800$ ✓

(c) $V^2 = u^2+2as$

$V^2 = 22^2+2\times(-12)\times 17.5$

$V^2 = 64$

$\therefore V = 8 m s^{-1}$

(d) Let the mass of Van = m

momentum = m × u, using→ Conservation of momentum:

$2300\times 8 + m\times 0 = 2300\times 2 + m\times 5$

$\Rightarrow m = 2760 kg$ ✓

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3. A particle P is projected vertically upwards with speed $5\,ms^{-1}$ from a point A which is 2.8 m above horizontal ground.

(a) Find the greatest height above the ground reached by P. ---[3]

(b) Find the length of time for which P is at a height of more than 3.6 m above the ground. ---[4]

[S-20/41/Q3]

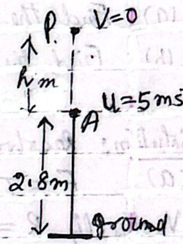
Solution

(a) At the greatest height $V=0$, given $U = 5\,ms^{-1}$

$$V^2 = u^2 + 2as$$

$$0 = 5^2 - 2gh$$

$$\Rightarrow h = \frac{25}{2g} = \frac{25}{2\times 10} = 1.25\,m$$



$\therefore$ Total height above the ground $= 2.8 + 1.25$

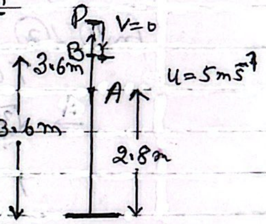
$= 4.05\,m$

(b) Now $h = 3.6m - 2.8m = 0.8$ m above A

or $AB = 0.8\,m$, when $u = 5\,m\,s^{-1}$

$$s = ut + \tfrac{1}{2}at^2$$

$$0.8 = 5t - \tfrac{1}{2}gt^2$$



or $0.8 = 5t - 5t^2$ $\quad (g = 10\,ms^{-2})$

or $5t^2 - 5t + .8 = 0$

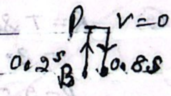
$5t^2 - 4t - t + .8 = 0$

$t(5t-4) - .2(5t-4) = 0$

$(5t-4)(t-.2) = 0$

$\Rightarrow t = \frac{4}{5}$ or $t = 0.2$

$\Rightarrow t = 0.8\,s$ or $t = .2\,s$

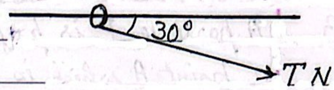


$\therefore$ length of time when P is above 3.6 m

$= 0.8 - 0.2$

$= \underline{0.6\text{ Sec.}}$

4. The diagram shows a ring of mass 0.1 kg threaded on a fixed horizontal rod. The rod is rough and the coefficient of friction between the ring and the rod is 0.8. A force of magnitude T N acts on the ring in a direction at 30° to the rod, downwards in the vertical plane containing the rod. Initially the ring is at rest.

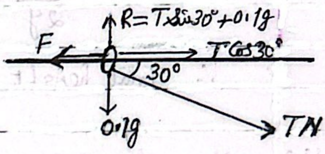


(a) Find the greatest value of T for which the ring remains at rest. ---[4]

(b) Find the acceleration of the ring when T = 3. ---[3]

[S-20/41/Q4]

Solution: Resolving forces Horizontally,



(a) $F = T\cos 30°$ —— ①

Vertically $R = T\sin 30° + 0.1g$ —— ②

$F = \mu R$; $\mu = 0.8$

from ① & ②

$$T\cos 30° = 0.8(T\sin 30° + 0.1g)$$

$$\Rightarrow \frac{\sqrt{3}}{2}T = 0.8 \times \frac{1}{2}T + 0.8 \times 1$$

$$T\left(\frac{\sqrt{3}}{2} - 0.4\right) = 0.8 \Rightarrow T = \frac{0.8}{\left(\frac{\sqrt{3}}{2} - 0.4\right)} = \frac{0.8}{0.466} = 1.712$$

$$\Rightarrow T = \underline{1.72\text{ N}}$$

(b) Now when T = 3 N,

from ② $R = T\sin 30° + 0.1g$

$= 3 \times \sin 30° + 0.1 \times 10 = 1.5 + 1 = 2.5$

$R = 2.5$ —— ③

Now horizontally ; $3\cos 30° - F = ma$

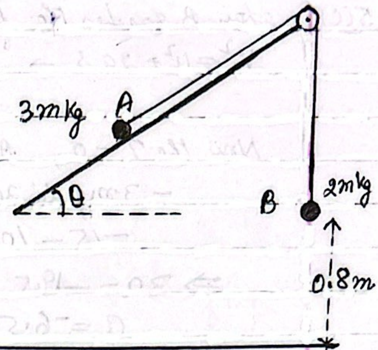
$3\cos 30 - 0.8 \times 2.5 = 0.1a$ ($F = \mu R$; $\mu = 0.8$, $R = 2.5$ from ③)

$\frac{3\sqrt{3}}{2} - 2 = .1a$

$\Rightarrow a = \frac{0.598}{0.1} = 5.98$

$a = 5.98\ ms^{-2}$

5. Two particles A and B, of masses 3m kg and 2m kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pully which is attached to the edge of a plane. The plane is inclined at an angle θ to the horizontal. A lies on the plane and B hangs vertically, 0.8 m above the floor.



The string between A and the pully is parallel to a line of greatest slope of the plane. Initially A and are at rest.

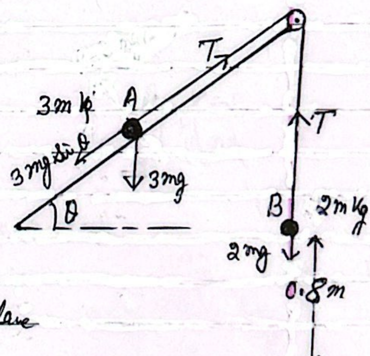
(a) Given that the plane is smooth, find the value of θ for which A remains at A. ---[3]

It is given instead that plane is rough, $\theta = 30°$ and the acceleration of A up the plane is $0.1\,m\,s^{-2}$

(b) Show that the coefficient of friction between A and the plane is $\frac{1}{10}\sqrt{3}$. ---[5]

(c) When B reaches the floor it comes to rest.
Find the length of time after B reaches the floor for which A is moving up the plane.

[S-20/43/ Q7]



Solution: for B, $T - 2mg = 0$ —— ①

(a) for A, $3mg\sin\theta - T = 0$ —— ②

fr ① and ② $3mg\sin\theta - 2mg = 0$

$mg(3\sin\theta - 2) = 0$

$\Rightarrow \sin\theta = 2/3 \Rightarrow \theta = \sin^{-1} 2/3 = 41.81°$

$\therefore \theta = 41.8°$ ✓

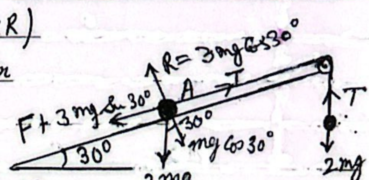
(b) Now $\theta = 30°$ and acc. $a = 0.1\,ms^{-2}$ up the plane

for B, $2mg - T = 0.1 \times 2m$ —— ③

for A, $T - 3mg\sin30° - \mu \times 3mg\cos30° = 0.1 \times 3m$ ④ $(F = \mu R)$

add ③ & ④ $2mg - 0.2m - 3mg\sin30° - \mu \times 3mg\cos30 = 0.3m$

$\Rightarrow 15\sqrt{3}\mu = 4.5 \Rightarrow \mu = \frac{\sqrt{3}}{10}$ ✓



(continued →)

5(C) when B reaches the floor let velocity $= v$, distance $s = 0.8$

$v^2 = u^2 + 2as \Rightarrow v^2 = 0 + 2\times 0.1\times 0.8 = 0.16$

$V = 0.4\ ms^{-1}$ ✓

Now the $T = 0$, A is moving up.

