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Pure Maths - 1

Coordinate Geometry.

Ex. 1, Solution (Revision)

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Coordinate Geometry - Revision

Example 1: The circle $x^2+y^2+4x-2y-20=0$ has centre C and passes through A and B.

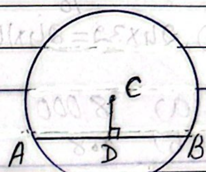
(a) State the coordinates of C. ---[1]

It is given that the mid point, D, of AB has coordinates $(1\frac{1}{2}, 1\frac{1}{2})$

(b) Find the equation of AB, giving your answer in the form $y=mx+c$ ---[4]

(c) Find, by calculation, the x-coordinates of A and B $\boxed{SP-20/01/Q10}$ ---[3]

Solution: Circle: $x^2+y^2+4x-2y-20=0$ —— ①



(a) $g=2,\ f=-1 \Rightarrow$ Centre $(-g,-f)$

$C(-2,1)$ ✓

(b) $D(1\frac{1}{2}, 1\frac{1}{2})$ or $D(\frac{3}{2},\frac{3}{2})$ is mid point of AB.

$(-2,1)$ C is the centre of circle and D is the mid point of chord AB

$\therefore CD \perp AB$, gradient of $CD = \frac{(3/2-1)}{(3/2+2)} = \frac{\frac{1}{2}}{7/2} = \frac{1}{7}$ ✓

$\therefore$ Gradient of $AB = -7$ ✓

Now AB passes through $D(3/2, 3/2)$ and Gradient $=-7$

$\therefore$ Equation of AB; $y-3/2 = -7(x-3/2)$

$\Rightarrow y = -7x+12$ —— ②

Solve ① and ② (To get the points of intersection A and B)

$x^2+(12-7x)^2+4x-2(12-x)-20=0$

$\Rightarrow 50(x^2-3x+2)=0$

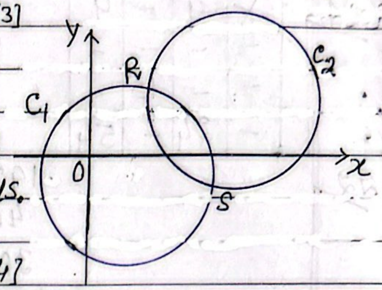
$(x-1)(x-2)=0 \Rightarrow x=1, 2$

$\therefore$ x-coordinates of A and B are $\rightarrow$ $\underline{1, 2}$ ✓

Example 2: A diameter of a circle C_1 has end points at $(-3,-5)$ and $(7,3)$

(a) Find an equation of the circle C_1. ---[3]

The circle C_1 is translated by $\begin{pmatrix}8\\4\end{pmatrix}$ to give circle C_2, as shown in the diagram.



(b) Find an equation of the circle C_2. ---[2]

The two circles intersect at points R and S.

(c) Show that the equation of the line RS is $y=-2x+13$ --- [4]

(d) Hence show that the x-coordinate of R and S satisfy the equation
$$5x^2-60x+159=0$$
[M-20 / 12 / Q 12] ---[2]

Solution: End points of the diameter of C_1 are $A(-3,-5)$ and $B(7,3)$.

(a) ∴ Centre of C_1 is the mid point of AB, $P(2,-1)$,

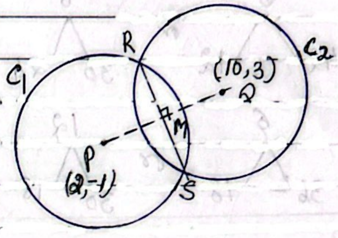
and $r^2 = CA^2 = [2-(-3)]^2+[-1-5]^2$

$\Rightarrow r^2 = 5^2+6^2 = 41$

∴ Equ^n of Circle C_1 is

$(x-2)^2+(y+1)^2=41$

or C_1: $x^2+y^2-4x+2y-36=0$ —— ①



(b) Centre of $C_2 = \begin{pmatrix}2\\-1\end{pmatrix}+\begin{pmatrix}8\\4\end{pmatrix}=\begin{pmatrix}10\\3\end{pmatrix}$

Centre of C_2, $Q(10,3)$, same $r^2=41$ as of ①

Equ^n of C_2; $(x-10)^2+(y-3)^2=41$

C_2: $x^2+y^2-20x-6y+68=0$ —— ②

(c) RS is the perp. bisector PQ the line joining the centres of the two circles. ⊛

Mid point of PQ is $M(6,1)$ ✓

gradient of $PQ = \frac{3+1}{10-2}=\frac{4}{8}=\frac{1}{2}$

∴ gradient of $RS=-2$

and passes through $M(6,1)$

Equ^n of RS: $y-1=-2(x-6)$

RS: or $y=-2x+13$ —— ③

(d) To get the x-coord. of R, S solving ② & ③

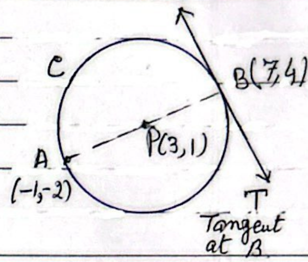
$x^2+(13-2x)^2-20x-6(13-2x)+68=0$

$\Rightarrow 5x^2-60x+159=0$ ✓

(⊛ The line joining the centres of two intersecting circles is perp. bisector of their common chord)

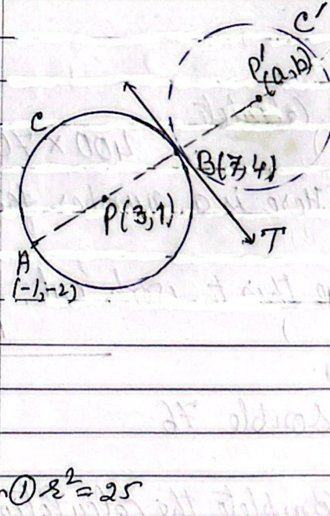
Example 3: The coordinates of the points A and B are $(-1,-2)$ and $(7,4)$ respectively.
(a) Find the equation of circle C, for which AB is a diameter. ---[4]
(b) Find the equation of the tangent, T, to the circle C at the point B. ---[4]
(c) Find the equation of the circle which is the reflection of circle C in the line T. [S-20/11/Q10] ---[3]

Solution: $A(-1,-2)$, $B(7,4)$



Mid point of AB is Centre of Circle –
Centre $P(3,1)$.
Radius r: $r^2 = PA^2 = (3-(-1))^2 + (1-(-2))^2$
$r^2 = 4^2 + 3^2 = 25$ ✓ —(1)
∴ Equ^n of circle $(x-a)^2 + (y-b)^2 = r^2$
C: $(x-3)^2 + (y-1)^2 = 25$

(b) Tangent $T \perp PB$ and passes through B.
gradient of $PB = \frac{4-1}{7-3} = 3/4$
∴ gradient of tangent $T = -4/3$
Point $(7,4)$
∴ Equ^n of tangent 'T'
$y-4 = -\frac{4}{3}(x-7)$
or $3y + 4x = 40$ ✓



(c) Let the centre of circle which is reflection of C in T, Centre $P'(a,b)$
B is mid point of PP'
$\Rightarrow \left(\frac{a+3}{2}, \frac{b+1}{2}\right) \equiv (7,4)$
$\Rightarrow \frac{a+3}{2} = 7$; $\frac{b+1}{2} = 4$
$a = 11$ & $b = 7$
∴ $P'(11,7)$, same radius from (1) $r^2 = 25$
∴ Equ^n of the reflected circle C',
C': $(x-11)^2 + (y-7)^2 = 25$ ✓

Example 4: The equation of a circle with centre C is $x^2+y^2-8x+4y-5=0$.

(a) Find the radius of the circle and the coordinate of C. --- [3]

The point P(1,2) lies on the circle.

