Date 27.12.23.
Page

P-1

Pure Math_1

Binomial Theorem
Exercise 1. Solution (Revision)

SP-20	M-20	M-22	S-20	S-22	W-20
W-22	M-21	M-23	S-21	S-23	W-21.

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Binomial Theorem - Revision

Example 1(a) Find the coefficients of x^2 and x^3 in the expansion of $(2-x)^6$. ---[3]

(b) Hence find the coefficient of x^3 in the expansion of, $(3x+1)(2-x)^6$. ---[2]

SP-20/01/Q6

Solution: $(2-x)^6$

(a) General term $= {}^6C_r\, 2^{6-r} \cdot (-x)^r$

$= (-1)^r\, {}^6C_r\, 2^{6-r} \cdot x^r$ ——①

[Gen. Term in $(x+y)^n = {}^nC_r\, x^{n-r} \cdot y^r$]

for $x^2 \Rightarrow r=2$ in ①

Coeff of $x^2 = (-1)^2 \cdot {}^6C_2 \cdot 2^{6-2}$

$= 15 \times 16 = \underline{240}$ ——②

for Coeff of x^3, put $r=3$ in ①

$= (-1)^3 \cdot {}^6C_3 \cdot 2^3$

Coeff of $x^3 = -20 \times 8 = \underline{-160}$ ——③

(b) In $(3x+1)(2-x)^6$

term with $x^3 = 3 \times$ coeff of x^2 in $(2-x)^6$

$+ 1 \times$ coeff of x^3 in $(2-x)^6$

$= 3 \times 240 + 1 \times (-160)$ from ② and ③

$= 720 - 160 = \underline{560}$

Example 2: The coefficient of $\frac{1}{x}$ in the expansion of $\left(2x + \frac{a}{x^2}\right)^5$ is 720.

(a) Find the possible values of the constant a. ---[3]

(b) Hence find the coefficient of $\frac{1}{x^7}$ in the expansion. ---[2]

M-20/12/Q6

Solution: General Term in $\left(2x+\frac{a}{x^2}\right)^5 = {}^5C_r\,(2x)^{5-r} \cdot \left(\frac{a}{x^2}\right)^r = {}^5C_r\, 2^{5-r} \cdot a^r \cdot x^{5-3r}$ ——①

(a) ∴ for the coeff $\frac{1}{x}$ or x^{-1}

from ① $5-3r=-1$

$\Rightarrow r=2$

from ① ∴ Coeff $\frac{1}{x} = {}^5C_2 \cdot 2^3 \times a^2 = 720$ (Given)

$10 \times 8 \times a^2 = 720 \Rightarrow a^2 = 9$

$a = \underline{\pm 3}$

(b) for Coeff of $\frac{1}{x^7}$ (or x^{-7})

from ① $5-3r=-7 \Rightarrow r=4$

∴ Req. Coeff of $\frac{1}{x^7}$ from ① for $\left\{\begin{matrix} r=4 \\ a=\pm 3\end{matrix}\right\}$

$= {}^5C_4 \times 2^1 \times (\pm 3)^4$

$= 5 \times 2 \times 81 = \underline{810}$

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3 (a) Find the first three terms in the expansion, in ascending power of x, $(1+x)^5$ --- [1]

(b) Find the first three terms in the expansion of $(1-2x)^6$ --- [2]

(c) Hence find the coefficient of x^2 in the expansion of $(1+x)^5 \cdot (1-2x)^6$. --- [2]

$\boxed{M-21/12/Q1}$

Solution (a) $(1+x)^5 = 1 + {}^5C_1 x + {}^5C_2 x^2 + \cdots = 1 + 5x + 10x^2 + \cdots$ ①

(b) $(1-2x)^6 = 1 + {}^6C_1(-2x) + {}^6C_2(-2x)^2 + \cdots = 1 - 12x + 60x^2 + \cdots$ ②

(C) Now consider $(1+x)^5 \times (1-2x)^6$

$= (1 + 5x + 10x^2 + \cdots)\cdot(1 - 12x + 60x^2 + \cdots)$

$\therefore x^2$ terms $= 1 \times (60x^2) + 5x(-12x) + 10x^2 \times 1.$

$= \cancel{60x^2} - \cancel{60x^2} + 10x^2 = 10x^2$

$\therefore$ Coeff of $x^2 = \underline{10}$

$[\,(1+x)^n = 1 + {}^nC_1 x + {}^nC_2 x^2 + \cdots {}^nC_r x^r + \cdots {}^nC_n x^n\,]$

4. Find the term independent of x in each of the following expansions.

(a) $\left(3x + \frac{2}{x^2}\right)^6$ --- [3]

