PURE MATHEMATICS - 1

9709

(March, June and November series 2020 – 2023 With marking scheme)

Coordinate Geometry

Exercise - 1

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NOIDA

1.

The circle $x^2 + y^2 + 4x - 2y - 20 = 0$ has centre C and passes through points A and B.

(a) State the coordinates of C. [1]

It is given that the midpoint, D, of AB has coordinates $(1\frac{1}{2}, 1\frac{1}{2})$.

(b) Find the equation of AB, giving your answer in the form $y = mx + c$. [4]

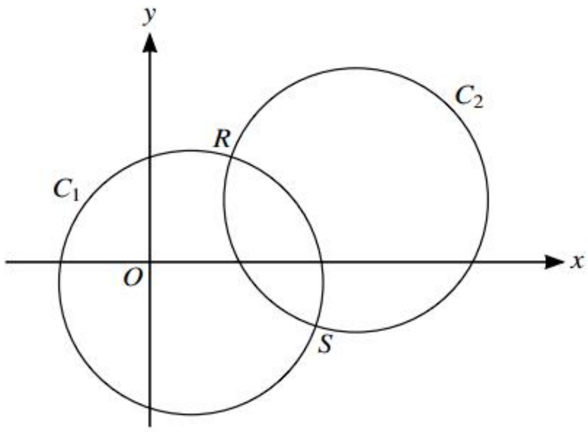
(c) Find, by calculation, the x-coordinates of A and B. [3]

QUESTION – 10: SP_20_01

2.

A diameter of a circle C_1 has end-points at $(-3, -5)$ and $(7, 3)$.

(a) Find an equation of the circle C_1. [3]



The circle C_1 is translated by $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$ to give circle C_2, as shown in the diagram.

(b) Find an equation of the circle C_2. [2]

The two circles intersect at points R and S.

(c) Show that the equation of the line RS is $y = -2x + 13$. [4]

(d) Hence show that the x-coordinates of R and S satisfy the equation $5x^2 - 60x + 159 = 0$. [2]

QUESTION – 12: QP_M20_12

3.

The coordinates of the points A and B are $(-1, -2)$ and $(7, 4)$ respectively.

(a) Find the equation of the circle, C, for which AB is a diameter. [4]

(b) Find the equation of the tangent, T, to circle C at the point B. [4]

(c) Find the equation of the circle which is the reflection of circle C in the line T. [3]

QUESTION – 10: QP_S20_11

4.

The equation of a circle with centre C is $x^2 + y^2 - 8x + 4y - 5 = 0$.

(a) Find the radius of the circle and the coordinates of C. [3]

The point $P(1, 2)$ lies on the circle.

(b) Show that the equation of the tangent to the circle at P is $4y = 3x + 5$. [3]

The point Q also lies on the circle and PQ is parallel to the x-axis.

(c) Write down the coordinates of Q. [2]

The tangents to the circle at P and Q meet at T.

(d) Find the coordinates of T. [3]

QUESTION – 11: QP_S20_12

5.

10 **(a)** The coordinates of two points A and B are $(-7, 3)$ and $(5, 11)$ respectively.

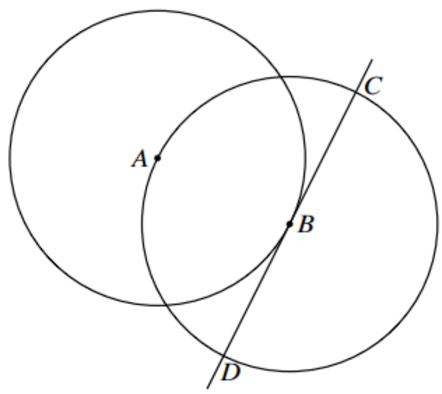
Show that the equation of the perpendicular bisector of AB is $3x + 2y = 11$. [4]

(b) A circle passes through A and B and its centre lies on the line $12x - 5y = 70$.

Find an equation of the circle. [5]

QUESTION – 10: QP_S20_13

6.



The diagram shows a circle with centre A passing through the point B. A second circle has centre B and passes through A. The tangent at B to the first circle intersects the second circle at C and D.

The coordinates of A are $(-1, 4)$ and the coordinates of B are $(3, 2)$.

(a) Find the equation of the tangent CBD. [2]

(b) Find an equation of the circle with centre B. [3]

(c) Find, by calculation, the x-coordinates of C and D. [3]

QUESTION – 9: QP_W20_11

7.

A circle has centre at the point $B(5, 1)$. The point $A(-1, -2)$ lies on the circle.

(a) Find the equation of the circle. [3]

Point C is such that AC is a diameter of the circle. Point D has coordinates $(5, 16)$.

(b) Show that DC is a tangent to the circle. [4]

The other tangent from D to the circle touches the circle at E.

(c) Find the coordinates of E. [2]

QUESTION – 9: QP_W20_12

8.

A circle with centre C has equation $(x-8)^2 + (y-4)^2 = 100$.

(a) Show that the point $T(-6, 6)$ is outside the circle. [3]

Two tangents from T to the circle are drawn.

(b) Show that the angle between one of the tangents and CT is exactly 45°. [2]

The two tangents touch the circle at A and B.

(c) Find the equation of the line AB, giving your answer in the form $y = mx + c$. [4]

(d) Find the x-coordinates of A and B. [3]

QUESTION – 11: QP_W20_13

9.

The points $A(7, 1)$, $B(7, 9)$ and $C(1, 9)$ are on the circumference of a circle.

(a) Find an equation of the circle. [5]

(b) Find an equation of the tangent to the circle at B. [2]

QUESTION – 8: QP_M21_12

10.

The equation of a circle is $x^2 + y^2 - 4x + 6y - 77 = 0$.

(a) Find the x-coordinates of the points A and B where the circle intersects the x-axis. [2]

(b) Find the point of intersection of the tangents to the circle at A and B. [6]

QUESTION – 10: QP_S21_11

11.

The equation of a curve is $y = (x - 3)\sqrt{x + 1} + 3$. The following points lie on the curve. Non-exact values are rounded to 4 decimal places.

$A\,(2, k)$ $B\,(2.9, 2.8025)$ $C\,(2.99, 2.9800)$ $D\,(2.999, 2.9980)$ $E\,(3, 3)$

(a) Find k, giving your answer correct to 4 decimal places. [1]

(b) Find the gradient of AE, giving your answer correct to 4 decimal places. [1]

The gradients of BE, CE and DE, rounded to 4 decimal places, are 1.9748, 1.9975 and 1.9997 respectively.

(c) State, giving a reason for your answer, what the values of the four gradients suggest about the gradient of the curve at the point E. [2]

QUESTION – 3: QP_S21_12

12.

Points A and B have coordinates (8, 3) and (p, q) respectively. The equation of the perpendicular bisector of AB is $y = -2x + 4$.

Find the values of p and q. [4]

QUESTION – 6: QP_S21_12

13.

The point A has coordinates (1, 5) and the line l has gradient $-\frac{2}{3}$ and passes through A. A circle has centre (5, 11) and radius $\sqrt{52}$.

(a) Show that l is the tangent to the circle at A. [2]

(b) Find the equation of the other circle of radius $\sqrt{52}$ for which l is also the tangent at A. [3]

QUESTION – 7: QP_S21_12

14.

Points $A\,(-2, 3)$, $B\,(3, 0)$ and $C\,(6, 5)$ lie on the circumference of a circle with centre D.

(a) Show that angle $ABC = 90°$. [2]

(b) Hence state the coordinates of D. [1]

(c) Find an equation of the circle. [2]

The point E lies on the circumference of the circle such that BE is a diameter.

(d) Find an equation of the tangent to the circle at E. [5]

QUESTION – 10: QP_S21_13

15.

A circle with centre (5, 2) passes through the point (7, 5).

