PURE MATHEMATICS -1

9709

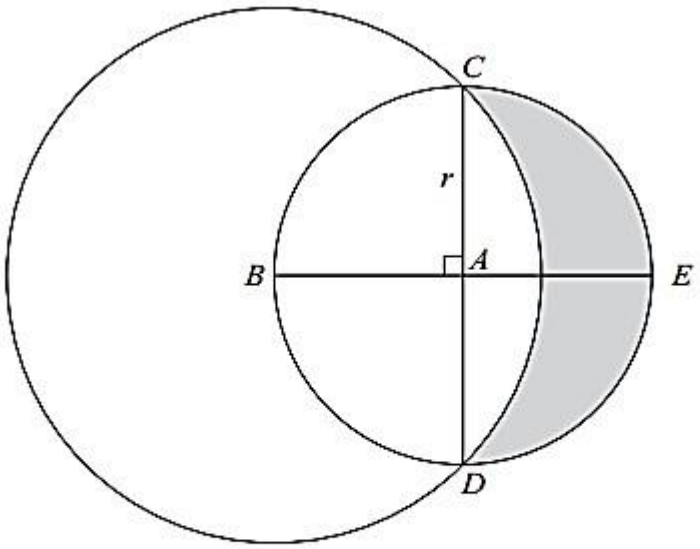
(March, June and November series 2020 – 2023 With marking scheme)

Circular Measure

EXERCISE -1

MANJULA BALAJI

1) SP-2020 _9709_1 Q9

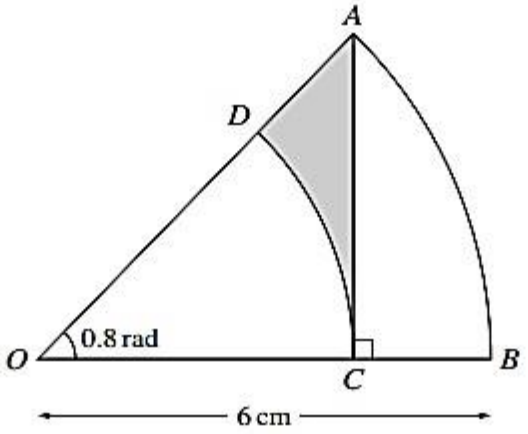


The diagram shows a circle with centre A and radius r. Diameters CAD and BAE are perpendicular to each other. A larger circle has centre B and passes through C and D.

(a) Show that the radius of the larger circle is $r\sqrt{2}$. [1]

(b) Find the area of the shaded region in terms of r. [6]

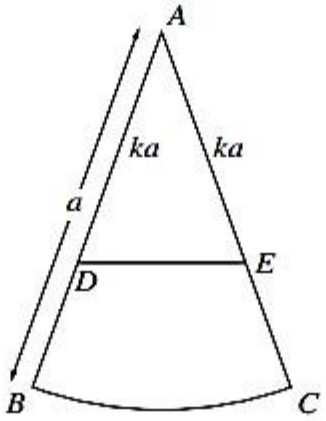
2) MARCH 2020_9709_12 Q7



The diagram shows a sector AOB which is part of a circle with centre O and radius 6 cm and with angle $AOB = 0.8$ radians. The point C on OB is such that AC is perpendicular to OB. The arc CD is part of a circle with centre O, where D lies on OA.

Find the area of the shaded region. [6]

3) MARCH 2021_9709_12 Q10



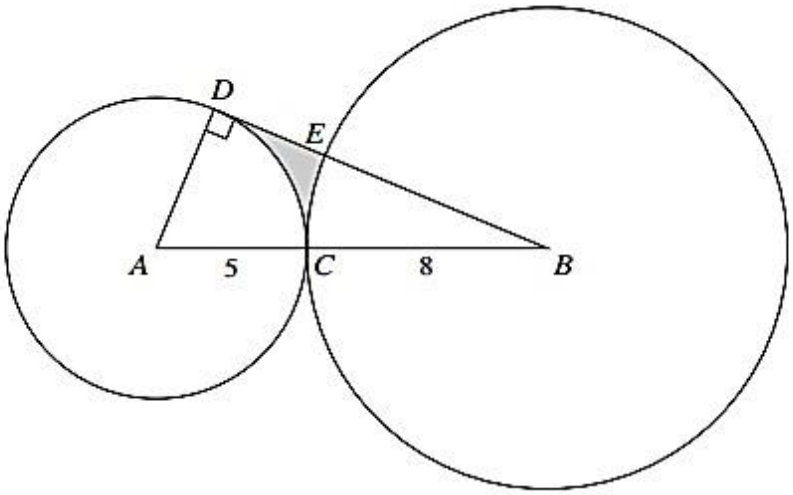
The diagram shows a sector ABC which is part of a circle of radius a. The points D and E lie on AB and AC respectively and are such that $AD = AE = ka$, where $k < 1$. The line DE divides the sector into two regions which are equal in area.

(a) For the case where angle $BAC = \frac{1}{6}\pi$ radians, find k correct to 4 significant figures. [5]

(b) For the general case in which angle $BAC = \theta$ radians, where $0 < \theta < \frac{1}{2}\pi$, it is given that $\dfrac{\theta}{\sin\theta} > 1$.

Find the set of possible values of k. [3]

4) MARCH 2022_9709_12 Q10

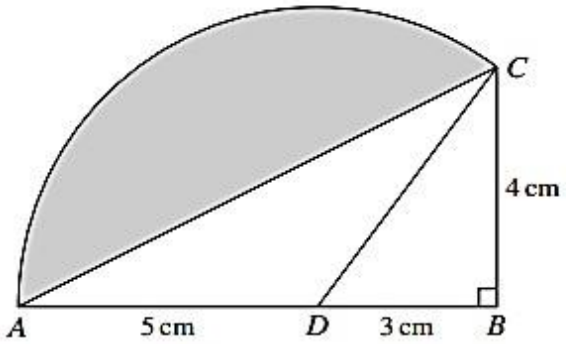


The diagram shows a circle with centre A of radius 5 cm and a circle with centre B of radius 8 cm. The circles touch at the point C so that ACB is a straight line. The tangent at the point D on the smaller circle intersects the larger circle at E and passes through B.

(a) Find the perimeter of the shaded region. [5]

(b) Find the area of the shaded region. [3]

5) MARCH 2023_9709_12 Q8

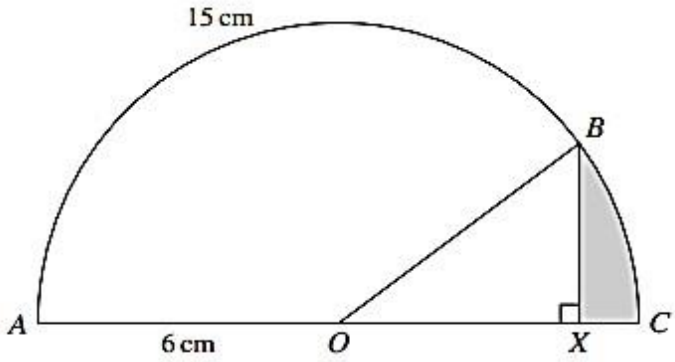


The diagram shows triangle ABC in which angle B is a right angle. The length of AB is 8 cm and the length of BC is 4 cm. The point D on AB is such that $AD = 5$ cm. The sector DAC is part of a circle with centre D.