(b) $\left(3x + \frac{2}{x^2}\right)^6 \cdot (1 - x^3)$ --- [3]

$\boxed{M-22/12/Q3}$

Solution (a) $\left(3x + 2/x^2\right)^6$ $\quad [\,(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \cdots\,]$

$= (3x)^6 + {}^6C_1 \cdot (3x)^5 \cdot \left(\frac{2}{x^2}\right)^1 + {}^6C_2 \cdot (3x)^4 \cdot \left(\frac{2}{x^2}\right)^2 + {}^6C_3 (3x)^3 \cdot \left(\frac{2}{x^2}\right)^3 + \cdots$ ①

from ①

Term independent of $x = {}^6C_2 \times (3x)^4 \cdot \left(\frac{6}{x^2}\right)^2 = 15 \times 3^4 \cdot \cancel{x^4} \times \frac{2^2}{\cancel{x^4}}$

$= 15 \times 81 \times 4 = \underline{4860}$ ②

(b) $\left(3x + \frac{2}{x^2}\right)^6 \cdot (1 - x^3)$

$= \left[(3x)^6 + {}^6C_1 (3x)^5 \cdot \left(\frac{2}{x^2}\right) + {}^6C_2 (3x)^4 \left(\frac{2}{x^2}\right)^2 + {}^6C_3 (3x)^3 \cdot \left(\frac{2}{x^2}\right)^3 \cdots\right] \times (1 - x^3)$

Term independent of $x = 1 \times {}^6C_2 \cdot (3x)^4 \cdot \left(\frac{2}{x^2}\right)^2 - x^3 \times {}^6C_3 (3x)^3 \cdot \left(\frac{2}{x^2}\right)^3$

$= 1 \times 15 \times 81 x^4 \times \frac{4}{x^4} - x^3 \times 20 \times 27 x^3 \times \frac{8}{x^6}$

$= 15 \times 81 \times 4 - 20 \times 27 \times 8 = 4860 - 4320 = \underline{540}$

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5. In the expansion of $\left(\frac{x}{a}+\frac{a}{x^2}\right)^7$, it is given that:

$\frac{\text{the coefficient of } x^4}{\text{the coefficient of } x} = 3$, find the possible values of constant a. ---[6]

$\boxed{M-23/12/Q6}$

Solution: $\left(\frac{x}{a}+\frac{a}{x^2}\right)^7 = {}^7C_0\left(\frac{x}{a}\right)^7 + \cdots + {}^7C_5\left(\frac{x}{a}\right)^5\cdot\left(\frac{a}{x^2}\right)^2 + {}^7C_6\left(\frac{x}{a}\right)^6\cdot\left(\frac{a}{x^2}\right)^1 + \cdots$

Term containing $x = {}^7C_5 \times \frac{x^5}{a^5} \times \frac{a^2}{x^4} = \frac{21}{a^3} \times x \Rightarrow$ Coeff of $x = \frac{21}{a^3}$ ✓

Term containing $x^4 = {}^7C_6 \cdot \left(\frac{x}{a}\right)^6 \cdot \frac{a}{x^2} = \frac{7}{a^5} \cdot x^4 \Rightarrow$ Coeff of $x^4 = 7/a^5$ ✓

Now given $\frac{\text{Coeff of } x^4}{\text{Coff } x} = \frac{7/a^5}{21/a^3} = 3 \Rightarrow \frac{7}{a^5} \times \frac{a^3}{21} = 3 \Rightarrow \frac{1}{a^2} = 9 \Rightarrow a^2 = \frac{1}{9}$

$\Rightarrow a = \pm 1/3$

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Example 6. The coefficient of $\frac{1}{x}$ in the expansion of $\left(kx+\frac{1}{x}\right)^5+\left(1-\frac{2}{x}\right)^8$ is 74. Find the value of the positive constant k. [S-20/11/Q2] --[5]

Solution: Gen. term in $\left(kx+\frac{1}{x}\right)^5 = {}^5C_r (kx)^{5-r}\cdot\left(\frac{1}{x}\right)^r = {}^5C_r k^{5-r}\cdot x^{5-2r}$ —①

from ① for the coeff of $\frac{1}{x}$; $5-2r=-1 \Rightarrow r=3$

$\therefore$ coeff of $\frac{1}{x} = {}^5C_3\cdot k^2 = 10k^2$ —②

Gen. Term in $\left(1-\frac{2}{x}\right)^8 = {}^8C_r\, 1^{8-r}\cdot\left(-\frac{2}{x}\right)^r = {}^8C_r(-2)^r\cdot x^{-r}$ —③

for coeff of $\frac{1}{x}$, $-r=-1 \Rightarrow r=1$

$\therefore$ Coeff of $\frac{1}{x}$ in ③ $= {}^8C_1(-2)^1 = -16$. —④

$\therefore$ Coeff of $\frac{1}{x}$ in $\left(kx+\frac{1}{x}\right)^5+\left(1-\frac{2}{x}\right)^8$