$-3mg\sin 30^\circ - \mu \times 3mg\cos 30^\circ = 3ma$

$-15 - \frac{\sqrt{3}}{10} \times 30 \times \frac{\sqrt{3}}{2} = 3a$

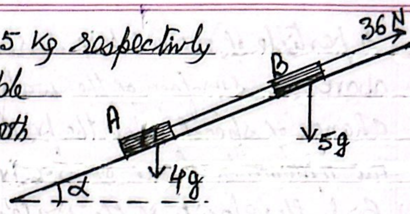
$\Rightarrow 3a = -19.5$

$a = -6.5$ ✓

Now $V = u + at \Rightarrow 0 = 0.4 - 6.5t$

A is moving up, till it stops, $V = 0$; $\Rightarrow t = \frac{0.4}{6.5} = \underline{0.0615\ sec}$ ✓

6. Two blocks of masses 4 kg and 5 kg respectively are joined by a light inextensible string. The blocks rests on a smooth plane inclined at an angle α to the horizontal, where $\tan\alpha = \frac{7}{24}$.



The string is parallel to line of greatest slope of the plane with B above A. A force of magnitude 36 N acts on B, parallel to a line of greatest slope of the plane.

(i) Find the acceleration of the blocks and the tension in the string. ---[5]

(ii) At a particular instant, the speed of the blocks is $1\,ms^{-1}$. Find the time, after this instant, that it takes for the blocks to travel 0.65 m. ---[2]

[W-19 / 41 / Q4]

Solution: (i) Along the plane, component of forces.

$\tan\alpha = \frac{7}{24}$

$\sin\alpha = \frac{7}{25}$

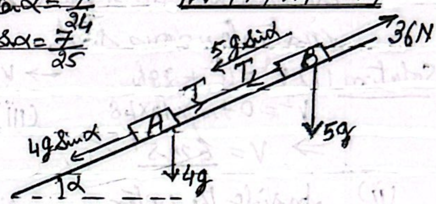
for Block A, $T - 4g\sin\alpha = 4a$ ——— ①

for Block B, $36 - T - 5g\sin\alpha = 5a$ — ②

add ① & ② $36 - 9\times 10\times \frac{7}{25} = 9a$

$\Rightarrow a = 1.2\ ms^{-2}$ ✓

from ① $T = 16\ N$ ✓



(ii) Now $0.65 = 1\times t + \frac{1}{2}\times 1.2\times t^2$

$\Rightarrow .6t^2 + t - 0.65 = 0$

$\Rightarrow 6t^2 + 10t - 6.5 = 0$

$\Rightarrow 12t^2 + 20t - 13 = 0$

$12t^2 + 26t - 6t - 13 = 0$

$2t(6t+13) - 1(6t+13) = 0$

$(6t+13)(2t-1) = 0$

$t = \frac{1}{2}$ or $-\frac{13}{6}$ ✗

$\therefore t = 0.5\ s$ ✓

7. A particle of mass 0.4 kg is released from rest at a height of 1.8 m above the surface of the water in a tank. There are no instantaneous change of speed when the particle enters the water. The water exerts an upwards force of 5.6 N on the particle when it is in the water.

(i) Find the velocity of the particle at the instant when it reaches the surface of the water. ---[2]

(ii) Find the time that it takes from the instant when the particle enters the water until it comes to instantaneous rest in the water. You may assume that the tank is deep enough so that the particle does not reach the bottom of the tank. ---[4]

(iii) Sketch a velocity time graph for the motion of the particle from the instant at which it is released until it come to instantaneous rest. [W-19/41/Q6] ---[3]

Solution (i) $v^2 = u^2 + 2gh$

$v^2 = 0 + 2 \times 10 \times 1.8$

$\Rightarrow v = \underline{6\ m s^{-1}}$ ✓

(ii) Inside the water

$0.4g - 5.6 = 0.4a$

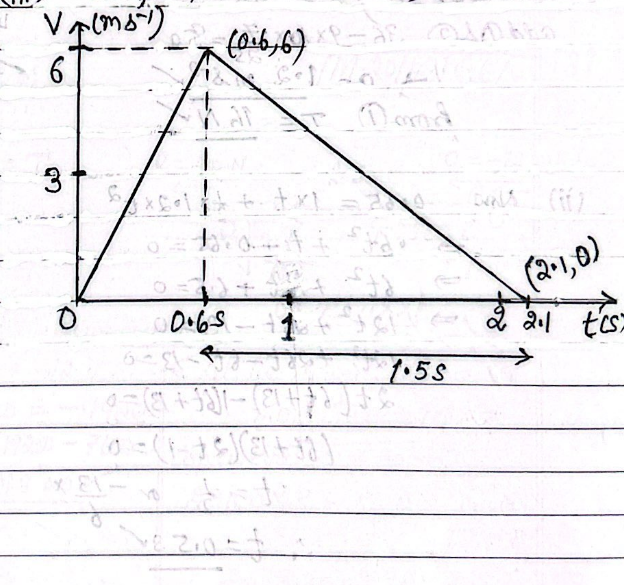
$\Rightarrow a = -4\ m s^{-2}$

$v = u + at$

$0 = 6 - 4t$

$\Rightarrow t = \underline{1.5 s}$ ✓

(iii) $\rightarrow v = u + gt \Rightarrow 6 = 0 + 10t \Rightarrow t = 0.6 s$ to reach the surface of water. at $t = 0.6$, $v = 6\ m s^{-1}$



M.1

8. A block of mass 3 kg is initially at rest on a rough horizontal plane. A force of magnitude 6N is applied to the block at angle of θ above the horizontal, where $\cos\theta = \frac{24}{25}$. The force is applied for a period of 5 s, during which time the block move a distance of 4.5 m.

(i) Find the magnitude of the frictional force on the block. ---[4]

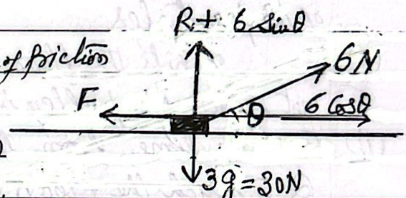
(ii) Show that the coefficient of friction between the block and the plane is 0.165, correct to 3 significant figures. ---[3]

(iii) When the block has moved a distance of 4.5 m, the force of magnitude 6N is removed and the block then decelerates to rest. Find the total time for which the block is in motion. ---[4]

W-19/42/Q6

Solution (i) $S = ut + \frac{1}{2}at^2$

$4.5 = 0 + \frac{1}{2}a \times 5^2 \Rightarrow a = 0.36$



Component of forces along the plane; F is force of friction

$6\cos\theta - F = ma$

$6 \times \frac{24}{25} - F = 3 \times 0.36 \Rightarrow F = \underline{4.68\,N}$ — ①

(ii) Components vertically,

$R + 6\sin\theta = 3g$ $\quad$ $\left[\cos\theta = \frac{24}{25} \Rightarrow \sin\theta = \frac{7}{25}\right.$

$\Rightarrow R = 3 \times 10 - 6 \times \frac{7}{25} = 28.32$ — ②

Now force of friction $F = \mu R \Rightarrow \mu = \frac{F}{R} = \frac{4.68}{28.32}$ from ① & ②

$\therefore \mu = 0.165$

(iii) $v = u + at \Rightarrow v = 0 + 0.36 \times 5 = 1.8$ — ③

Now when the force 6N is removed. $R = 3g \Rightarrow F = \mu R$
$= 0.165 \times 3g$

$\therefore$ along the plane:

$0 - F = 3a \Rightarrow -0.165 \times 3g = 3a \Rightarrow a = -1.65$ — ④

Now particle comes to rest.