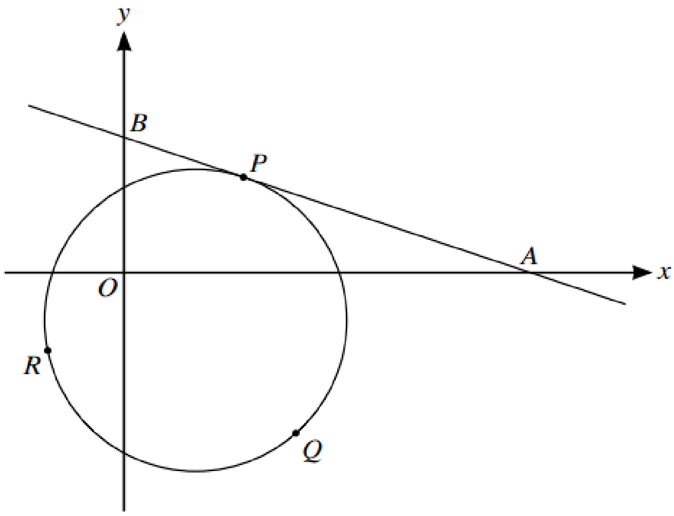
(a) Find an equation of the circle. [2]

The line $y = 5x - 10$ intersects the circle at A and B.

(b) Find the exact length of the chord AB. [7]

QUESTION – 7: QP_W21_11

16.



The diagram shows the circle with equation $x^2 + y^2 - 6x + 4y - 27 = 0$ and the tangent to the circle at the point $P(5, 4)$.

(a) The tangent to the circle at P meets the x-axis at A and the y-axis at B.

Find the area of triangle OAB, where O is the origin. [5]

(b) Points Q and R also lie on the circle, such that PQR is an equilateral triangle.

Find the exact area of triangle PQR. [3]

QUESTION – 12: QP_W21_12

17.

The line $y = 2x + 5$ intersects the circle with equation $x^2 + y^2 = 20$ at A and B.

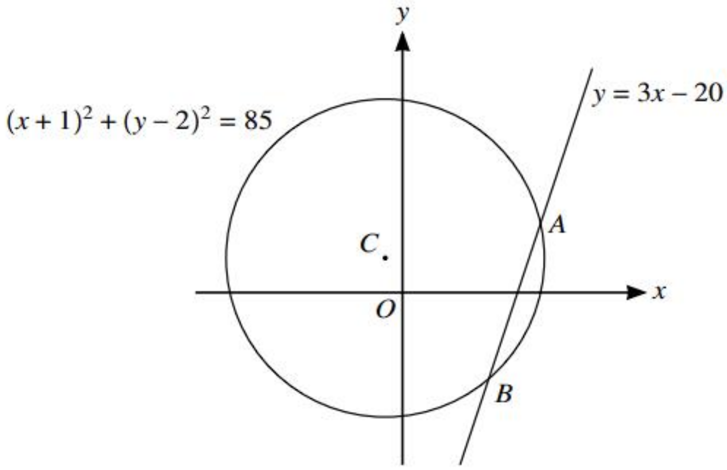
(a) Find the coordinates of A and B in surd form and hence find the exact length of the chord AB. [7]

A straight line through the point $(10, 0)$ with gradient m is a tangent to the circle.

(b) Find the two possible values of m. [5]

QUESTION – 9: QP_W21_13

18.



The circle with equation $(x + 1)^2 + (y - 2)^2 = 85$ and the straight line with equation $y = 3x - 20$ are shown in the diagram. The line intersects the circle at A and B, and the centre of the circle is at C.

(a) Find, by calculation, the coordinates of A and B. [4]

(b) Find an equation of the circle which has its centre at C and for which the line with equation $y = 3x - 20$ is a tangent to the circle. [4]

QUESTION – 6: QP_M22_12

19.

The equation of a circle is $x^2 + y^2 + 6x - 2y - 26 = 0$.

(a) Find the coordinates of the centre of the circle and the radius. Hence find the coordinates of the lowest point on the circle. [4]

(b) Find the set of values of the constant k for which the line with equation $y = kx - 5$ intersects the circle at two distinct points. [6]

QUESTION – 9: QP_S22_11

20.

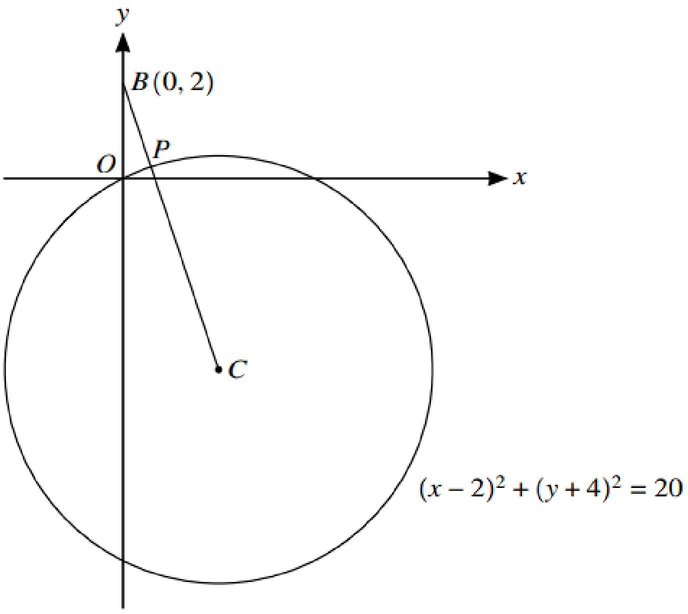
The equation of a circle is $x^2 + y^2 + ax + by - 12 = 0$. The points $A\,(1,\ 1)$ and $B\,(2,\ -6)$ lie on the circle.

(a) Find the values of a and b and hence find the coordinates of the centre of the circle. [4]

(b) Find the equation of the tangent to the circle at the point A, giving your answer in the form $px + qy = k$, where p, q and k are integers. [4]

QUESTION – 8: QP_S22_12

21.



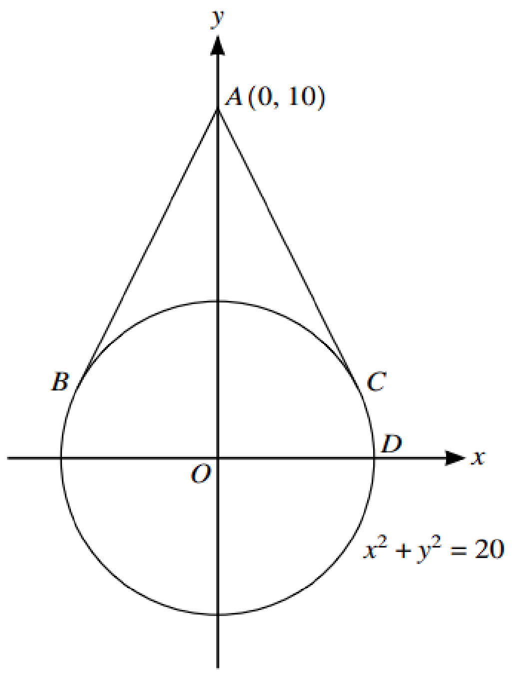
The diagram shows the circle with equation $(x-2)^2 + (y+4)^2 = 20$ and with centre C. The point B has coordinates $(0, 2)$ and the line segment BC intersects the circle at P.

(a) Find the equation of BC. [2]

(b) Hence find the coordinates of P, giving your answer in exact form. [5]

QUESTION – 7: QP_S22_13

22.



The diagram shows the circle with equation $x^2 + y^2 = 20$. Tangents touching the circle at points B and C pass through the point $A\ (0,\ 10)$.

(a) By letting the equation of a tangent be $y = mx + 10$, find the two possible values of m. [4]

(b) Find the coordinates of B and C. [3]

The point D is where the circle crosses the positive x-axis.

(c) Find angle BDC in degrees. [3]

QUESTION – 11: QP_W22_11

23.

Points A and B have coordinates $(5,\ 2)$ and $(10,\ -1)$ respectively.

(a) Find the equation of the perpendicular bisector of AB. [3]

(b) Find the equation of the circle with centre A which passes through B. [3]

QUESTION – 1: QP_W22_12

24.

The coordinates of points A, B and C are $A\ (5,\ -2)$, $B\ (10,\ 3)$ and $C\ (2p,\ p)$, where p is a constant.

(a) Given that AC and BC are equal in length, find the value of the fraction p. [3]

(b) It is now given instead that AC is perpendicular to BC and that p is an integer.