$= 10k^2-16 = 74$ (Given)

$\Rightarrow k^2=9 \Rightarrow k=3$ ✓ $(k>0)$

Example 7.(a) Find the coeff of x^2 in the expansion of $\left(x-\frac{2}{x}\right)^6$ --[2]

(b) Find the Coefficient of x^2 in the expansion of $(2+3x^2)\left(x-\frac{2}{x}\right)^6$ --[3]

[S-20/12/Q1]

Solution: Gen. term in $\left(x-\frac{2}{x}\right)^6 = {}^6C_r x^{6-r}\cdot\left(-\frac{2}{x}\right)^r = {}^6C_r(-2)^r\cdot x^{6-2r}$ —①

(a) for the coeff of x^2, from ① $6-2r=2 \Rightarrow r=2$

$\therefore$ from ① coeff of $x^2 = {}^6C_2(-2)^2 = 15\times4 = 60$ ✓ —②

(b) from for constant term $6-2r=0 \Rightarrow r=3$

$\therefore$ Constant term in $\left(x-\frac{2}{x}\right)^6 = {}^6C_3(-2)^3 = 20\times(-8) = -160$ —③

Now In $(2+3x^2)\left(x-\frac{2}{x}\right)^6$

for the coeff of x^2 = $2\times$ coeff of x^2 in $\left(x-\frac{2}{x}\right)^6$

$+\ 3\times$ constant term in $\left(x-\frac{2}{x}\right)^6$

$= 2\times60 + 3\times(-160)$ from ② & ③

$= 120-480$

$= -360$ ✓

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Example 8 (a) Expand $(1+a)^5$ in ascending powers of a upto and including the term in a^3. --[1]

(b) Hence expand $[1+(x+x^2)]^5$ in ascending powers of x upto and including the term in x^3, simplify your answer. --[3]

[S-20/13/Q4]

Solution (a) $(1+a)^5 = {}^5C_0 + {}^5C_1\, a + {}^5C_2\, a^2 + {}^5C_3\, a^3 + \cdots$

$= 1 + 5a + 10a^2 + 10a^3 + \cdots$ —— (1)

(b) Using (1)

$[1+(x+x^2)]^5 = 1 + 5(x+x^2) + 10(x+x^2)^2 + 10(x+x^2)^3 + \cdots$

$= 1 + 5(x+x^2) + 10(x^2+2x^3+\cdots) + 10(x^3+\cdots)$

$= \underline{1 + 5x + 15x^2 + 30x^3 + \cdots}$ ✓

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9 (a) Find the three terms in the expansion of $(3-2x)^5$ in ascending powers of x. ---[3]

(b) Hence find the coefficient of x^2 in the expansion of $(4+x)^2 \cdot (3-2x)^5$ --[3]

[S-21/11/Q3]

Solution (a) $(3-2x)^5 = {}^5C_0 \cdot 3^5 + {}^5C_1 \cdot 3^4 \cdot (-2x)^1 + {}^5C_2 \cdot 3^3 (-2x)^2 + ---$

$= 1 \times 243 + 5 \times 81 \times (-2x) + 10 \times 27 \times 4x^2 + \cdots$

$= 243 - 810x + 1080x^2 + \cdots$ ✓

(b) $(4+x)^2 \cdot (3-2x)^5$

$(16 + x^2 + 8x) \cdot (243 - 810x + 1080x^2 + \cdots)$

Coefficient of $x^2 = 16 \times 1080 + 1 \times 243 + 8 \times (-810) = 11043$ ✓

10. The coefficient of x in the expansion of $\left(4x + \frac{10}{x}\right)^3$ is p. The coefficient of $\frac{1}{x}$ in the expansion of $\left(2x + \frac{k}{x^2}\right)^5$ is q. Given that $p = 6q$, find the possible values of k. ---[5]

[S-21/12/Q4]

Solution: $\left(4x + \frac{10}{x}\right)^3 = {}^3C_0 (4x)^3 + {}^3C_2 (4x)^2 \times \left(\frac{10}{x}\right) + ---$

The term with x in $\left(4x + \frac{10}{x}\right)^3 = 3 \times 16x^2 \times \frac{10}{x} = 480x \rightarrow$ Coeff of $x = 480 = p$

