PURE MATHEMATICS -1

9709

(March, June and November series 2020 – 2023 With marking scheme)

DIFFERENTIATION

EXERCISE -1

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1) SP-2020 _9709_1 Q1

The following points

$$A(0, 1), \quad B(1, 6), \quad C(1.5, 7.75), \quad D(1.9, 8.79) \quad \text{and} \quad E(2, 9)$$

lie on the curve $y = \text{f}(x)$. The table below shows the gradients of the chords AE and BE.

Chord	AE	BE	CE	DE
Gradient of chord	4	3		

(a) Complete the table to show the gradients of CE and DE. [2]

(b) State what the values in the table indicate about the value of $\text{f}'(2)$. [1]

2) SP-2020 _9709_1 Q8

A curve has equation $y = \dfrac{12}{3-2x}$.

(a) Find $\dfrac{dy}{dx}$. [2]

A point moves along this curve. As the point passes through A, the x-coordinate is increasing at a rate of 0.15 units per second and the y-coordinate is increasing at a rate of 0.4 units per second.

(b) Find the possible x-coordinates of A. [4]

3) MARCH 2020_9709_12 Q1

The function f is defined by $\text{f}(x) = \dfrac{1}{3x+2} + x^2$ for $x < -1$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

4) MARCH 2020_9709_12 Q4

A curve has equation $y = x^2 - 2x - 3$. A point is moving along the curve in such a way that at P the y-coordinate is increasing at 4 units per second and the x-coordinate is increasing at 6 units per second.

Find the x-coordinate of P. [4]

5) MARCH 2020_9709_12 Q10(a)(b)

The gradient of a curve at the point (x, y) is given by $\dfrac{dy}{dx} = 2(x+3)^{\frac{1}{2}} - x$. The curve has a stationary point at $(a, 14)$, where a is a positive constant.

(a) Find the value of a. [3]

(b) Determine the nature of the stationary point. [3]

(c) Find the equation of the curve. [4]

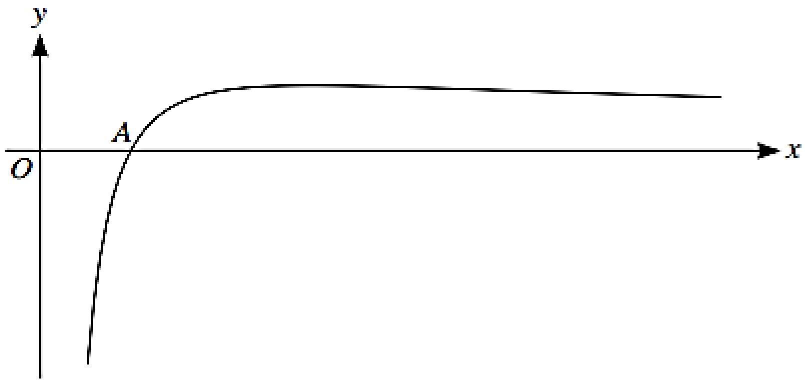
6) MARCH 2021_9709_12 Q6a

A curve is such that $\frac{dy}{dx} = \frac{6}{(3x-2)^3}$ and $A\,(1, -3)$ lies on the curve. A point is moving along the curve and at A the y-coordinate of the point is increasing at 3 units per second.

(a) Find the rate of increase at A of the x-coordinate of the point. [3]

(b) Find the equation of the curve. [4]

7) MARCH 2021_9709_12 Q11



The diagram shows the curve with equation $y = 9(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}})$. The curve crosses the x-axis at the point A.

(a) Find the x-coordinate of A. [2]

(b) Find the equation of the tangent to the curve at A. [4]

(c) Find the x-coordinate of the maximum point of the curve. [2]

(d) Find the area of the region bounded by the curve, the x-axis and the line $x = 9$. [4]

8) MARCH 2022_9709_12 Q11

It is given that a curve has equation $y = k(3x-k)^{-1} + 3x$, where k is a constant.

(a) Find, in terms of k, the values of x at which there is a stationary point. [4]

The function f has a stationary value at $x = a$ and is defined by

$$f(x) = 4(3x-4)^{-1} + 3x \quad \text{for } x \geqslant \tfrac{3}{2}.$$

(b) Find the value of a and determine the nature of the stationary value. [3]

(c) The function g is defined by $g(x) = -(3x+1)^{-1} + 3x$ for $x \geqslant 0$.

