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P-1

# Pure Maths-1

## Circular Measure

## Exercise 1 Solution (Revision)

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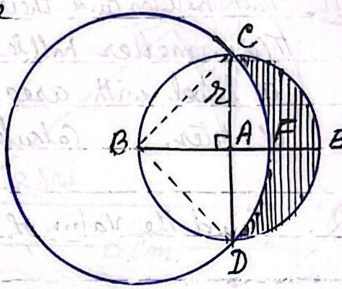
# Circular measure _ Revision



Example 1: The diagram shows a circle with centre A and radius r. Diameters CAD and BAE are perpendicular to each other. A large circle has centre B and passes through C and D.




(a) Show that the radius of the larger circle is $r\sqrt{2}$. ---[1]

(b) Find the area of the shaded region. ---[6]

[SP-20/01/Q9]

Solution (a) In rt triangle BAC,

$BC^2 = r^2 + r^2 = 2r^2 \rightarrow BC = r\sqrt{2}$, the radius of larger circle.

(b) Shaded area = Area of semicircle CADE − area of segment of CADF —①

Area of segment CADF = area of sector BCFD − ar $\Delta$ BCD —②

Area of Sector BCFD $= \frac{1}{2}(r\sqrt{2})^2 \times \frac{\pi}{2}$

$= \frac{\pi}{4} \times 2r^2 = \frac{1}{2}\pi r^2$ ✓ —③

$\because \left( A = \frac{1}{2} \times R^2 \times \theta \right.$

$R = BC = r\sqrt{2}$

$\theta$ = angle BCD $= \frac{\pi}{2}^c$

Area rt triangle BCD $= \frac{1}{2} \times$ base $\times$ Alt $= \frac{1}{2} \times 2r \times r$

$= r^2$ —④

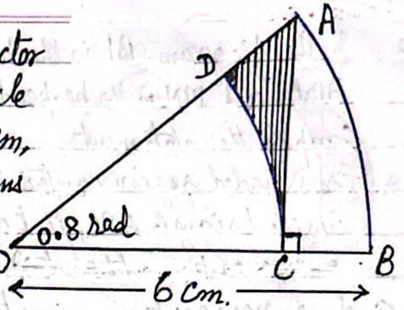
$\therefore$ Area of Segment CADF $= \left(\frac{1}{2}\pi r^2 - r^2\right)$ —⑤ [from ②, ③ and ④]

from ① & ⑤

Area of Shaded region $= \frac{1}{2}\pi r^2 - \left(\frac{1}{2}\pi r^2 - r^2\right)$

Required Area $A = r^2$ ✓

<u>Example 2:</u> The diagram shows a sector AOB which is a part of a circle with centre O and radius 6 cm, and with angle AOB = 0.8 radians. The point C on OB is such that AC is perpendicular to OB. The arc CD is part of a circle with centre O, where D lies on OA. Find the area of the shaded region. [M-20/12/Q 7] --[6]




<u>Solution:</u> Required shaded area = area of $\triangle AOC$ − area of sector DOC —(1)

Now area of $\triangle AOC = \frac{1}{2} \times OC \times AC$
$= \frac{1}{2} \times 4.18 \times 4.30$
$= 8.987$ —(2)

$OC = OA \cdot \cos 0.8$
$= 6 \times 0.6967$
$= 4.18$

$AC = OA \cdot \sin 0.8$
$= 6 \times 0.7173 = 4.30$

Area of sector DOC $= \frac{1}{2} \times OC^2 \times 0.8$ $\left[\frac{1}{2} r^2 \theta\right]$
$= \frac{1}{2} \times (4.18)^2 \times 0.8$
$= 6.988$ —(3)

$\therefore$ Shaded area $= 8.987 - 6.988$ (from (1), (2) & (3))
$= 1.999$
$= 2$ sq. cm.

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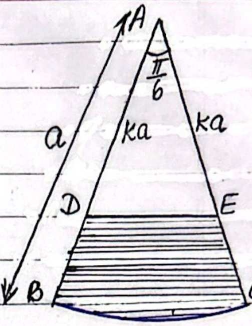
3. The diagram shows a sector ABC, which is part of a circle of radius $a$. The points D and E lie on AB and AC respectively and are such that $AD = AE = ka$, where $k < 1$. The line DE divides the sector into two regions which are equal in area.




(a) For the case where angle $BAC = \frac{1}{6}\pi$ radians, find $k$ correct to 4 significant figures. --- [5]

(b) For the general case in which angle $BAC = \theta$ radians, where $0 < \theta < \frac{1}{2}\pi$, it is given that $\frac{\theta}{\sin\theta} > 1$. Find the possible values of $k$. --- [3]

[M-21/12/ Q10]

Solution (a) angle $BAC = \frac{\pi}{6}$, and area of $\triangle ADE$ = area $DEBC$

$$\Rightarrow \frac{1}{2} \times ka \times ka \times \sin\frac{\pi}{6} = \left[\frac{1}{2}a^2 \cdot \frac{\pi}{6} - \frac{1}{2} \cdot ka \cdot ka \sin\frac{\pi}{6}\right] \quad (1)$$

$$\Rightarrow k^2 a^2 \times \frac{1}{2} = \frac{1}{2}a^2 \cdot \frac{\pi}{6} \Rightarrow k^2 \cancel{a^2} \times \cancel{\tfrac{1}{2}} = \cancel{\tfrac{1}{2}}\cancel{a^2} \times \frac{\pi}{6}$$

$$\Rightarrow k^2 = \frac{\pi}{6} \Rightarrow k = \sqrt{\frac{\pi}{6}} = \underline{0.7236}$$

(b) Now when angle $BAC = \theta$ rad, $0 < \theta < \frac{\pi}{2}$ and $\frac{\theta}{\sin\theta} > 1$ following like in equation (1)

$$k^2 a^2 \sin\theta = \frac{1}{2}a^2\theta \Rightarrow k^2 = \frac{1}{2} \cdot \frac{\theta}{\sin\theta} > \frac{1}{2} \quad \left[\because \frac{\theta}{\sin\theta} > 1\right]$$

$$k^2 > \left(\frac{1}{\sqrt{2}}\right)^2$$

$$k^2 - \left(\frac{1}{\sqrt{2}}\right)^2 > 0$$

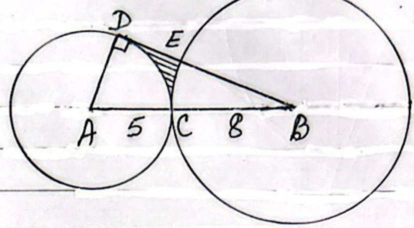
$$\left(k - \frac{1}{\sqrt{2}}\right)\left(k + \frac{1}{\sqrt{2}}\right) > 0$$

$$\Rightarrow k < -\frac{1}{\sqrt{2}} \times \; ; \; k > \frac{1}{\sqrt{2}} \quad \text{(also given } k < 1)$$

$(\because k > 0)$

$$\Rightarrow \underline{\frac{1}{\sqrt{2}} < k < 1}$$

4. The diagram shows a circle with centre A of radius 5 cm and a circle with centre B of radius 8 cm. The circles touch at the point C so that ACB is a straight line.