$\Rightarrow p = 480$ — ①

$\left(2x + \frac{k}{x^2}\right)^5 = {}^5C_0 \cdot (2x)^5 + {}^5C_1 \times (2x)^4 \times \left(\frac{k}{x^2}\right) + {}^5C_2 \cdot (2x)^3 \cdot \left(\frac{k}{x^2}\right)^2 + --$

Term containing $\frac{1}{x} = 10 \times 8x^3 \times \frac{k^2}{x^4} = 80k^2 \cdot \frac{1}{x} \Rightarrow$ Coeff of $\frac{1}{x} = 80k^2 = q$ --②

Now given $p = 6q \Rightarrow 480 = 6 \times 80k^2$ [from ① & ②]

$\Rightarrow k^2 = 1$

$\Rightarrow k = \pm 1$ ✓

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11. (a) Write down the first four terms in the expansion, in ascending powers of x, of $(a-x)^6$. ---[2]

(b) Given that the coefficient of x^2 in the expansion of $\left(1+\frac{2}{ax}\right)(a-x)^6$ is -20, find in exact form the possible values of the constant a.

$\boxed{S-21/13/Q7}$ ---[5]

Solution (a) $(a-x)^6 = {}^6C_0 a^6 + {}^6C_1 a^5(-x)^1 + {}^6C_2 a^4 \cdot (-x)^2 + {}^6C_3 a^3 \cdot (-x)^3 + \cdots$

$\rightarrow (a-x)^6 = a^6 - 6a^5x + 15a^4x^2 - 20a^3x^3$ --- (1)

(b) Now $\left(1+\frac{2}{ax}\right)\cdot(a-x)^6 = \left(1+\frac{2}{ax}\right)\cdot\left(a^6 - 6a^5x + 15a^4x^2 - 20a^3x^3 \cdots\right)$

Term containing $x^2 = 1 \times 15a^4x^2 + \frac{2}{ax} \times -20a^3x^3$.

Coeff. of $x^2 = (15a^4 - 40a^2) = -20$ (given)

$\Rightarrow 15a^4 - 40a^2 + 20 = 0$

$\Rightarrow 3a^4 - 8a^2 + 4 = 0 \Rightarrow (3a^2-2)(a^2-2) = 0$

$a^2 = 2/3, \; a^2 = 2$

$\Rightarrow a = \pm\sqrt{\frac{2}{3}}, \; a = \pm\sqrt{2}$

12. The coefficient of x^4 in the expansion of $\left(2x^2+\frac{k^2}{x}\right)^5$ is a, and the coefficient of x^2 in the expansion of $(2kx-1)^4$ is b.

(a) Find a and b in terms of the constant k. ---[3]

(b) Given that $a+b=216$, find the possible values of k. ---[3]

[S-22/11/Q3]

Solution: (a)

$\left(2x^2+\frac{k^2}{x}\right)^5 = {}^5C_0(2x^2)^5 + {}^5C_1(2x^2)^4\cdot\left(\frac{k^2}{x}\right)^1 + {}^5C_2(2x^2)^3\cdot\left(\frac{k^2}{x}\right)^2 + \cdots$

term with $x^4 = {}^5C_2 \cdot 8x^6 \times \frac{k^4}{x^2} = 10\times 8\cdot x^6 \cdot \frac{k^4}{x^2} = 80k^4x^4$

$\therefore$ Coeff of $x^4 = a = 80k^4$ ✓ ——— (1)

Again $(2kx-1)^4 = {}^4C_0(2kx)^4 + {}^4C_1(2kx)^3\cdot(-1)^1 + {}^4C_2(2kx)^2(-1)^2 + \cdots$

Term in $x^2 = {}^4C_2(2kx)^2(-1)^2 = 6\times 4k^2x^2\times 1$

$\therefore$ Coeff of $x^2 = b = 24k^2$ ✓ ——— (2)

(b) Given $a+b=216 \Rightarrow 80k^4+24k^2=216$ (from (1) & (2))

$\Rightarrow 10k^4+3k^2-27=0 \Rightarrow (2k^2-3)(5k^2+9)=0$

$\Rightarrow k^2 = 3/2$ or $k^2 = -\frac{9}{5}$ ✗ <0

$\therefore k = \pm\sqrt{3/2}$ ✓

13. The coefficient of x^4 in the expansion of $(3+x)^5$ is equal to the coefficient of x^2 in the expansion of $\left(2x+\frac{a}{x}\right)^6$. Find the value of the positive constant a. ---[4]

[S-22/12/Q1]