(i) Find the value of p. [4]

(ii) Find the equation of the circle which passes through A, B and C, giving your answer in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are constants. [4]

QUESTION – 11: QP_W22_13

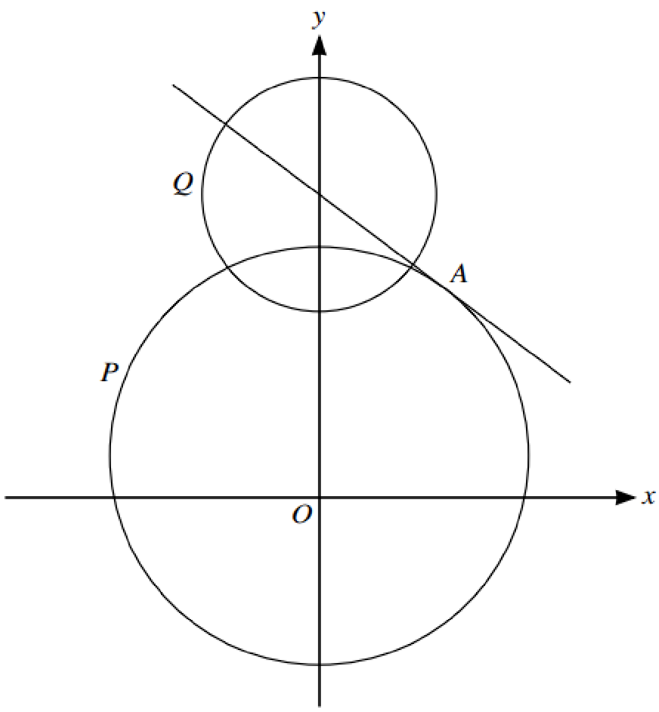
25.

Points $A\ (7,\ 12)$ and B lie on a circle with centre $(-2,\ 5)$. The line AB has equation $y = -2x + 26$.

Find the coordinates of B. [6]

QUESTION – 5: QP_M23_12

26.



The diagram shows a circle P with centre $(0,\ 2)$ and radius 10 and the tangent to the circle at the point A with coordinates $(6,\ 10)$. It also shows a second circle Q with centre at the point where this tangent meets the y-axis and with radius $\frac{5}{2}\sqrt{5}$.

(a) Write down the equation of circle P. [1]

(b) Find the equation of the tangent to the circle P at A. [2]

(c) Find the equation of circle Q and hence verify that the y-coordinates of both of the points of intersection of the two circles are 11. [3]

(d) Find the coordinates of the points of intersection of the tangent and circle Q, giving the answers in surd form. [3]

QUESTION – 12: QP_S23_11

27.

The equation of a circle is $(x - a)^2 + (y - 3)^2 = 20$. The line $y = \frac{1}{2}x + 6$ is a tangent to the circle at the point P.

(a) Show that one possible value of a is 4 and find the other possible value. [5]

(b) For $a = 4$, find the equation of the normal to the circle at P. [4]

(c) For $a = 4$, find the equations of the two tangents to the circle which are parallel to the normal found in **(b)**. [4]

QUESTION – 10: QP_S23_12

28.

A circle has equation $(x - 1)^2 + (y + 4)^2 = 40$. A line with equation $y = x - 9$ intersects the circle at points A and B.

(a) Find the coordinates of the two points of intersection. [4]

(b) Find an equation of the circle with diameter AB. [3]

QUESTION – 5: QP_S23_13

MARK SCHEME

1.

(a)	(–2, 1)	1	B1	
(b)	Gradient of $CD = \frac{1}{2} \div 3\frac{1}{2} = \frac{1}{7}$	1	B1	
	Gradient of $AB = -7$	1	M1	With gradient –1/their m
	Equation of AB is $y - 1\frac{1}{2} = -7(x - 1\frac{1}{2})$	1	M1	
	$y = -7x + 12$	1	A1	
		4		
(c)	$x^2 + (12 - 7x)^2 + 4x - 2(12 - 7x) - 20\ (= 0)$	1	M1	Substituting their AB equation into circle equation
	$(50)(x^2 - 3x + 2)\ (= 0)$	1	A1	
	$x = 1, 2$	1	A1	Dependent on method seen for solving quadratic equation
		3		

2.

(a)	Centre = (2, –1)	B1	
	$r^2 = [2-(-3)]^2 + [-1-(-5)]^2$ or $[2-7]^2 + [-1-3]^2$ OE	M1	OR $\frac{1}{2}\left[(-3-7)^2 + (-5-3)^2\right]$ OE
	$(x-2)^2 + (y+1)^2 = 41$	A1	Must not involve surd form **SCB3** $(x+3)(x-7)+(y+5)(y-3)=0$
		3	
(b)	Centre = *their* (2, –1) + $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$ = (10, 3)	B1FT	SOI FT on *their* (2, –1)
	$(x-10)^2 + (y-3)^2 =$ *their* 41	B1FT	FT on *their* 41 even if in surd form **SCB2** $(x-5)(x-15)+(y+1)(y-7)=0$
		2	

3.

(a)	Centre is (3, 1)	B1
	Radius = 5 (Pythagoras)	B1
	Equation of C is $(x-3)^2 + (y-1)^2 = 25$ (**FT** on *their* centre)	M1 A1FT
		4
(b)	Gradient from (3, 1) to (7, 4) = ¾ (this is the normal)	B1
	Gradient of tangent $= -\frac{4}{3}$	M1
	Equation is $y - 4 = -\frac{4}{3}(x-7)$ or $3y + 4x = 40$	M1A1
		4
(c)	B is centre of line joining centres → (11, 7)	B1
	Radius = 5 New equation is $(x-11)^2 + (y-7)^2 = 25$ (**FT** on coordinates of B)	M1 A1FT
		3

4.

(a)	Express as $(x-4)^2+(y+2)^2=16+4+5$	M1
	Centre $C(4, -2)$	A1
	Radius = $\sqrt{25}$ = 5	A1
		3
(b)	$P(1,2)$ to $C(4, -2)$ has gradient $-\frac{4}{3}$ (**FT** on coordinates of C)	B1FT
	Tangent at P has gradient = $\frac{3}{4}$	M1
	Equation is $y-2=\frac{3}{4}(x-1)$ or $4y = 3x + 5$	A1
		3
(c)	Q has the same coordinate as P $y = 2$	B1
	Q is as far to the right of C as P $x = 3 + 3 + 1 = 7$ Q (7, 2)	B1
		2
(d)	Gradient of tangent at $Q = -\frac{3}{4}$ by symmetry (**FT** from part (**b**))	B1FT
	Eqn of tangent at Q is $y-2=-\frac{3}{4}(x-7)$ or $4y + 3x = 29$	M1
	$T(4, \frac{17}{4})$	A1
		3

5.

(a)	Mid-point is (–1, 7)	B1
	Gradient, m, of AB is 8/12 OE	B1
	$y-7=-\frac{12}{8}(x+1)$	M1
	$3x+2y=11$ **AG**	A1
		4
(b)	Solve simultaneously $12x-5y=70$ and *their* $3x+2y=11$	M1
	$x=5,\ y=-2$	A1
	Attempt to find distance between *their* (5, –2) and either (–7,3) or (5, 11)	M1
	$(r)=\sqrt{12^2+5^2}$ or $\sqrt{13^2+0}=13$	A1
	Equation of circle is $(x-5)^2+(y+2)^2=169$	A1
		5

6.

(a)	$m_{AB}=\frac{4-2}{-1-3}=-\frac{1}{2}$	B1	
	Equation of tangent is $y-2=2(x-3)$	B1 FT	(3, 2) with *their* gradient $-\frac{1}{m_{AB}}$
		2	
(b)	$AB^2=4^2+2^2=20$ or $r^2=20$ or $r=\sqrt{20}$ or $AB=\sqrt{20}$	B1	
	Equation of circle centre B is $(x-3)^2+(y-2)^2=20$	M1 A1	FT *their* 20 for M1
		3	
(c)	$(x-3)^2+(2x-6)^2=$ *their* 20	M1	Substitute *their* $y-2=2x-6$ into *their* circle, centre B
	$5x^2-30x+25=0$ or $5(x-3)^2=20$	A1	
	$[(5)(x-5)(x-1)$ or $x-3=\pm2]$ $x=5,\ 1$	A1	
		3	

7.