(a) Find the perimeter of the shaded region. [5]

(b) Find the area of the shaded region. [3]

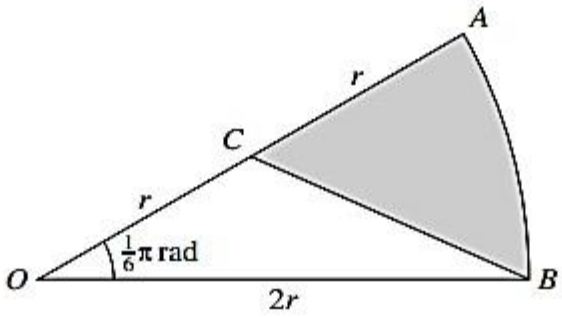
6) JUNE 2020_9709_11 Q8



In the diagram, ABC is a semicircle with diameter AC, centre O and radius 6 cm. The length of the arc AB is 15 cm. The point X lies on AC and BX is perpendicular to AX.

Find the perimeter of the shaded region BXC. [6]

7) JUNE 2020_9709_12 Q7



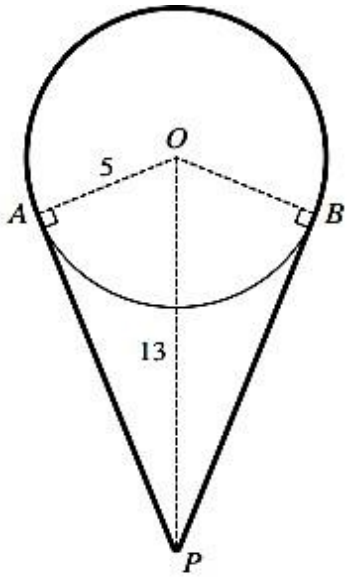
In the diagram, OAB is a sector of a circle with centre O and radius $2r$, and angle $AOB = \frac{1}{6}\pi$ radians. The point C is the midpoint of OA.

(a) Show that the exact length of BC is $r\sqrt{5-2\sqrt{3}}$. [2]

(b) Find the exact perimeter of the shaded region. [2]

(c) Find the exact area of the shaded region. [3]

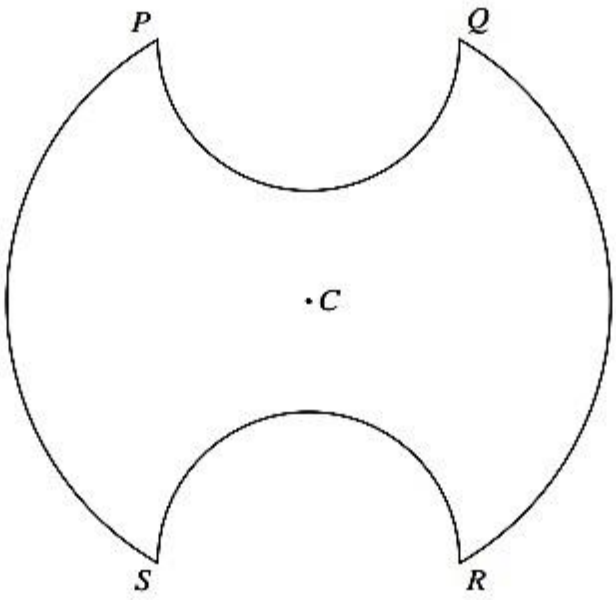
8) JUNE 2020_9709_13 Q5



The diagram shows a cord going around a pulley and a pin. The pulley is modelled as a circle with centre O and radius 5 cm. The thickness of the cord and the size of the pin P can be neglected. The pin is situated 13 cm vertically below O. Points A and B are on the circumference of the circle such that AP and BP are tangents to the circle. The cord passes over the major arc AB of the circle and under the pin such that the cord is taut.

Calculate the length of the cord. [6]

9) JUNE 2021_9709_11 Q8



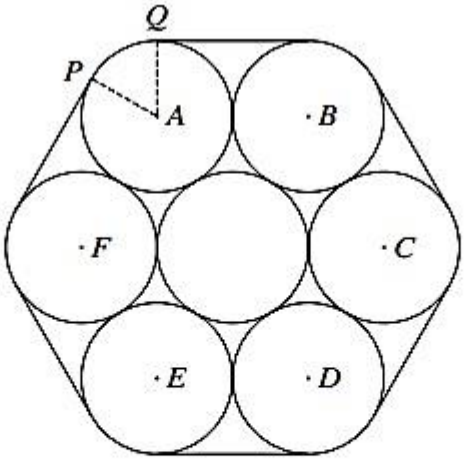
The diagram shows a symmetrical metal plate. The plate is made by removing two identical pieces from a circular disc with centre C. The boundary of the plate consists of two arcs PS and QR of the original circle and two semicircles with PQ and RS as diameters. The radius of the circle with centre C is 4 cm, and $PQ = RS = 4$ cm also.

(a) Show that angle $PCS = \frac{2}{3}\pi$ radians. [2]

(b) Find the exact perimeter of the plate. [3]

(c) Show that the area of the plate is $\left(\frac{20}{3}\pi + 8\sqrt{3}\right)$ cm^2. [5]

10) JUNE 2021_9709_12 Q12



The diagram shows a cross-section of seven cylindrical pipes, each of radius 20 cm, held together by a thin rope which is wrapped tightly around the pipes. The centres of the six outer pipes are A, B, C, D, E and F. Points P and Q are situated where straight sections of the rope meet the pipe with centre A.

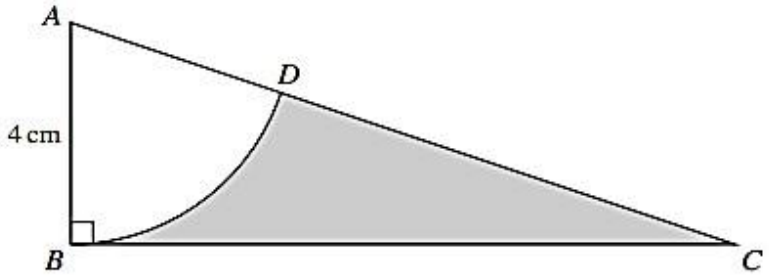
(a) Show that angle $PAQ = \frac{1}{3}\pi$ radians. [2]

(b) Find the length of the rope. [4]

(c) Find the area of the hexagon $ABCDEF$, giving your answer in terms of $\sqrt{3}$. [2]

(d) Find the area of the complete region enclosed by the rope. [3]

11) JUNE 2021_9709_13 Q5

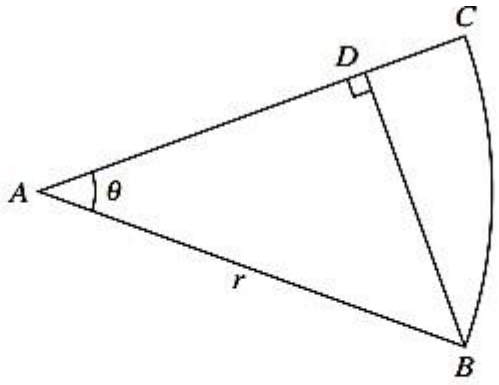


The diagram shows a triangle ABC, in which angle $ABC = 90°$ and $AB = 4$ cm. The sector ABD is part of a circle with centre A. The area of the sector is $10\,\text{cm}^2$.