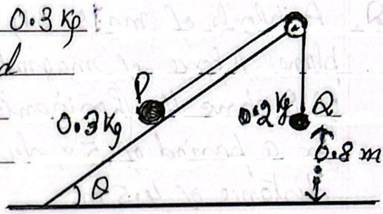
$V = u + at$

$\Rightarrow 0 = 1.8 - 1.65 \times t$ (from ③ & ④)

$\Rightarrow t = 1.09$

$\therefore$ Total time for particle in motion $= 5 + 1.09 = \underline{6.09\,s}$

9. Two particles P and Q, of masses 0.3 kg and 0.2 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed pully which is attached to the edge of a smooth plane. The plane is inclined at an angle θ to the horizontal, where $\sin\theta = 3/5$, P lies on the plane and Q hangs vertically below the pulley at a height of 0.8 m above the floor. The string between P and the pulley is parallel to a line of greatest slope of plane. P is released from rest and Q moves vertically downwards.



(i) Find the tension in the string and the magnitude of the acceleration of the particles. ---[5]

Q hits the floor and does not bounce. It is given that P does not reach the pulley in the subsequent motion.

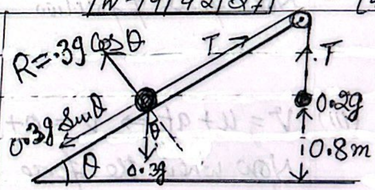
(ii) Find the time, from the instant at which P is released, for Q to reach the floor. --[2]

(iii) When Q reach the floor the string becomes slack. Find the time, from the instant at which P is released, for the string to become taut again. --[4]

W-19/42/Q7

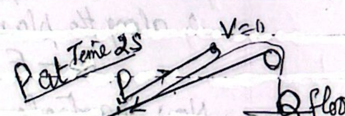
Solution: (i) for P, $T - 0.3g\sin\theta = 0.3a$
for Q, $0.2g - T = 0.02a$

Solving $a = 0.4\ ms^{-2}$, $T = 1.92N$



(ii) $S = ut + \frac{1}{2}at^2$

$\Rightarrow .8 = 0 + \frac{1}{2}\times 0.4\, t^2 \Rightarrow t = 2s$



(iii) Speed of Q when it hits the floor
$V = u + at \Rightarrow V = 0 + 2\times 0.4 = 0.8\ ms^{-1}$

Now the tension at P = 0

at P, $-0.3g\times\frac{3}{5} = 0.3a \Rightarrow a = -6$

to reach the highest point $V = 0$

$0 = 0.8 - 6t \Rightarrow t = \frac{0.8}{6} = 0.133$

string taut after $= 2\times 0.133 = 0.266$ after Q hits the floor.

∴ Total time of P to become taut $= 2 + 2\times 0.133 = 2.266\ s$

10. A crate of mass 500 kg is being pulled along rough horizontal ground by a horizontal rope attached to a winch. The winch produces a constant pulling force of 2500 N and the crate is moving at a constant speed. Find the coefficient of friction between the crate and the ground. [W-19/43/Q1] --[3]

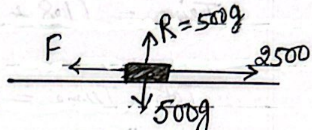
Solution: $F = \mu \times mg = \mu \times 500g$

$F - 2500 = 0$ (constant speed ⇒ No acceleration)

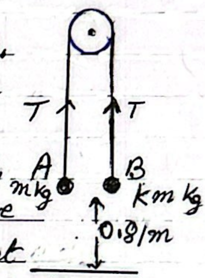
$\mu \times 500g - 2500 = 0$

$\Rightarrow \mu = \frac{2500}{500 \times 10} = 0.5$ ✓

$\Rightarrow \underline{\mu = 0.5}$ ✓



11. Two particles A and B have masses m kg and km kg, resp. where $k > 1$. The particles are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley and the particles hang vertically below it. Both particles are at a height of 0.81 m above horizontal ground. The system is released from rest and the particle B reaches the ground 0.9 s later. The particle A does not reaches the pulley in its subsequent motion.



(i) Find the value of k and show that the tension in the string before B reaches the ground is equal to 12m N. ---[7]

At the instant when B reaches the ground, the string breaks.

(ii) Show that the speed of A when it reaches the ground is $5.97\,ms^{-1}$, correct to 3 s.f figures, and find the time taken after the string breaks, for A to reach the ground. --[4]

(iii) Sketch a velocity-time graph for the motion of particle A from the instant when the system is released until A reaches the ground. --[2]

[W-19/43/Q7]

Solution: (i) $s = ut + \frac{1}{2}at^2$

$0.81 = 0 + \frac{1}{2} a \times (0.9)^2$

$\Rightarrow a = 2$

for A,
Now $T - mg = ma$

$T = 10m + 2m = \underline{12m\ N}$ ✓

and for B,

$kmg - T = kma$

$10mk - 12m = k \times m \times 2$

$k(10m - 2m) = 12m$

$k = \frac{12m}{8m} = \underline{1.5}$ ✓ (Continued→)

(Continued→)

11 (ii) Velocity of A, when string breaks

$V = u + at \Rightarrow V = 0 + 2 \times 0.9 = 1.8\ ms^{-1}$ upwards

$V^2 = u^2 + 2gh$

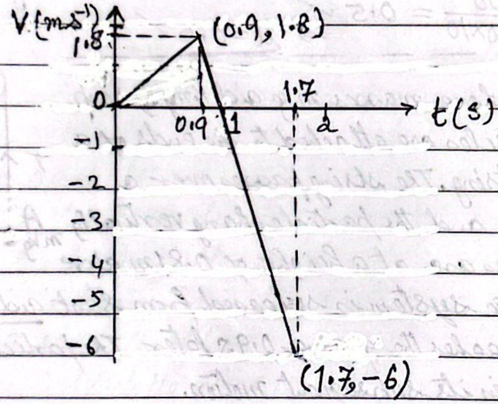
$= 1.8^2 + 2g \times 1.62$

$\Rightarrow V = 5.97\ ms^{-1}$

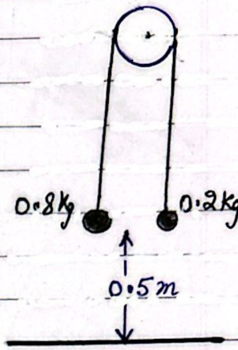
Time $= \frac{(1.8 + 5.97)}{g} = \underline{0.777\ s}$

Total Time $= 0.9 + 0.777 = 1.677\ s$

(iii)



12. Two particles of masses 0·8 kg and 0·2 kg are connected by a light inextensible string that passes over a fixed smooth pully. The system is released from rest with both particles 0·5 m above a horizontal floor. In the subsequent motion the 0·2 kg particle does not reach the pulley.