(a)	$r=\sqrt{(6^2+3^2)}$ or $r^2=45$	B1	Sight of r = 6.7 implies B1
	$(x-5)^2+(y-1)^2=r^2$ or $x^2-10x+y^2-2y=r^2-26$	M1	Using centre given and *their* radius or r in correct formula
	$(x-5)^2+(y-1)^2=45$ or $x^2-10x+y^2-2y=19$	A1	Do not allow $(\sqrt{45})^2$ for r^2
		3	
(b)	C has coordinates (11, 4)	B1	
	0.5	B1	OE, Gradient of AB, BC or AC.
	Grad of CD $=-2$	M1	Calculation of gradient needs to be shown for this M1.
	($\frac{1}{2}\times-2=-1$) then states + perpendicular → hence shown or tangent	A1	Clear reasoning needed.
	Alternative method for question 9(b)		
	C has coordinates (11, 4)	B1	
	0.5	B1	OE, Gradient of AB, BC or AC.
	Gradient of the perpendicular is -2 → Equation of the perpendicular is $y-4=-2(x-11)$	M1	Use of $m_1m_2=-1$ with *their* gradient of AB, BC or AC and correct method for the equation of the perpendicular. Could use $D(5, 16)$ instead of $C(11,4)$.
	Checks $D(5, 16)$ **or** checks gradient of CD and then states D lies on the line **or** CD has gradient -2→ hence shown or tangent	A1	Clear check and reasoning needed. Checks that the other point lies on the line or checks gradient.

(b)	**Alternative method for question 9(b)**		
	C has coordinates (11, 4) **or** Gradient of AB, BC or $AC = 0.5$	**B1**	Only one of AB, BC or AC needed.
	Equation of the perpendicular is $y-4=-2(x-11)$	**B1**	Finding equation of CD.
	$(x-5)^2+(-2x+26-1)^2=45 \rightarrow (x^2-22x+121=0)$	**M1**	Solving simultaneously with the equation of the circle.
	$(x-11)^2=0$ or $b^2-4ac=0 \rightarrow$ repeated root $\rightarrow$ hence shown or tangent	**A1**	Must state repeated root.
	Alternative method for question 9(b)		
	C has coordinates (11, 4)	**B1**	
	Finding $CD=\sqrt{180}$ and $BD=\sqrt{225}$	**B1**	OE. Calculated from the co-ordinates of B, C & D without using r.
	Checking (their BD)^2 – (their CD)^2 is the same as (their r)^2	**M1**	
	$\therefore$ Pythagoras valid $\therefore$ perpendicular $\rightarrow$ hence shown or tangent	**A1**	Triangle ACD could be used instead.
	Alternative method for question 9(b)		
	C has coordinates (11, 4)	**B1**	
	Finding vectors $\overrightarrow{AC}$ and $\overrightarrow{CD}$ **or** $\overrightarrow{BC}$ and $\overrightarrow{CD}$ $\left(= \begin{pmatrix}6\\3\end{pmatrix} and \begin{pmatrix}-6\\12\end{pmatrix}\right.$ **or** $\left.\begin{pmatrix}12\\6\end{pmatrix} and \begin{pmatrix}-6\\12\end{pmatrix}\right)$	**B1**	Must be correct pairing.
	Applying the scalar product to one of these pairs of vectors	**M1**	Accept *their* $\overrightarrow{AC}$ and $\overrightarrow{CD}$ or *their* $\overrightarrow{BC}$ and $\overrightarrow{CD}$
	Scalar product = 0 then states $\therefore$ perpendicular $\rightarrow$ hence shown or tangent	**A1**	
		4	
(c)	$E(-1, 4)$	**B1 B1**	WWW B1 for each coordinate Note: Equation of DE which is $y=2x+6$ may be used to find E
		2	

8.

(a)	$(-6-8)^2+(6-4)^2$	**M1**	OE
	$=200$	**A1**	
	$\sqrt{200}>10$, hence outside circle	**A1**	AG (‘Shown’ not sufficient). Accept equivalents of $\sqrt{200}>10$
	Alternative method for question 11(a)		
	Radius = 10 and $C = (8, 4)$	**B1**	
	Min(x) on circle = 8 – 10 = –2	**M1**	
	Hence outside circle	**A1**	AG
		3	
(b)	$\text{angle} = \sin^{-1}\left(\dfrac{their\,10}{their\,10\sqrt{2}}\right)$	**M1**	Allow decimals for $10\sqrt{2}$ at this stage. If cosine used, angle ACT or BCT must be identified, or implied by use of 90°– 45°.
	$\text{angle} = \sin^{-1}\left(\dfrac{1}{\sqrt{2}} \text{ or } \dfrac{\sqrt{2}}{2} \text{ or } \dfrac{10}{10\sqrt{2}} \text{ or } \dfrac{10}{\sqrt{200}}\right) = 45°$	**A1**	AG Do not allow decimals
	Alternative method for question 11(b)		
	$(10\sqrt{2})^2=10^2+TA^2$	**M1**	
	$TA = 10 \rightarrow 45°$	**A1**	AG
		2	

(c)	Gradient, m, of $CT = -\frac{1}{7}$	B1	OE
	Attempt to find mid-point (M) of CT	*M1	Expect (1, 5)
	Equation of AB is $y-5=7(x-1)$	DM1	Through *their* (1, 5) with gradient $-\frac{1}{m}$
	$y=7x-2$	A1	
		4	
(d)	$(x-8)^2+(7x-2-4)^2=100$ or equivalent in terms of y	M1	Substitute *their* equation of AB into equation of circle.
	$50x^2-100x\ (=0)$	A1	
	$x=0$ and 2	A1	WWW
	Alternative method for question 11(d)		
	MC $= \begin{pmatrix}7\\-1\end{pmatrix}$	M1	
	$\begin{pmatrix}1\\5\end{pmatrix}+\begin{pmatrix}-1\\-7\end{pmatrix}=\begin{pmatrix}0\\-2\end{pmatrix}$, $\begin{pmatrix}1\\5\end{pmatrix}+\begin{pmatrix}1\\7\end{pmatrix}=\begin{pmatrix}2\\12\end{pmatrix}$	A1	
	$x=0$ and 2	A1	
		3	

9.

(a)	Centre of circle is (4, 5)	B1 B1	
	$r^2=(7-4)^2+(1-5)^2$	M1	OE. Either using *their* centre and A or C **or** using A and C and dividing by 2.
	$r=5$	A1 FT	FT on *their* (4, 5) if used.
	Equation is $(x-4)^2+(y-5)^2=25$	A1	OE. Allow 5^2 for 25.
		5	
(b)	Gradient of radius $=\frac{9-5}{7-4}=\frac{4}{3}$	B1 FT	FT for use of *their* centre.
	Equation of tangent is $y-9=-\frac{3}{4}(x-7)$	B1	or $y=\frac{-3x}{4}+\frac{57}{4}$
		2	

10.

(a)	When $y=0$ $x^2-4x-77=0$ [$\Rightarrow (x+7)(x-11)=0$ or $(x-2)^2=81$]	M1	Substituting $y=0$
	So x-coordinates are -7 and 11	A1	
		2	

(b)	Centre of circle C is $(2, -3)$	B1	
	Gradient of AC is $-\frac{1}{3}$ or Gradient of BC is $\frac{1}{3}$	M1	For either gradient (M1 sign error, M0 if x-coordinate(s) in numerator)
	Gradient of tangent at A is 3 or Gradient of tangent at B is -3	M1	For either perpendicular gradient
	Equations of tangents are $y = 3x + 21$, $y = -3x + 33$	A1	For either equation
	Meet when $3x + 21 = -3x + 33$	M1	OR: (centre of circle has x coordinate 2) so x coordinate of point of intersection is 2
	Coordinates of point of intersection $(2, 27)$	A1	
	Alternative method for Question 10(b)		
	Implicit differentiation: $2y\frac{dy}{dx}$ seen	B1	
	$2x-4+2y\frac{dy}{dx}+6\frac{dy}{dx}=0$	M1	Fully differentiated $=0$ with at least one term involving y differentiated correctly
	Gradient of tangent at A is 3 or Gradient of tangent at B is -3	M1	For either gradient
	Equations of tangents are $y = 3x + 21$, $y = -3x + 33$	A1	For either equation
	Meet when $3x + 21 = -3x + 33$	M1	OR: (centre of circle has x coordinate 2) so x coordinate of point of intersection is 2
	Coordinates of point of intersection $(2, 27)$	A1	
		6	

11.

(a)	1.2679	B1	AWRT. ISW if correct answer seen. $3-\sqrt{3}$ scores B0
		1	
(b)	1.7321	B1	AWRT. ISW if correct answer seen.
		1	
(c)	Sight of 2 or 2.0000 or two in reference to the gradient	*B1	
	This is because the gradient at E is the limit of the gradients of the chords as the x-value tends to 3 or δx tends to 0.	DB1	Allow it gets nearer/approaches/tends/almost/approximately 2
		2	

12.