Solution: $(3+x)^5 = {}^5C_0 3^5 + {}^5C_1 3^4\cdot x^1 + {}^5C_2 3^3\cdot x^2 + {}^5C_3 3^2\cdot x^3 + {}^5C_4 3^1\cdot x^4 + \cdots$

$\therefore$ Coeff. of x^4 in $(3+x)^5 = {}^5C_4 \times 3 = 5\times 3 = 15$ ---- (1)

Now $\left(2x+\frac{a}{x}\right)^6 = (2x)^6 + {}^6C_1(2x)^5\cdot\frac{a}{x} + {}^6C_2(2x)^4\cdot\left(\frac{a}{x}\right)^2 + \cdots$

The term of x^2 in $\left(2x+\frac{a}{x}\right)^6 = {}^6C_2\cdot(2x)^4\cdot\left(\frac{a}{x}\right)^2 = 15\times 16x^4 \times \frac{a^2}{x^2} = 240x^2a^2$

$\therefore$ Coeff of x^2 in $\left(2x+\frac{a}{x}\right)^6 = 240a^2$ ——— (2)

Given Coeff of x^4 in $(3+x)^5$ = Coeff of x^2 in $\left(2x+\frac{a}{x}\right)^6$

$\Rightarrow 15 = 240a^2$ from (1) & (2)

$\Rightarrow a^2 = \frac{15}{240} = \frac{1}{16}$

$a = \frac{1}{4}$ ✓ $(a>0)$

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14 The coefficient of x^3 in the expansion of $\left(p+\frac{1}{p}x\right)^4$ is 144. Find the possible values of the constant p. ---[4]

$\boxed{S\text{-}22/13/Q1}$

Solution: $\left(p+\frac{1}{p}x\right)^4 = {}^4C_0\,p^4 + {}^4C_1\,p^3\cdot\left(\frac{1}{p}x\right)^1 + {}^4C_2\,p^2\left(\frac{1}{p}x\right)^2 + {}^4C_3\,p\cdot\left(\frac{1}{p}x\right)^3 + \cdots$

Term containing $x^3 = {}^4C_3\cdot p\cdot\frac{1}{p^3}\cdot x^3 = \frac{4}{p^2}\cdot x^3$

Coeff. of $x^3 \Rightarrow \frac{4}{p^2} = 144$ (Given)

$\Rightarrow p^2 = \frac{4}{144} = \frac{1}{36}$

$p = \pm\frac{1}{6}$ ✓

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15 (a) Find the first three terms in the expansion, in ascending powers of x, of $(2+3x)^4$. ---[2]

(b) Find the first three terms in the expansion, in ascending powers of x, of $(1-2x)^5$. ---[2]

(c) Hence find the coefficient of x^2 in the expansion of $(2+3x)^4 \cdot (1-2x)^5$ ---[2]

[S-23/ 11/Q2]

<u>Solution</u> $(a+x)^n = a^n + {}^nC_1 a^{n-1}x + {}^nC_2 a^{n-2}x^2 + \cdots$

(a) $(2+3x)^4 = 2^4 + {}^4C_1 \cdot 2^3 \cdot (3x) + {}^4C_2 \cdot 2^2 \cdot (3x)^2 + \cdots$ $\left\{ \begin{array}{l} {}^4C_1 = 4 \\ {}^4C_2 = 6 \end{array} \right.$

$= 16 + 96x + 216x^2 + \cdots$ ①

(b) $(1-2x)^5 = 1^5 + {}^5C_1 \cdot 1^4 \cdot (-2x)^1 + {}^5C_2 \cdot 1^3 \cdot (-2x)^2 + \cdots$ $\left\{ \begin{array}{l} {}^5C_1 = 5 \\ {}^5C_2 = 10 \end{array} \right.$

$= 1 - 10x + 40x^2 + \cdots$ ②

(c) $(2+3x)^4 \cdot (1-2x)^5 = (16 + 96x + 216x^2 + \cdots) \times (1 - 10x + 40x^2 + \cdots)$

$\therefore$ In this the terms with $x^2 = 16 \times 40x^2 - 96x \times 10x + 216x^2 \times 1$.