(a) Find angle BAD in radians. [2]

(b) Find the perimeter of the shaded region. [4]

12) JUNE 2022_9709_11 Q5

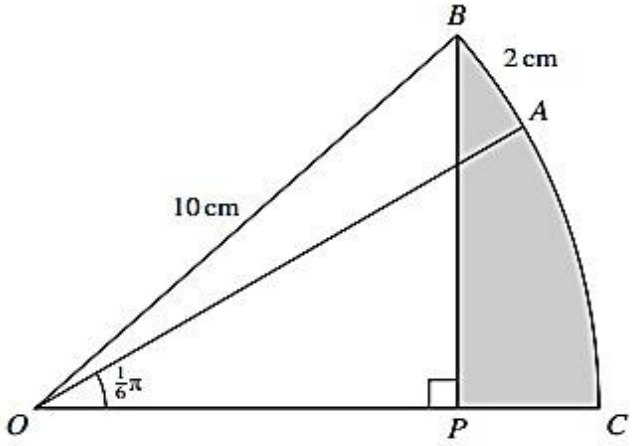


The diagram shows a sector ABC of a circle with centre A and radius r. The line BD is perpendicular to AC. Angle CAB is θ radians.

(a) Given that $\theta = \frac{1}{6}\pi$, find the exact area of BCD in terms of r. [3]

(b) Given instead that the length of BD is $\frac{\sqrt{3}}{2}r$, find the exact perimeter of BCD in terms of r. [4]

13) JUNE 2022_9709_12 Q7

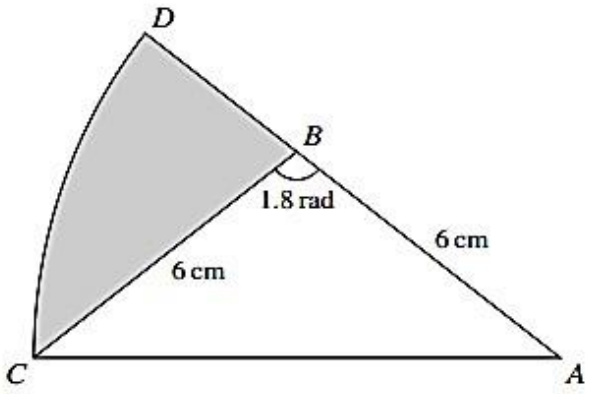


The diagram shows a sector $OBAC$ of a circle with centre O and radius 10 cm. The point P lies on OC and BP is perpendicular to OC. Angle $AOC = \frac{1}{6}\pi$ and the length of the arc AB is 2 cm.

(a) Find the angle BOC. [2]

(b) Hence find the area of the shaded region BPC giving your answer correct to 3 significant figures. [4]

14) JUNE 2022_9709_13 Q9

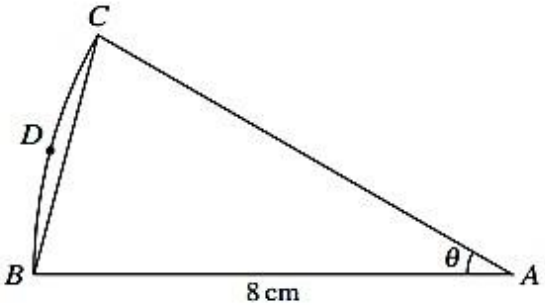


The diagram shows triangle ABC with $AB = BC = 6$ cm and angle $ABC = 1.8$ radians. The arc CD is part of a circle with centre A and ABD is a straight line.

(a) Find the perimeter of the shaded region. [5]

(b) Find the area of the shaded region. [3]

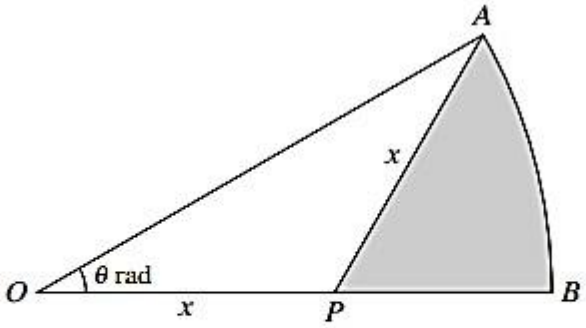
15) JUNE 2023_9709_11 Q4



The diagram shows a sector ABC of a circle with centre A and radius 8 cm. The area of the sector is $\frac{16}{3}\pi$ cm^2. The point D lies on the arc BC.

Find the perimeter of the segment BCD. [4]

16) JUNE 2023_9709_12 Q6

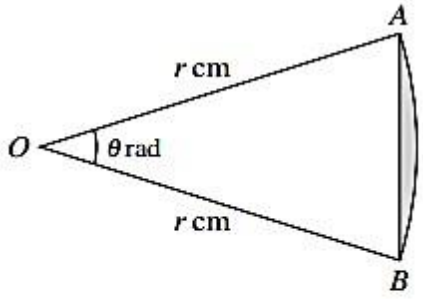


The diagram shows a sector OAB of a circle with centre O. Angle $AOB = \theta$ radians and $OP = AP = x$.

(a) Show that the arc length AB is $2x\theta \cos \theta$. [2]

(b) Find the area of the shaded region APB in terms of x and θ. [4]

17) JUNE 2023_9709_13 Q6

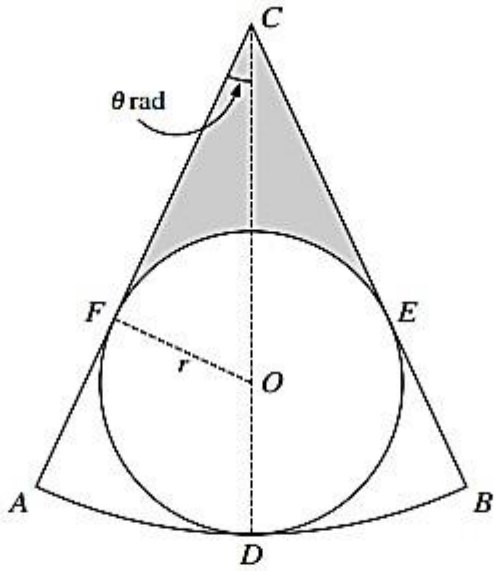


The diagram shows a sector OAB of a circle with centre O and radius r cm. Angle $AOB = \theta$ radians. It is given that the length of the arc AB is 9.6 cm and that the area of the sector OAB is 76.8 cm^2.

(a) Find the area of the shaded region. [5]

(b) Find the perimeter of the shaded region. [2]

18) OCT 2020_9709_11 Q10



The diagram shows a sector CAB which is part of a circle with centre C. A circle with centre O and radius r lies within the sector and touches it at D, E and F, where COD is a straight line and angle ACD is θ radians.

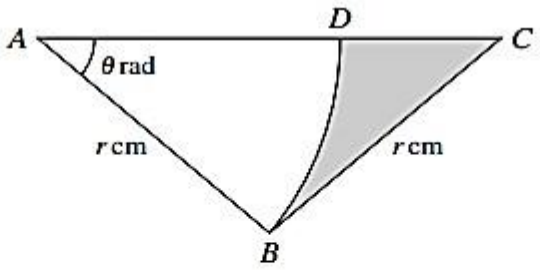
(a) Find CD in terms of r and $\sin\theta$. [3]

It is now given that $r = 4$ and $\theta = \frac{1}{6}\pi$.

(b) Find the perimeter of sector CAB in terms of π. [3]

19) OCT2020_9709_12 Q8

(c) Find the area of the shaded region in terms of π and $\sqrt{3}$. [4]

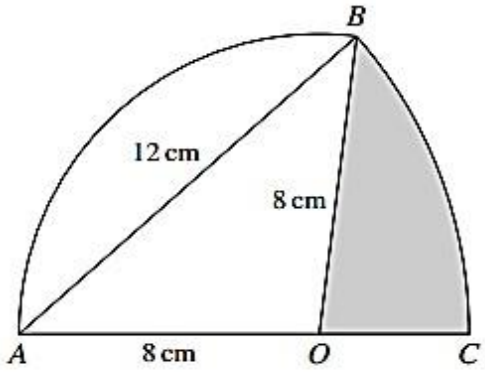


In the diagram, ABC is an isosceles triangle with $AB = BC = r$ cm and angle $BAC = \theta$ radians. The point D lies on AC and ABD is a sector of a circle with centre A.