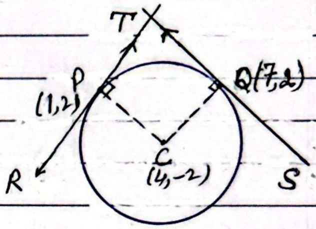
(b) Show that the equation of the tangent to the circle at P is $4y=3x+5$ --- [3]

The point Q also lies on the circle and PQ is parallel to the x-axis.

(c) Write down the coordinates of Q. -- [2]

The tangents to the circle at P and Q meet at T

(d) Find the coordinates of T. S-20/12/Q11 --- [3]



Solution: Circle: $x^2+y^2-8x+4y-5=0$ — ①

(a) $g=-\frac{8}{2}=-4,\ f=\frac{4}{2}=2,\ c=-5$

Centre $C(-g,-f)=C(4,-2)$ ✓

radius $r=\sqrt{g^2+f^2-c}=\sqrt{(-4)^2+2^2-(-5)}$
$=\sqrt{25}=5$

$\therefore r=5$ ✓

(b) Tangent at $PR \perp CP$ ✓

gradient of $CP=\frac{2-(-2)}{1-4}=-\frac{4}{3}$

$\therefore$ gradient of Tangent $=3/4$, Point P(1,2)

$\therefore$ Equation of the tangent PR,

$y-2=\frac{3}{4}(x-1)$ or $4y-3x=5$ — ②

(c) Q also lies on the circle

PQ ∥ x-axis

$\therefore$ y-coordinate Q = 2 (same as the y-coord of P (1,2))

Q lies on the circle for ① put $y=2$

$x^2+2^2-8x+4\times2-5=0$.

$x^2-8x+7=0$

$(x-1)(x+7)=0$

$x=7$ or $x=\cancel{1}$ for P

$\therefore Q(7,2)$ ✓

(d) Tangent at $Q(7,2)\rightarrow QS\perp CQ$

gradient of $CQ=\frac{4}{3}$

$\therefore$ gradient of tangent $QS=-\frac{3}{4}$

$\therefore$ Equⁿ of QS,

$y-2=-\frac{3}{4}(x-7)$

or $4y+3x=29$ — ③

Tangent PR; $4y-3x=5$ — ②

Tangent QS; $4y+3x=29$ — ③

Solving ② and ③ to get the point of intersection;

$T\left(4,\frac{17}{4}\right)$ ✓

Example 5.(a) The coordinates of two points A and B are $(-7,3)$ and $(5,11)$ respectively. Show that the equation of the perpendicular bisector of AB is $3x+2y=11$ ---[4]

(b) A circle passes through A and B and its centre lies on the line $12x-5y=70$. Find the equation of the circle. | S-20 | 13 | Q 10 | ---[5]

Solution: $A(-7,3)$, $B(5,11)$

(a) Gradient of $AB = \frac{11-3}{5+7} = \frac{8}{12} = \frac{2}{3}$

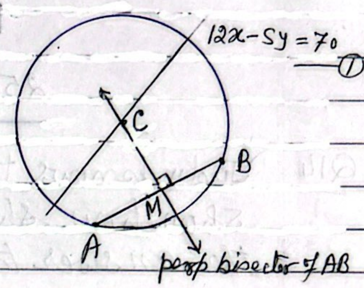
$\therefore$ Grad. of perp bisector $= -3/2$

Mid point $M(-1,7)$ of AB.

$\therefore$ Eqn of perp. bisector of AB.

$y-7 = -\frac{3}{2}(x+1)$

$\Rightarrow 3x+2y=11$ ✓ —①



(b) C, Centre lies on the line $12x-5y=70$ —②

Eqn of perp bisector of AB $3x+2y=11$ —①

① & ② both pass through Centre

{ The perp bisector of chord passes through Centre.

Solving ① & ② we get $C(5,-2)$ centre of circle.

Now radius of circle $r = CA = \sqrt{(5+7)^2+(-2-3)^2}$ $\{C(5,-2), A(-7,3)\}$

$= \sqrt{12^2+5^2}$

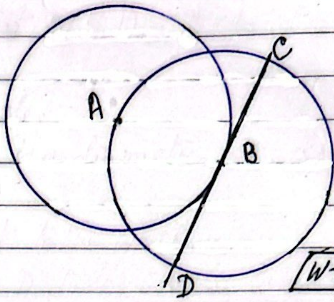
$r = 13$

$\therefore$ Eqn of circle with centre $(5,-2)$, $r=13$.

$(x-5)^2+(y+2)^2 = 13^2$ $[(x-a)^2+(y-b)^2=r^2]$ ✓

or $(x-5)^2+(y+2)^2 = 169$ ✓

6. The diagram shows a circle with centre A passing through the point B. A second circle has centre B and passes through A. The tangent at B to the first circle intersects the second circle at C and D.



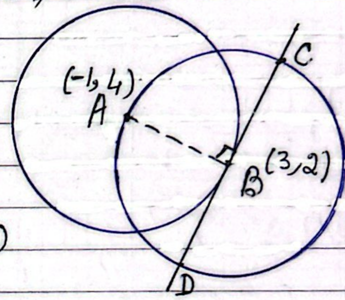
[W-20/11/Q9]

The coordinates of A are $(-1,4)$ and coordinate of B are $(3,2)$.

(a) Find the equation of the tangent CBD. --- [2]

(b) Find the equation of the circle with centre at B. --- [3]

(c) Find, by calculation, the x-coordinate of C and D. --- [3]



Solution (a) Gradient of AB $= \frac{4-2}{-1-3} = -\frac{1}{2}$

tangent CBD is perp. to radius AB

$\therefore$ Gradient of CBD $= 2$ ✓ $\left(-\frac{1}{m}\right)$

$\therefore$ Equation of tangent CBD, $B(3,2)$

$(y-2) = 2(x-3)$ —— ①

(b) Circle with centre $B(3,2)$

$r =$ radius $= AB = \sqrt{(3+1)^2+(2-4)^2} = \sqrt{16+4} = \sqrt{20}$

Equation of circle $(x-a)^2+(y-b)^2 = r^2$ [Centre (a,b)]

$\therefore$ Eqn of circle with centre B, $(x-3)^2+(y-2)^2 = 20$ —— ②

(c) for getting the x-coord. of C & D, Solving ① & ②

$(x-3)^2 + [2(x-3)]^2 = 20$ [from ① $(y-2) = 2(x-3)$]

$\Rightarrow x^2-6x+9+4x^2+36-24x = 20$

$\Rightarrow 5x^2-30x+25 = 0$

$\Rightarrow x^2-6x+5 = 0$

$(x-5)(x-1) = 0$

$\Rightarrow x = 5, 1.$

$\therefore$ x-coord of C & D are <u>1 and 5.</u> ✓

7 A circle has centre at the point $B(5,1)$. The point $A(-1,-2)$ lies on the circle.

(a) Find the equation of the circle. ---[3]

Point C is such that AC is a diameter of the circle.

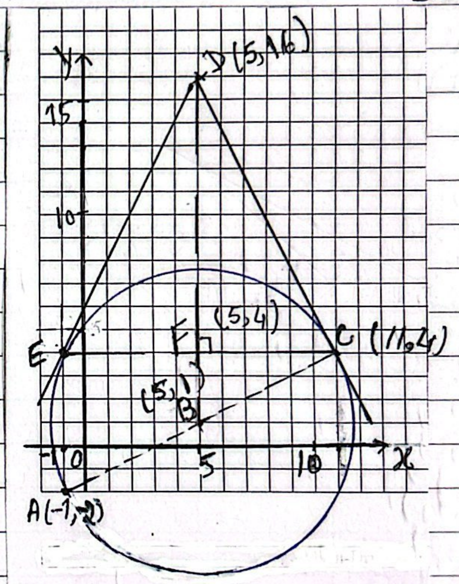
Point D has coordinates $(5,16)$.

(b) Show that DC is a tangent to the circle. ---[4]

The other tangent from D to the circle touches the circle at E.