The tangent at the point D on the smaller circle intersect the larger circle at E and passes through B.

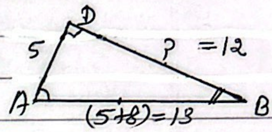
(a) Find the perimeter of the shaded region. ---[5]

(b) Find the area of the shaded region. ---[3]

[M-22/12/Q 10]

<u>Solution:</u> In rt $\Delta$ ADB, $DB^2 = 13^2 - 5^2 = 144$

$\Rightarrow \underline{DB = 12}$ ✓




$\tan A = 12/5$, $\cos A = 5/13$ and $\sin A = \frac{12}{13}$

angle $\underline{A} = \tan^{-1} 12/5 = \underline{1.176 \text{ rad}}$ ✓ ; $\tan B = 5/12 \Rightarrow \underline{B} = \tan^{-1}\frac{5}{12} = \underline{0.3948}$ ✓

(a) $DE = DB - BE = 12 - 8 = 4$ ✓

Arc $DC = r\theta = 5 \times 1.176 = 5.880$ and Arc $EC = 8 \times 0.3948 = 3.158$ ✓

$\therefore$ Perimeter of the shaded region $= 4 + 5.88 + 3.158 = 13.038$

$= \underline{13.0}$ ✓

(b) Area of the shaded region = ar$\Delta$ ABD − (ar sector ADC + ar sect BCE) (1)

ar $\Delta$ ABD $= \frac{1}{2} \times 5 \times 12 = 30$ ✓

area of sector ADC $= \frac{1}{2} r^2 \theta = \frac{1}{2} \times 5^2 \times 1.176 = 14.7$ ✓

area of sector BCE $= \frac{1}{2} r^2 \theta = \frac{1}{2} \times 8^2 \times 0.3948 = 12.6336$ ✓

$\therefore$ from (1) Area of the shaded region $= 30 - (14.7 + 12.63)$

$= \underline{2.67}$ ✓

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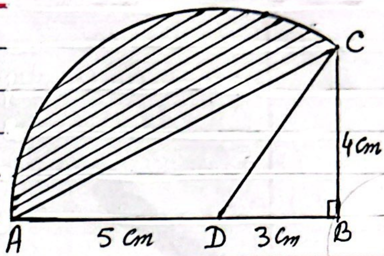
5. The diagram shows triangle ABC in which angle B is a right angle. The length is 8 cm, and the length BC is 4 cm. The point D on AB is such that AD = 5 cm. The sector DAC is a part of a circle with centre D.

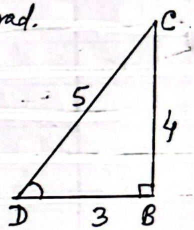



(a) Find the perimeter of the shaded region. --[5]

(b) Find the area of the shaded region. ---[3]

[M-23 / 12 / Q8]

Solution (a) In rt $\Delta$ CDB $\rightarrow$ $\tan BDC = \frac{4}{3} \Rightarrow$ angle $BDC = \tan^{-1}\frac{4}{3} = 0.927$ rad.




(AD = DC = 5 cm)

$\Rightarrow$ angle $ADC = \pi - 0.927 = 2.214$ rad.

$\Rightarrow$ length of arc $\widehat{AC} = r\theta = 5 \times 2.214 = 11.07$ --- (1)

rt $\Delta ABC \rightarrow AC = \sqrt{8^2+4^2} = 8.94$ cm ——— (2)

$\therefore$ perimeter of the shaded region = arc $\widehat{AC}$ + AC

{from (1) & (2)} $= 11.07 + 8.94 = 20.01$

$= \underline{20.0 \text{ cm}}$

(b) Area of the shaded region = ar sector ADC − ar $\Delta$ ADC --- (3)

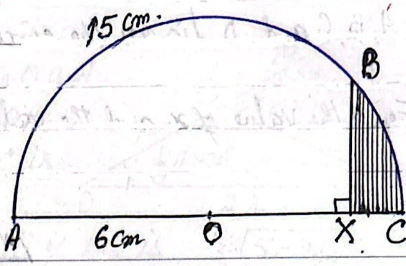
Area of sector ADC $= \frac{1}{2} r^2\theta = \frac{1}{2} \times 5^2 \times 2.214 = 27.7$ — (4)

Area of $\Delta$ ADC $= \frac{1}{2} r^2 \sin\theta = \frac{1}{2} \times 5^2 \times \sin 2.214 = 10.00$ — (5)

from (4) and (5) in (3) area of shaded region $= 27.7 - 10 = \underline{17.7 \text{ cm}^2}$

Example 6:

In the diagram, ABC is a semicircle, with diameter AC, centre O and radius 6 cm. The length of the arc AB is 15 cm.

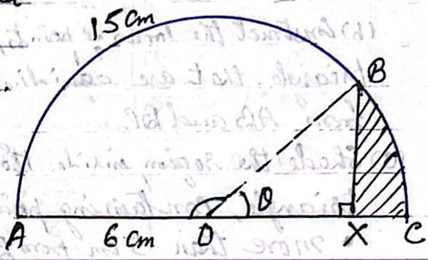



The point X lies on AC and BX is perpendicular to AX.
Find the perimeter of the shaded region BXC. ---[6]

[S-20/11/Q 8]

Solution: Perimeter $= BX + XC +$ arc $BC$ ——— ①

angle $AOB = \frac{l}{r} = \frac{15}{6} = 2.5$ rad




$\theta =$ angle $BOC = \pi - 2.5 = 0.6416$ rad.

length of arc $BC = r \cdot \theta$
$= 6 \times 0.6416$
$= 3.8495$ ✓ ②

In rt $\Delta BOX$

$BX = OB \sin\theta = 6 \sin 0.6416 = 3.59$ cm ——③

and $OX = OB \cos\theta = 6 \times \cos 0.6416 = 4.806$

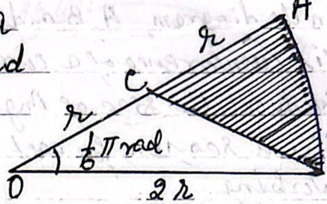
$\therefore XC = 6 - OX = 6 - 4.806 = 1.194$ ——④

for ②, ③ and ④ in ①

$P = 3.59 + 1.194 + 3.8495$
$= 8.6335$

$\therefore$ Required Perimeter $= 8.63$ cm ✓

Example 7. In the diagram, OAB is a sector of a circle with centre O and radius $2r$, and angle AOB is $\frac{1}{6}\pi$ radians. The point C is the mid point of OA.