(a) Express the area of the shaded region in terms of r and θ. [3]

(b) In the case where $r = 10$ and $\theta = 0.6$, find the perimeter of the shaded region. [4]

20) OCT 2020_9709_13 Q9



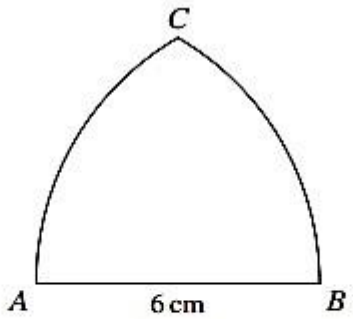
In the diagram, arc AB is part of a circle with centre O and radius 8 cm. Arc BC is part of a circle with centre A and radius 12 cm, where AOC is a straight line.

(a) Find angle BAO in radians. [2]

(b) Find the area of the shaded region. [4]

(c) Find the perimeter of the shaded region. [3]

21) OCT 2021_9709_11 Q6

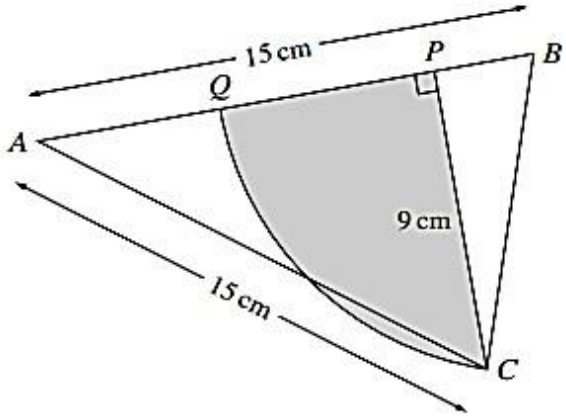


The diagram shows a metal plate ABC in which the sides are the straight line AB and the arcs AC and BC. The line AB has length 6 cm. The arc AC is part of a circle with centre B and radius 6 cm, and the arc BC is part of a circle with centre A and radius 6 cm.

(a) Find the perimeter of the plate, giving your answer in terms of π. [3]

(b) Find the area of the plate, giving your answer in terms of π and $\sqrt{3}$. [4]

22) OCT 2021_9709_12 Q7

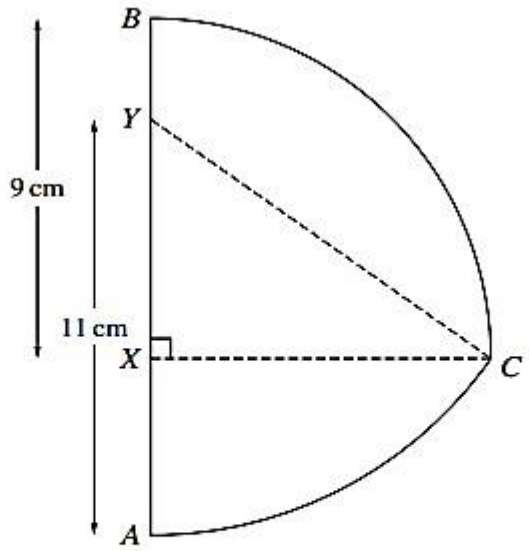


In the diagram the lengths of AB and AC are both 15 cm. The point P is the foot of the perpendicular from C to AB. The length $CP = 9$ cm. An arc of a circle with centre B passes through C and meets AB at Q.

(a) Show that angle $ABC = 1.25$ radians, correct to 3 significant figures. [2]

(b) Calculate the area of the shaded region which is bounded by the arc CQ and the lines CP and PQ. [4]

23) OCT 2021_9709_13 Q5

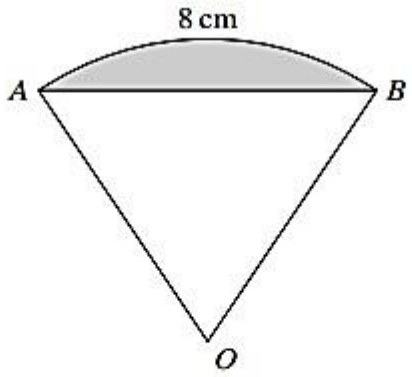


In the diagram, X and Y are points on the line AB such that $BX = 9$ cm and $AY = 11$ cm. Arc BC is part of a circle with centre X and radius 9 cm, where CX is perpendicular to AB. Arc AC is part of a circle with centre Y and radius 11 cm.

(a) Show that angle $XYC = 0.9582$ radians, correct to 4 significant figures. [1]

(b) Find the perimeter of ABC. [6]

24) OCT 2022_9709_11 Q5

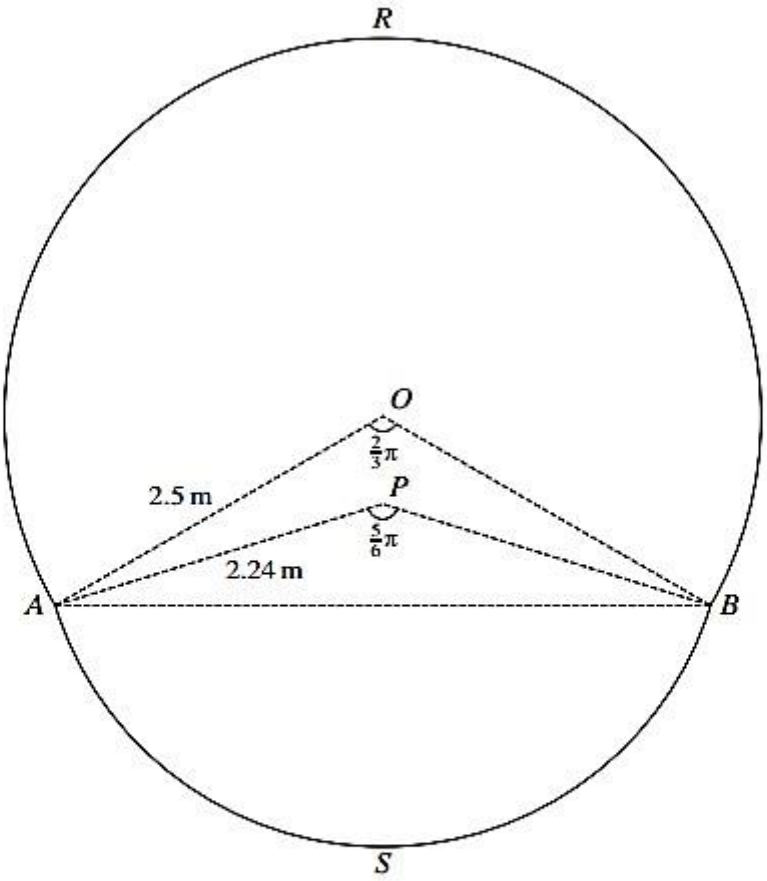


The diagram shows a sector OAB of a circle with centre O. The length of the arc AB is 8 cm. It is given that the perimeter of the sector is 20 cm.