(a) Show that the magnitude of the acceleration of the particles is $6\,ms^{-2}$ and find the tension in the string. ---[4]

(b) When the 0·8 kg particle reaches the floor it comes to rest. Find the greatest height of the 0·2 kg particle above the floor. ---[3]

W-20 / 41 / Q5

Solution (a) $0{\cdot}8g - T = 0{\cdot}8a$ —— ① using Newton's laws

$T - 0{\cdot}2g = 0{\cdot}2a$ —— ② of Motion

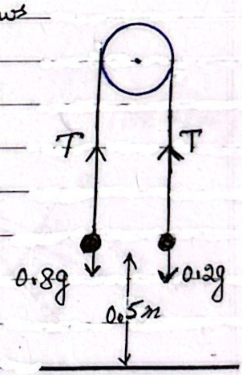
add ① & ②

$(0{\cdot}8g - 0{\cdot}2g) = (0{\cdot}8 + 0{\cdot}2)a$

$\Rightarrow 0{\cdot}6g = a \Rightarrow \underline{a = 6\,ms^{-2}}$ ✓

from ② $T - 0{\cdot}2g = 0{\cdot}2 \times 6$

$T = 2 + 1{\cdot}2 = \underline{3{\cdot}2\,N}$ ✓



(b) Velocity v, when 0·8 kg Particle reaches the ground, $V^2 = 0 + 2 \times 0{\cdot}5 = 6$ —— ③

Now for 0·2 kg particle moving upwards initial speed u: $u^2 = 6$ from ③

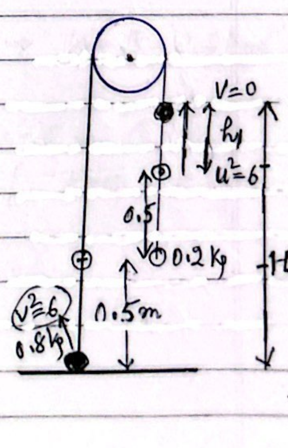
for the greatest height h_1: $v = 0$

$v^2 = u^2 - 2gh$

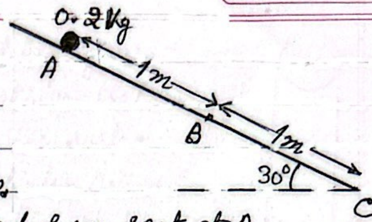
$0 = 6 - 2 \times 10 \times h_1 \quad h_1 = \frac{6}{20} = 0{\cdot}3\,m$

∴ Greatest height $H = 0{\cdot}5 + 0{\cdot}5 + 0{\cdot}3$

$= \underline{1{\cdot}3\,m}$ ✓



13. Three points A, B and C lie on a line of greatest slope of a plane inclined at an angle of 30° to the horizontal, with AB = 1m and BC = 1m.



A particle of mass 0.2 Kg is released from rest at A, and slide down the plane. The part of plane from A to B is smooth. The part of the plane from B to C is rough, with coefficient of friction μ between the plane and the particle.

(a) Given that $\mu = \frac{1}{2}\sqrt{3}$, find the speed of the particle at C. ---[8]

(b) Given instead that the particle comes to rest at C, find the exact value of μ. ---[4]

$\boxed{W-20/41/Q7}$

Solution (a) from A to B,

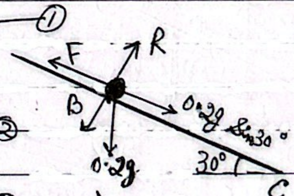
$0.2g \sin 30^\circ = 0.2a \Rightarrow$ acc $a = 5$

Let its velocity at B is V_B : $V_B^2 = 0 + 2 \times a s$

$= 2 \times 5 \times 1$ (AB = 1m)

$\Rightarrow V_B^2 = 10$ —— ①

Now at B



$R = 0.2g \cos 30^\circ = \sqrt{3}$

$\therefore$ force of friction $F = \mu R = \sqrt{3} \times \frac{1}{2}\sqrt{3} = \frac{3}{2}$ ✓ —— ②

$0.2g \sin 30^\circ - F = 0.2a$

$1 - 3/2 = 0.2a$ (from ② $F = 3/2$)

$\Rightarrow a = -2.5$

$\therefore V_C^2 = V_B^2 + 2as \Rightarrow V_C^2 = 10 - 2 \times (2.5) \times 1 = 5 \Rightarrow \underline{V_C = \sqrt{5} = 2.24 \, ms^{-1}}$ ✓

(b) $V_C^2 = V_B^2 + 2as$

$0 = 10 + 2a \times 1 \Rightarrow a = -5$ —— ③

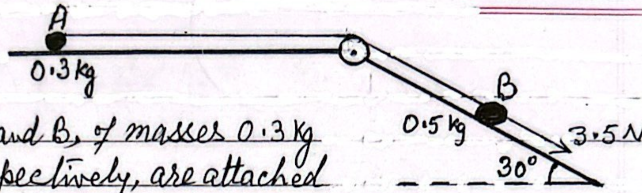
Now $0.2g \sin 30^\circ - F = 0.2 \times (-5)$ [Force = ma]

$\Rightarrow F = 2$ —— ④

$F = \mu R = \sqrt{3}\mu = 2$ from ④

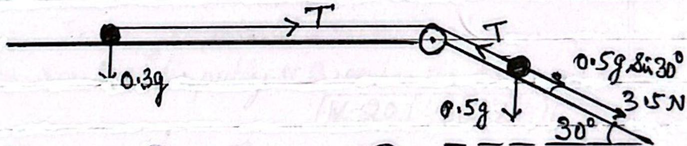
$\therefore \mu = \frac{2}{\sqrt{3}}$ ✓

14.



Two particles A and B, of masses 0.3 kg and 0.5 kg respectively, are attached to the ends of a light inextensible string.
The string passes over a fixed smooth pulley which is attached to a horizontal plane and to the top of an inclined plane. The particles are intitially at rest with A on the horizontal plane and B on the inclined plane, which makes an angle of 30° with the horizontal. The string is taut and B can move on a line of greatest slope of the inclined plane. A force of magnitude 3.5 N is applied to B acting down the plane.
Given that both planes are smooth, find the tension in the string and the acceleration of B. [W-20/42/Q8(a)] ---[5]

Solution:

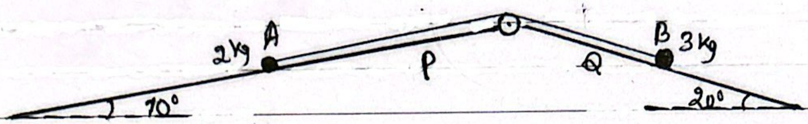


for A, $T = 0.3a$ ——— ①
for B. $3.5 + 0.5g\sin 30° - T = 0.5a$ ——— ②
add ① & ② $3.5 + \frac{5}{2} = 0.8a$

$\Rightarrow \quad a = \frac{6}{0.8} = \underline{7.5\ ms^{-2}}$ ✓

from ① $T = 0.3 \times 7.5 = \underline{2.25}$ ✓

15.

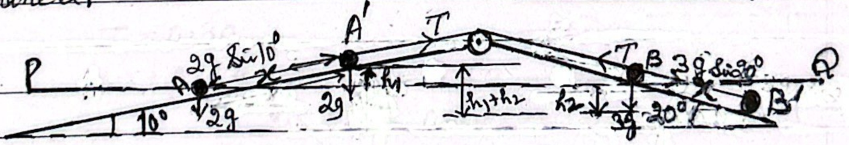


Particles A and B of masses 2kg and 3kg respectively are attached to the ends of a light inextensible string. The string passes over a small fixed smooth pully which is attached to the top of two inclined planes. Particle A is on plane P, which is inclined at an angle of 10° to the horizontal. Particle B is on plane Q, which is inclined at an angle of 20° to the horizontal. The string is taut, and the ... of the string are parallel to lines of greatest slope of their respective planes.