Determine, making your reasoning clear, whether g is an increasing function, a decreasing function or neither. [2]

9) MARCH 2023_9709_12 Q3

A curve has equation $y = \frac{1}{60}(3x+1)^2$ and a point is moving along the curve.

Find the x-coordinate of the point on the curve at which the x- and y-coordinates are increasing at the same rate. [4]

10) MARCH 2023_9709_12 Q10

At the point $(4, -1)$ on a curve, the gradient of the curve is $-\frac{3}{2}$. It is given that $\frac{dy}{dx} = x^{-\frac{1}{2}} + k$, where k is a constant.

(a) Show that $k = -2$. [1]

(b) Find the equation of the curve. [4]

(c) Find the coordinates of the stationary point. [3]

(d) Determine the nature of the stationary point. [2]

11) JUNE 2020_9709_11 Q9

The equation of a curve is $y = (3 - 2x)^3 + 24x$.

(a) Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(b) Find the coordinates of each of the stationary points on the curve. [3]

(c) Determine the nature of each stationary point. [2]

12) JUNE 2020_9709_12 Q3

A weather balloon in the shape of a sphere is being inflated by a pump. The volume of the balloon is increasing at a constant rate of $600\,\text{cm}^3$ per second. The balloon was empty at the start of pumping.

(a) Find the radius of the balloon after 30 seconds. [2]

(b) Find the rate of increase of the radius after 30 seconds. [3]

13) JUNE 2020_9709_12 Q10

The equation of a curve is $y = 54x - (2x - 7)^3$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(b) Find the coordinates of each of the stationary points on the curve. [3]

(c) Determine the nature of each of the stationary points. [2]

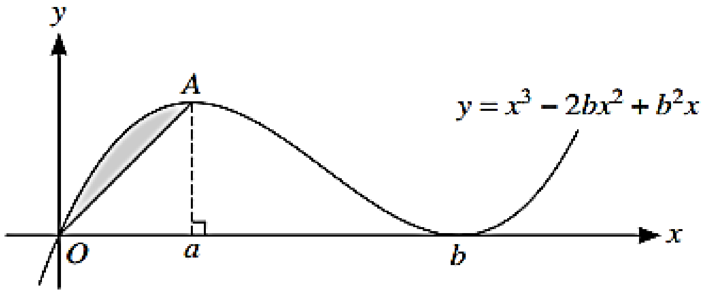
14) JUNE 2020_9709_13 Q6

A point P is moving along a curve in such a way that the x-coordinate of P is increasing at a constant rate of 2 units per minute. The equation of the curve is $y = (5x - 1)^{\frac{1}{2}}$.

(a) Find the rate at which the y-coordinate is increasing when $x = 1$. [4]

(b) Find the value of x when the y-coordinate is increasing at $\frac{5}{8}$ units per minute. [3]

15) JUNE 2020_9709_13 Q11(a)



The diagram shows part of the curve with equation $y = x^3 - 2bx^2 + b^2x$ and the line OA, where A is the maximum point on the curve. The x-coordinate of A is a and the curve has a minimum point at $(b, 0)$, where a and b are positive constants.

(a) Show that $b = 3a$. [4]

16) JUNE 2021_9709_11 Q11

The equation of a curve is $y = 2\sqrt{3x+4} - x$.

(a) Find the equation of the normal to the curve at the point $(4, 4)$, giving your answer in the form $y = mx + c$. [5]

(b) Find the coordinates of the stationary point. [3]

(c) Determine the nature of the stationary point. [2]

(d) Find the exact area of the region bounded by the curve, the x-axis and the lines $x = 0$ and $x = 4$. [4]

17) JUNE 2021_9709_12 Q11

The gradient of a curve is given by $\frac{dy}{dx} = 6(3x-5)^3 - kx^2$, where k is a constant. The curve has a stationary point at $(2, -3.5)$.