(a) Show that the exact length of BC is $r\sqrt{5-2\sqrt{3}}$ --- [2]

(b) Find the exact perimeter of the shaded region. --- [2]

(c) Find the exact area of the shaded region. --- [3]

[S-20/12/Q7]

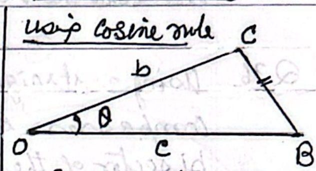
Solution (a) $BC^2 = r^2 + (2r)^2 - 2r\cdot 2r \cos\frac{\pi}{6}$

$BC^2 = 5r^2 - 2r^2\sqrt{3}$

$= r^2(5-2\sqrt{3})$

$\therefore BC = r\sqrt{5-2\sqrt{3}}$

using cosine rule




$BC^2 = b^2 + c^2 - 2bc\cos\theta$

(b) Perimeter $= BC + CA + \text{arc } AB$

$= r\sqrt{5-2\sqrt{3}} + r + 2r\cdot\frac{\pi}{6}$

$= r\left(\sqrt{5-2\sqrt{3}} + 1 + \frac{\pi}{3}\right)$

arc $AB = 2r\cdot\theta$

(c) Area = Area of Sector OAB − area $\Delta OCB$ —— ①

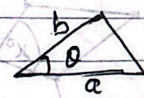
Area of Sector OAB $= \frac{1}{2}(2r^2)\times\frac{\pi}{6} = \frac{\pi}{3}r^2$ —— ②

Area of $\Delta OCB = \frac{1}{2}\times OB\times OC\cdot\sin\frac{\pi}{6}$

$= \frac{1}{2}\times 2r\times r\times\frac{1}{2}$

$= \frac{1}{2}r^2$ —— ③

$\because$



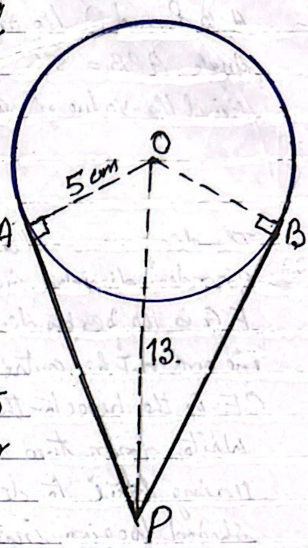

Area $= \frac{1}{2}ab\sin\theta$

put ② & ③ in ①

Required Area $= \frac{\pi}{3}r^2 - \frac{1}{2}r^2$

$= r^2\left(\frac{\pi}{3} - \frac{1}{2}\right)$

**Example 8:** The diagram shows a cord going round a pulley and a pin. The pully is modelled as a circle with centre O and radius 5 cm. The thickness of the cord and the size of pin P can be neglected. The pin is situated 13 cm vertically below O. Points A and B are on the circumference of the circle such that AP and BP are tangents to the circle. The cord passes over the major arc AB of the circle and under the pin such that the cord is taut. Calculate the length of the cord. ---[6]

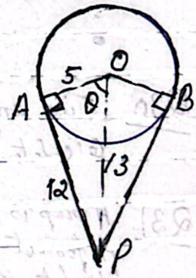



[S-20/13/Q5]

**Solution:** In rt $\Delta OAP$, $PA = \sqrt{13^2 - 5^2} = 12$ ✓

Let angle $POA = \theta$,

$\cos\theta = \frac{5}{13}$, $\sin\theta = \frac{12}{13}$




$\theta = \cos^{-1}\frac{5}{13} = 1.176$ rad.

Reflex $AOB = 2\pi - 2\times1.176 = 3.931$ rad.

Major arc $= r\times\theta = 5\times3.931 = 19.656$ ✓

$\therefore$ length of cord = Major arc + $2\times AP$

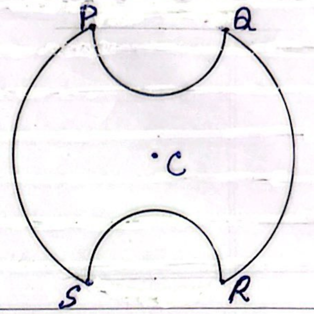
$= 19.656 + 2\times12$

$= 43.656$

$\therefore$ Req. Length $= 43.7$

43.7 cm.

9. The diagram shows a symmetrical metal plate. The plate is made by removing two identical pieces from a circular disc with centre C. The boundary of the plate consists of two arcs PS and QR of the original circle and two semicircles with PQ and RS diameters. The radius of the circle with centre C is 4 cm, and PQ = RS = 4 cm also.




(a) Show that angle $PCS = \frac{2}{3}\pi$ radians. --- [2]

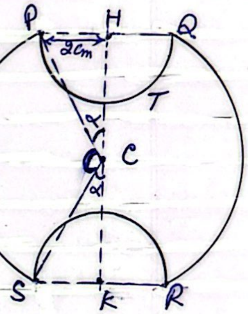
(b) Find the exact perimeter of the plate. -- [3]

(c) Show that the area of the plate is $\left(\frac{20}{3}\pi + 8\sqrt{3}\right)cm^2$. -- [5]

[S-21 | 11 | Q8]

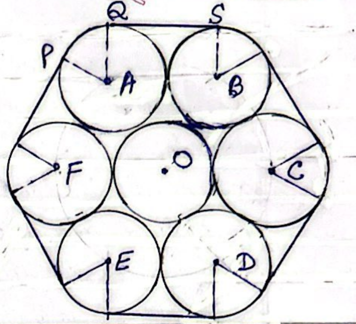
Solution (a) PQ = 4 cm, Let H is the mid point PQ, $CH \perp PH$,
PC = 4 cm (radius) Let angle $HCP = \alpha$,
$\sin\alpha = \frac{2}{4} \Rightarrow \alpha = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$ = angle SCK (Similarly)

Hence angle $PCS = \pi - \left(\frac{\pi}{6} + \frac{\pi}{6}\right) = \frac{2\pi}{3}$ ✓




(b) Perimeter = 2 [length of arc PS + length of semicircle PTQ]
$= 2[r_1\theta + \pi r_2]$ $[r_1 = 4,\ r_2 = 2]$
$= 2[4 \times \frac{2\pi}{3} + \pi \times 2]$
$= \frac{28}{3}\pi$ ✓

(c) Area of the plate = 2 [Area of sector CPS + ar $\Delta$CPQ − ar semicircle PQT]
$= 2\left[\frac{1}{2} \times 4^2 \times \frac{2\pi}{3} + \frac{1}{2} \times 4^2 \times \sin\frac{\pi}{3} - \frac{1}{2} \times \pi \times 2^2\right]$
$= 2\left[\frac{16\pi}{3} + 4\sqrt{3} - 2\pi\right]$
$= \left(\frac{20\pi}{3} + 8\sqrt{3}\right)$ ✓




S-21 | 12 | Q 12

10. The diagram shows a cross-section of seven cylindrical pipes, each of radius 20 cm, held together by a thin rope, which is wrapped tightly around the pipes. The centres of the six outer pipes are A, B, C, D, E and F. Points P and Q are situated where straight sections of the rope meet the pipe with centre A.