[W-20 | 12 | Q9]

(c) Find the coordinates of E. ---[2]



Solution (a) radius of circle $r = AB$

$= \sqrt{(5+1)^2+(1+2)^2} = \sqrt{45}$

or $r^2 = 45$ ✓

Centre at $B(5,1)$

Equation of circle:

$(x-5)^2+(y-1)^2 = 45$

or $x^2+y^2-10x-2y=19$ —— ①

(b) Coordinate of $C(11,4)$

$\left\{\begin{array}{l} C(c_1,c_2) \\ \Rightarrow \dfrac{c_1+(-1)}{2} = 5 \\ \dfrac{c_2+(-2)}{2} = 1 \end{array}\right.$ ✓

$m_1 =$ Gradient of $BC = \dfrac{4-1}{11-5} = \dfrac{1}{2}$ ✓

$m_2 =$ Grad. of $CD = \dfrac{16-4}{5-11} = -2$

$m_1 \times m_2 = \frac{1}{2} \times (-2) = -1$

$\therefore$ CD is perp to BC $\Rightarrow$ CD is tangent to the circle at C.

(C) Equation of line BD is $x=5$ —— ②

$\left\{\begin{array}{l}\text{BD is perp. bisector of} \\ \text{EC.}\end{array}\right.$

EC ⊥ to BD → gradient of EC = 0

EC Passes through (11,4), Eqn of EC is $y=4$ —— ③

Solving ② & ③ coordinate of the intersection of EC and BD

$F(5,4)$. Now F is the mid point of CE

Let $E(x_1,y_1) \Rightarrow \dfrac{x_1+11}{2} = 5 \Rightarrow x_1 = -1$

$\dfrac{y_1+4}{2} = 4 \Rightarrow y_1 = 4$

$\therefore$ Coord of $E(-1,4)$ ✓

8. A circle with centre C has equation $(x-8)^2+(y-4)^2=100$.

(a) Show that $T(-6,6)$ is out side the circle. ---[3]

Two tangents from T to circle are drawn.

(b) Show that the angle between one of the tangents and CT is exactly $45°$. ---[2]

The two tangents touch the circle at A and B.

(c) Find the equation of line AB, giving your answer in the form $y=mx+c$ --[4]

(d) Find the x-coordinates of A and B. [W-20/13/Q 11] --[3]

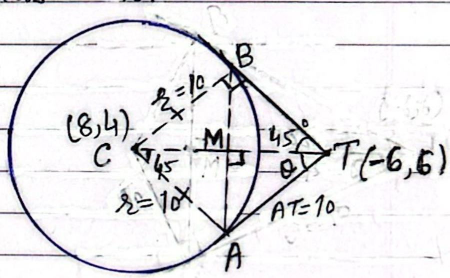
Solution (a) Circle: $(x-8)^2+(y-4)^2=100$ — ①

with centre $C(8,4)$ and radius $r=10$.

Distance $CT=\sqrt{(8+6)^2+(4-6)^2}$
$=\sqrt{196+4}$
$=\sqrt{200}=10\sqrt{2}$

$\therefore CT > r$ [$\because 10\sqrt{2}>10$]

$\therefore$ T lies outside the circle. ✓



(b) $\sin\theta=\dfrac{10}{10\sqrt{2}}=\dfrac{1}{\sqrt{2}} \Rightarrow \theta=45°$

$\therefore$ angle between the tangent AT and CT is $45°$. ✓

(c) Grad. of $CT=\dfrac{6-4}{-6-8}=\dfrac{2}{-14}=-\dfrac{1}{7} \Rightarrow$ Grad. of $AB=7$ ($\because AB\perp CT$)

$\because \Delta ACT$ is isosceles. $\therefore$ M is the mid point of CT

Now ACBT is a square. $M\left(\dfrac{8-6}{2},\dfrac{4+6}{2}\right)=M(1,5)$

$\therefore$ AB passes through $M(1,5)$ and grad. 7

$\therefore$ Equ^n of AB

$y-5=7(x-1)$

$\Rightarrow y=7x-2$ — ②

(d) Solving ① & ②

$(x-8)^2+(7x-2-4)^2=100$

$x^2-16x+64+49x^2-$ [illegible] $x+36=100$

$\Rightarrow 50x^2-100x=0$

$50x(x-2)=0$

$x=0$ and $x=2$ ✓

are the x-coord. of A and B.

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9. The points A(7,1), B(7,9) and C(1,9) are on the circumference of a circle.

(a) Find an equation of the circle. --[5]

(b) Find an equation of the tangent to the circle at B. --[2]

M-21 / 12 / Q8

Solution (a) Let the equation of the circle is:

$(x-a)^2+(x-y)^2=r^2$ --- ①

Point A(7,1) lies on the circle ①.

$\Rightarrow (7-a)^2+(1-b)^2=r^2$ -- ②

B(7,9) lies on ① $\Rightarrow (7-a)^2+(9-b)^2=r^2$ - ③

C(1,9) lies on ① $\Rightarrow (1-a)^2+(9-b)^2=r^2$ -- ④

Subtract ③ for ② $\Rightarrow (1-b)^2-(9-b)^2=0$

$\Rightarrow (1-b-9+b)(1-b+9-b)=0$

$\Rightarrow -8(10-2b)=0 \Rightarrow b=5$ ✓ — ⑤

Subract ④ for ③ $\Rightarrow (7-a)^2-(1-a)^2=0$

$\Rightarrow (7-a-1+a)(7-a+1-a)=0$

$6(8-2a)=0 \Rightarrow a=4$ ✓ -- ⑥

Put $a=4$ and $b=5$ in ②

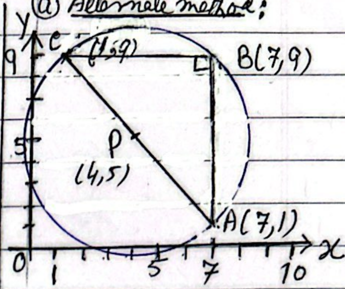
$(7-4)^2+(1-5)^2=r^2 \Rightarrow r^2=3^2+4^2=25$ - ⑦

Now $a=4$, $b=5$ and $r^2=25$, (or $r=5$)

∴ Eqn of circle ① $\rightarrow (x-4)^2+(y-5)^2=25$ ✓

<u>Centre at P(4,7) and rad $r=5$</u> ✓

ⓐ <u>Alternate method:</u>



AB ⊥ BC ⇒ angle ABC = 90°

for the circle through the points A, B and C,

AC is is diameter.

Mid point of AC, $P\left(\frac{1+7}{2}, \frac{9+1}{2}\right)$

P(4,5) is the Centre.

radius r = AP

$AP^2 = (7-4)^2+(1-5)^2$

$r^2 = 3^2+4^2=25$

∴ Eqn of circle.

$(x-4)^2+(y-5)^2=25$ ✓

(b) Let BT is the tangent to the circle, B(7,9)

⇒ BT is perp. to PB.