(a) Find the value of k. [2]

(b) Find the equation of the curve. [4]

(c) Find $\frac{d^2y}{dx^2}$. [2]

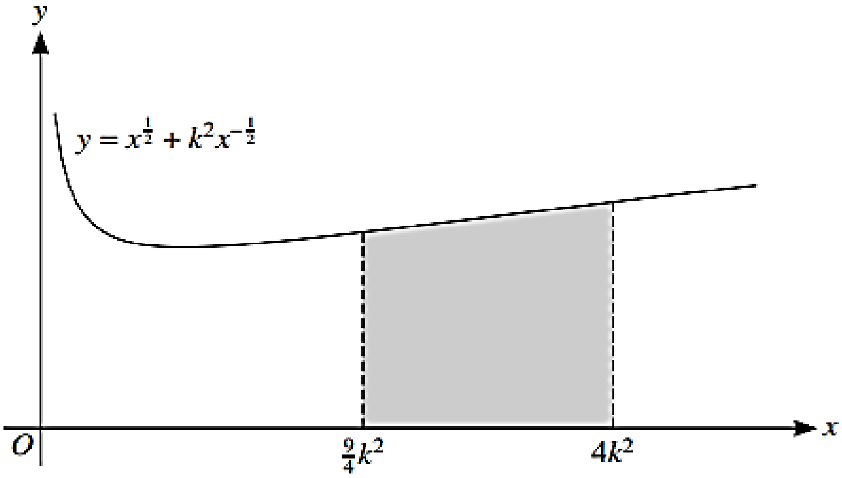
(d) Determine the nature of the stationary point at $(2, -3.5)$. [2]

18) JUNE 2021_9709_13 Q2

The function f is defined by $f(x) = \frac{1}{3}(2x-1)^{\frac{3}{2}} - 2x$ for $\frac{1}{2} < x < a$. It is given that f is a decreasing function.

Find the maximum possible value of the constant a. [4]

19) JUNE 2021_9709_13 Q11



The diagram shows part of the curve with equation $y = x^{\frac{1}{2}} + k^2x^{-\frac{1}{2}}$, where k is a positive constant.

(a) Find the coordinates of the minimum point of the curve, giving your answer in terms of k. **[4]**

The tangent at the point on the curve where $x = 4k^2$ intersects the y-axis at P.

(b) Find the y-coordinate of P in terms of k. **[4]**

The shaded region is bounded by the curve, the x-axis and the lines $x = \frac{9}{4}k^2$ and $x = 4k^2$.

(c) Find the area of the shaded region in terms of k. **[3]**

20) JUNE 2022_9709_11 Q10

The equation of a curve is such that $\frac{d^2y}{dx^2} = 6x^2 - \frac{4}{x^3}$. The curve has a stationary point at $\left(-1, \frac{9}{2}\right)$.

(a) Determine the nature of the stationary point at $\left(-1, \frac{9}{2}\right)$. **[1]**

(b) Find the equation of the curve. **[5]**

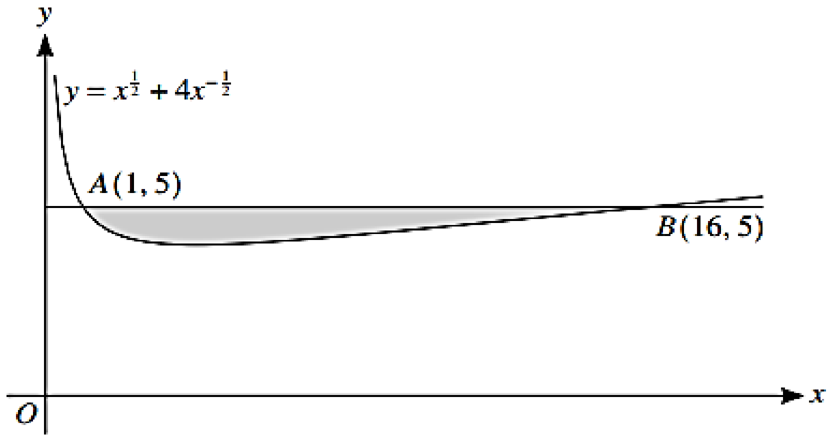
21) JUNE 2022_9709_12 Q9

The equation of a curve is $y = 3x + 1 - 4(3x + 1)^{\frac{1}{2}}$ for $x > -\frac{1}{3}$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. **[3]**

(b) Find the coordinates of the stationary point of the curve and determine its nature. **[4]**

22) JUNE 2022_9709_13 Q8a



The diagram shows the curve with equation $y = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$. The line $y = 5$ intersects the curve at the points $A\,(1, 5)$ and $B\,(16, 5)$.