(a) Show that angle $PAQ = \frac{1}{3}\pi$ radians --[2]

(b) Find the length of the rope. --[4]

(c) Find the area of the hexagon ABCDEF, giving your answer in terms of $\sqrt{3}$. --[2]

(d) Find the area of the complete region enclosed by the rope. --[3]

**Solution:** (a) angle $P\hat{A}Q = 2\pi \div 6 = \frac{\pi}{3}$ ($\because$ There are six sectors around the diagram that make up a complete circle)

(b) length of arc $PQ = r\theta = 20 \times \frac{\pi}{3}$ and length $QS = AB = 2r = 40$ cm

$\therefore$ Total length of rope $= 6\left(\frac{20\pi}{3} + 40\right) = (40\pi + 240\text{cm})$

(c) Area of hexagon (ABCDEF) $= 6 \times$ area of $\Delta OAB$ —— ①

area of $\Delta OAB = \frac{1}{2} \times 40 \times 40 \times \sin\frac{\pi}{3} = 400\sqrt{3}$ —— ②

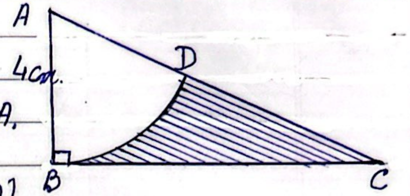
from ① & ② area of hexagon (ABCDEF) $= 6 \times 400\sqrt{3} = 2400\sqrt{3}$ —— ③

(d) Area of complete region = area of hexagon ABCD + 6× ar sector APQ + 6× ar rectangle ABSQ

$= 2400\sqrt{3} + 6\times\left(\frac{1}{2}\times 20^2 \times \frac{\pi}{3}\right) + 6\times(20\times 40)$

$= (2400\sqrt{3} + 400\pi + 4800) = 10200\text{ cm}^2$

11 The diagram shows a triangle ABC, in which angle ABC = 90° and AB = 4 cm. The ABD is part of a circle with centre A. The area of the sector is 10 cm².




(a) Find the angle BAD in radians.....[2]

(b) Find the perimeter of the shaded region. ....[4]

S-21 | 13 | Q5

Solution (a) Let angle $BAD = \theta \rightarrow$ Area of Sector $= \frac{1}{2} r^2 \theta = 10$ (Given)

$$\Rightarrow \frac{1}{2} \times 4^2 \theta = 10 \Rightarrow \theta = \frac{10}{8} = 1.25$$

$\therefore$ angle $BAD = 1.25$ radians

(b) Length of arc $BD = r \cdot \theta = 4 \times 1.25 = 5$ cm — ①

In $\Delta ABC$, $\frac{BC}{AB} = \tan\theta \rightarrow BC = 4 \times \tan 1.25 = 4 \times 3.009 = 12.04$ — ②

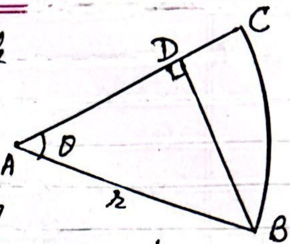
Now $\frac{BC}{AC} = \cos\theta \Rightarrow AC = \frac{BC}{\cos\theta} = \frac{4}{\cos 1.25} = 12.69$

$\therefore CD = AC - AD$
$= 12.69 - 4 = 8.69$ — ③

$\therefore$ The perimeter of the shaded region $= BD + BC + CD$

from ①, ② & ③ $= 5 + 12.04 + 8.69 = 25.7$

12. The diagram shows a sector ABC of a circle with centre A and radius $r$. The line BD is perpendicular to AC. Angle CAB is $\theta$ radians.




(a) Given that $\theta = \frac{1}{6}\pi$, find the exact area of BCD in terms of $r$. ---[3]

(b) Given instead that the length BD is $\frac{\sqrt{3}}{2}r$, find the exact perimeter of BCD in terms of $r$. ---[4]

[S-22 | 11 | Q5]

Solution (a) Area of sector $ABC = \frac{1}{2}r^2\theta$

$= \frac{1}{2}r^2 \cdot \frac{\pi}{6} = \frac{\pi}{12}r^2$ --- ①

In rt $\Delta BAD$, $\begin{cases} BD = r\cdot\sin\frac{\pi}{6} = \frac{1}{2}r \\ AD = r\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}r \end{cases}$

$\therefore$ Area of $\Delta ABD = \frac{1}{2}\cdot BD \times AD$

$= \frac{1}{2}\times\frac{1}{2}r\times\frac{\sqrt{3}}{2}r = \frac{\sqrt{3}}{8}r^2$ --- ②

from ① and ②

Area of $BCD = \frac{1}{12}\pi r^2 - \frac{\sqrt{3}}{8}r^2$ ✓

(b) Given $BD = \sqrt{3}/2\, r$ --- ③

In rt $\Delta ABD = \frac{BD}{AB} = \sin\theta$

$\Rightarrow \sin\theta = \frac{\frac{\sqrt{3}}{2}r}{r} = \frac{\sqrt{3}}{2}$

$\therefore \theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ ✓

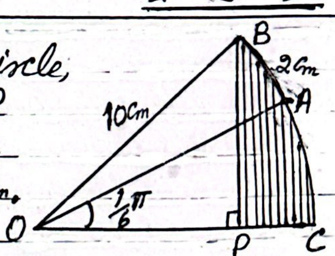
Length of arc $BC = r\theta = \frac{\pi}{3}\cdot r$ --- ④

Length $AD = r\cdot\cos\frac{\pi}{3} = \frac{1}{2}r$ --- ⑤

from ③, ④ and ⑤

Perimeter of $BCD = \frac{\sqrt{3}}{2}r + \frac{\pi}{3}r + \frac{1}{2}r$ ✓

13. The diagram shows a sector OBAC of a circle, with centre O and radius 10 cm. The point P lies on OC and BP is perpendicular to OC. Angle $AOC = \frac{1}{6}\pi$ and the length of arc AB is 2 cm.