It is given that both planes are smooth and that the particles are released from rest at the same horizontal level.

Find the time taken until the difference in the vertical height of the particles is 1m. [you should assume that this occours before A reaches the pully or B reaches the bottom of plane Q. [W-20/43/Q7(b)] --- [6]

Solution:



for Particle A, $T - 2g\sin 10° = 2a$ —— ①

for Particle B, $3g\sin 20° - T = 3a$ —— ②

add ① & ② $\quad 5a = 3g\sin 20 - 2g\sin 10$

$\Rightarrow a = \dfrac{(30\sin 20 - 20\sin 10°)}{5} = 1.3575$ —— ①

Initially Particles A and B are on the same horizontal level $\overline{PQ}$

In new positions $AA' = BB' = x$ along the Planes

Let A moves vertical up by h_1, and B moves vertically down $\rightarrow h_2$ $\quad h_1 = x\sin 10°$, $h_2 = x\sin 20°$ $\quad \therefore$ difference between their level $\Rightarrow h_1 + h_2 = 1$ —— ③

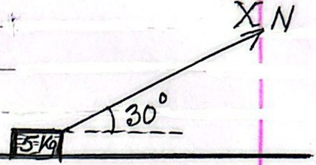
from ③ $\therefore x\sin 10° + x\sin 20° = 1 \Rightarrow x = \dfrac{1}{(\sin 10° + \sin 20°)}$ —— ④

distance Also $x = 0 + \frac{1}{2}at^2$

or $x = \frac{1}{2} \times 1.3575t^2$ —— ⑤ $\Rightarrow \dfrac{1}{(\sin 10° + \sin 20°)} = \frac{1}{2} \times 1.3575t^2$ from ④ & ⑤

$\Rightarrow t^2 = \dfrac{2}{(\sin 10° + \sin 20°) \times 1.3575} = 2.8571 \Rightarrow t = 1.69s$ ✓

16. A block of mass 5 Kg is being pulled along a rough horizontal floor by a force of magnitude X N acting at 30° above the horizontal. The block starts from rest and travels 2 m in the first 5 s of its motion.



(a) Find the acceleration of the block. ---[2]

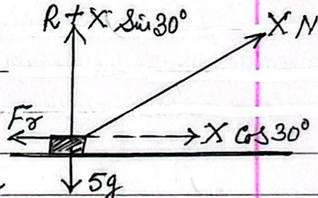
(b) Given that the coefficient of friction between the block and the floor is 0.4, find X. --[4]

The block is now placed on a part of the floor where the coefficient of friction between the block and the floor has a different value. The value of X is changed to 25, and the block is now in limiting equilibrium.

(c) Find the value of the coefficient of friction between the box and the floor. --[3]

[M-21/42/Q5]

Solution: Let the acceleration $= a$, $s = 2m$, $t = 5s$



(a) $s = ut + \frac{1}{2}at^2 \Rightarrow 2 = 0 + \frac{1}{2}a \times 5^2$

$\Rightarrow a = \frac{4}{25} = \underline{0.16\, ms^{-2}}$ ✓

(b) Now coeff of friction $\mu = 0.4$, force of frict $=$ Fr

Vertical Component: $R + X\sin 30° = 5g \Rightarrow R = (50 - X\sin 30°)$ ---①

Horizontal Component: $X\cos 30° - Fr = 5a$ $\quad [Fr = \mu R = 0.4(50 - X\sin 30°)]$

$\Rightarrow \frac{\sqrt{3}}{2}X - 0.4(50 - X\sin 30°) = 5 \times 0.16$ $\quad [a = 0.16]$

$\Rightarrow \frac{\sqrt{3}}{2}x - 20 + \frac{X}{5} = 0.8 \Rightarrow \frac{\sqrt{3}}{2}X + \frac{X}{5} = 20.8 \Rightarrow \frac{20.8}{1.066} = 19.512$ ✓

$\Rightarrow \underline{X = 19.5}$ ✓

(c) For limiting equilibrium: $X = 25N$, $\mu = ?$, $a = 0$

Horizontal: $X\cos 30° - Fr = 5 \times 0$

$\Rightarrow \frac{\sqrt{3}}{2}X - \mu(50 - X\sin 30°) = 0$

$\frac{\sqrt{3}}{2} \times 25 - \mu(50 - 25 \times \frac{1}{2}) = 0$

$\Rightarrow 37.5\mu = \frac{\sqrt{3}}{2} \times 25$

$\Rightarrow \mu = \frac{\sqrt{3}}{2} \times 25 \times \frac{1}{37.5} = 0.577$

$\therefore \underline{\mu = 0.577}$

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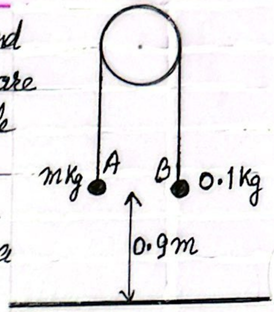
17. Two particles A and B have masses m kg and 0.1 kg respectively, where $m > 0.1$. The particles are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley and the particles hang vertically below it. Both particles are at a height of 0.9 m above horizontal ground. The system is released from rest, and while both particles are in motion the tension in the string is 1.5 N. Particle B does not reach the pulley.



(a) Find m. ---[4]

(b) Find the speed at which A reaches the ground. [S-21/41/Q2]

Solution (a) ($F = ma$)

for Particle B; $T - 0.1g = 0.1a$ --①

for Particle A; $mg - T = ma$ --②

add ① & ② $mg - 0.1g = 0.1a + ma$

$\Rightarrow mg - ma = 0.1a + 0.1g$

$10m - 5m = 0.1 \times 5 + 1$

$5m = 1.5$

$\Rightarrow m = \underline{0.3}$ ✓

[for ① $T = 1.5$

$1.5 - 1 = 0.1a$

$\Rightarrow a = 5$ ✓]

(b) $u = 0,\ v = ?,\ s = 0.9,\ a = 5$

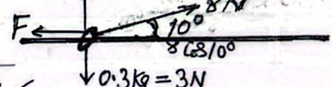
$v^2 = u^2 + 2as$

$\Rightarrow v^2 = 0 + 2 \times 5 \times 0.9 = 9$

$v = \sqrt{9} = 3$

$\underline{v = 3\,ms^{-1}}$

18. The ring of mass 0.3 kg is threaded on a horizontal rough rod. The coefficient of friction between the ring and the rod is 0.8. A force of magnitude 8 N acts on the ring. The force acts at an angle of 10° above the horizontal in the vertical plane containing the rod. Find the time taken for the ring to move, from rest, 0.6 m along the rod. [S-21/42/Q3] --[6]



Solution: Resolving perp. to rod:

$8\sin 10° + R = 0.3g \Rightarrow R = 3 - 8 \times 0.1736 = 1.61$ ✓

Resolving horizontally: $8\cos 10 - F = 0.3a$

$\Rightarrow 8 \times 0.9848 - \mu R = 0.3a$

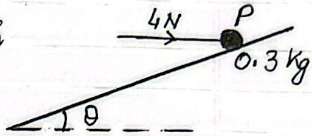
$\Rightarrow 7.8784 - 0.8 \times 1.61 = 0.3a$

$\Rightarrow a = \frac{6.59}{0.3} = 21.966$ ✓

$s = ut + \frac{1}{2}at^2$

$0.6 = 0 + \frac{1}{2} \times 21.966 \times t^2$

$\Rightarrow t^2 = 0.05462 \Rightarrow \underline{t = 0.234}$ ✓



19. A particle P of mass 0.3 Kg rests on a rough plane inclined at an angle θ to the horizontal, whose $\sin\theta = \frac{7}{25}$. A horizontal force of magnitude 4N, acting in a vertical plane containing a line of greatest slope of the plane is applied to P. The particle is on the point of sliding up the plane.