Gradient AB = $\frac{1}{2}$	B1	SOI
Lines meet when $-2x+4=\frac{1}{2}(x-8)+3$ Solving as far as $x =$	*M1	Equating given perpendicular bisector with the line through (8, 3) using *their* gradient of AB (but not -2) and solving. Expect $x = 2$, $y = 0$.
Using mid-point to get as far as $p =$ or $q =$	DM1	Expect $\frac{8+p}{2}=2$ or $\frac{3+q}{2}=0$
$p=-4$, $q=-3$	A1	Allow coordinates of B are $(-4, -3)$.
Alternative method for Question 6		
Gradient AB = $\frac{1}{2}$	B1	SOI
$\frac{q-3}{p-8}=\frac{1}{2}$ [leading to $2q=p-2$], $\frac{q+3}{2}=-2\left(\frac{8+p}{2}\right)+4$ [leading to $q=-11-2p$]	*M1	Equating gradient of AB with *their* gradient of AB (but not -2) and using mid-point in equation of perpendicular bisector.
Solving simultaneously *their* 2 linear equations	DM1	Equating and solving 2 correct equations as far as $p =$ or $q =$.
$p=-4$, $q=-3$	A1	Allow coordinates of B are $(-4, -3)$.

Alternative method for Question 6		
Gradient AB = $\frac{1}{2}$	B1	
$\frac{q-3}{p-8}=\frac{1}{2}$ [leading to $p=2q+2$], $y-\frac{q+3}{2}=-2\left(x-(q+5)\right)$ $\left[\text{leading to } y=-2x+\frac{5q+23}{2}\right]$	*M1	Equating gradient of *AB* with *their* gradient of *AB* (but not -2) and using mid-point in equation of perpendicular bisector.
their $\frac{5q+23}{2}=4 \Rightarrow q=$	DM1	Equating and solving as far as q or $p=$
$p=-4$, $q=-3$	A1	Allow coordinates of *B* are $(-4, -3)$.
	4	

13.

(a)	$(5-1)^2+(11-5)^2=52$ or $\frac{11-5}{5-1}$	M1	For substituting (1,5) into circle equation or showing gradient $=\frac{3}{2}$.
	For both circle equation and gradient, and proving line is perpendicular and stating that *A* lies on the circle	A1	Clear reasoning.
	Alternative method for Question 7(a)		
	$(x-5)^2+(y-11)^2=52$ and $y-5=-\frac{2}{3}(x-1)$	M1	Both equations seen and attempt to solve. May see $y=-\frac{2}{3}x+\frac{17}{3}$
	Solving simultaneously to obtain $(y-5)^2=0$ or $(x-1)^2=0 \Rightarrow 1$ root or tangent or discriminant $=0 \Rightarrow 1$ root or tangent	A1	Clear reasoning.
	Alternative method for Question 7(a)		
	$\frac{dy}{dx}=\frac{10-2x}{2y-22}=\frac{10-2}{10-22}$	M1	Attempting implicit differentiation of circle equation and substitute $x=1$ and $y=5$.
	Showing gradient of circle at A is $-\frac{2}{3}$	A1	Clear reasoning.
		2	
(b)	Centre is $(-3, -1)$	B1 B1	B1 for each correct co-ordinate.
	Equation is $(x+3)^2+(y+1)^2=52$	B1 FT	FT *their* centre, but not if either (1, 5) or (5, 11). Do not accept $\sqrt{52^2}$.
		3	

14.

(a)	Gradient of $AB=-\frac{3}{5}$, gradient of $BC=\frac{5}{3}$ or lengths of all 3 sides or vectors	**M1**	Attempting to find required gradients, sides or vectors
	$m_{ab}m_{bc}=-1$ or Pythagoras or $\overrightarrow{AB}.\overrightarrow{BC}=0$ or $\cos ABC=0$ from cosine rule	**A1**	WWW
		2	
(b)	Centre = mid-point of $AC=(2,4)$	**B1**	
		1	

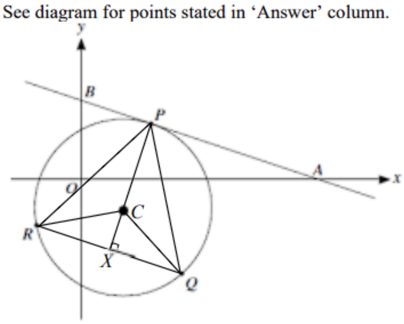
(c)	$(x - their\, x_c)^2 + (y - their\, y_c)^2 [= r^2]$ or $(their\, x_c - x)^2 + (their\, y_c - y)^2 = [r^2]$	M1	Use of circle equation with *their* centre
	$(x-2)^2 + (y-4)^2 = 17$	A1	Accept $x^2 - 4x + y^2 - 8y + 3 = 0$ OE
		2	
(d)	$\left(\frac{x+3}{2}, \frac{y+0}{2}\right) = (2,4)$ or $\mathbf{BE} = 2\mathbf{BD} = 2\begin{pmatrix}-1\\4\end{pmatrix}$ Or Equation of *BE* is $y = -4(x-3)$ or $y - 4 = -4(x-2)$ leading to $y = -4x + 12$ Substitute equation of *BE* into circle and form a 3-term quadratic.	M1	Use of mid-point formula, vectors, steps on a diagram May be seen to find *x* coordinate at *E*
	$(x, y) = (1, 8)$ or $\mathbf{OE} = \begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}-2\\8\end{pmatrix} = \begin{pmatrix}1\\8\end{pmatrix}$	A1	$E = (1, 8)$ Accept without working for both marks **SC B2**
	Gradient of *BD*, *m*, $= -4$ or gradient $AC = \frac{1}{4}$ = gradient of tangent	B1	Or gradient of *BE* = -4
	Equation of tangent is $y - 8 = ¼(x-1)$ OE	M1 A1	For M1, equation through *their* E or (1, 8) (not, *A*, *B* or *C*) and with gradient $\frac{-1}{their - 4}$
		5	

15.

(a)	$r^2 \left[= (5-2)^2 + (7-5)^2\right] = 13$	B1	$r^2 = 13$ or $r = \sqrt{13}$
	Equation of circle is $(x-5)^2 + (y-2)^2 = 13$	B1 FT	OE. FT on *their* 13 but LHS must be correct.
		2	
(b)	$(x-5)^2 + (5x - 10 - 2)^2 = 13$	M1	Substitute $y = 5x - 10$ into *their* equation.
	$26x^2 - 130x + 156\ [= 0]$	A1 FT	OE 3-term quadratic with all terms on one side. FT on *their* circle equation.
	$[26](x-2)(x-3)\ [= 0]$	M1	Solve 3-term quadratic in *x* by factorising, using formula or completing the square. Factors must expand to give *their* coefficient of x^2.
	(2, 0), (3, 5)	A1 A1	Coordinates must be clearly paired; A1 for each correct point. A1 A0 available if two *x* or *y* values only. If M0 for solving quadratic, **SC B2** can be awarded for correct coordinates, **SC B1** if two *x* or *y* values only.
	$(AB)^2 = (3-2)^2 + (5-0)^2$	M1	SOI. Using *their* points to find length of *AB*.
	$AB = \sqrt{26}$	A1	ISW. Dependent on final M1 only.
(b)	**Alternative method for question 7(b)**		
	$\left(\frac{y+10}{5} - 5\right)^2 + (y-2)^2 = 13$	M1	Substitute $x = \frac{y+10}{5}$ into *their* equation.
	$\frac{26y^2}{25} - \frac{26y}{5}\ [= 0]$	A1 FT	OE 2-term quadratic with all terms on one side. FT on *their* circle equation.
	$[26]y(y-5)\ [= 0]$	M1	Solve 2-term quadratic in *y* by factorising, using formula or completing the square. Factors must expand to give *their* coefficient of y^2.
	(2, 0), (3, 5)	A1 A1	Coordinates must be clearly paired; A1 for each correct point. A1 A0 available if two *x* or *y* values only. If M0 for solving quadratic, **SC B2** can be awarded for correct coordinates, **SC B1** if two *x* or *y* values only.
	$(AB)^2 = (3-2)^2 + (5-0)^2$	M1	SOI. Using *their* points to find length of *AB*.
	$AB = \sqrt{26}$	A1	ISW. Dependent on final M1 only.
		7	

16.