(a) Find angle $\angle BOC$. ----[2]

(b) Hence find the area of the shaded region BPC, giving your answer to 3 s.f. ----[4]

[S-22 | 12 | Q7]

Solution (a) angle $AOB = \frac{2}{10}$ $\left[\begin{matrix} l = r\theta \\ \theta = l/r \end{matrix}\right.$

$\therefore$ angle $BOC = \frac{2}{10} + \frac{\pi}{6} = \left(\frac{5\pi+6}{30}\right)$ ✓

(b) In rt $\Delta BOP$; $BP = 10\cdot\sin\left(\frac{5\pi+6}{30}\right)$

$= 6.6208$ ✓

and $OP = 10\cdot\cos\left(\frac{5\pi+6}{30}\right) = 7.494$

Area of $\Delta OBP = \frac{1}{2}\times BP\times OP$

$= \frac{1}{2}\times 6.6208\times 7.494 = 24.809$ -- ①

Area of Sector $BOC = \frac{1}{2}r^2\theta$

$= \frac{1}{2}\times 10^2\times\left(\frac{5\pi+6}{30}\right) = 36.1799$ --- ②

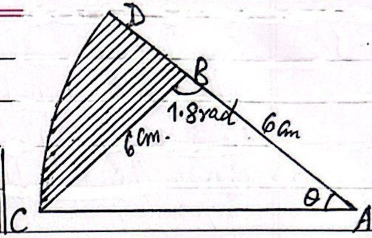
$\therefore$ Area of the shaded region BOC $= 36.1799 - 24.809$

$= 11.4$ ✓ [from ① and ②]

14. The diagram shows triangle ABC with AB = BC = 6 cm. and angle ABC = 1·8 radians. The arc CD is part of a circle with centre A and ABD is a straight line.

(a) Find the perimeter of the shaded region. ---[5]

(b) Find the area of the shaded region. ---[5]




S-22 | 13 | Q9

Solution (a) In $\triangle ABC$, using Cosine rule;

$AC^2 = 6^2 + 6^2 - 2\times6\times6\cos1{\cdot}8 = 88{\cdot}3584$

$r = AC = 9{\cdot}40$ ✓

angle $CAB = \theta = \frac{1}{2}(\pi - 1{\cdot}8) = 0{\cdot}6708$ ✓

∴ length of arc $CD = r\theta = 9{\cdot}4\times0{\cdot}6708$
$= 6{\cdot}306$ ✓

$BD = AD - AB = AC - AB = 9{\cdot}4 - 6 = 3{\cdot}4$

$BC = 6$ ✓

∴ Perimeter of the shaded region BCD

$= \overset{\frown}{CD} + BD + BC = 6{\cdot}306 + 3{\cdot}4 + 6 = \underline{15{\cdot}7}$ ✓

(b) Area of $\triangle ABC = \frac{1}{2}\times6^2\times\sin1{\cdot}8$
$= 17{\cdot}53$ --- ①

Area of Sector $ACD = \frac{1}{2}r^2\theta$
$= \frac{1}{2}\times AC^2\times$ angle $CAB$
$= \frac{1}{2}\times9{\cdot}4^2\times0{\cdot}6708 = 29{\cdot}64$ --- ②

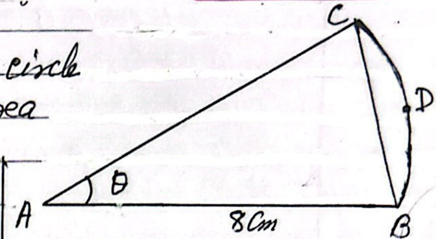
from ① and ②

Shaded area:

ar Sector ACD − ar $\triangle ABC$

$= 29{\cdot}64 - 17{\cdot}53 = \underline{12{\cdot}1}$ ✓

15. The diagram shows a sector ABC of a circle with centre A and radius 8 cm. The area of the sector is $\frac{16}{3}\pi\ cm^2$. The point D lies on the arc BC. Find the perimeter of the segment BCD. ...[4]




S-23 | 11 | Q4

Solution: Area of the sector ABC $= \frac{1}{2} r^2 \theta = \frac{16}{3}\pi$

$\Rightarrow \frac{1}{2} \times 8^2 \times \theta = \frac{16}{3}\pi \Rightarrow \theta = \frac{\pi}{6}$ ✓

Length of arc BCD $= r\theta = 8 \times \frac{\pi}{6} = 4.188$ Cm.

Chord BC $= 2 \cdot r \sin\frac{\theta}{2} = 2 \times 8 \sin\left(\frac{1}{2} \times \frac{\pi}{6}\right)$

$= 16 \sin\frac{\pi}{12} = 4.141$ cm

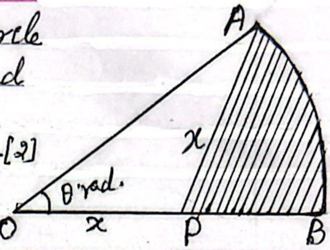
∴ Perimeter of segment BCD = Chord BC + arc BCD = 4.188 + 4.144

= 8.33 cm.

16 The diagram shows a sector OAB of a circle with centre O. Angle AOB = θ radians and OP = AP = $x$.

(a) Show that the arc length AB is $2x\theta\cos\theta$ ... [2]

(b) Find the area of the shaded region APB in terms of $x$ and $\theta$. --- [4]




S-23 | 12 | Q6

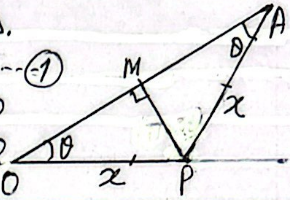
Solution: Draw PM ⊥ OA, ΔOPA is isosceles Δ.

(a) ⇒ M is the mid point of OA; OA = 2OM --(1)

In rt Δ OPM, $\frac{OM}{OP} = \cos\theta \Rightarrow OM = x\cos\theta$ --(2)

from (1) & (2) $r = OA = 2x\cos\theta$ ✓ --(3)

Hence arc length $AB = r\cdot\theta = 2x\cdot\theta\cos\theta$ ✓




(b) Shaded area = area of sector OAB − area ΔOAP ---(4)

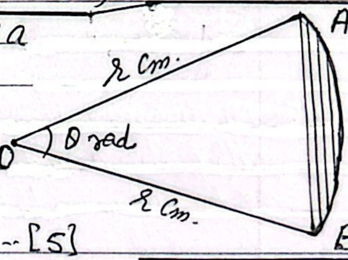
Area of sector OAB $= \frac{1}{2}r^2\theta = \frac{1}{2}\times(2x\cos\theta)^2\times\theta = 2x^2\theta\cos^2\theta$ --(5)

Area of Δ OPA $= \frac{1}{2}OA\cdot PM = \frac{1}{2}\times 2x\cos\theta\times x\sin\theta$
$= x^2\sin\theta\cdot\cos\theta$ --- (6)

ΔOPM $\left[\frac{PM}{OP} = \sin\theta\right.$

from (4) Area of shaded region $= 2x^2\theta\cdot\cos^2\theta - x^2\sin\theta\cos\theta$ [from (5) & (6)]
$= x^2\cos\theta(2\theta\cos\theta - \sin\theta)$ ✓

17 The diagram shows a sector OAB of a circle with centre O and radius r cm. angle AOB = θ radians. It is given that the length of arc AB = 9.6 cm and that area of the sector OAB = 76.8 cm².