$= (640x^2 - 960x^2 + 216x^2)$

$= -104x^2$

$\therefore$ Coeff of $x^2 = \underline{-104}$ ✓

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Q16 The coefficient of x^4 in the expansion of $(x+a)^6$ is p and the coeff. of x^2 in the expansion of $(ax+3)^4$ is q. It is given that $p+q=276$; Find the possible values of the constant a. --[4]

S-23 | 12 | Q2

Solution: $(x+a)^n = x^n + {}^nC_1 x^{n-1} a + {}^nC_2 x^{n-2} a^2 + ---$

$(x+a)^6 = x^6 + {}^6C_1 x^5 a + {}^6C_2 x^4 a^2 + ---$

$\therefore$ Coeff of x^4 in $(x+a)^6 = {}^6C_2\, a^2 = 15a^2 = p$ --- ① Given,

and

$(ax+3)^4 = (ax)^4 + {}^4C_1 (ax)^3 \cdot 3 + {}^4C_2 (ax)^2 \cdot 3^2 + ---$

Coeff of x^2 in $(ax+3)^4 = {}^4C_2 \cdot a^2 \times 9 = 54a^2 = q$ --- ② Given.

Also given $p+q=276$

$\Rightarrow 15a^2 + 54a^2 = 276 \Rightarrow 69a^2 = 276 \Rightarrow a^2 = 4 \Rightarrow a = \pm 2$ ✓

Q17. (a) Give the complete expansion of $\left(x+\frac{2}{x}\right)^5$ ---[2]

(b) In the expansion of $(a+bx^2)\left(x+\frac{2}{x}\right)^5$, the coefficient of x is zero and the coefficient of $\frac{1}{x}$ is 80. Find the values of the constants a and b. ---[4]

S-23 | 13 | Q3

Solution (a) $\left(x+\frac{2}{x}\right)^5 = x^5 + {}^5C_1 \cdot x^4 \cdot \frac{2}{x} + {}^5C_2 \cdot x^3 \cdot \left(\frac{2}{x}\right)^2 + {}^5C_3 x^2 \cdot \left(\frac{2}{x}\right)^3 + {}^5C_4 x \left(\frac{2}{x}\right)^4 + {}^5C_5 \left(\frac{2}{x}\right)^5$

$= x^5 + 10x^3 + 40x + \frac{80}{x} + \frac{80}{x^3} + \frac{32}{x^5}$ ✓ --- ①

(b) $(a+bx^2)\left(x+\frac{2}{x}\right)^5$

$= (a+bx^2)\left(x^5 + 10x^3 + 40x + \frac{80}{x} + \frac{80}{x^3} + \frac{32}{x^5}\right)$ --- ② from ①

In ② coeff. of $x = a \times 40 + b \times 80 = 0$ given

$40a + 80b = 0 \Rightarrow a + 2b = 0$ --- ③

Also from ② coeff. of $\frac{1}{x} = 80a + 80b = 80 \Rightarrow a+b=1$ --- ④

Solving ③ & ④

$a = 2$ and $b = -1$ ✓

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18. In the expansion of $\left(2x^2+\frac{a}{x}\right)^6$, the coefficients of x^6 and x^3 are equal.

(a) Find the value of the non-zero constant a. --- [4]

(b) Find the coefficient of x^6 in the expansion of $(1-x^3)\left(2x^2+\frac{a}{x}\right)^6$ -- [1]

[W-20 | 11 | Q5]

Solution: Binomial theorem:

(a) $(x+y)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} y + {}^nC_2 x^{n-2} y^2 + ---$

$\therefore \left(2x^2+\frac{a}{x}\right)^6 = {}^6C_0 (2x^2)^6 + {}^6C_1 (2x^2)^5 \cdot \frac{a}{x} + {}^6C_2 (2x^2)^4 \cdot \left(\frac{a}{x}\right)^2 + {}^6C_3 (2x^2)^3 \cdot \left(\frac{a}{x}\right)^3 + --$

$= 64x^{12} + 6\times 32 \times x^9 . a + \underline{15\times 16\, x^6 a^2} + \underline{20\times 8\, x^3 a^3} + ----$

Coefficient of $x^6 = 15\times 16\times a^2 = 240a^2$ —— ①

Coefficient of $x^3 = 20\times 8\, a^3$ —— ②

Given Coeff of x^6 = Coeff of $x^3 \Rightarrow 160a^3 = 240a^2$

$\Rightarrow a = \frac{240}{160} = \underline{3/2}$ ✓

(b) Coeff of x^6 in –

$(1-x^3)\left(2x^6+\frac{a}{x}\right)^6 = 1\times$ Coeff of $x^6 - 1\times$ Coeff of x^3

$= 0$ ✓ (as given that In $\left(2x^2+\frac{a}{x}\right)^6$, Coeff of x^6 = Coeff x^3)

---

19. The coefficient of x^3 in the expansion of $(1+kx)(1-2x)^5$ is 20. Find the value of the constant k. [W-20 | 12 | Q1] --[4]

Solution: $(1-2x)^5 = {}^5C_0 + {}^5C_1(-2x) + {}^5C_2(-2x)^2 + {}^5C_3(-2x)^3 + ---$