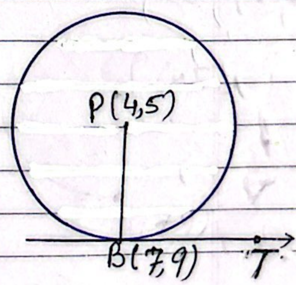
gradient of $PB = \frac{(9-5)}{(7-4)} = \frac{4}{3}$ ✓

∴ gradient of the tangent $= -\frac{3}{4}$

∴ Equation of tangent at B(7,9)

$y-9 = -\frac{3}{4}(x-7)$

$\Rightarrow$ <u>$3x+4y=57$</u> ✓



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10. The equation of a circle is $x^2+y^2-4x+6y-77=0$

(a) Find the x-coordinates of the points A and B where the circle intersects the x-axis. ---[2]

(b) Find the point of intersection of the tangents to the circle at A and B. ---[6]

$\boxed{S\text{-}21 \mid .11 \mid Q10}$

Solution: Circle: $x^2+y^2-4x+6y-77=0$ —①

(a) Circle ① intersects x-axis for $y=0$

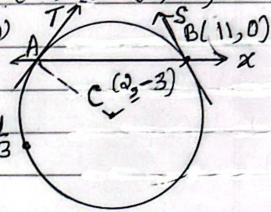
$\Rightarrow x^2-4x-77=0$

$(x+7)(x-11)=0$

$\Rightarrow x=-7,\ x=11.$

(b) for circle ① $g=-2, f=3$

Centre $(-g,-f)=(2,-3)$ C ✓



Grad of BC $=\frac{-3}{-9}=\frac{1}{3}$

Now gradient of AC $=\frac{-3-0}{2+7}=-\frac{1}{3}$ ✓

$\therefore$ Gradient of tangent AT $=3$; Gradient of tangent BS $=-3$

Equation of tangent AT, $y=3(x+7)$

$y=3x+21$ —②

and Eqn of tangent BS,

$y=-3(x-11)$

$y=-3x+33$ —③

Solving equations ② & ③ $x=2, y=27$

$\therefore$ The point of intersection of the tangents is $(2,27)$ ✓

11 The equation of a curve is $y=(x-3)\sqrt{x+1}+3$. The following points lie on the curve. Non-exact values are rounded to 4 decimal places.
$A(2,k)$, $B(2.9, 2.8025)$, $C(2.99, 2.9800)$, $D(2.999, 2.9980)$, $E(3,3)$

(a) Find k, giving your answer correct to 4 decimal places. ---[1]

(b) Find the gradient of AE, giving your answer to 4 decimal places. ---[1]

The gradients of BE, CE and DE, rounded to four decimal places, are 1.9748, 1.9975 and 1.9997 respectively.

(c) State, giving a reason for your answer, what values of the four gradients suggest about the gradient of the curve at the point E. ---[2]

[S-21/12/Q3]

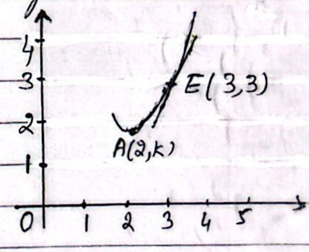
Solution: $y=(x-3)\sqrt{x+1}+3$ ----①

(a) $A(2,k)$ lies on ① $\Rightarrow k=(2-3)\sqrt{2+1}+3 \Rightarrow k=-\sqrt{3}+3 \Rightarrow k=\underline{1.2679}$ ✓

(b) Gradient of AE $=\dfrac{3-1.2679}{3-2}$ $\left\{\begin{array}{l}A(2, 1.2679)\\ E(3,3)\end{array}\right.$

$=\underline{1.7321}$ ✓

(C) Sight of 2 (2.0000) or two in reference to the gradient.
This is because the gradient at E is the limit of the gradients of the chords as the x-value tends to 3 or δx tends to 0.



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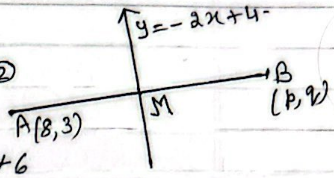
12 The points A and B have coordinates (8,3) and (p, q) respectively. The equation of the perpendicular bisector of AB is $y = -2x + 4$. Find the values of p and q. [S-21/12/Q6] ---[4]

Solution: A(8, 3), B(p, q); perp bisector AB $y = -2x + 4$ ① $\Rightarrow$ Gradient of perp bisector $= -2$

Gradient of Line AB $= +\frac{1}{2}$

Equation of line AB, passing through A(8,3)

$y - 3 = \frac{1}{2}(x-8) \Rightarrow y = \frac{1}{2}(x-8) + 3$ ②



Solve ① and ②

$-2x + 4 = \frac{1}{2}(x-8) + 3 \Rightarrow -4x + 8 = x - 8 + 6$

$\Rightarrow 5x = 10 \Rightarrow x = 2,\ y = 0$ (from ① put $x=2$)

$\therefore M(2,0) \equiv \left(\frac{p+8}{2}, \frac{q+3}{2}\right)$ (Mid point of AB)

$\Rightarrow \frac{p+8}{2} = 2 ;\ \frac{q+3}{2} = 0$

$\Rightarrow p = -4;\ q = -3$

13. The point A has coordinates (1, 5) and the line l has gradient $-\frac{2}{3}$ and passes through A. A circle has centre (5, 11) and radius $\sqrt{52}$.

(a) Show that l is the tangent to the circle at A. ---[2]

(b) Find the equation of the other circle of radius $\sqrt{52}$ for which l is also the tangent at A. [S-21/12/Q7] --[3]

Solution:

(a) Gradient of radius $CA = \frac{11-5}{5-1} = \frac{6}{4} = \frac{3}{2}$

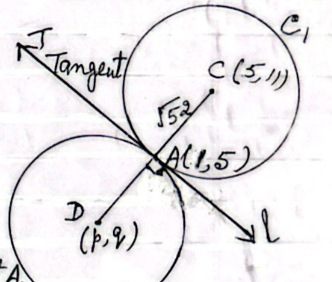
Gradient of line line $= -\frac{2}{3}$

Equation of circle C_1, $(x-5)^2 + (y-11)^2 = (\sqrt{52})^2$ ①

Put A(1,5) in ① $(1-5)^2 + (5-11)^2 = 52$

$16 + 36 = 52$ True.

$\therefore$ A lies on the circle, and grad of $l \times$ grad CA $= -\frac{2}{3} \times \frac{3}{2} = -1$



$\therefore l \perp$ rad CA $\Rightarrow$ l is tangent to the circle at A.

(b) Centre D(p, q) of the other circle lie on line CA, such that A is mid point of CD

$\left(\frac{5+p}{2}, \frac{11+q}{2}\right) \equiv (1, 5) \Rightarrow p = -3, -1$, Centre D(-3, -1), $r = \sqrt{52}$

$\therefore$ Eqn of second circle.

$[(x-a)^2 + (y-b)^2 = r^2] \longrightarrow (x+3)^2 + (y+1)^2 = 52$

14. Points $A(-2,3)$, $B(3,0)$, and $C(6,5)$ lie on the circumference of a circle with centre D.

(a) Show that angle $ABC = 90°$ --- [2]

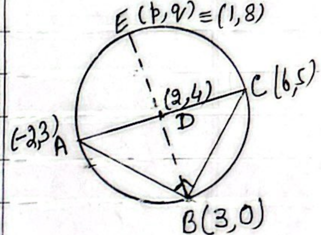
(b) Hence find the coordinate of D. --- [1]

(c) Find the equation of circle. --- [2]

The point E lies on the circumference of the circle such that BE is a diameter.

(d) Find the equation of the tangent to the circle at E. --- [4]

S-21/13/Q 10



Solution: $A(-2,3)$, $B(3,0)$ and $C(6,5)$

(a) Gradient of AB, $m_1 = \frac{0-3}{3+2} = -3/5$

Gradient of BC, $m_2 = \frac{5-0}{6-3} = \frac{5}{3}$

$m_1 \times m_2 = -\frac{3}{5} \times \frac{5}{3} = -1$

$\therefore$ angle $ABC = 90°$

(b) angle ABC is right angle $\rightarrow$ AC is the diameter.