(a) Find the area of the shaded region. ---- [5]

(b) Find the perimeter of the shaded region. ---[2]

S-23 | 13 | Q 6

Solution: (a) Area of Sector OAB; $\frac{1}{2}r^2\theta = 76.8$ --(1)

Length of arc $AB = r\theta = 9.6$ --(2)

from (1) & (2) $\frac{\frac{1}{2}r^2\theta}{r\theta} = \frac{76.8}{9.6} \Rightarrow r = 16$ ✓ --(3)

from (2) & (3) $16\times\theta = 9.6 \Rightarrow \theta = 0.6$ rad. --(4)

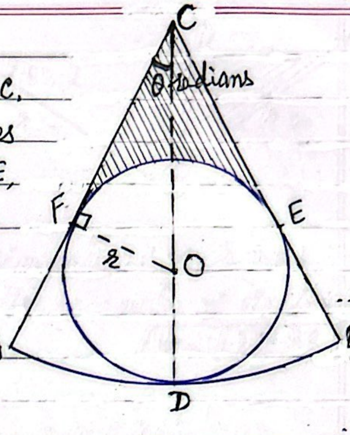
Area of ΔAOB $= \frac{1}{2}r^2\sin\theta = \frac{1}{2}\times16^2\times\sin 0.6$
$= 72.27$ --- (5)

Area of Shaded region
= area of Sector − ar ΔOAB
$= 76.8 - 72.27 = \underline{4.53}$ cm² ✓

(b) Length Chord $AB = 2r\sin\theta/2$
$= 2\times16\times\sin\frac{0.6}{2} = 9.46$ --- (6)

∴ Perimeter of Shaded region
= Chord AB + arc AB
$= 9.46 + 9.6 = \underline{19.1}$ ✓
$= 19.06$

18 The diagram shows a sector CAB which is a part of a circle with centre C. A circle with centre O and radius r lies within the sector and touches it at D, E, and F, where COD is a straight line and angle ACD is $\theta$ radians.




(a) Find CD in terms of r and $\sin\theta$. ...[3]

It is now given that $r=4$ & $\theta=\frac{\pi}{6}$;

(b) Find the perimeter of sector CAB in term of $\pi$ ...[3]

(c) Find the area of the shaded region in terms of $\pi$ and $\sqrt{3}$. ...[4]

| W-20 | 11 | Q 10 |
|---|---|---|

Solution (a) $CD = OC + OD = OC + r$ —— ①

In rt $\Delta OFC$, $\frac{OF}{OC} = \sin\theta \Rightarrow \frac{r}{OC} = \sin\theta \Rightarrow OC = \frac{r}{\sin\theta}$

$\therefore$ from ① $CD = r + \frac{r}{\sin\theta}$ —— ②

(b) radius 'R' of arc AB, $R = CD = 4 + \frac{4}{\sin\frac{\pi}{6}}$ [$\because$ from ② and $r=4$, $\theta=\frac{\pi}{6}$

$R = 12$ ✓

$\therefore$ length of arc $\widehat{AB} = R \times 2\theta = 12 \times 2 \times \frac{\pi}{6} = 4\pi$

$\therefore$ Perimeter of sector CAB $= CA + CB + \widehat{AB} = R + R + 4\pi$

$= 2R + 4\pi$

$= 2\times 12 + 4\pi$

$= 24 + 4\pi$ ✓

(c) area of $\Delta FOC = \frac{1}{2} \times OF \times FC = \frac{1}{2} \times r \times OC \cdot \cos\theta$

$= \frac{1}{2} \times 4 \times \frac{4}{\sin\frac{\pi}{6}} \times \cos\frac{\pi}{6}$

$= 2 \times 4 \times 2 \times \frac{\sqrt{3}}{2} = 8\sqrt{3}$

$\therefore$ area of $\Delta FOC = 8\sqrt{3}$ —— ③

area of sector OEF $= \frac{1}{2} r^2 \cdot \phi$ [Angle $FOE = 2\times\left(\frac{\pi}{2} - \theta\right)$

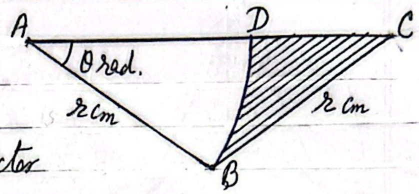
$= \frac{1}{2} \times 4^2 \times \frac{2\pi}{3}$ $= 2\times\left(\frac{\pi}{2} - \frac{\pi}{6}\right)$

$= \frac{16\pi}{3}$ —— ④ $\phi = 2 \times \frac{\pi}{3}$

for ④ ③ Area of Shaded region $= 2\times 8\sqrt{3} - \frac{16\pi}{3}$

$= \left(16\sqrt{3} - \frac{16\pi}{3}\right)$ ✓

19. ABC is an isosceles triangle, with $AB = BC = r$ cm and angle $BAC = \theta$ radians. The point D lies on AC and ABD is a sector of a circle with centre A.




(a) Express the area of the shaded region in terms of $r$ and $\theta$ ...[3]

(b) In case $r = 10$ and $\theta = 0.6$, find the perimeter of the shaded region.

W-20/12/Q8 ...[4]

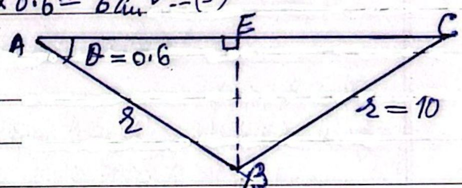
Solution (a) Area of shaded area = area $\Delta ABC$ − area of sector OBD. —— ①

Area of $\Delta ABC = \frac{1}{2} r^2 \sin(\pi - 2\theta) = \frac{1}{2} r^2 \sin 2\theta$ —— ②

Area of sector $OBD = \frac{1}{2} r^2 \theta$ —— ③

∴ Area of shaded region $= \frac{1}{2} r^2 \sin 2\theta - \frac{1}{2} r^2 \theta$ ✓ (from ①, ② & ③)

(b) length of arc $BD = r\theta = 10 \times 0.6 = 6$ cm ✓ --(3′)




Draw $AE \perp AC$

$AE = r \cos\theta$

∴ $AC = 2AE = 2r\cos\theta$

$= 2 \times 10 \times \cos 0.6 = 16.506$ —— ④

$DC = AC - AD = AC - r$

$= 16.506 - 10 = 6.506$ —— ⑤

∴ Perimeter of shaded area $= 10 + 6.506 + 6 = \underline{22.5 \text{ cm}}$ ✓

from (3′), ④ and ⑤. $(BC + DC + \overset{\frown}{BD})$

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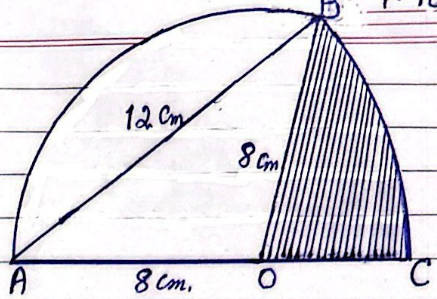
20. In the diagram, arc AB is part of a circle with centre O and radius 8 cm. Arc BC is part of a circle with centre A and radius 12 cm, where AOC is a straight line.