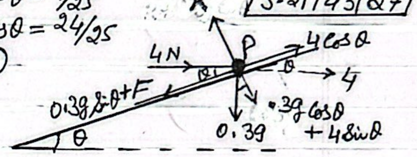
(a) Show that the coefficient of friction between the particle and the plane is $\frac{3}{4}$. ---[4]

The force acting horizontally is replaced by a force of magnitude 4N acting up the plane parallel to a line of greatest slope.

(b) Find the acceleration of P. ---[3]

(c) Starting with P at rest, the force 4N parallel to the plane acts for 3 seconds and then removed. Find the total distance travelled until P comes to instantaneous rest. --[3]

[S-21/43/Q7]



Solution (a) vertically $R = 0.3g\cos\theta + 4\sin\theta$ $\left\{ \begin{array}{l} \sin\theta = 7/25 \\ \cos\theta = 24/25 \end{array} \right.$

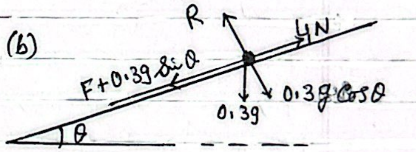
or $R = 3 \times \frac{24}{25} + 4 \times \frac{7}{25} = 4$ ---(1)

horizontally: $F + 0.3g\sin\theta = 4\cos\theta$

$\Rightarrow F = 4 \times \frac{24}{25} - 3 \times \frac{7}{25} = 3$ ---(2)

$\therefore \mu = \frac{F}{R} = \frac{3}{4}$ (from (1) & (2))

(b)



(b) $R = 0.3 \cdot g\cos\theta$

$F = \mu R = \frac{3}{4} \times 3 \times \frac{24}{25} = \frac{54}{25}$ ---(3)

Horizontally: $4 - (F + 0.3g\sin\theta) = 0.3a$ ---(4)

$\Rightarrow 4 - \frac{54}{25} - 3 \times \frac{7}{25} = 0.3a \Rightarrow a = \frac{10}{3}$ ---(5)

(c) distance travelled by P in 3 seconds $s_1 = 0 + \frac{1}{2} \times \frac{10}{3} \times 3^2$ ($s = ut + \frac{1}{2}at^2$)

$\Longrightarrow s_1 = 15m$ ---(6)

and $v = 0 + \frac{10}{3} \times 3 = 10$ ($v = u + at$) ---(7)

Now when the 4N is removed after 3 seconds.

from (4) $0 - (F + 0.3g\sin\theta) = 0.3a$

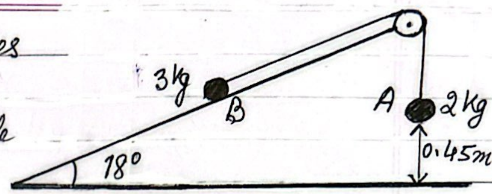
$\Rightarrow -\frac{54}{25} - 3 \times \frac{7}{25} = 0.3a \Rightarrow a = -10\,ms^{-2}$;

Let let the distance travelled before P come to rest is S_2

$v^2 = u^2 + 2as \Rightarrow 0 = 10^2 + 2 \cdot (-10) \cdot S_2 \Rightarrow S_2 = 5m$ ---(8)

$\therefore$ Total distance $= S_1 + S_2 = 15 + 5 = 20m$ [from (7) & (8)]

20. Two particles A and B of masses 2 kg and 3 kg respectively are connected by a light inextensible string. Particle B is on a smooth fixed plane which is at an angle of 18° to the horizontal ground. The string passes over a fixed smooth pulley at the top of the plane. Particle A hangs vertically below the pulley and is 0.45 m above the ground. The system is released from rest with the string taut. When A reaches the ground, the string breaks.



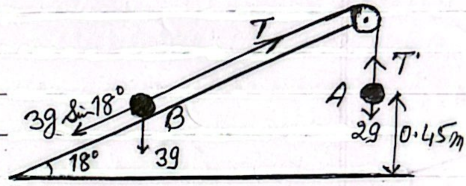
Find the total distance travelled by B before coming to instantaneous rest. You may assume that B does not reach the pulley. ---[8]

[W-21/41/Q7]

Solution: For Particle A: $2g - T = 2a$ ---①

for Particle B; $T - 3g\sin 18^\circ = 3a$ ---②

add ① and ②



$2g - 3g\sin 18 = 5a$

$\Rightarrow 5a = 20 - 9.27 = 10.729$

$\Rightarrow a = 2.14589$ ---③

Now A reaches the ground after distance $s_1 = 0.45$ ✓ ---④

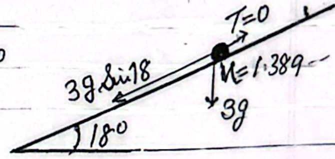
Velocity of B when A reaches the ground, $V^2 = 0 + 2 \times 2.1489 \times 0.45$

$\Rightarrow V^2 = 1.931308$ $\quad (V^2 = u^2 + 2as_1)$

$V = 1.38971$ ✓

Now when the thread breaks;

acc. $a_2 = ?$, $T = 0$



$0 - 3g\sin 18^\circ = 3a_2$

$\Rightarrow a_2 = -\frac{9.27}{3} = -3.09$

Now $u = 1.38971$ Let dis $= S_2$ when final $V = 0$

$\therefore 0 = (1.38971)^2 - 2 \times 3.09 \times S_2$ $\quad (V^2 = u^2 + 2aS)$

$S_2 = \frac{1.9313}{6.18} = 0.312\,m$ ---⑤

$\therefore$ Total distance travelled by $B = S_1 + S_2 = 0.45 + 0.312 = 0.762\,m$

[from ④ & ⑤]

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21. A van of mass 3600 kg is towing a trailer of mass 1200 kg along a straight horizontal road using a light horizontal rope. There are resistance forces of 700 N on the van and 300 N on the trailer.

(a) The driving force exerted by the van is 2500 N. Find the tension in the rope. ---[4]

The driving force is now removed and the van driver applies a break force which acts only on the van. The resistance forces remain unchaged.

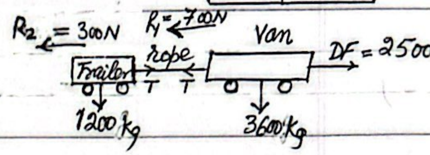
(b) Find the least possible value of the breaking force which will cause the rope to become slack. ---[2]

[W-21/42/Q2]

Solution (a) For Van: $2500 - 700 - T = 3600a$ --- (1)

for Trailer: $T - 300 = 1200a$ --- (2)

add (1) & (2)



$$2500 - 700 - 300 = (3600 + 1200)a$$

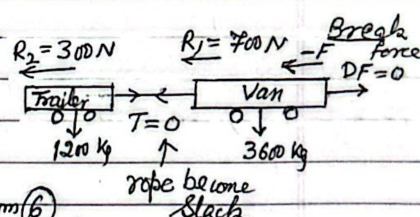
$$\Rightarrow 4800a = 1500 \Rightarrow a = 0.3125 \text{ --- (3)}$$

from (2) and (3) $T = 300 + 1200 \times 0.3125 = \underline{675\,N}$ --- (4)

(b) For Van; $-F - 700 = 3600a_1$ --- (5)

For Trailer $-300 = 1200a_1$ --- (6)

add (5) and (6)



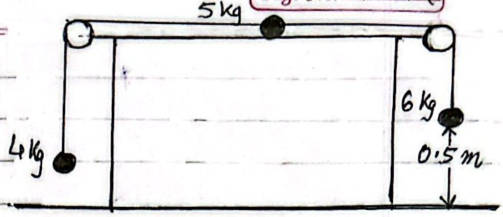
$$-F - 700 - 300 = (3600 + 1200)a_1$$

$$F = -1000 - 4800a_1$$

$$= -1000 - 4800 \times \left(-\frac{1}{4}\right) \quad \left[\text{from (6)} \; a_1 = -\frac{1}{4}\right.$$

$$= -1000 + 1200$$

$$F = \underline{200\,N}$$ Breaking force.