(a) Find the perimeter of the shaded segment. [4]

(b) Find the area of the shaded segment. [2]

25) OCT 2022_9709_12 Q10



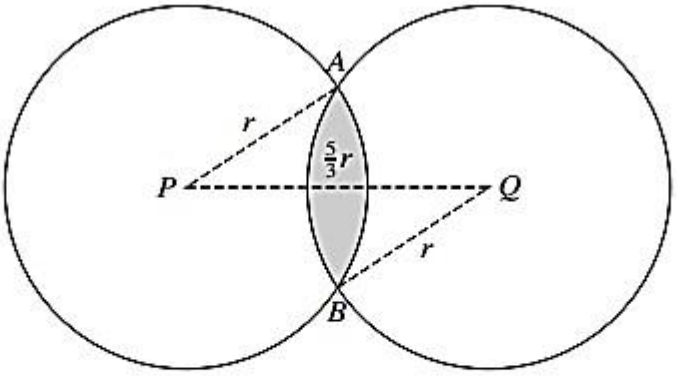
The diagram shows a cross-section $RASB$ of the body of an aircraft. The cross-section consists of a sector $OARB$ of a circle of radius 2.5 m, with centre O, a sector $PASB$ of another circle of radius 2.24 m with centre P and a quadrilateral $OAPB$. Angle $AOB = \frac{2}{3}\pi$ and angle $APB = \frac{5}{6}\pi$.

(a) Find the perimeter of the cross-section $RASB$, giving your answer correct to 2 decimal places. [3]

(b) Find the difference in area of the two triangles AOB and APB, giving your answer correct to 2 decimal places. [2]

(c) Find the area of the cross-section $RASB$, giving your answer correct to 1 decimal place. [3]

26) OCT 2022_9709_13 Q8



The diagram shows two identical circles intersecting at points A and B and with centres at P and Q. The radius of each circle is r and the distance PQ is $\frac{5}{3}r$.

(a) Find the perimeter of the shaded region in terms of r. [4]

(b) Find the area of the shaded region in terms of r. [3]

MARKING SCHEME

1) SP-2020 _9709_1 Q9

(a)	$BC^2 = r^2 + r^2 = 2r^2 \rightarrow BC = r\sqrt{2}$	**1**
(b)	Area sector $BCFD = \frac{1}{4}\pi\left(r\sqrt{2}\right)^2$ seen or implied	1
	Area $\Delta BCD = \frac{1}{2}(2r)r$	1
	Area segment $CFDA = \frac{1}{2}\pi r^2 - r^2$	1
	Area semi circle $CADE = \frac{1}{2}\pi r^2$	1
	Shaded area $= \frac{1}{2}\pi r^2 - (\frac{1}{2}\pi r^2 - r^2)$ **OR** $\pi r^2 - \left[\frac{1}{2}\pi r^2 + \left(\frac{1}{2}\pi r^2 - r^2\right)\right]$	1
	$= r^2$	1
		6

2) MARCH 2020_9709_12 Q7

$OC = 6\cos 0.8 = 4.18(0)$	**M1A1**
Area sector $OCD = \frac{1}{2}(their\,4.18)^2 \times 0.8$	***M1**
$\Delta OCA = \frac{1}{2} \times 6 \times their\,4.18 \times \sin 0.8$	**M1**
Required area = *their* ΔOCA – *their* sectorOCD	**DM1**
2.01	**A1**
	6

3) MARCH 2021_9709_12 Q10

(a)	$\Delta ADE = \frac{1}{2}(ka)^2 \sin\frac{\pi}{6}$	**M1**
	$\frac{1}{4}k^2a^2$	**A1**
	Sector $ABC = \frac{1}{2}a^2\frac{\pi}{6}$	**B1**
	$2 \times \frac{1}{4}k^2a^2 = \frac{1}{2}a^2\frac{\pi}{6}$	**M1**
	$k = \left(\sqrt{\frac{\pi}{6}}\right) = 0.7236$	**A1**
		5
(b)	$2 \times \frac{1}{2}(ka)^2 \sin\theta = \frac{1}{2}a^2\theta$	**M1**
	$k^2 = \frac{\theta}{2\sin\theta}$	**A1**
	$k^2 > \frac{1}{2}$ leading to $\frac{1}{\sqrt{2}} < k < 1$	**A1**

4) MARCH 2022_9709_12 Q10

a)	$\tan A = \frac{12}{5}$ or $\cos A = \frac{5}{13}$ or $\sin A = \frac{12}{13}$	M1
	$A = 1.176$ $B = 0.3948$	A1
	$DE = 4$	B1
	Arcs = $5 \times their 1.176$ *and* $8 \times their 0.3948$	M1
	[Perimeter = 5.880 + 3.158 + 4 =] 13.0	A1
		5
b)	Area of triangle = $\frac{1}{2} \times 5 \times their 12$ [= 30]	B1 FT
	Area of sectors = $\frac{1}{2} \times 5^2 \times their\ 1.176 + \frac{1}{2} \times 8^2 \times their\ 0.3948$	M1
	[Area = 30 – 14.70 –12.63 =] 2.67	A1

5) MARCH 2023_9709_12 Q8

(a)	$\tan BDC = \frac{4}{3}$ or $\sin BDC = \frac{4}{5}$ or $\cos BDC = \frac{3}{5}$ used to find ADC	M1
	$BDC = 0.927[3] \rightarrow ADC = \pi - 0.927[3]$ [= 2.214 to 2.215]	A1
	Arc $AC = 5 \times their 2.214$	M1
	$AC = \sqrt{8^2 + 4^2}$ *or* $2 \times 5 \times \sin 1.107$	M1
	[Perimeter = 11.07 + 8.94 =] 20.0	A1
		5
(b)	Sector $ACD = ½ \times 5^2 \times their 2.214$	M1
	Subtracting the area of $\Delta ADC = ½ \times 5 \times 4$ or $\frac{1}{2} 5^2 \sin their\ 2.214$ or $\frac{1}{2} \times 8 \times 4 - \frac{1}{2} \times 3 \times 4$	M1
	Shaded area = 27.7 – 10 = 17.7	A1

6) JUNE 2020_9709_11 Q8

Angle $AOB = 15 \div 6 = 2.5$ radians	B1
Angle $BOC = \pi - 2.5$ (FT on angle AOB)	B1FT
$BC = 6(\pi - 2.5)$ $(BC = 3.850)$	M1
$\sin(\pi - 2.5) = BX \div 6$ $(BX = 3.59)$	M1
Either $OX = 6\cos(\pi - 2.5)$ or Pythagoras $(OX = 4.807)$	M1
$XC = 6 - OX$ $(XC = 1.193) \rightarrow P = 8.63$	A1
	6

7) JUNE 2020_9709_12 Q7

(a)	$BC^2 = r^2 + 4r^2 - 2r.2r \times \cos\left(\frac{\pi}{6}\right) = 5r^2 - 2r^2\sqrt{3}$	M1
	$BC = r\sqrt{(5-2\sqrt{3})}$	A1
		2
(b)	Perimeter $= \frac{2\pi r}{6} + r + r\sqrt{(5-2\sqrt{3})}$	M1 A1
		2
(c)	Area = sector – triangle	
	Sector area $= \frac{1}{2}4r^2\frac{\pi}{6}$	M1
	Triangle area = ½ r. $2r \sin\frac{\pi}{6}$	M1
	Shaded area $= r^2\left(\frac{\pi}{3} - \frac{1}{2}\right)$	A1
		3