(a) Find the equation of the tangent to the curve at the point A. [4]

23) JUNE 2022_9709_13 Q10b

The function f is defined by $f(x) = (4x + 2)^{-2}$ for $x > -\frac{1}{2}$.

A point is moving along the curve $y = f(x)$ in such a way that, as it passes through the point A, its y-coordinate is **decreasing** at the rate of k units per second and its x-coordinate is **increasing** at the rate of k units per second.

(b) Find the coordinates of A. [6]

24) JUNE 2023_9709_11 Q9

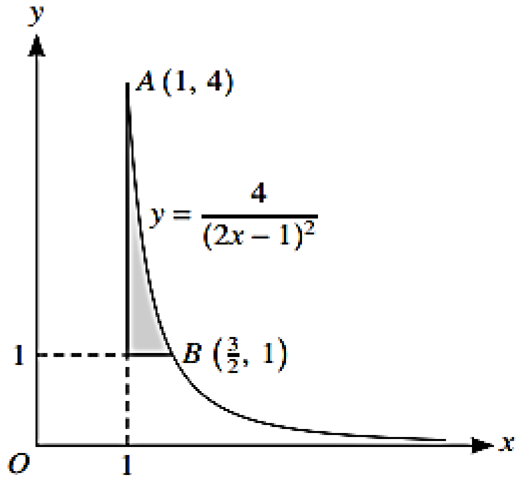
Water is poured into a tank at a constant rate of $500\,\text{cm}^3$ per second. The depth of water in the tank, t seconds after filling starts, is h cm. When the depth of water in the tank is h cm, the volume, $V\,\text{cm}^3$, of water in the tank is given by the formula $V = \frac{4}{3}(25 + h)^3 - \frac{62500}{3}$.

(a) Find the rate at which h is increasing at the instant when $h = 10$ cm. [3]

(b) At another instant, the rate at which h is increasing is 0.075 cm per second.

Find the value of V at this instant. [3]

25) JUNE 2023_9709_11 Q10b



The diagram shows part of the curve with equation $y = \frac{4}{(2x-1)^2}$ and parts of the lines $x = 1$ and $y = 1$. The curve passes through the points A (1, 4) and B, $\left(\frac{3}{2}, 1\right)$.

A triangle is formed from the tangent to the curve at B, the normal to the curve at B and the x-axis.

Find the area of this triangle. [6]

26) JUNE 2023_9709_11 Q11

The equation of a curve is such that $\frac{dy}{dx} = 6x^2 - 30x + 6a$, where a is a positive constant. The curve has a stationary point at $(a, -15)$.

(a) Find the value of a. [2]

(b) Determine the nature of this stationary point. [2]

(c) Find the equation of the curve. [3]

(d) Find the coordinates of any other stationary points on the curve. [2]

27) JUNE 2023_9709_12 Q 11

The equation of a curve is

$$y = k\sqrt{4x+1} - x + 5,$$

where k is a positive constant.

(a) Find $\frac{dy}{dx}$. [2]

(b) Find the x-coordinate of the stationary point in terms of k. [2]

(c) Given that $k = 10.5$, find the equation of the normal to the curve at the point where the tangent to the curve makes an angle of $\tan^{-1}(2)$ with the positive x-axis. [4]

28) JUNE 2023_9709_13 Q9(b)(c)