22. The diagram shows a particle of mass 5kg on a rough horizontal table, and two light inextensible strings attached to it passing over smooth pulleys fixed at the edges of the table. Particles of masses 4 kg and 6 kg hang freely at the ends of the strings. The particle of mass 6 kg is 0.5m above the ground. The system is in limiting equilibrium.

(a) Show that the coefficient of friction between the 5 kg particle and the table is 0.4. ---[2]

The 6kg particle is now replaced by a particle of mass 8kg and the system is released from rest.

(b) Find the acceleration of the 4 kg particle and the tensions in strings. ---[5]

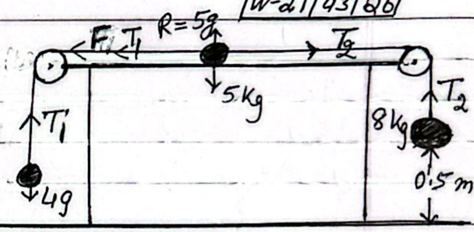
(c) In the subsequent motion the 8kg particle hits the ground and does not rebound. Find the time that elapses after the 8 kg particle hit the ground before the two particles come to instantaneous rest. ---[5]

[W-21/43/Q6]

Solution (a) $R = 5g$, $F = 6g - 4g = 2g$

$\therefore \mu = \frac{F}{R} = \frac{2g}{5g} = 0.4$

(b)



(b) $T_1 - 4g = 4a$ — ①

$8g - T_2 = 8a$ — ②

$T_2 - T_1 - F = 5a$ ($F = \mu R = 0.4 \times 5g = 2g$ force of friction)

$\Rightarrow T_2 - T_1 - 2g = 5a$ -- ③

add ①, ② and ③

$\Rightarrow 8g - 4g - 2g = 17a$

$\Rightarrow a = \frac{2g}{17} = \frac{20}{17} = 1.18\,ms^{-2}\ (1.176)$ — ④

(C) $T_1 - 4g = 4a$ --- ⑤

$-T_1 - F = 5a$ [In ③ $T_2 = 0$]

or $-T_1 - 2g = 5a$ -- ⑥ ($F = 2g$)

add ⑥ and ⑤ $-6g = 9a \Rightarrow a = -\frac{60}{9}$

Now when 8kg particle reaches the ground.

$\Rightarrow v^2 = 0 + 2aS$

$\Rightarrow v^2 = 2 \times \frac{20}{17} \times 0.5$

$v^2 = \frac{20}{17} \Rightarrow v = \sqrt{\frac{20}{17}}$

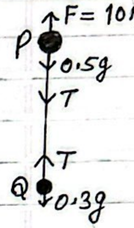
Time before 4 kg particle comes to rest

$0 = \sqrt{\frac{20}{17}} - \frac{60}{17}t$ { $v = 0$, $u = \sqrt{\frac{20}{17}}$ } ($v = u + at$)

$\Rightarrow t = 0.163\,s$

23 Two particles P and Q, of masses 0.5 kg and 0.3 kg respectively, are connected by a light inextensible string. The string is taut and P is vertically above Q. A force of magnitude 10N is applied to P vertically upwards. Find the acceleration of the particle and the tension in the string connecting them. ---[5]

[S-22/41/Q2]



Solution: For P: $F - 0.5g - T = 0.5a$ (using Newtons second law)

$\Rightarrow 10 - 0.5g - T = 0.5a$ --- ① ($F = 10N$)

for Q: $T - 0.3g = 0.3a$ --- ②

add ① and ② $\quad 10 - 0.8g = 0.8a$

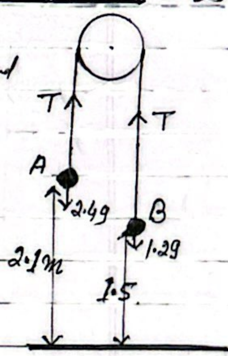
$\Rightarrow 0.8a = 2 \Rightarrow a = \underline{2.5\,ms^{-2}}$ ✓

from ② $T - 0.3g = 0.3 \times 2.5 \Rightarrow T = 3 + 0.75 = \underline{3.75N}$

24. Two particles A and B, of masses 2.4 kg and 1.2 kg respectively, are connected by a light inextensible string which passes over a fixed smooth pully. A is held at a distance of 2.1m above a horizontel plane and B is 1.5 m above the plane. The particles hang vertically and are released from rest. In the subsequent motion A reaches the plane and does not rebound and B reach the pulley.

(a) Show that the tension in the string before A reaches the plane is 16N and find the magnitude of the acceleration of the particles before A reaches the plane. ---[4]

(b) Find the greatest height of B above the plane. [S-22/42/Q3] ---[3]



Solution(a) for A: $2.4g - T = 2.4a$ --- ①

for B: $T - 1.2g = 1.2a$ --- ②

add ① and ②

$2.4g - 1.2g = (2.4 + 1.2)a$

$\Rightarrow 3.6a = 1.2 \times 10 \Rightarrow a = \frac{12}{3.6} = \frac{10}{3}$

$\underline{a = \frac{10}{3}\,ms^{-1}}$, from ② $T = \underline{16N}$ ✓

(b) when A reaches the ground

$V^2 = 0 + 2 \times \frac{10}{3} \times 2.1 = 14$

$\Rightarrow V = \sqrt{14} = 3.741\,ms^{-1}$

(b) continued →

for B when A reaches the ground

$u = \sqrt{14}$, for greatest height $V = 0$

$0 = u^2 - 2gs$

$\Rightarrow 0 = 14 - 2 \times 10s$

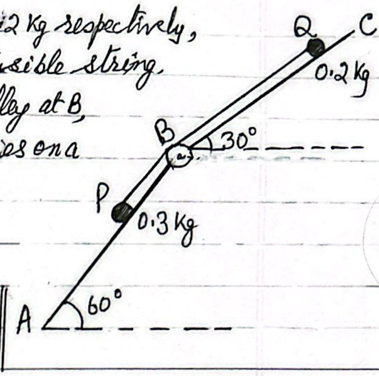
$s = \frac{14}{20} = 0.7$

$\therefore$ greatest height of B from the ground

$= 1.5 + 2.1 + 0.7$

$= \underline{4.3m}$ ✓

25. Two particles P and Q, of masses 0.3 kg and 0.2 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley at B, which is attached to two inclined planes. P lies on a smooth plane AB which is inclined at 60° to the horizontal. Q lies on a plance BC which is inclined at 30° to the horizontal. The string is taut and the particles can move on lines of greatest slope of the two planes.