(a)	Centre is (3, – 2)	B1	
	Gradient of radius = $\frac{(their -2)-4}{(their\,3)-5}$[= 3]	*M1	Finding gradient using *their* centre (not (0, 0)) and *P* (5,4).
	Equation of tangent $y-4=-\frac{1}{3}(x-5)$	DM1	Using *P* and the negative reciprocal of *their* gradient to find the equation of *AB*.
	Sight of [x =]17 and [y =] $\frac{17}{3}$	A1	
	$\left[\text{Area}=\frac{1}{2}\times\frac{17}{3}\times 17=\right]\ \frac{289}{6}$	A1	Or $48\frac{1}{6}$ or AWRT 48.2.
	Alternative method for question 12(a)		
	$2x+2y\frac{dy}{dx}-6+4\frac{dy}{dx}=0$	B1	
	At *P*: $10+8\frac{dy}{dx}-6+4\frac{dy}{dx}=0\left[\Rightarrow\frac{dy}{dx}=-\frac{1}{3}\right]$	*M1	Find the gradient using *P* (5,4) in *their* implicit differential (with at least one correctly differentiated *y* term).
	Equation of tangent $y-4=-\frac{1}{3}(x-5)$	DM1	Using *P* and *their* value for the gradient to find the equation of *AB*.
	Sight of [x =]17 and [y =] $\frac{17}{3}$	A1	
	$\left[\text{Area}=\frac{1}{2}\times\frac{17}{3}\times 17=\right]\ \frac{289}{6}$	A1	Or $48\frac{1}{6}$ or AWRT 48.2.
(a) nt'd	**Alternative method for question 12(a)**		
	$\left[y=-2\pm\left(40-(x-3)^2\right)^{\frac{1}{2}}\text{ OE leading to}\right]\frac{dy}{dx}=(3-x)\left(31+6x-x^2\right)^{-\frac{1}{2}}$	B1	OE. Correct differentiation of rearranged equation.
	$\frac{dy}{dx}=(3-5)\left(31+6(5)-(5)^2\right)^{-\frac{1}{2}}\left[\Rightarrow\frac{dy}{dx}=-\frac{1}{3}\right]$	*M1	Find the gradient using x = 5 in *their* differential (with clear use of chain rule).
	Equation of tangent $y-4=-\frac{1}{3}(x-5)$	DM1	Using *P* and *their* value for the gradient to find the equation of *AB*.
	Sight of [x =]17 and [y =] $\frac{17}{3}$	A1	
	$\left[\text{Area}=\frac{1}{2}\times\frac{17}{3}\times 17=\right]\ \frac{289}{6}$	A1	Or $48\frac{1}{6}$ or AWRT 48.2.
		5	

(b)	Radius of circle = $\sqrt{40}$,	B1	Or $2\sqrt{10}$ or 6.32 AWRT or $r^2 = 40$.
	Area of $\Delta CRQ = \frac{1}{2}\times(their\ r)^2 \sin 120\left[=\frac{1}{2}\times 40\times\frac{\sqrt{3}}{2}\right]$ OR Area of $\Delta CQX = \frac{1}{2}\times\sqrt{40}\cos 30\times\sqrt{40}\cos 60$ OE $\left[=\frac{1}{2}\times\sqrt{30}\times\sqrt{10}\right]$ OR Area of circle – 3× Area of segment = $40\pi - 3\times(40\frac{\pi}{3} - 10\sqrt{3})$ OR $QR = \sqrt{120}$ or $2\sqrt{30}$ and area = $\frac{1}{2}QR^2\sin 60$	M1	Using $\frac{1}{2}r^2\sin\theta$ with *their* r and 120 or 60 $[\times 3]$ Using $\frac{1}{2}$ ×base×height in a correct right-angled triangle $[\times 6]$. Use of cosine rule and area of large triangle
	$30\sqrt{3}$	A1	AWRT 52[.0] implies B1M1A0.
		3	See diagram for points stated in 'Answer' column. 

17.

(a)	$x^2 + (2x+5)^2 = 20$ leading to $x^2 + 4x^2 + 20x + 25 = 20$	M1	Substitute $y = 2x + 5$ and expand bracket.
	$(5)(x^2 + 4x + 1)[=0]$	A1	3-term quadratic.
	$x = \frac{-4 \pm \sqrt{16-4}}{2}$	M1	OE. Apply formula or complete the square.
	$A = (-2+\sqrt{3}, 1+2\sqrt{3})$	A1	Or 2 correct x values.
	$B = (-2-\sqrt{3}, 1-2\sqrt{3})$	A1	Or all values correct. **SC B1** all 4 values correct in surd form without working. **SC B1** all 4 values correct in decimal form from correct formula or completion of the square
	$AB^2 = their\,(x_2 - x_1)^2 + their\,(y_2 - y_1)^2$	M1	Using *their* coordinates in a correct distance formula. Condone one sign error in $x_2 - x_1$ or $y_2 - y_1$
	$\left[AB^2 = 48 + 12 \text{ leading to}\right] AB = \sqrt{60}$	A1	OE. CAO. Do not accept decimal answer. Answer must come from use of surd form in distance formula.
		7	

(b)	$x^2+m^2(x-10)^2=20$	*M1	Finding equation of tangent and substituting into circle equation.
	$x^2(m^2+1)-20m^2x+20(5m^2-1)$ [=0]	DM1	OE. Brackets expanded and all terms collected on one side of the equation.
	$[b^2-4ac=]400m^4-80(m^2+1)(5m^2-1)$	M1	Using correct coefficients from *their* quadratic equation.
	$400m^4-80(5m^4+4m^2-1)=0 \rightarrow (-80)(4m^2-1)=0$	A1	OE. Must have '=0' for A1.
	$m=\pm\frac{1}{2}$	A1	
	Alternative method for question 9(b)		
	Length, l of tangent, is given by $l^2=10^2-20$	M1	
	$l=\sqrt{80}$	A1	
	$\tan\alpha=\frac{\sqrt{20}}{\sqrt{80}}=\frac{1}{2}$	M1 A1	Where α is the angle between the tangent and the x-axis.
	$m=\pm\frac{1}{2}$	A1	
		5	

18.

(a)	$(x+1)^2+(3x-22)^2=85$	M1	OE. Substitute equation of line into equation of circle.
	$10x^2-130x+400$ [=0]	A1	Correct 3-term quadratic
	$[10](x-8)(x-5)$ leading to $x=8$ or 5	A1	Dependent on factors or formula or completing of square seen.
	(8, 4), (5, –5)	A1	If M1A1A0A0 scored, then **SC B1** for correct final answer only.
		4	
(b)	Mid-point of AB = $\left(6\frac{1}{2},-\frac{1}{2}\right)$	M1	Any valid method
	Use of C = (–1, 2)	B1	SOI
	$r^2=\left(-1-6\frac{1}{2}\right)^2+\left(2+\frac{1}{2}\right)^2$	M1	Attempt to find r^2. Expect $r^2=62\frac{1}{2}$.
	Equation of circle is $(x+1)^2+(y-2)^2=62\frac{1}{2}$	A1	OE.
		4	

19.

(a)	Express as $(x+3)^2+(y-1)^2=26+9+1[=36]$	M1	Completing the square on x and y or using the form $x^2+y^2+2gx+2fy+c=0$, centre $(-g,-f)$ and radius $\sqrt{g^2+f^2-c}$. SOI by correct answer.
	Centre (−3, 1)	B1	
	Radius 6	B1	
	So lowest point is (−3, −5)	A1 FT	FT on *their* centre and *their* radius.
		4	
(b)	Intersects when $x^2+(kx-5)^2+6x-2(kx-5)-26=0$ **or** $(x+3)^2+(kx-5-1)^2=36$	*M1	Substituting $y=kx-5$ into *their* circle equation or rearranging and equating y.
	$x^2+k^2x^2-10kx+25+6x-2kx+10-26=0$ **or** $x^2+6x+9+k^2x^2-12kx+36=36$ leading to $k^2x^2+x^2+6x-12kx+9[=0]$ **or** $(k^2+1)x^2+(6-12k)x+9[=0]$	DM1 A1	Rearranging to 3-term quadratic (terms grouped, all on one side). Allow 1 error. Correct quadratic (need to see 9 as constant term).
	$(6-12k)^2-4(k^2+1)\times 9\ [>0]$ $\left[\text{leading to } 144k^2-144k+36-36k^2-36>0\right]$	DM1	Using discriminant $b^2-4ac[>0]$ with *their* values. Allow if in square root.
	$[108k^2-144k=0$ leading to$]$ $k=0$ or $k=\frac{4}{3}$	A1	Need not see method for solving.
	$k<0, k>\frac{4}{3}$	A1	Do not accept $\frac{4}{3}<k<0$.
		6	

20.