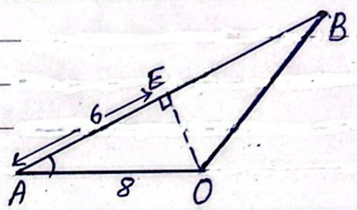

(a) Find angle BAO in radians. ---[2]

(b) Find the area of the shaded region. ---[4]

(c) Find the perimeter of the shaded region. ---[3]

W-20 | 13 | Q9

<u>Solution:</u> Draw OE ⊥ AB




(a) $\cos BAO = \frac{6}{8} = 0.75$

$\therefore$ angle $BAO = \cos^{-1} 0.75 = 0.7227$
$= 0.723$ rad.

(b) area sector $ABC = \frac{1}{2} r^2 \theta = \frac{1}{2} \times 12^2 \times 0.7227 = 52$ —— ①

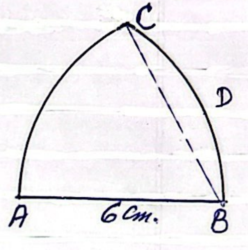
area $\Delta AOB = \frac{1}{2} \times AB \times EO$
$= \frac{1}{2} \times 12 \times 8 \sin(0.7227) = 31.7$ —— ②

$\therefore$ area of the shaded region $= 52 - 31.7$ (from ① & ②)
$= \underline{20.3 \text{ cm}^2}$ ✓

(c) Arc $AB = r\theta = 12 \times 0.7227 = 8.67$
$OC = 12 - 8 = 4$

$\therefore$ Perimeter of the shaded region $= 8 + 4 + 8.67$
$= \underline{20.7 \text{ cm}}$ ✓

21. The diagram shows a metal plate ABC, in which the sides are the straight line AB and arcs AC and BC. The line AB has length 6cm. The arc AC is a part of circle with centre B and radius 6 cm, and the arc BC is a part of a circle with centre A and radius 6cm.




(a) Find the perimeter of the plate, giving your answer in terms of $\pi$. ---[3]

(b) Find the area of the plate, giving answer in terms of $\pi$ and $\sqrt{3}$. --[4]

| W-21 | 11 | Q6 |

Solution (a) $AB = BC = AC = 6\,cm \rightarrow$ angle $C\hat{A}B = \frac{\pi}{3}$

Length of arc $AC = BC = r\theta = 6 \times \frac{\pi}{3} = 2\pi$ ✓

$\therefore$ Perimeter = arc AC + arc BC + AB

$= 2\pi + 2\pi + 6$

$= \underline{(6+4\pi)\,cm}$.

(b) Area sector $ABC = \frac{1}{2} r^2 \theta = \frac{1}{2} \times 6^2 \times \frac{\pi}{3}$

$= 6\pi$ — ①

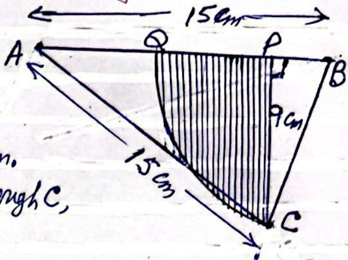
Area of segment BCD

= area sector CAB − ar $\Delta ABC$

$= 6\pi - \frac{1}{2} \times 6^2 \times \sin\frac{\pi}{3} = (6\pi - 9\sqrt{3})$

$\therefore$ Area of plate $= 6\pi + (6\pi - 9\sqrt{3})$

$= \underline{(12\pi - 9\sqrt{3})}$ ✓




22. In the diagram the lengths AB and AC are both 15 cm. The point P is the foot of perpendicular from C to AB. The length CP = 9cm. An arc of a circle with centre B passes through C, and meets AB at Q.

(a) Show that angle ABC = 1.25 radians, correct to 3sf. --- [2]

(b) Calculate the area of the shaded region which is bounded by the arc CQ and lines CP and PQ.

W-21/12/Q7 --[4]

Solution: (a) In rt $\Delta$ APC, $\sin B\hat{A}C = \frac{9}{15} = 0.6$

angle $B\hat{A}C = \sin^{-1} 0.6 = 0.6435$ rad.

$\therefore$ In Iso $\Delta$ ABC, angle $ABC = \frac{(\pi - 0.6435)}{2}$

angle $ABC = \underline{1.25}$ radians

(b) In rt $\Delta$ ACP, using Pythagoras theo

$AP^2 + 9^2 = 15^2 \Rightarrow AP^2 = 225 - 81 = 144$

$AP = \sqrt{144} = 12$; $BP = 15 - 12 = 3$ ✓

Now In rt $\Delta$ CPB, $\frac{PC}{BC} = \sin A\hat{B}C$

$\Rightarrow \frac{9}{BC} = \sin 1.25 \Rightarrow BC = \frac{9}{\sin 1.25} = 9.4838$

Area of sector $QCB = \frac{1}{2} r^2 \theta = \frac{1}{2}(9.4838)^2 \times 1.25$

$= 56.21$ ✓ — 1

Area of $\Delta$ PBC $= \frac{1}{2} \cdot PB \times PC$

$= \frac{1}{2} \times 3 \times 9 = 13.5$ ✓ — 2

$\Rightarrow$ Area of shaded region

= Area of sector − area of $\Delta$ PBC

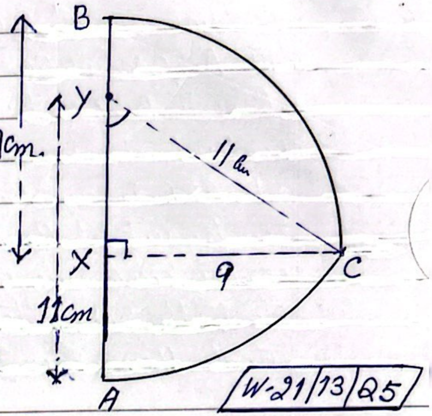
from 1 & 2

$= 56.21 - 13.5 = \underline{42.71}$ ✓

23. In the diagram, X and Y are points on the line AB, such that BX = 9 cm and AY = 11 cm. Arc BC is part of a circle with centre X and radius 9 cm, where CX is perpendicular to AB. Arc AC is part of a circle with centre Y and radius 11 cm.

(a) Show that angle XYC = 0.9582 rad. ---[1]

(b) Find the perimeter of ABC. ---[6]




W-21/13/Q5

Solution: (a) CX = BX = 9 cm and YC = YA = 11 cm.

In rt $\Delta CYX$, $\sin CYX = \frac{9}{11} \Rightarrow$ angle $XYC = \sin^{-1}\left(\frac{9}{11}\right) = 0.9582$ rad.