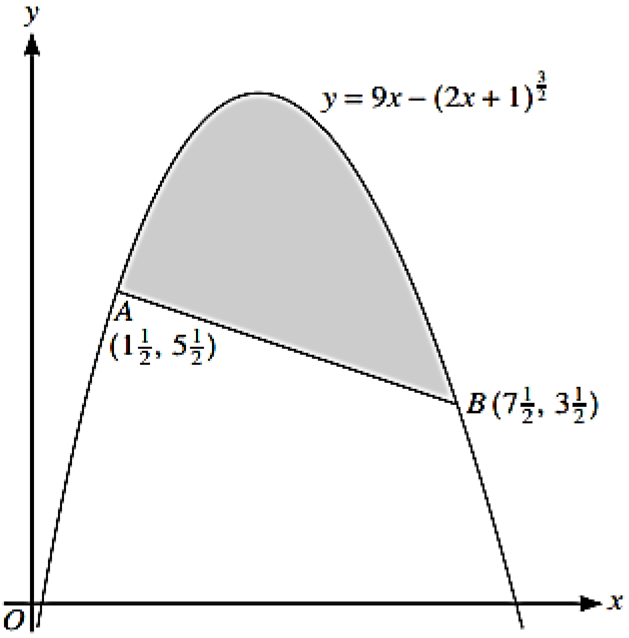
A curve which passes through (0, 3) has equation $y = f(x)$. It is given that $f'(x) = 1 - \frac{2}{(x-1)^3}$.

The tangent to the curve at (0, 3) intersects the curve again at one other point, P.

(b) Show that the x-coordinate of P satisfies the equation $(2x+1)(x-1)^2 - 1 = 0$. [4]

(c) Verify that $x = \frac{3}{2}$ satisfies this equation and hence find the y-coordinate of P. [2]

29) JUNE 2023_9709_13 Q10(a)(b)



The diagram shows the points $A\,(1\frac{1}{2}, 5\frac{1}{2})$ and $B\,(7\frac{1}{2}, 3\frac{1}{2})$ lying on the curve with equation $y = 9x - (2x+1)^{\frac{3}{2}}$.

(a) Find the coordinates of the maximum point of the curve. [4]

(b) Verify that the line AB is the normal to the curve at A. [3]

30) OCT 2020_9709_11 Q3

Air is being pumped into a balloon in the shape of a sphere so that its volume is increasing at a constant rate of $50\,\text{cm}^3\,\text{s}^{-1}$.

Find the rate at which the radius of the balloon is increasing when the radius is 10 cm. [3]

31) OCT 2020_9709_11 Q6

The equation of a curve is $y = 2 + \sqrt{25 - x^2}$.

Find the coordinates of the point on the curve at which the gradient is $\frac{4}{3}$. [5]

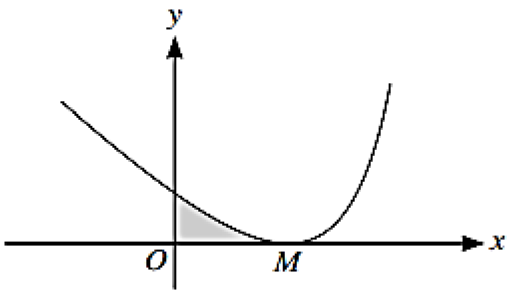
32) OCT2020_9709_12 Q7(a)

The point (4, 7) lies on the curve $y = f(x)$ and it is given that $f'(x) = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$.

(a) A point moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.12 units per second.

Find the rate of increase of the y-coordinate when $x = 4$. [3]

33) OCT2020_9709_12 Q10(a)(b)



The diagram shows part of the curve $y = \frac{2}{(3-2x)^2} - x$ and its minimum point M, which lies on the x-axis.

(a) Find expressions for $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\int y\,dx$. [6]

(b) Find, by calculation, the x-coordinate of M. [2]

34) OCT 2020_9709_13 Q8

The equation of a curve is $y = 2x + 1 + \frac{1}{2x+1}$ for $x > -\frac{1}{2}$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(b) Find the coordinates of the stationary point and determine the nature of the stationary point. [5]

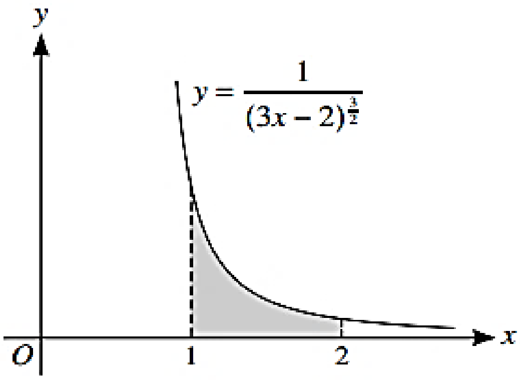
35) OCT 2020_9709_13 Q10(a)

A curve has equation $y = \frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2}$ where $x > 0$ and k is a positive constant.