$\Rightarrow$ Centre is the mid point of AC; $D\left(\frac{-2+6}{2}, \frac{3+5}{2}\right)$

Centre $D(2,4)$

(c) radius of the circle: $r^2 = DA^2 = (2-3)^2 + (4-0)^2 = 1+16 = 17 = r^2$

$\therefore$ Equation of the circle $(x-a)^2 + (y-b)^2 = r^2$ — Centre (a,b)

$(x-2)^2 + (y-4)^2 = 17$

(d) Let the coordinate of $E(p,q)$, Centre $D(2,4)$ is the mid point of diameter BE

$\Rightarrow \left(\frac{p+3}{2}, \frac{q+0}{2}\right) = D(2,4) \Rightarrow p=1,\ q=8 \Rightarrow E(1,8)$

Now gradient of $DE = \frac{8-4}{1-2} = -4 \Rightarrow$ Grad of tangent at $E = \frac{1}{4}$

$\therefore$ Equation of the tangent at $E(1,8)$

$y-8 = \frac{1}{4}(x-1)$

$\Rightarrow 4y - 32 = x - 1$

$4y = x + 31$

15. A circle with centre $(5,2)$ passes through the point $(7,5)$.

(a) Find the equation of the circle. --- [2]

The line $y=5x-10$ intersects the circle at A and B.

(b) Find the exact length of the chord AB. --- [7]

W-21/11/Q7

Solution: Circle: Centre $(5,2)$, Point $(7,5)$ lies on

(a) it: $r^2=(7-5)^2+(5-2)^2=13$

Eqn of Circle $(x-a)^2+(y-b)^2=r^2$

$(x-5)^2+(y-2)^2=13$ —— (1)

(b) Given line $y=(5x-10)$ —— (2)

from (1) & (2)

$(x-5)^2+(5x-10-2)^2=13$

$26x^2-130x+156=0$

(dividing by 26)

$x^2-5x+6=0$

$(x-2)(x-3)=0$

$x=2, \quad x=3$

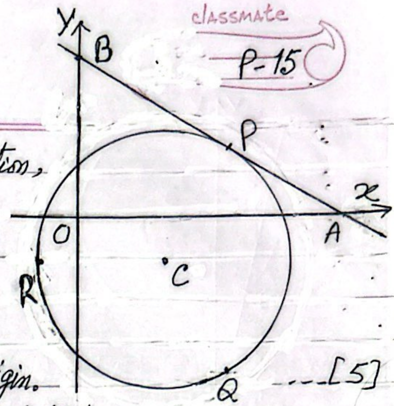
from (2) $x=2$, $y=0$ and $x=3$, $y=5$

$A(2,0)$, $B(3,5)$

Length of Chord AB

$=\sqrt{(3-2)^2+(5-0)^2}=\sqrt{26}$

$AB=\sqrt{26}$ ✓



16. The diagram shows the circle with equation, $x^2+y^2-6x+4y-27=0$ and the tangent to the circle at point $P(5,4)$

(a) The tangent to the circle at P meets the x-axis at A and y-axis at B.
Find the area of triangle OAB, where O is origin. ---[5]

(b) Points Q and R also lie on the circle, such that PQR is an equileteral triangle. Find the exact area of triangle PQR. --[3]

[W-21/12/ Q12]

Solution:

(a) Circle: $x^2+y^2-6x+4y-27=0$ —①

Centre $(-g,-f)=(3,-2)C$ & $P(5,4)$

Gradient of $PC=\dfrac{4+2}{5-3}=3$

$\therefore$ Grad. of tangent at $P=-\frac{1}{3}$

Equ^n of tangent $y-4=-\frac{1}{3}(x-5)$

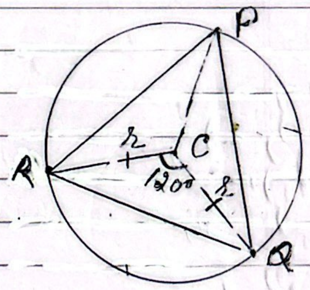
or $3y+x=17$ —②

② intersects X-axis for $y=0$; $x=17$
$A(17,0)$ ✓

and ② Intersects Y-axis, $x=0$, $y=\frac{17}{3}$
$B(0,17/3)$ ✓

Area of $\Delta OAB=\frac{1}{2}\times OA\times OB=\frac{1}{2}\times 17\times\frac{17}{3}$
$=\dfrac{289}{6}$ ✓

Radius r:
$r^2=g^2+f^2-c$
$=(-3)^2+2^2+27=40$



(b) Join CP, CQ, CR, PQ, QR, PR

In Equilateral triangle angle $QCR=120°$

area of $\Delta QCR=\frac{1}{2}r^2\sin 120°$
$=\frac{1}{2}\times 40\times\frac{\sqrt{3}}{2}=10\sqrt{3}$

ar of $\Delta PQR=3\times$ ar ΔQCR
$=3\times 10\sqrt{3}$
$=\underline{30\sqrt{3}}$ ✓

17. The line $y=2x+5$ intersects the circle with equation $x^2+y^2=20$ at A and B

(a) Find the coordinates of A and B in surd form and hence find the exact length of the chord AB. --- [7]

A straight line through the point (10,0) with gradient m is a tangent to the circle.

(b) Find the two possible values of m. --- [5]

[W-21/13/Q9]

Solution: circle : $x^2+y^2=20$ — ①

(a) line : $y=2x+5$ — ②

Solving ① & ② for Points of intersections,

$$x^2+(2x+5)^2=20$$

$$5x^2+20x+5=0$$

or $$x^2+4x+1=0$$

$$x=\frac{-4\pm\sqrt{16-4}}{2}$$

$$x=-2\pm\sqrt{3}$$

from ② $A(-2+\sqrt{3}, 1+2\sqrt{3})$, $B(-2-\sqrt{3}, 1-2\sqrt{3})$

$AB^2=(2\sqrt{3})^2+(4\sqrt{3})^2=60$

$\Rightarrow$ $AB=\sqrt{60}$ ✓

(b) Equation of line through Point (10,0) and gradient m is $y=m(x-10)$ — ③

Solving ① & ③

$$x^2+[m(x-10)]^2=20$$

$x^2(1+m^2)-20m^2x+20(5m^2-1)=0$ — ④

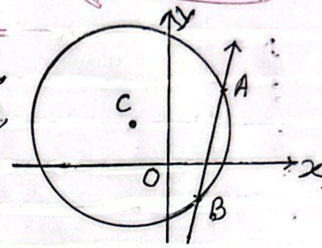
for line ③ be tangent to circle ① $B^2-4AC=0$ for Eqn ④

$$(20m^2)^2-4(1+m^2)\times 20(5m^2-1)=0$$

$\Rightarrow$ $-80(4m^2-1)=0$ $\Rightarrow$ $4m^2-1=0$

$\Rightarrow$ $m=\pm\frac{1}{2}$ ✓

18. The circle with equation $(x+1)^2+(y-2)^2=85$, and the straight line with equation $y=3x-20$, are shown in the diagram. The line intersects the circle at A and B, and the centre of the circle is at C.



(a) Find by calculation, the coordinates of A and B. ---[4]

(b) Find the equation of the circle which has its centre at C and for which the line with equation $y=3x-20$ is a tangent to the circle. [M-22/12/Q6] --[4]

Solution: Circle: $(x+1)^2+(y-2)^2=85$ ---①

(a) Line AB: $y=3x-20$ ---②

To find A and B from ② in ①

$(x+1)^2+(3x-20-2)^2=85$

$x^2+2x+1+9x^2-132x+484$

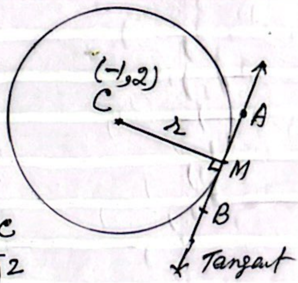
$\Rightarrow 10x^2-130x+400=0$

$\Rightarrow x^2-13x+40=0$

$(x-8)(x-5)=0 \Rightarrow \begin{cases} x=8, & x=5 \\ y=4, & y=-5 \end{cases}$ from ②

$\therefore$ A(8,4), B(5,−5) ✓

(b) from ① C(−1,2)



Now let CM ⊥ AB ⇒ M is mid point of AB.