8) JUNE 2020_9709_13 Q5

$\cos POA = \frac{5}{13} \rightarrow POA = 1.17(6)$ Allow 67.4° or $\sin = \frac{12}{13}$ or $\tan = \frac{12}{5}$	M1 A1
Reflex $AOB = 2\pi - 2\times$*their* 1.17(6) OE in degrees or minor arc AB = 5×2×*their*1.17(6)	M1
Major arc = 5 × *their* 3.93(1) or 2π×5 - *their* 11.7(6)	M1
AP (or BP) = $\sqrt{13^2 - 5^2}$ = 12	B1
Cord length = 43.7	A1
	6

9) JUNE 2021_9709_11 Q8

(a)	**Either** Let midpoint of PQ be H: $\sin HCP = \frac{2}{4} \Rightarrow$ Angle $HCP = \frac{\pi}{6}$ **Or** $\sin PSQ = \frac{4}{8} \Rightarrow$ Angle $PSQ = \frac{\pi}{6}$ **Or** using cosine rule: angle $PCQ = \frac{\pi}{3}$ **Or** by inspection: triangle PCQ or PCT is equilateral so angle $PCQ = \frac{\pi}{3}$	M1
	Angle $PCS = \pi - \frac{\pi}{6} - \frac{\pi}{6} = \frac{2}{3}\pi$	A1
		2
(b)	Perimeter $= 2\times4\times\frac{2\pi}{3}$ or $8\pi - \frac{8\pi}{3}$	M1
	$+2\pi\times2$	M1
	$\frac{28\pi}{3}$	A1
		3

(c)	Area sector $CPQ = \frac{1}{2}\times 4^2 \times \frac{\pi}{3} = \frac{8\pi}{3}$	M1
	Area of segment of large circle beyond CPQ $= \frac{8\pi}{3} - \frac{1}{2}\times 4^2 \times \sin\left(\frac{\pi}{3}\right) = \frac{8\pi}{3} - 4\sqrt{3}$	M1
	Area of small semicircle = $\pi\times 2$ or area of small circle = $\pi\times 2^2$	M1
	Area of plate = Large circle – [2 ×] small semicircle – [2 ×] segment area	M1
	$\pi\times 4^2 - \pi\times 2^2 - 2\times\left(\frac{8\pi}{3} - 4\sqrt{3}\right) = \frac{20\pi}{3} + 8\sqrt{3}$	A1
	Alternative method for Question 8(c)	
	Area of sector $PCS = \frac{1}{2}\times 4^2 \times \frac{2\pi}{3} = \frac{16\pi}{3}$	M1
	Area of triangle $PCQ = \frac{1}{2}\times 4^2 \times \sin\frac{\pi}{3} = 4\sqrt{3}$	M1
	Area of small semicircle = $\pi\times 2$ or area of circle = $\pi\times 2^2$	M1
	Area of plate = [2 ×] large sector + [2 ×] triangle – [2 ×] small semicircle	M1
	$2\left(\frac{16\pi}{3}\right) + 2\left(4\sqrt{3}\right) - \pi\times 2^2 = \frac{20\pi}{3} + 8\sqrt{3}$	A1
		5

10) JUNE 2021_9709_12 Q12

(a)	[By symmetry] [6 × $P\hat{A}Q = 2\pi$], [$P\hat{A}Q$ =] $2\pi\div 6$,	M1
	Explaining that there are six sectors around the diagram that make up a complete circle.	A1
	Alternative method for Question 12(a)	
	Using area or circumference of circle centre $A \div 6$	M1
	Justification for dividing by 6 followed by comparison with the sector area or arc length.	A1
	Alternative method for Question 12(a)	
	Explain why ΔPAQ is an equilateral triangle	M1
	Using ΔPAQ is an equilateral triangle $\therefore P\hat{A}Q = \frac{\pi}{3}$	A1
	Alternative method for Question 12(a)	
	Using the internal angle of a regular hexagon = $\frac{2\pi}{3}$ Or $F\hat{A}O + O\hat{A}B = \frac{2\pi}{3}$, equilateral triangles	M1
	$P\hat{A}Q = 2\pi - \left(\frac{\pi}{2} + \frac{2\pi}{3} + \frac{\pi}{2}\right) = \frac{\pi}{3}$	A1

a)	**Alternative method for Question 12(a)**	
	$Sin\theta = \frac{20}{40}$, with θ clearly identified	M1
	$\theta = \frac{\pi}{6}, 2\theta = \frac{\pi}{3} = F\hat{A}O$ and by similar triangles $= P\hat{A}Q$	A1
		2
b)	Each straight section of rope has length 40 cm	B1
	Each curved section round each pipe has length $r\theta = 20\times\frac{\pi}{3}$	*M1
	Total length $= 6\times((their\ 40)+k\pi)$	DM1
	$240+40\pi$ or 366 (AWRT) (cm)	A1
		4

11) JUNE 2021_9709_13 Q5

(a)	$\frac{1}{2}\times 4^2 \times$ angle BAD $= 10$	M1
	Angle BAD $= 1.25$	A1
		2
(b)	Arc $BD = 4\times their 1.25$	M1
	$BC = 4\tan(their 1.25)$	M1
	$CD = \frac{4}{\cos(their 1.25)} - 4$ or $\sqrt{4^2+(their\ BC)^2} - 4$	M1
	Perimeter $= 5 + 12.0(4) + 8.69 = 25.7$ (cm)	A1
		4

12) JUNE 2022_9709_11 Q5

(a)	Sector area $= \frac{1}{2}r^2\left(\frac{\pi}{6}\right)\left[=\frac{\pi}{12}r^2\right]$	B1
	$BD = \sin\frac{\pi}{6}r\left[=\frac{1}{2}r\right]$ and $AD = \cos\frac{\pi}{6}r\left[=\frac{\sqrt{3}}{2}r\right]$ so triangle area $= \frac{1}{2}\left(\sin\frac{\pi}{6}r\right)\left(\cos\frac{\pi}{6}r\right)\left[=\frac{1}{2}\times\frac{1}{2}r\times\frac{\sqrt{3}}{2}r\right]$ **or** $\frac{1}{2}r\left(\cos\frac{\pi}{6}r\right)\left(\sin\frac{\pi}{6}\right)\left[=\frac{1}{2}r\times\frac{\sqrt{3}}{2}r\times\frac{1}{2}\right]$	B1
	Area of $BCD = \frac{1}{12}\pi r^2 - \frac{\sqrt{3}}{8}r^2$	B1
		3

(b)	Angle $BAC = \sin^{-1}\left(\frac{\frac{\sqrt{3}}{2}r}{r}\right)\left[=\frac{\pi}{3}\right]$	B1
	Length $AD = \cos\frac{\pi}{3}r\left[=\frac{1}{2}r\right]$ [so length $CD = \frac{1}{2}r$]	M1
	Length of arc $BC = r \times \frac{\pi}{3}$	M1
	Perimeter of $BCD = \frac{\sqrt{3}}{2}r + \frac{1}{2}r + \frac{\pi}{3}r$	A1