$= 1 + 5(-2x) + 10\times 4x^2 + 10(-8x^3)$

$\Rightarrow (1-2x)^5 = 1 - 10x + 40x^2 - 80x^3 + --$

Now In $(1+kx)(1-2x)^5$

The coeff of $x^3 = 1\times(-80) + k(40) = 20$ (Given)

$40k = 100$

$k = \frac{100}{40} = 5/2$

$\therefore k = \underline{\frac{5}{2}}$ ✓

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20. In the expansion of $(a+bx)^7$, where a and b are non-zero constants, the coefficients of x, x^2 and x^4 are the first, second and third terms respectively of a geometric progression. Find the value of a/b. --[5]

W-20/13/Q5

Solution: Binomial theorem: $(a+b)^n = {}^nC_0\, a^n + {}^nC_1\, a^{n-1} b + {}^nC_2\, a^{n-2} b^2 + {}^nC_3\, a^{n-3} . b^3 + \dots$

$\Rightarrow (a+bx)^7 = {}^7C_0\, a^7 + {}^7C_1\, a^6 . bx + {}^7C_2\, a^5 . (bx)^2 + {}^7C_3\, a^4 . (bx)^3 + {}^7C_4\, a^3 . (bx)^4 + \dots$

Coefficient of $x = {}^7C_1\, a^6 b = 7a^6 b$ —— (1)

Coeff. of $x^2 = {}^7C_2 \times a^5 . b^2 = 21a^5b^2$ —— (2)

Coeff of $x^4 = {}^7C_4 . a^3 . b^4 = 35a^3b^4$ —— (3)

Given $7a^6b$, $21a^5b^2$ and $35a^3b^4$ are in G.P

$\Rightarrow \dfrac{21a^5b^2}{7a^6b} = \dfrac{35a^3b^4}{21a^5b^2}$ [$\because$ a, b, c are in G.P $\Rightarrow \dfrac{b}{a} = \dfrac{c}{b}$]

$\Rightarrow \dfrac{3b}{a} = \dfrac{5b^2}{3a^2} \Rightarrow \dfrac{b}{a} \times \dfrac{a^2}{b^2} = \dfrac{5}{3} \times \dfrac{1}{3} \Rightarrow \dfrac{a}{b} = \dfrac{5}{9}$ ✓

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21 (a) Expand $\left(1-\frac{1}{2x}\right)^2$ ---[1]

(b) Find the first four terms in the expansion, in ascending powers of x, of $(1+2x)^6$. ---[2]

(c) Hence find the coefficient of x in the expansion of $\left(1-\frac{1}{2x}\right)^2(1+2x)^6$ ---[2]

$\boxed{W-21/11/Q1}$

Solution (a) $\left(1-\frac{1}{2x}\right)^2 = 1-\frac{1}{x}+\frac{1}{4x^2}$ --- ①

(b) $(1+2x)^6 = {}^6C_0 + {}^6C_1\cdot(2x) + {}^6C_2\cdot(2x)^2 + {}^6C_3\times(2x)^3 + \ldots$

$= 1+6\times 2x + 15\times 4x^2 + 20\times 8x^3$

$= 1+12x+60x^2+160x^3$ ---- ②

(c) Consider $\left(1-\frac{1}{2x}\right)^2(1+2x)^6 = \left(1-\frac{1}{x}+\frac{1}{4x^2}\right)\left(1+12x+60x^2+160x^3 \ldots\right)$

from ① & ②

$\therefore$ Term containing $x = 1\times 12x - \frac{1}{x}\times 60x^2 + \frac{1}{4x^2}\times 160x^3 \ldots$

$\therefore$ Coeff of $x = (12-60+40) = \underline{-8}$ ✓

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22(a) It is given that in the expansion of $(4+2x)(2-ax)^5$, the coefficient of x^2 is -15. Find the possible values of a. ---[4]

(b) It is given instead that in the expansion of $(4+2x)(2-ax)^5$, the coefficient x^2 is k. It is also given that there is only one value of a which leads to this value of k. Find the values of k and a. ---[4]

[W-21/12/Q8]

Solution (a) $(4+2x)(2-ax)^5 = (4+2x)[2^5 + {}^5C_1 2^4(-ax) + {}^5C_2 \cdot 2^3(-ax)^2 + --]$

Terms with $x^2 = 4 \times 80a^2x^2 + 2x \times 80(-ax) = (320a^2 - 160a)x^2$

$\therefore$ Coeff. of $x^2 = (320a^2 - 160a) = -15$ (Given)