$M\left(\frac{8+5}{2}, \frac{4-5}{2}\right) = M\left(\frac{13}{2}, -\frac{1}{2}\right)$

AB is tangent to the new circle with centre at C

$r = CM = \sqrt{\left(-1-\frac{13}{2}\right)^2+\left(2+\frac{1}{2}\right)^2}$

$= \sqrt{\frac{250}{4}} = \sqrt{\frac{125}{2}} \Rightarrow r^2 = \frac{125}{2}$ ✓

$\therefore$ Equation of required circle, C(−1,2), $r=\sqrt{\frac{125}{2}}$

$(x+1)^2+(y-2)^2 = \left(\sqrt{\frac{125}{2}}\right)^2$

or $(x+1)^2+(y-2)^2 = 62\frac{1}{2}$ ✓

19. The equation of a circle is $x^2+y^2+6x-2y-26=0$

(a) Find the coordinates of the centre of the circle and the radius. Hence find the coordinates of the lowest point on the circle. ---[4]

(b) Find the set of values of the constant k for which the line with equation $y=kx-5$ intersects the circle at two distinct points. --[6]

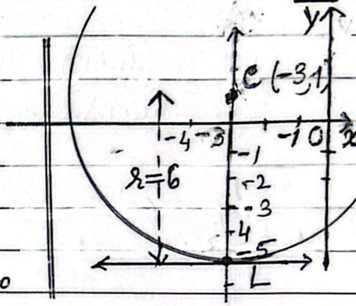
S-22 | 11 | Q9

Solution: Circle: $x^2+y^2+6x-2y-26=0$ --(1) $[x^2+y^2+2gx+2fy+c=0]$

(a) $g=\frac{6}{2}=3$, $f=\frac{-2}{2}=-1$, $c=-26$

Centre $(-g,-f)=(-3,1)$, $r=\sqrt{g^2+f^2-c}=\sqrt{3^2+(-1)^2-(-26)}=\sqrt{36}=6$

Lowest point L $(-3,-5)$.



(b) Line $y=kx-5$ --- (2)

for the intersection of circle (1) & line (2)

$x^2+(kx-5)^2+6x-2(kx-5)-26=0$

$x^2+k^2x^2-10kx+25+6x-2kx+10-26=0$

$(k^2+1)x^2+(6-12k)x+9=0$

for two points of Intersection $B^2-4AC>0$

$(6-12k)^2-4(k^2+1)\times 9>0$

$\cancel{36}+144k^2-144k-36k^2-\cancel{36}>0$

$\Rightarrow 108k^2-144k>0$

$\Rightarrow 36k\cdot(3k-4)>0$

[Critical values, $k=0$, $k=4/3$]

$\therefore \quad k<0; k>4/3$

20Q The equation of a circle is $x^2+y^2+ax+by-12=0$. The points $A(1,1)$ and $B(2,-6)$ lie on the circle.

(a) Find the values of a and b and hence find find the coordinates of the centre of the circle. ---[4]

(b) Find the equation of the tangent to the circle at the point A, giving the answer in the form $px+qy=k$, p, q, k are integers

[S-22/12/Q8] --[4]

Solution: circle: $x^2+y^2+ax+by-12=0$ ----- ①

(a) $A(1,1)$ lies on ① $\Rightarrow 1+1+a+b-12=0 \Rightarrow a+b=10$ --- ②

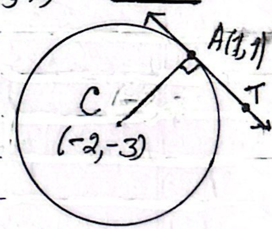
$B(2,-6)$ lies on ① $\Rightarrow 4+36+2a-6b-12=0 \Rightarrow 2a-6b=-28$ -- ③

Solving ② and ③ $\Longrightarrow$ $\underline{a=4 ; b=6}$ ✓

from ① equation of circle; $x^2+y^2+4x+6y-12=0$ --- ④

from ④ $g=4/2=2$, $f=6/2=3 \Rightarrow$ Centre $(-g,-f) = C(\underline{-2,-3})$ ✓

(b) Tangent at AT, AT is perp to CA.



$A(1,1)$, $C(-2,-3)$; gradient of $CA = \frac{1+3}{1+2} = \frac{4}{3}$ ✓

$\therefore$ Gradient of tangent $AT = -\frac{3}{4}$

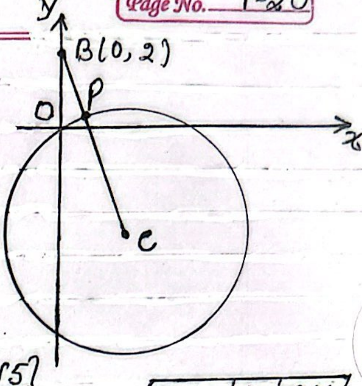
$\therefore$ Equation of tangent at $A(1,1)$

$$y-1 = -\frac{3}{4}(x-1)$$

$$\Rightarrow 4y-4 = -3x+3$$

$$\Rightarrow \underline{3x+4y=7}$$ ✓

21. The diagram shows the circle with equation: $(x-2)^2+(y+4)^2=20$, and with centre C. The point B(0,2) and the line segment BC intersects the circle at P.



(a) Find the equation of BC. ---- [2]

(b) Hence find the coordinates of P, giving your answer in exact form. ---- [5]

S-22/13/Q7

Solution: Circle: $(x-2)^2+(y+4)^2=20$ --- (1)

(a) or $(x-2)^2+(y-(-4))^2=20$ $\left\{ (x-a)^2+(y-b)^2=r^2 \right.$ Centre (a,b)

Coordinates of Centre $C(2,-4)$ and $B(0,2)$

$\therefore$ Equation of line BC: $y-2=\frac{2+4}{0-2}(x-0)$

BC: $\Rightarrow y-2=-3x \Rightarrow y=-3x+2$ --- (2)

(b) P is the point of intersection of line BC and circle.

from (1) & (2) $\Rightarrow (x-2)^2+(2-3x+4)^2=20$

$(x-2)^2+(6-3x)^2-20=0$

$x^2-4x+4+36+9x^2-36x-20=0$

$10x^2-40x+20=0$

$x^2-4x+2=0$

$x=\frac{4\pm\sqrt{8}}{2}$ $\left\{ b^2-4ac=(-4)^2-4\times1\times2 = 8 \right.$

$=\frac{4\pm2\sqrt{2}}{2}=2\pm\sqrt{2}$

$x=2-\sqrt{2}$ or $(2+\sqrt{2})^{\times}$

from $y=2-3(2-\sqrt{2})=3\sqrt{2}-4$

$\therefore$ $P(2-\sqrt{2}, 3\sqrt{2}-4)$ ✓

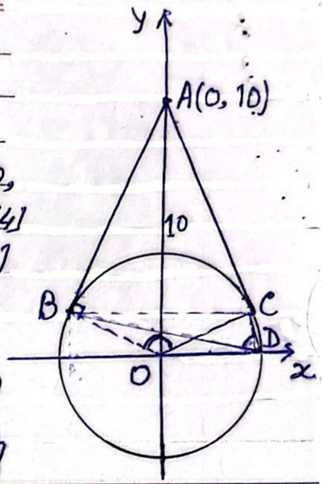
22. The diagram shows the circle with equation $x^2+y^2=20$. Tangents touching the circle at points B and C pass through the point A(0, 10).

(a) By letting the equation of tangent be $y=mx+10$, find the two possible values of m. ---[4]

(b) Find the coordinates of B and C. ---[3]

The point D is where the circle crosses the positive x-axis.