(b) In rt $\Delta CYX$, $XY = \sqrt{11^2 - 9^2} = \sqrt{40} =$

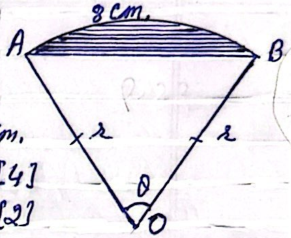
$AB = 9 + 11 - XY = 20 - 6.3245 = 13.6754$

Length of Arc $BC = r_1\theta = 9 \times \frac{\pi}{2} = 14.14$

Length of Arc $AC = r_2\theta = 11 \times 0.9582 = 10.54$

$\therefore$ Perimeter $= 13.675 + 14.14 + 10.54 = 38.3552 = 38.4$

24. The diagram shows a sector OAB of a circle with centre O. The length of the arc AB is 8cm. It is given that the perimeter of the sector is 20cm.




(a) Find the perimeter of the shaded segment. ---[4]

(b) Find the area of the shaded segment. ---[2]

W-22/11/Q5

**Solution:** (a) Perimeter of sector OAB = 20

$\Rightarrow 2r + 8 = 20$

$\Rightarrow r = 6$ ✓

Length of arc AB $= r\theta = 8$ (given)

$\longrightarrow 6 \times \text{angle } \angle AOB = 8$

$\angle AOB = 8/6 = \frac{4}{3}$ rad.

Using cosine rule

$AB = \sqrt{6^2 + 6^2 - 2\times 6\times 6 \cos 4/3} = \sqrt{55.062}$

$AB = 7.42$

$\therefore$ Perimeter of shaded region $= AB + \widehat{AB} = 7.42 + 8 = \underline{15.42\,cm}$ ✓

(b) Area of shaded segment

$= \text{Area of sector } OAB - \text{ar } \Delta OAB$

$= \frac{1}{2} \times r^2 \times \theta - \frac{1}{2} r^2 \sin\theta$

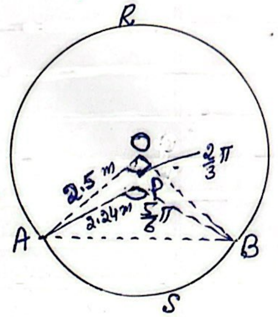
$= \frac{1}{2} r^2 (\theta - \sin\theta)$

$= \frac{1}{2} \times 6^2 \left(\frac{4}{3} - \sin 4/3\right)$

$= 18\left(\frac{4}{3} - 0.972\right)$

$= \underline{6.51\ cm^2}$

25. The diagram shows a cross-section RASB of the body of an aircraft. The cross-section consists of a sector OARB of a circle of radius 2.5m, with centre O, a sector PASB of another circle of radius 2.24 m with centre P and a quadrileteral OAPB. Angle AOB $= \frac{2}{3}\pi$ and angle APB $= \frac{5}{6}\pi$.




(a) Find the perimeter of cross-section RASB, giving your answer to 2dp --- [3]

(b) Find the difference in area of the two triangles AOB and APB. --[2]

(c) Find the area of cross-section RASB, giving the answer to 1d.p. ---[3]

W-22/12/Q10

**Solution:**

(a) Perimeter of crossection RASB

= length of arc ARB + length of arc ASB

$= 2.5\times(2\pi - \frac{2\pi}{3}) + 2.24\times\frac{5}{6}\pi$ $[l = r\theta]$

$= 10.472 + 5.864 = 10.336$

$= \underline{10.34\ m}$ (2 decimal places).

(b) Area of $\Delta AOB = \frac{1}{2}r^2\sin\theta =$

$= \frac{1}{2}(2.5)^2 \times \sin\frac{2\pi}{3}$

$= 2.706$ --- (1)

Area of $\Delta APB = \frac{1}{2}(2.24)^2 \times \sin\frac{5\pi}{6}$

$= 1.254$ -- (2)

from (1) and (2)

$\therefore$ Difference in areas $= 2.706 - 1.254$

$= 1.452$

$= \underline{1.45\ m^2}$ (2 d.p.)

= Area of quad OAPB $= 1.45\ m^2$ -- (3)

(C) Area of Cross-section RASB

= Area of sector OARB + Area of sector PASB

+ area of quad OAPB. --- (4)

Now ar sector RASB $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}\times 2.5^2\times(2\pi - \frac{2}{3}\pi) = 13.09$ -(5)

ar sector PASB $= \frac{1}{2}\times 2.24^2\times\frac{5\pi}{6}$

$= 6.57\ m^2$ -- (6)

Area of quad OAPB $= 1.45$ from (3) -- (7)

$\therefore$ from (4) and (5), (6), (7)

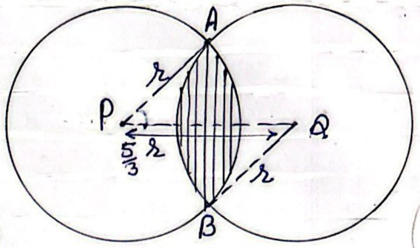
Required area

$= 13.09 + 6.57 + 1.45$

$= 21.11$

$= \underline{21.1\ m^2}$ (upto 1d.p)

26, The diagram shows two identical circles intersecting at points A and B; with centres at P and Q. The radius of each circle is $r$, and the distance PQ is $\frac{5}{3}r$.




(a) Find the perimeter of the shaded region in terms of $r$. ---[4]

(b) Find the area of the shaded region in terms of $r$. ---[3]

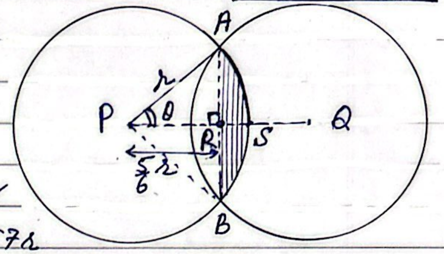
W-22/13/Q8

**Solution:** Join AB, let PQ and AB intersect

(a) at R; $AR \perp PQ$; $PR = \frac{1}{2} \times PQ$ $\left( = \frac{1}{2} \times \frac{5}{3}r = \frac{5}{6}r \right.$

$\cos APQ = \frac{PR}{PA} = \frac{\frac{5}{6}r}{r} = \frac{5}{6}$

$\therefore \theta = \text{angle } APQ = \cos^{-1}\frac{5}{6} = 0.5857$ ✓




length of arc AS $= r\theta = 0.5857r$

$\therefore$ perimeter of shaded region $= 4 \times \text{arc } AS$

$= 4 \times 0.5857r = \underline{2.34r}$ ✓

(b) Area of the shaded region $= 2 \times$ area of segment of circle ASBR.

$= 2[\text{ar sector } PASB - \text{ar } \Delta PAB)$ — ①

area of sector $PASB = \frac{1}{2}r^2 \times (2\theta) = r^2\theta = 0.5857r^2$ — ②

area of $\Delta PAB = \frac{1}{2}r^2 \sin 2\theta = \frac{1}{2}r^2 \times \sin(2 \times .5857)$

$= \frac{1}{2}r^2 \times 0.9213 = 0.4606r^2$ — ③

from ② & ③ in ①

The required shaded region $= 2[0.5857r^2 - 0.4606r^2]$

$= 2 \times 0.1251r^2$

$= \underline{0.250r^2}$ ✓