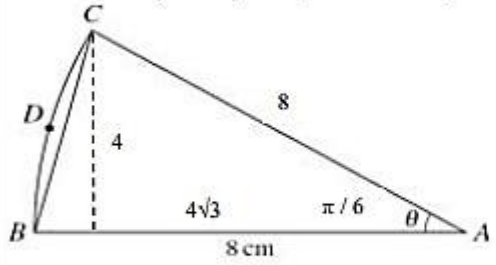
13) JUNE 2022_9709_12 Q7

(a)	$\left[A\hat{O}B=\right]\ \frac{2}{10}$	B1
	$[B\hat{O}C =]\frac{5\pi+6}{30}$ or $\frac{1}{6}\pi+0.2$	B1
	Alternative method for question 7(a)	
	OR [Arc AC =] $\frac{10\pi}{6}$ or [Arc BC =] $\frac{10\pi}{6}+2$ or 7.2	B1
	$[B\hat{O}C =]\frac{5\pi+6}{30}$ or $\frac{1}{6}\pi+0.2$	B1
(b)	$[BP] = 10\sin\left(\frac{5\pi+6}{30}\right)$ and $[OP] = 10\cos\left(\frac{5\pi+6}{30}\right)$ [= 6.6208…] and [= 7.494…] **OR** [BP] = $10\sin\left(\frac{5\pi+6}{30}\right)$ and $[O\hat{B}P] = \left(\frac{5\pi-3}{15}\right)$ [= 6.6208…] and [= 0.84719…]	M1
	Area of $\Delta OBP = \frac{1}{2}\times\ 10\sin\left(\frac{5\pi+6}{30}\right)\times 10\cos\left(\frac{5\pi+6}{30}\right)$ or $\frac{1}{2}\times 10\times 10\sin\left(\frac{5\pi+6}{30}\right)\times\sin\left(\left(\frac{5\pi-3}{15}\right)\right)$ [=24.809]	A1
	[Sector BOC] $= \frac{1}{2}\times 10^2 \times their\ \left(\frac{5\pi+6}{30}\right)$ $\left[=50\left(\frac{5\pi+6}{30}\right)=36.1799...\right]$	M1
	Area of region BPC = 11.4	A1

14) JUNE 2022_9709_13 Q9

(a)	$6\sin 0.9 = \frac{AC}{2}$ or $AC^2 = 6^2 + 6^2 - 2\times 6\times 6\cos 1.8$	M1
	$AC = 9.40$	A1
	Angle $CAB = ½(\pi - 1.8)$	M1
	Arc CD = *their* 9.40 × *their* 0.6708	M1
	[Perimeter = 6 + 3.40 + 6.306 =] 15.7	A1
(b)	Sector $ADC - \Delta ABC = ½ \times$ *their* $9.40^2 \times$ *their* $0.6708 - ½ \times 6^2 \times \sin 1.8$	M1 M1
	[29.64 – 17.53 =] 12.1	A1
		3

15) JUNE 2023_9709_11 Q4

$\frac{1}{2}\times 8^2 \times \theta = \frac{16\pi}{3} \Rightarrow \theta = \frac{\pi}{6}$	B1
Arc length = $8 \times$ *their* $\frac{\pi}{6}$ [= 4.1887…]	M1
[BC =] $2\times 8\sin\left(\frac{1}{2}\times \text{their}\frac{\pi}{6}\right)$ [= 4.1411…]	M1
Perimeter = 8.33	A1
Alternative methods for Question 4: 2nd M1 mark (use normal scheme for the other	
ALT 1 $BC^2 = 8^2 + 8^2 - 2\times 8\times 8\cos\left(\text{their}\frac{\pi}{6}\right)$ [$\Rightarrow BC = 4.14\ldots$] **ALT 2** $BC^2 = \left(8 - 4\sqrt{3}\right)^2 + 4^2$ [$\Rightarrow BC = 4.14\ldots$]  **ALT 3** $\frac{BC}{\sin\left(\frac{\pi}{6}\right)} = \frac{8}{\sin\left(\frac{5\pi}{12}\right)}$ [$\Rightarrow BC = 4.14\ldots$]	
	4

16) JUNE 2023_9709_12 Q6

(a)	$\frac{1}{2}OA = x\cos\theta$ or $\frac{OA}{\sin(\pi-2\theta)} = \frac{x}{\sin\theta}$ or $OA^2 = x^2 + x^2 - 2x^2\cos(\pi-2\theta)$ or $x^2 = r^2 + x^2 - 2rx\cos\theta$ or other valid method.	*B1
	$OA = 2x\cos\theta$ leading to Arc length $= 2x\theta\cos\theta$	DB1
		2
(b)	Sector area $= \frac{1}{2}(2x\cos\theta)^2 \times 0$	M1
	Triangle area $= \frac{1}{2} \times 2x\cos\theta x\sin\theta$ OR $\frac{1}{2}x^2\sin(\pi-2\theta)$	M1
	[Area APB =] *Their* sector area – *their* triangle area	M1
	[Area APB =] $\frac{1}{2}(2x\cos\theta)^2 \times \theta - \frac{1}{2}x^2\sin(\pi-2\theta)$ $[= x^2(2\theta\cos^2\theta - \frac{1}{2}\sin 2\theta)$ or $x^2\cos\theta(2\theta\cos\theta - \sin\theta)]$	A1
		4

17) JUNE 2023_9709_13 Q6

(a)	$\frac{½r^2\theta}{r\theta} = \frac{76.8}{9.6}$ or $\frac{1}{2}\left(\frac{9.6^2}{\theta^2}\right)\theta = 76.8$	M1
	$r = 16$	A1
	$\theta = 0.6$	A1
	ΔOAB = ½ × *their* 16^2 × sin *their* 0.6	M1
	[Area = 76.8 – 72.27 =] 4.53	A1
		5
(b)	$AB = 2\times16\times\sin 0.3$ OR $AB^2 = 16^2 + 16^2 - 2\times16^2\cos 0.6$	M1
	Perimeter = 9.6 + 9.46 = 19.1	A1
		2

18) OCT 2020_9709_11 Q10

)(a)	$\left(\sin\theta = \frac{r}{OC} \rightarrow\right) OC = \frac{r}{\sin\theta}$	M1 A1
	$CD = r + \frac{r}{\sin\theta}$	A1
		3
)(b)	Radius of arc $AB = 4 + \frac{4}{\sin\frac{\pi}{6}} = 4 + 8 = 12$	B1
	(Arc AB =) *their* $12 \times \frac{2\pi}{6}$ or $\left(\frac{1}{2}AB = \right)\left(\textit{their } 12 \times \frac{\pi}{6}\right)$	M1
	Perimeter = $24 + 4\pi$	A1
		3
c)	Area $FOC = \frac{1}{2} \times 4 \times \textit{their } OC \times \sin\frac{\pi}{3}$	M1
	$8\sqrt{3}$	A1
	Area sector $FOE = \frac{1}{2} \times \frac{2\pi}{3} \times 4^2 = \frac{16\pi}{3}$	B1
	Shaded area = $16\sqrt{3} - \frac{16\pi}{3}$	A1
	Alternative method for question 10(c)	
	$FC = \sqrt{(\textit{their } OC)^2 - 4^2}$	M1
	Area $FOC = \frac{1}{2} \times 4 \times 4\sqrt{3} = 8\sqrt{3}$	A1
	Area of half sector $FOE = \frac{1}{2} \times \frac{\pi}{3} \times 4^2 = \frac{8\pi}{3}$	B1
	Shaded area = $16\sqrt{3} - \frac{16\pi}{3}$	A1
		4

19) OCT 2020_9709_12 Q8

a)	Use of correct formula for the area of triangle ABC	M1
	$\frac{1}{2}r^2\sin(\pi-2\theta)$ **or** $\frac{1}{2}r^2\sin 2\theta$ **or** $2\times\frac{1}{2}r\times r\cos\theta\times\sin\theta$ **or** $2\times\frac{1}{2}r\cos\theta\times r\sin\theta$	A1
	[Shaded area = triangle – sector] = *their* triangle area $-\frac{1}{2}r^2\theta$	B1 FT
		3
b)	Arc $BD = r\theta = 6$ cm	B1
	$AC = 2r\cos\theta = (2\times10\cos0.6 = 20\cos0.6 = 16.506)$ or $\sqrt{\left(2r^2-2r^2\cos(\pi-2\theta)\right)}$ or $\frac{r\times\sin(\pi-2\theta)}{\sin\theta}$	*M1
	$DC = 2r\cos\theta - r$ or $\sqrt{\left(2r^2-2r^2\cos(\pi-2\theta)\right)} - r$ $(= 6.506)$	DM1
	(Perimeter = 10 + 6 + 6.506 =) 22.5	A1
		4