(a)	$1+1+a+b-12=0[\Rightarrow a+b=10]$ $4+36+2a-6b-12=0[\Rightarrow 2a-6b=-28]$	B1 B1	B1 for each equation. Allow unsimplified. Can be implied by correct values for a and b.
	$a=4, b=6$	B1	
	Centre is $\left(-\frac{their\ a}{2}, -\frac{their\ b}{2}\right)$ $[-2,-3]$	B1 FT	Or $x=-2, y=-3$
		4	

(b)	Gradient of AC is $\frac{1-\text{their } y}{1-\text{their } x}$ $[=\frac{1--3}{1--2}=\frac{1+3}{1+2}=\frac{4}{3}]$	*M1	Using *their* centre correctly.
	Gradient of tangent is $=\frac{-1}{\text{their }\frac{4}{3}}\left[=-\frac{3}{4}\right]$	A1 FT	Use of $m_1m_2=-1$ to obtain the gradient of the tangent.
	Equation: $y-1=$ '*their* $-\frac{3}{4}$'$(x-1)$ or $y=-\frac{3}{4}x+\frac{7}{4}$	DM1	Using $(1,1)$ with *their* gradient of the tangent at *A*.
	$3x+4y=7$ or $4y+3x=7$. or integer multiples of these	A1	
	Alternative method for question 8(b)		
	$2x+2y\frac{dy}{dx}+4+6\frac{dy}{dx}=0$	*M1	Implicit differentiation with at least one *y* term differentiated correctly.
	$8\frac{dy}{dx}=-6\Rightarrow\frac{dy}{dx}=-\frac{6}{8}$	A1	
	Equation: $y-1=$ '*their* $-\frac{3}{4}$'$(x-1)$ or $y=-\frac{3}{4}x+\frac{7}{4}$	DM1	Using $(1,1)$ with *their* gradient of the tangent at *A*.
	$3x+4y=7$ or $4y+3x=7$. or integer multiples of these	A1	
	Alternative method for question 8(b)		
	$\frac{dy}{dx}=\frac{1}{2}\{25-(x+2)^2\}^{-\frac{1}{2}}(-2x-4)$	*M1	Rearranging to form $y=$ and differentiating using the chain rule.
	$\frac{dy}{dx}=\frac{1}{2}(25-9)^{-\frac{1}{2}}(-6)=-\frac{6}{8}$	A1	
(b)	Equation: $y-1=$ '*their* $-\frac{3}{4}$'$(x-1)$ or $y=-\frac{3}{4}x+\frac{7}{4}$	DM1	Using $(1,1)$ with *their* gradient of the tangent at *A*.
	$3x+4y=7$ or $4y+3x=7$. or integer multiples of these	A1	
		4	

21.

(a)	Equation of BC is $\{y=\}\{2\}\{-3x\}$	B2, 1, 0	OE forms $y+4=-3(x-2)$ or $y-2=-3(x-0)$.
		2	
(b)	$(x-2)^2+(2-3x+4)^2=20$	*M1	OE Sub line equation into equation of circle to eliminate *y*.
	$10(x-2)^2=20$ or $[10](x^2-4x+2)[=0]$	A1	OE Accept $(10x^2-40x+20)$.
	$x-2=[\pm]\sqrt{2}$ or $x=\frac{4[\pm]\sqrt{16-8}}{2}$	DM1	Correctly solving *their* quadratic.
	$x=2-\sqrt{2}$	A1	OE only solution. Answer only **SC B1** If DM1 not scored.
	$y=3\sqrt{2}-4$	A1	OE only solution. Answer only **SC B1** If DM1 not scored.
		5	

22.

(a)	$x^2+(mx+10)^2=20$ **or** $y^2+\left(\frac{y-10}{m}\right)^2=20$ **or** $mx+10=\sqrt{20-x^2}$	*M1	Substitute equation of line into equation of circle.
	$x^2(1+m^2)+20mx+80\ [=0]$ **or** $y^2(m^2+1)-20y+(100-20m^2)[=0]$	A1	Collect terms into a 3 term quadratic.
	$(20m)^2-4(1+m^2)\times80[=0\Rightarrow 80m^2-320=0\Rightarrow[80](m^2-4)=0]$ **or** $(-20)^2-4(m^2+1)(100-20m^2)[=0\Rightarrow\ [80](m^4-4m^2)=0]$	DM1	Use $b^2-4ac[=0]$.
	$m=\pm2$	A1	Two values for m.
		4	

(b)	**Method 1: Use of quadratic**		
	$(1+2^2)x^2 \pm 20(2)x+80\ [=0 \Rightarrow 5x^2 \pm 40x+80=0]$ **or** $y^2(2^2+1)-20y+(100-20(2^2))[=0 \Rightarrow [5](y^2-4y+4)=0]$	M1	Sub *their* m into *their* quadratic in x or y **or** restart with *their* tangent equation and equation of circle.
	$[5](x \pm 4)^2=0 \Rightarrow x=\pm 4$ **or** $y=2$	A1	Correct solutions or one correct pair (x, y).
	$(-4,2),\ (4,2)$	A1	Two correct points with x and y paired correctly.
	Method 2: Using equation of normal		
	$2x+10=-\frac{1}{2}x$ **or** $-2x+10=\frac{1}{2}x$	M1	Equate tangent and normal and solve for x.
	$x=\pm 4$	A1	Two correct x-values or one correct pair (x, y).
	$(-4,2),\ (4,2)$	A1	Two correct points with x and y paired correctly.
		3	
(c)	**Method 1: Using angle at circumference**		
	$\cos BOA=\frac{\sqrt{20}}{10}$ or $\sin BOA=\frac{\sqrt{80}}{10}$ or $\tan BOA=\frac{\sqrt{80}}{\sqrt{20}}[=2]$	*M1	Use a trig function in triangle *AOB*.
	$BOA=63.4° \Rightarrow BOC=126.8°\text{ or }126.9°$	DM1	Strategy involving doubling
	$[BDC=]63.4°$	A1	AWRT
	Metho 2: Using cosine rule		
	$BC=8,\ BD=\sqrt{(\sqrt{20}+4)^2+2^2},\ CD=\sqrt{(\sqrt{20}-4)^2+2^2}$	*M1	Calculate two lengths in triangle *BCD*.
	$64=80-16\sqrt{5}\cos BDC$	DM1	Use cosine rule with *their* lengths
	$\cos BDC=\frac{\sqrt{5}}{5} \Rightarrow [BDC=]63.4°$	A1	AWRT
	Method 3: Subtract angles from 90°		
	Calculate one angle at D $[=13.28]$	*M1	*ODB* or angle between *CD* and the vertical from *D*
	Calculate a second angle at D $[=13.28]$ and subtract both from 90°	DM1	
	$[BDC=]63.4°$	A1	AWRT
		3	

23.

(a)	Mid-point AB is $\left(\frac{10+5}{2},\frac{2-1}{2}\right)\left[=\left(\frac{15}{2},\frac{1}{2}\right)\right]$	B1	Accept unsimplified.
	Gradient of $AB = \frac{-1-2}{10-5}=\frac{-3}{5}$ Gradient perpendicular $=\frac{5}{3}$	M1	For use of $\frac{\text{Change in } y}{\text{Change in } x}$, condone inconsistent order of x and y, and $m_1m_2=-1$.
	$\frac{y-\frac{1}{2}}{x-\frac{15}{2}}=\frac{5}{3}$ $\left[y-\frac{1}{2}=\frac{5}{3}\left(x-\frac{15}{2}\right)\right]$	A1	OE ISW Any correct version e.g. $y=\frac{5}{3}x-12$ or $5x-3y=36$.
		3	
(b)	[Radius =] $\sqrt{34}$ or 5.8 AWRT or [(radius)^2 =] 34	B1	Sight of $\sqrt{34}$ or 34. Condone confusion of r and r^2.
	$(x-5)^2+(y-2)^2$	B1	Sight of $(x-5)^2+(y-2)^2$
	$(x-5)^2+(y-2)^2=34$	B1	CAO ISW
	Alternative method for Question 1(b)		
	$x^2+y^2-10x-4y$	B1	
	$[c=]5$ or $[c=]-5$	B1	Substitution of (10, –1) into $x^2+y^2-10x-4y+c=0$.
	$x^2+y^2-10x-4y-5=0$	B1	
		3	

24.