(a) It is given that the plane BC is smooth and that the particle are released from rest. Find the tension in the string and the magnitude of the acceleration of the particles. ---[5]

(b) It is given instead that the plane BC is rough. A force of magnitude 3N is applied at Q directly up the plane along a line of greatest slope of the plane. Find the least value of the coefficient of friction between Q and the plane BC for which the particles remain at rest. [S-22/43/Q 6]-- [5]

~~(using Newton's second law)~~

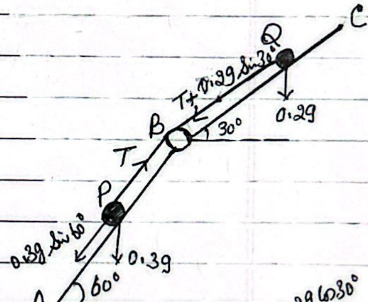
Solution: along the planes;

(a) for P; $0.3g\sin 60 - T = 0.3a$ --- (1)

for Q, $T + 0.2g\sin 30° = 0.2a$ --- (2)

add (1) & (2) $0.3g\sin 60° + 0.2g\sin 30° = 0.5a$

$\Rightarrow a = \underline{7.20\ \text{ms}^{-1}}$; for (1) $\underline{T = 0.439\,\text{N}}$.



(b)

$R = 0.2g\cos 30°$

for P; $0.3g\sin 60° - T = 0$ (P is at rest)

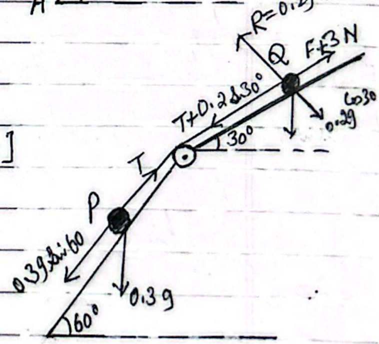
$\Rightarrow T = 0.3g\sin 60 = \frac{3\sqrt{3}}{2}$ ✓

for Q, $T + 0.2g\sin 30° = F + 3$

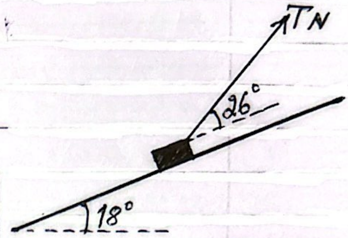
$\Rightarrow \frac{3\sqrt{3}}{2} + 1 = \mu(0.2g\cos 30) + 3$ $[F = \mu R]$

$3.598 = 1.732\mu + 3 \Rightarrow \mu = \frac{0.598}{1.732}$

$\Rightarrow \underline{\mu = 0.345}$ ✓



26. A block of mass 8 kg is placed on a rough plane which is inclined at an angle of 18° to the horizontal. The block is pulled up the plane by light string that makes an angle of 26° above a line of greatest slope. The tension in the string is T N.



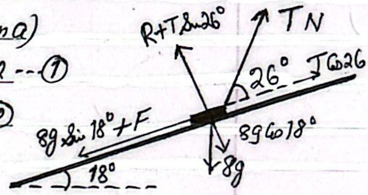
The coefficient of friction between the block and plane is 0.65.

(a) The acceleration of the block is $0.2\ ms^{-2}$. Find T. ---[7]

(b) The block is initially at rest. Find the distance travelled by the block during the fourth second of motion. ---[2]

[W-22/41/Q4]

Solution: Resolving the forces: (Force = ma)



(a) along the plane: $T\cos 26 - 8g\sin 18^\circ - F = 8\times 0.2$ --- ①

Perp. to the plane: $R + T\sin 26 = 8g\cos 18^\circ$ --- ②

$F = \mu R \Rightarrow F = 0.65R$ --- ③ $[\mu = 0.65]$

from ① and ③ $T\cos 26 - 8g\sin 18^\circ - 0.65R = 1.6$ --- ④

from ④ and ⑤ [from ② $R = 8g\cos 18^\circ - T\sin 26^\circ$] --- ⑤

$$T\cos 26 - 8g\sin 18^\circ - 0.65(8g\cos 18^\circ - T\sin 26^\circ) = 1.6$$

$$T(\cos 26^\circ + 0.65\sin 26^\circ) = 1.6 + 8g\sin 18 + 0.65\times 8g\cos 18$$

$$1.1837\,T = 75.7762 \Rightarrow T = 64.01$$

$$\therefore T = \underline{64\ N}$$

(b) Distance in fourth second:

$= $ Dis. in 4 S $-$ Dis in 3 S

$= \frac{1}{2}\times 0.2\times 4^2 - \frac{1}{2}\times 0.2\times 3^2$

$= \frac{1}{2}\times 0.2(16-9)$

$= 0.1\times 7$

$= \underline{0.7m}$

$$\left\{\begin{array}{l} S = ut + \frac{1}{2}at^2 \\ u = 0 \\ S = \frac{1}{2}at^2 \end{array}\right.$$

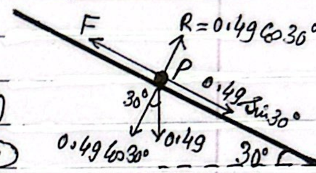
27. A particle P of mass 0.4 kg is in limiting equilibrium on a plane inclined at 30° to the horizontal.

(a) Show that the coefficient of friction between the particle and the plane is $\frac{1}{3}\sqrt{3}$ -- [3]

A force of magnitude 7.2 N is now applied to P directly up the line of greatest slope of the plane.

(b) Given that P starts from rest, find the time that it takes P to move 1 m up the plane.

[W-22/42/Q2] -- [4]



Solution: Normal reaction at P; $R = 0.4g \cos 30°$

(a) Force of friction $F = \mu R = \mu \times 0.4g \cos 30$ -- ①

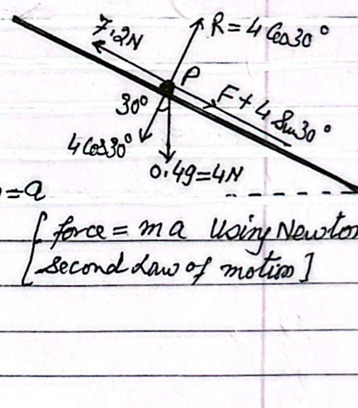
downward force along the plane $= 0.4g \sin 30$ -- ②

Under the limiting equilibrium: from ① and ②

$$F = \mu \cdot 0.4g \cos 30 = 0.4g \sin 30° \Rightarrow \mu = \frac{\sin 30°}{\cos 30°} = \frac{1}{2} \Big/ \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}} = \frac{1}{3}\sqrt{3}$$

Hence $\mu = \frac{1}{3}\sqrt{3}$

(b) $R = 4\cos 30°$, $\mu = \frac{1}{3}\sqrt{3}$



Force of friction $F = \mu R = \frac{1}{3}\sqrt{3} \times 4\cos 30°$

$= \frac{1}{3}\sqrt{3} \times 4 \times \frac{\sqrt{3}}{2}$

$\therefore F = 2$ --- ①

Now along the plane: Let the acceleration $= a$

$7.2 - 4\sin 30° - F = 0.4a$

$7.2 - 4 \times \frac{1}{2} - 2 = 0.4a$

[force = ma Using Newton's second law of motion]

$\Rightarrow a = \frac{3.2}{0.4} \Rightarrow a = 8$

Now $s = ut + \frac{1}{2}at^2$

$1 = 0 + \frac{1}{2} \times 8 \times t^2$ $\quad (u = 0,\ s = 1\,m,\ a = 8,\ t = ?)$

$\Rightarrow t^2 = \frac{1}{4} \Rightarrow t = \frac{1}{2}$

$\therefore$ Time $= 0.5$ s