20) OCT 2020_9709_13 Q9

(a)	$\cos BAO = \frac{6}{8}$ or $\frac{8^2+12^2-8^2}{2\times8\times12}$	M1
	$BAO = 0.723$	A1
		2
(b)	Sector ABC = ½×12² × *their* 0.7227	*M1
	Triangle AOB = ½×8×12 sin(*their* 0.7227) or ½×12×√28	*M1
	Shaded area = *their* 52.0 – *their* 31.7 = 20.3	DM1 A1
		4
(c)	Arc BC = 12 × *their* 0.7227	*M1
	Perimeter = 8 + 4 + *their* 8.67 = 20.7	DM1 A1
		3

21) OCT 2021_9709_11 Q6

(a)	Recognise that at least one of angles A, B, C is $\frac{\pi}{3}$	B1
	One arc $6\times$ *their* $\frac{\pi}{3}$ leading to two arcs $2\times6\times$ *their* $\frac{\pi}{3}$	M1
	Perimeter = $6 + 4\pi$	A1
	Alternative method for question 6(a)	
	Calculate circumference of whole circle = 12π	B1
	One arc $\frac{1}{6}\times12\pi$ leading to two arcs $2\times\frac{1}{6}\times12\pi$	M1
	Perimeter = $6 + 4\pi$	A1
		3

b)	Sector $= \frac{1}{2}\times 6^2 \times their\left(\frac{\pi}{3}\right)$	M1
	$\frac{1}{2}\times(6^2)\times their\left(\frac{\pi}{3}\right)-\frac{1}{2}\times(6^2)\times\sin\left(their\left(\frac{\pi}{3}\right)\right)+6\pi\ \left[=6\pi-9\sqrt{3}+6\pi\right]$	M1 A1
	Area $= 12\pi - 9\sqrt{3}$	A1
	Alternative method for question 6(b)	
	Sector $= \frac{1}{2}\times 6^2 \times their\left(\frac{\pi}{3}\right)$	M1
	$2\times\left(\frac{1}{2}\times 6^2 \times their\left(\frac{\pi}{3}\right)\right)-\frac{1}{2}\times(6^2)\times\sin\left(their\left(\frac{\pi}{3}\right)\right)$	M1 A1
	Area $= 12\pi - 9\sqrt{3}$	A1
	Alternative method for question 6(b)	
	Sector $= \frac{1}{2}\times 6^2 \times their\left(\frac{\pi}{3}\right)$	M1
	$2\times\left(\frac{1}{2}\times(6^2)\times their\left(\frac{\pi}{3}\right)-\frac{1}{2}\times(6^2)\times\sin\left(their\left(\frac{\pi}{3}\right)\right)\right)+$ $\frac{1}{2}\times(6^2)\times\sin\left(their\left(\frac{\pi}{3}\right)\right)\ \left[=12\pi-18\sqrt{3}+9\sqrt{3}\right]$	M1 A1
	Area $\left[=6\pi-9\sqrt{3}+6\pi\right] = 12\pi - 9\sqrt{3}$	A1
		4

22) OCT 2021_9709_12 Q7

a)	**EITHER** By using trigonometry: $B\hat{A}C = 0.6435\ldots$ and $A\hat{B}C = \frac{\pi - 0.6435}{2}$ **OR** By Pythagoras: $AP = 12 \Rightarrow BP = 3$ so $\tan A\hat{B}C = \frac{9}{3}$ **OR** Using ΔPBC and either the sine or cosine rule $\sin A\hat{B}C = \frac{3}{\sqrt{10}}$ or $\cos A\hat{B}C = \frac{\sqrt{10}}{10}$	M1
	$A\hat{B}C = \frac{\pi - 0.6435}{2}$ or $\tan^{-1}\frac{9}{3}$ or $\sin^{-1}\frac{3}{\sqrt{10}}$ or $\cos^{-1}\frac{\sqrt{10}}{10}$ or $1.249(04\ldots)$ or $71.56°$ $= 1.25$ radians (3 sf)	A1
		2
b)	$BC = \sqrt{(their\,3)^2 + 9^2}$ or $\frac{9}{\sin 1.25}$ $[= \sqrt{90}\ , 3\sqrt{10}$ or $9.48697\ldots]$	M1
	Area of sector $= \frac{1}{2}\times(their\,BC)^2 \times \tan^{-1}3\ [=56.207\ or\ 56.25]$	M1
	Area of triangle PBC $= 13.4$ to 13.6 or $\frac{1}{2}\times 9\times 3$	B1
	[Area = (56.207 or 56.25) – *their* 13.5 =] 42.7 or 42.8	A1
		4

23) OCT 2021_9709_13 Q5

(a)	Angle $XYC = \sin^{-1}\left(\frac{9}{11}\right) = 0.9582$ or $\sin XYC = \frac{9}{11}$ leading to $XYC = 0.9582$	B1
		1
(b)	$XY = \sqrt{11^2 - 9^2} = \sqrt{40}$ or using 0.9582 and trigonometry	*M1 A1
	$AB = 9 + 11 - \text{their}XY$	B1 FT
	Arc $AC = 11 \times 0.9582$	M1
	Arc $BC = 9 \times \frac{\pi}{2}$	M1
	Perimeter = [13.6(8) + 10.5(4) +14.1(4) =] 38.4	A1

24) OCT 2022_9709_11 Q5

(a)	$a = 5$	B1
	$b = 2$	B1
	$c = 3$	B1
		3
(a)	$a = 5$	B1
	$b = 2$	B1
	$c = 3$	B1
		3
(b)(i)	3	B1
		1
(b)(ii)	2	B1
		1

25) OCT 2022_9709_12 Q10

(a)	[f'(x) =] $2x - \frac{k}{x^2}$	B1
	$f'(2) = 0 \left[2 \times 2 - \frac{k}{2^2} = 0\right] \Rightarrow k = \ldots$	M1
	$k = 16$	A1
		3
(b)	f'(2) = e.g. $2 + \frac{2k}{2^3}$	M1
	$\left[2 + \frac{2k}{2^3}\right] > 0 \Rightarrow$ minimum or $= 6 \Rightarrow$ minimum	A1 FT
		2
(c)	When $x = 2$, f(x) = 14	B1
	[Range is or y or f(x)] $\geqslant$ *their* f(2)	B1 FT
		2

26) OCT 2022_9709_13 Q8

(a)	$\int\left(\frac{5}{2}-x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)\mathrm{d}x$	M1
	$\left\{\frac{5}{2}x-\frac{2}{3}x^{\frac{3}{2}}\right\}\{-\}\left\{2x^{\frac{1}{2}}\right\}$	A1 A1 A1
	$\left(10-\frac{16}{3}-4\right)-\left(\frac{5}{8}-\frac{1}{12}-1\right)$	DM1
	$\frac{9}{8}$ or 1.125	A1
		6
(b)	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\right]-\frac{1}{2}x^{-\frac{3}{2}}$	B1
	When $x=1$, $m=-\frac{1}{2}$	M1
	[Equation of normal is] $y-1=2(x-1)$	M1
	[When $x=0$,] $p=-1$	A1
		4