(c) Find the angle BDC. --[3]



W-22 / 11 / Q11

Solution: Circle: $x^2+y^2=20$ ---- ①

(a) Tangent: $y=mx+10$ —— ②

Solving ① and ②

$\Rightarrow x^2+(mx+10)^2=20$

$\Rightarrow x^2+m^2x^2+20mx+100=20$

$(1+m^2)x^2+20mx+80=0$ --- ③

for ② is tangent to circle ①, only one point of intersection ($b^2-4ac=0$)

$\Rightarrow (20m)^2-4(1+m^2)\times 80=0$

$\Rightarrow 80m^2-320=0 \Rightarrow m^2-4=0$

$\underline{m=\pm 2}$ ✓

(b) Put $m=\pm 2$ in ③

$(1+2^2)x^2 \pm 2(20)x+80=0$

$5x^2 \pm 40x+80=0$

$x^2 \pm 8x+16=0$

$(x\pm 4)^2=0$

$\Rightarrow x=-4 ; 4$

from ① B(-4, 2), C(4, 2) ✓

(c) from ① put $y=0$

$D(\sqrt{20}, 0)$

In rt Δ ABO

$\cos BOA=\frac{OB}{OA}=\frac{2}{10}=\frac{\sqrt{20}}{10}$

$\therefore \angle BOA=\cos^{-1}\left(\frac{\sqrt{20}}{10}\right)=\cos^{-1}0.4472$

$=63.4°$

$\therefore \angle BOC=2\cdot\angle BOA$

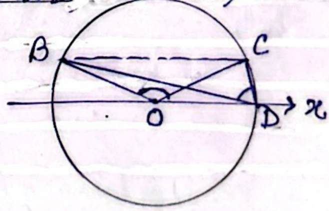
$=2\times 63.4=126.8°$

Now $\angle BDC=\frac{1}{2}\angle BOC$

$=\frac{1}{2}\times 126.8°$

$=63.4°$

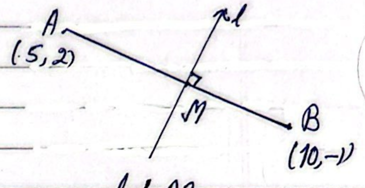
$\because$ (Angle BOC, subtended by chord BC the centre of circle is double angle at the remaining part of circle at D. → angle BDC)



23. Points A and B have coordinates (5, 2) and (10, −1) respectively.
(a) Find the equation of the perpendicular bisector of AB. --- [3]
(b) Find the equation of the circle with centre A which passes through B. --- [3]

[W-22 | 12 | Q 1]

Solution: $A(5, 2)$ and $B(10, -1)$



(a) Mid point of AB, $M\left(\frac{5+10}{2}, \frac{2+(-1)}{2}\right) = \left(\frac{15}{2}, \frac{1}{2}\right)$

Gradient of AB $= \frac{-1-2}{10-5} = -\frac{3}{5}$

$\therefore$ Gradient of line l, perp. to AB $= \frac{5}{3}$

$\left\{ \begin{array}{l} l \perp AB \\ m_1 \times m_2 = -1 \\ m_2 = -\frac{1}{m_1} \end{array} \right.$

$\therefore$ Equation of line l, perp. bisector of AB

$M\left(\frac{15}{2}, \frac{1}{2}\right)$, Grad. $= \frac{5}{3}$ $\quad$ ($y - y_1 = m(x - x_1)$)

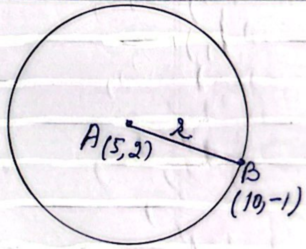
$y - \frac{1}{2} = \frac{5}{3}\left(x - \frac{15}{2}\right) \Rightarrow \frac{2y-1}{2} = \frac{5}{3}x - \frac{75}{6}$

$\Rightarrow \frac{6(2y-1)}{2} = 10x - 75$

$\Rightarrow 6y - 3 = 10x - 75 \Rightarrow 10x - 6y = 72$

or $\underline{5x - 3y = 36}$ ✓

(b) Centre of Circle $A(5, 2)$,
$B(10, -1)$ lies on the circle,
Let the rad. of circle $= r$



$r^2 = AB^2$
$= (10-5)^2 + (-1-2)^2$
$r^2 = 25 + 9 = 34$

$\therefore$ Equation of the circle:

$(x-5)^2 + (y-2)^2 = 34$

$x^2 - 10x + 25 + y^2 - 4y + 4 = 34$

or $\underline{x^2 + y^2 - 10x - 4y - 5 = 0}$ ✓

$\left\{ \begin{array}{l} \text{Equation of Circle,} \\ \text{Centre } (a, b)\text{; rad} = r \\ (x-a)^2 + (y-b)^2 = r^2 \end{array} \right.$

24, The coordinates of the points A, B and C are $A(5,-2), B(10,3), C(2p,p)$, where p is a constant.

(a) Given that AC and BC are equal in length, find the value of the fraction p. [3]

(b) It is given instead that AC is perpendicular to BC and p is an integer.

(i) Find the value of p. [4]

(ii) Find the equation of the circle which passes through A, B and C, giving your answer in the form $x^2+y^2+ax+bx+c=0$, where a, b and c are constant. [4]

[W-22/13/Q11]

Solution: $A(5,-2)$, $B(10,3)$ and $C(2p,p)$

(a) Given $AC = BC \Rightarrow AC^2 = BC^2$

$\Rightarrow (2p-5)^2+(p+2)^2 = (2p-10)^2+(3-p)^2$

$\Rightarrow 25-20p+4p^2+p^2+4p+4$

$= 100+4p^2-40p+9+p^2-6p$

$\Rightarrow 30p = 80 \Rightarrow p = 8/3$ ✓

(b)(i) $m_{AC} = \frac{p+2}{2p-5}$, $m_{BC} = \frac{p-3}{2p-10}$

$AC \perp BC \Rightarrow m_1 \times m_2 = -1$

$\Rightarrow \frac{p+2}{2p-5} \times \frac{p-3}{2p-10} = -1$

$\Rightarrow p^2-p-6 = -(4p^2-30p+50)$

$\Rightarrow 5p^2-31p+44 = 0$

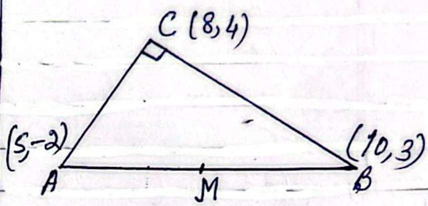
$\Rightarrow (p-4)(5p-11) = 0$

$\Rightarrow p = 4$ ✓ or $p = \frac{11}{5}$ ✗

(Given p is integer)

$p = 4$ ✓

(b)(ii) $p = 4 \Rightarrow C(2p,p) \equiv C(8,4)$



Since AC is perp to BC.
If we draw AB as diameter, the circle will pass threw the point C,
Centre of the circle thraw A, B and C is the mid point of AB, $M\left(\frac{5+10}{2}, \frac{-2+3}{2}\right)$

$= M\left(\frac{15}{2}, \frac{1}{2}\right)$

$r = AM \Rightarrow r^2 = AM^2$

$= \left(\frac{15}{2}-5\right)^2+\left(\frac{1}{2}+2\right)^2$

$= \left(\frac{5}{2}\right)^2+\left(\frac{5}{2}\right)^2 = \frac{50}{4}$ ✓

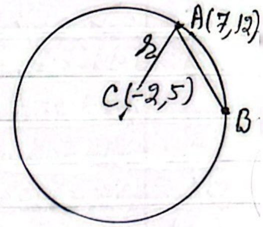
Equation of circle through A, B, C

$\left(x-\frac{15}{2}\right)^2+\left(y-\frac{1}{2}\right)^2 = \frac{50}{4}$

$x^2-15x+y+44=0$

25. Points $A(7,12)$ and B lie on a circle with centre $(-2,5)$. The line AB has equation $y=-2x+26$. Find the coordinates of B. ---[6]