$\Rightarrow 320a^2 - 160a + 15 = 0 \Rightarrow 64a^2 - 32a + 3 = 0$

$\Rightarrow (8a-3)(8a-1) = 0 \Rightarrow a = \frac{1}{8}, a = 3/8$ ✓

(b) Coeff of x^2: $320a^2 - 160a = k$ (Given)

$\Rightarrow 320a^2 - 160a - k = 0$ --- ①

has only one value of $a \Rightarrow (B^2 - 4AC = 0) \Rightarrow 160^2 - 4 \times 320(-k) = 0$

$\Rightarrow k = -20$ ✓

for $k = -20$ in ① $320a^2 - 160a + 20 = 0$

$\Rightarrow 16a^2 - 8a + 1 = 0 \Rightarrow (4a-1)^2 = 0$

$\Rightarrow a = \frac{1}{4}$ ✓

23(a) Find the first three terms, in the ascending powers of x, in the expansion of $(1+ax)^6$. ---[1]

(b) Given that the coefficient of x^2 in the expansion of $(1-3x)(1+ax)^6$ is -3, find the possible values of the constant a. ---[4]

[W-21/13/Q2]

Solution (a) $(1+ax)^6 = 1 + {}^6C_1 \cdot ax + {}^6C_2(ax)^2 + -$

$\Rightarrow (1+ax)^6 = 1 + 6ax + 15a^2x^2 + ---$ ✓

(b) $(1-3x)(1+ax)^6 = (1-3x)(1 + 6ax + 15a^2x^2 --)$

Coeff of $x^2 = 1 \times 15a^2 - 3 \times 6a = -3$ (Given)

$\Rightarrow 15a^2 - 18a + 3 = 0$

$\Rightarrow 5a^2 - 6a + 1 = 0$

$(a-1)(5a-1) = 0$

$\Rightarrow a = 1, \frac{1}{5}$ ✓

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Note: $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_r x^r + \dots$

24. The coefficient of x^2 in the expansion of $\left(1+\frac{2}{p}x\right)^5 + (1+px)^6$ is 70. Find the possible values of the constant p. ---[6]

$\boxed{W\text{-}22 / 11 / Q4}$

Solution: $\left(1+\frac{2}{p}x\right)^5 = 1 + {}^5C_1 \cdot \frac{2}{p}x + {}^5C_2\left(\frac{2}{p}x\right)^2 + \dots$

$\Rightarrow$ Coeff of x^2 in $\left(1+\frac{2}{p}x\right)^5 = {}^5C_2 \cdot \frac{4}{p} = \frac{40}{p^2}$ --- ①

Now $(1+px)^6 = 1 + {}^6C_1\, px + {}^6C_2 (px)^2 + \dots$

$\Rightarrow$ Coeff of x^2 in $(1+px)^6 = {}^6C_2\, p^2 = 15p^2$ — ②

$\therefore$ Coeff of x^2 in $\left[\left(1+\frac{2}{p}x\right)^5 + (1+px)^6\right] = \frac{40}{p^2} + 15p^2 = 70$ Given

$\Rightarrow 15p^4 - 70p^2 + 40 = 0 \Rightarrow 3p^4 - 14p^2 + 8 = 0$

$\Rightarrow (p^2-4)(3p^2-2) = 0$ $\quad p = \pm 2$ or $\pm\sqrt{2/3}$

$\Rightarrow p^2 = 4$ or $2/3$

25(a) Find the first three terms in the ascending powers of x of the expansion of $(1+2x)^5$. --[2]

(b) Find the first three terms in the ascending powers of x of the expansion of $(1-3x)^4$ ---[2]

(c) Hence find the coefficient of x^2 in the expansion of $(1+2x)^5 \cdot (1-3x)^4$ --[2]

$\boxed{W\text{-}22 / 13 / Q3}$

Solution: (a) $(1+2x)^5 = {}^5C_0 + {}^5C_1 \cdot 2x + {}^5C_2 \cdot (2x)^2 + \dots$

$= 1 + 10x + 40x^2 + \dots$ -- ①

(b) $(1-3x)^4 = {}^4C_0 + {}^4C_1(-3x) + {}^4C_2(-3x)^2 + \dots$

$= 1 - 12x + 54x^2 + \dots$ --- ②

(c) $(1+2x)^5 \cdot (1-3x)^4 = (1 + 10x + 40x^2 + \dots)(1 - 12x + 54x^2 + \dots)$

Terms of x^2 in $(1+2x)^5 \cdot (1-3x)^4 = 1 \times 54x^2 + 10x(-12x) + 40x^2 \times 1$

$\therefore$ Coeff of $x^2 = (54 - 120 + 40)$

$= -26$

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