(a)	$(5-2p)^2+(p+2)^2=(10-2p)^2+(3-p)^2$	M1 A1	Allow one sign error for M mark only.
	$25-20p+4p^2+p^2+4p+4=100-40p+4p^2+9-6p+p^2$ $30p=80 \rightarrow p=\frac{8}{3}$ oe	A1	Allow 2.67 AWRT.
		3	
(b)(i)	$m_{AC}=\frac{p+2}{2p-5}$ $m_{BC}=\frac{p-3}{2p-10}$	M1	Allow a sign error.
	$\frac{p+2}{2p-5}\times\frac{p-3}{2p-10}=-1$	M1	Use of $m_1m_2=-1$ with their m_{AC} and m_{BC}.
	$p^2-p-6=-(4p^2-30p+50) \rightarrow 5p^2-31p+44\ (=0)$	A1	
	$p=4$ (Ignore $p=\frac{11}{5}$)	A1	Factors $(p-4)(5p-11)$, or formula or completing square must be seen.
		4	
(b)(ii)	Mid-point of AB = (7½, ½)	B1	SOI
	$r^2=2½^2+2½^2$ $\left[=\frac{50}{4}\right]$ or $r=\sqrt{(2½^2+2½^2)}$ $\left[=\frac{5\sqrt{2}}{2}\right]$	*M1	Or $r^2=\frac{1}{4}(5^2+5^2)$ $\left[=\frac{50}{4}\right]$ etc.
	Equation of circle is $(x-\textit{their}\,7½)^2+(y-\textit{their}\,½)^2=\textit{their}\,\frac{50}{4}$	DM1	Must use r^2 not r or d or d^2
	$x^2+y^2-15x-y+44=0$	A1	CAO
		4	

25.

$r^2=(7+2)^2+(12-5)^2$	**B1**	Expect 130, may use *AC* rather than *r*.
Equation of circle is $(x+2)^2+(y-5)^2=130$	**B1 FT**	OE FT *their* 130, may use distance *BC* rather than circle.
$(x+2)^2+(-2x+21)^2=130$	**M1**	Substitute $y=-2x+26$ into a circle equation.
$5x^2-80x+315\,[=0]$ leading to $[5](x-9)(x-7)$	**M1**	Factorisation OE must be seen.
$x=9$	**A1**	With or without $x=7$.
$y=8$ OR (9, 8)	**A1**	$y=8$ or $(9,8)$ only. Both A1's dependent on the first M1.
	6	

26.

(a)	$x^2+(y-2)^2=100$	**B1**	OE e.g. $(x-0)^2+(y-2)^2=10^2$ ISW.
		1	
(b)	Gradient of radius $=\left[\frac{10-2}{6-0}=\right]\frac{4}{3}$ or gradient of tangent $=\frac{-3}{4}$	**M1**	OE SOI Use coordinates to find gradient of radius or differentiate to find m_T e.g. $2x+2(y-2)\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}=\frac{-3}{4}$ at (6, 10) $y=2+\sqrt{100-x^2}\Rightarrow\frac{dy}{dx}=\frac{1}{2}(100-x^2)^{-\frac{1}{2}}(-2x)=-\frac{3}{4}$.
	Equation of tangent is $y-10=-\frac{3}{4}(x-6)$ $\left[\Rightarrow y=-\frac{3}{4}x+\frac{29}{2}\right]$	**A1**	OE ISW Allow e.g. $\frac{58}{4}$.
		2	
(c)	Coordinates of centre of circle *Q* are $\left(0, their\frac{29}{2}\right)$	**M1**	SOI From a linear equation in **(b)**.
	Equation of circle *Q* is $x^2+\left(y-their\frac{29}{2}\right)^2=\left(\frac{5\sqrt{5}}{2}\right)^2\left[=\frac{125}{4}\right]$	**A1FT**	OE e.g. $(x-0)^2+(y-14.5)^2=31.25$ ISW.
	$x^2+(11-2)^2=100\Rightarrow x^2=19$ and $x^2+\left(11-\frac{29}{2}\right)^2=\frac{125}{4}\Rightarrow x^2=19$ OR e.g. $\frac{125}{4}-\left(y-\frac{29}{2}\right)^2+(y-2)^2=100\Rightarrow 25y=275\Rightarrow y=11$	**B1**	OE e.g. $x=[\pm]\sqrt{19}$, $x^2-19=x^2-19$ Correct argument to verify both y-coords are 11 ISW.
		3	
(d)	$x^2+\left(-\frac{3}{4}x+\frac{29}{2}-\frac{29}{2}\right)^2=\frac{125}{4}\left[\Rightarrow\frac{25}{16}x^2=\frac{125}{4}\Rightarrow x^2=20\right]$ or $y^2-29y+199[=0]$	**M1**	Substitute equation of *their* tangent into equation of *their* circle. May see $y=\sqrt{31.25-x^2}+14.5$.
	$x=\pm2\sqrt{5}$ or $y=\frac{29\mp3\sqrt{5}}{2}$	**A1**	OE e.g. $x=\pm\sqrt{20}$ For 2 x-values or 2 y-values or correct (x,y) pair.
	$y\left[=\left(-\frac{3}{4}\times\pm\sqrt{20}\right)+\frac{29}{2}\right]=\frac{29\mp3\sqrt{5}}{2}$	**A1**	OE e.g. $\frac{58}{4}+\frac{3\sqrt{20}}{4}$, $\frac{58}{4}-\frac{3\sqrt{20}}{4}$ Correct (x,y) pairs.
		3	

27.

(a)	$(x-a)^2+\left(\frac{1}{2}x+6-3\right)^2=20$ or using $x=2y-12$	*M1	Obtaining an unsimplified equation in x or y only.
	$\frac{5}{4}x^2+(3-2a)x+a^2-11[=0]$	A1	OE e.g. $5x^2+4(3-2a)x+4a^2-44$ Rearranging to get a correct 3-term quadratic on one side. Condone terms not grouped together. $5y^2-y(54+4a)+133+a^2+24$.
	$(3-2a)^2-4\times\frac{5}{4}(a^2-11)[=0]$	DM1	OE Using b^2-4ac on *their* 3 term quadratic $[=0]$.
	Method 1 for final 2 marks		
	Using $a=4$: $(3-8)^2-5(5)=0$	A1	Clearly substituting $a=4$.
	$a=-16$	B1	Condone no method shown for this value.
	Method 2 for final 2 marks		
	$-a^2-12a+64=0 \Rightarrow (a-4)(a+16)=0 \Rightarrow a=4$	A1	AG Full method clearly shown.
	$a=-16$	B1	Condone no method shown for this value.
		5	If M0, **SCB1** available for substituting $a=4$, finding P(2, 7) and verifying that $CP^2=20$.
(b)	Centre (4, 3) identified or used or the point P is (2, 7)	B1	
	∴ gradient of normal $=-2$	B1	SOI
	Forming normal equation using their gradient (not 0.5) and their centre or P	M1	Condone use of $(\pm4,\pm3)$.
	$\frac{y-3}{(x-4)}=-2$ or $y-7=-2(x-2)$	A1	OE Condone $f(x)=$.
		4	

28.

(a)	$(x-1)^2+(x-9+4)^2=40$	M1	Substitute line into circle.
	$x^2-6x-7\ [=0]$ leading to $(x+1)(x-7)\ [=0]$	M1	Simplify to 3-term quadratic and factorise OE.
	(–1, –10), (7, –2) or $x=-1$ and 7, $y=-10$ and –2	A1 A1	Answers only **SC B1**, **SC B1** but must see a correct quadratic equation.
		4	
(b)	[C is mid-point =] $\left(\frac{their\ x_1+their\ x_2}{2}, \frac{their\ y_1+their\ y_2}{2}\right)$	M1	Expect (3, –6).
	Radius = $\sqrt{(their\ x-their\ 3)^2+(their\ y-their\ (-6))^2}$ OR $their\sqrt{\left((7-(-1))^2+(-2-(-10))^2\right)}/2$	M1	Expect $\sqrt{32}$.
	$(x-3)^2+(y+6)^2=32$	A1	OE
		3	