M-23/12/Q5



Solution: $A(7,12)$ lies on the circle with centre $C(-2,5)$

radius r : $r^2=(7+2)^2+(12-5)^2=9^2+7^2=1\ 0$ ✓

$\therefore$ Equation of circle centre $(-2,5)$ and $r^2=1\ 0$

$(x+2)^2+(y-5)^2=120$ --(1) $[(x-a)^2+(y-b)^2=r^2]$

Equation of line AB: $y=-2x+26$ --(2)

from (1) and (2) $(x+2)^2+[-2x+26-5)^2=1\ 0$

$(x+2)^2+(-2x+21)^2=1\ 0$

$\Rightarrow x^2+4x+4+4x^2-84x+441-1\ 0=0$

$\Rightarrow 5x^2-80x+315=0 \Rightarrow x^2-16x+63=0 \Rightarrow (x-9)(x-7)=0$

$\Rightarrow x=9$ or $x=7$ ✗ (for Point A)

from (2) $y=8$ $\therefore$ $B(9,8)$ ✓

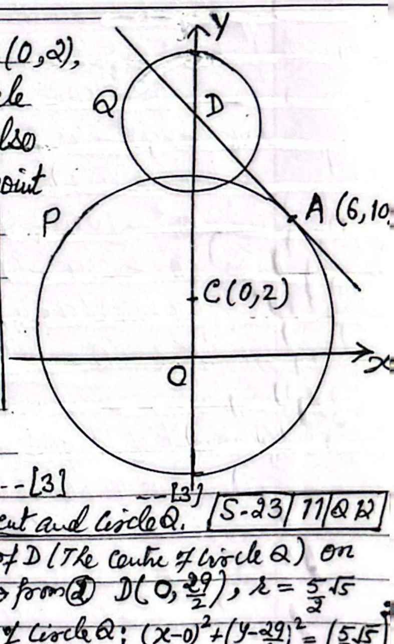
26. The diagram shows a circle P with centre $(0,2)$, and radius 10 and the tangent to the circle at the point A with coordinates $(6,10)$. It also shows a second circle Q with centre at the point where this tangent meets the y-axis and with radius $\frac{5}{2}\sqrt{5}$.

(a) Write down the equation of circle P. ---[1]

(b) Find the equation of the tangent to circle P at A. ---[2]

(c) Find the equation of circle Q and hence verify that the y-coord. of both the points of intersection of the two circles are 11. ---[3]

(d) Find the coord. of the points of intersection of the tangent and circle Q. ---[3]



S-23/11/Q12

Solution (a) Circle P: Centre $(0,2)$, rad $=10$.

Equ^n of Circle P: $x^2+(y-2)^2=100$ ✓ --(1)

(b) Gradient of radius $CA=\frac{10-2}{6-0}=\frac{4}{3}$ ✓

Equation of tangent to circle P at A.

(Tangent ⊥ rad) $y-10=-\frac{3}{4}(x-6)$

$\Rightarrow y=-\frac{3}{4}x+\frac{29}{2}$ ✓ --(2)

(c) Coord. of D (The centre of circle Q) on y-axis → from (2) $D\left(0,\frac{29}{2}\right)$, $r=\frac{5}{2}\sqrt{5}$

Equation of circle Q: $(x-0)^2+\left(y-\frac{29}{2}\right)^2=\left(\frac{5\sqrt{5}}{2}\right)^2$ --(3)

from (1) & (3) equating value of x^2:

$100-(y-2)^2=\frac{125}{4}-\left(y-\frac{29}{2}\right)^2$

True for $y=11$ ✓

(d) Solving (2) & (3) $x=\pm 2\sqrt{5}$, $y=\frac{29\mp 3\sqrt{5}}{2}$ ✓

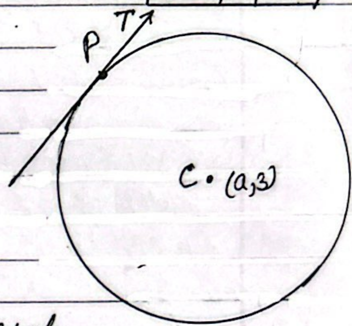
27. The equation of a circle is $(x-a)^2+(y-3)^2=20$. The line $y=\frac{1}{2}x+6$ is a tangent to the circle at the point P.

(a) Show that one possible value of a is 4 and find the other value. --[5]

(b) For $a=4$, find the equation of the normal to the circle at P. --[4]

(c) For $a=4$, find the equations of the two tangents to the circle which are parallel to the normal found in (b). --[4]

S-23/12/Q10



Solution: Circle: $(x-a)^2+(y-3)^2=20$ ---①

(a) Tangent PT: $y=\frac{1}{2}x+6$ --②

Solving ①&②

$(x-a)^2+\left(\frac{1}{2}x+6-3\right)^2=20$

$(x-a)^2+\left(\frac{1}{2}x+3\right)^2=20$

$x^2-2ax+a^2+\frac{1}{4}x^2+3x+9-20=0$

$\frac{5}{4}x^2+(3-2a)x+a^2-11=0$ ---①

Tangent has only one point of intersection with circle

$B^2-4AC=0$ $\left\{A=\frac{5}{4},\ B=(3-2a),\ C=a^2-11\right.$

$(3-2a)^2-4\times\frac{5}{4}\times(a^2-11)=0$

$4a^2-12a+9-5a^2+55=0$

$\Rightarrow -a^2-12a+64=0$

$\Rightarrow a^2+12a-64=0$

$(a+16)(a-4)=0$

$a=4$; $\underline{a=-16}$

(b) Gradient of tangent at P $=\frac{1}{2}$

Gradient of Normal at P $=-2$

for $a=4$, from ①

$\frac{5}{4}x^2-5x+5=0$

$x^2-4x+4=0$

$(x-2)^2=0\Rightarrow x=2$

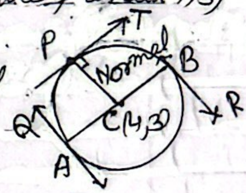
from ② $x=2\Rightarrow y=7$

$\therefore$ P(2,7), gradient of Normal $=-2$

$\therefore$ Equation of normal at P, $y-7=-2(x-2)\Rightarrow \underline{y=-2x+11}$

(c) for $a=4$, Centre of circle (4,3)

Equation of the diameter (AB) parallel to the tangent;



$y-3=\frac{1}{2}(x-4)$

$\Rightarrow y=\frac{1}{2}x+1$ ——③

Solving ① and ③ for $a=4$

$(x-4)^2+\left(\frac{1}{2}x+1-3\right)^2=20$

$\Rightarrow \frac{5}{4}x^2-10x=0\Rightarrow x=0,\ x=8$

from ③ A(0,1), B(8,5)

gradient of Normal at P $=-2$

$\therefore$ Equations of lines parallel to normal,

through A ; $y-1=-2(x-0)\Rightarrow \underline{y=-2x+1}$

Through B, $y-5=-2(x-8)\Rightarrow \underline{y=-2x+21}$

28. A circle has equation $(x-1)^2+(y+4)^2=40$. A line with equation $y=x-9$ intersects the circle at points A and B.

(a) Find the coordinates of the two point of intersection. ---[4]

(b) Find an equation of the circle with diameter AB. ---[3]

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Solution: Circle: $(x-1)^2+(y+4)^2=40$ --- ①

Line: $y=x-9$ ——— ②

Solving ① & ②

$(x-1)^2+(x-9+4)^2=40$

$\Rightarrow x^2-6x-7=0$

$(x+1)(x-7)=0$

$x=-1 \quad x=7$

from ② $x=-1, y=-10; x=7, y=-2$

$\therefore$ Points of Intersection $(-1,-10), (7,-2)$

(b) $A(-1,-10)$, $B(7,-2)$ as diameter.

Centre is the mid point of AB.

$C\left(\frac{-1+7}{2}, \frac{-10-2}{2}\right) \equiv C(3,-6)$

Radius r: $r^2=CA^2=(3+1)^2+(-6+10)^2$

$r^2 = 16+16=32$

$\therefore$ Equ^n of Circle:

$(x-a)^2+(y-b)^2=r^2$ [Centre (a,b)]

$\Rightarrow (x-3)^2+(y+6)^2=32$

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