(a) It is given that when $x = \frac{1}{4}$, the gradient of the curve is 3.

Find the value of k. [4]

36) OCT 2021_9709_11 Q10(c)



The diagram shows the curve with equation $y = \dfrac{1}{(3x-2)^{\frac{3}{2}}}$. The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$. The shaded region is rotated through 360° about the x-axis.

The normal to the curve at the point $(1,\ 1)$ crosses the y-axis at the point A.

(c) Find the y-coordinate of A. [4]

37) OCT 2021_9709_12 Q9

The volume $V\,\text{m}^3$ of a large circular mound of iron ore of radius r m is modelled by the equation $V = \frac{3}{2}\left(r - \frac{1}{2}\right)^3 - 1$ for $r \geqslant 2$. Iron ore is added to the mound at a constant rate of $1.5\,\text{m}^3$ per second.

(a) Find the rate at which the radius of the mound is increasing at the instant when the radius is 5.5 m. [3]

(b) Find the volume of the mound at the instant when the radius is increasing at 0.1 m per second. [3]

38) OCT 2021_9709_12 Q10

The function f is defined by $f(x) = x^2 + \dfrac{k}{x} + 2$ for $x > 0$.

(a) Given that the curve with equation $y = f(x)$ has a stationary point when $x = 2$, find k. [3]

(b) Determine the nature of the stationary point. [2]

(c) Given that this is the only stationary point of the curve, find the range of f. [2]

39) OCT 2021_9709_13 Q3

(a) Express $5y^2 - 30y + 50$ in the form $5(y + a)^2 + b$, where a and b are constants. [2]

(b) The function f is defined by $f(x) = x^5 - 10x^3 + 50x$ for $x \in \mathbb{R}$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

40) OCT 2021_9709_13 Q8(b)

The normal to the curve $y = x^{-\frac{1}{2}}$ at the point $(1,\ 1)$ intersects the y-axis at the point $(0,\ p)$.

Find the value of p. [4]

41) OCT 2022_9709_11 Q2(a)

The equation of a curve is such that $\frac{dy}{dx} = 12(\frac{1}{2}x - 1)^{-4}$. It is given that the curve passes through the point $P(6,\ 4)$.

(a) Find the equation of the tangent to the curve at P. [2]

42) OCT 2022_9709_11 Q3

A curve has equation $y = ax^{\frac{1}{2}} - 2x$, where $x > 0$ and a is a constant. The curve has a stationary point at the point P, which has x-coordinate 9.

Find the y-coordinate of P. [5]

43) OCT 2022_9709_11 Q 8(a)

The function f is defined by $f(x) = 2 - \frac{3}{4x - p}$ for $x > \frac{p}{4}$, where p is a constant.

(a) Find $f'(x)$ and hence determine whether f is an increasing function, a decreasing function or neither. [3]

44) OCT 2022_9709_12 Q8(b)(c)

The equation of a curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$. The curve passes through the point $(3,\ 5)$.

(a) Find the equation of the curve. [4]

(b) Find the x-coordinate of the stationary point. [2]

(c) State the set of values of x for which y increases as x increases. [1]

45) OCT 2022_9709_12 Q11(c)

A point P is moving along the curve $y = 18 - \frac{3}{8}x^{\frac{5}{2}}$ in such a way that the x-coordinate of P is increasing at a constant rate of 2 units per second.

Find the rate at which the y-coordinate of P is changing when $x = 4$. [3]

46) OCT 2022_9709_13 Q4

A large industrial water tank is such that, when the depth of the water in the tank is x metres, the volume $V\,\text{m}^3$ of water in the tank is given by $V = 243 - \frac{1}{3}(9 - x)^3$. Water is being pumped into the tank at a constant rate of $3.6\,\text{m}^3$ per hour.

Find the rate of increase of the depth of the water when the depth is 4 m, giving your answer in cm per minute. [5]

47) OCT 2022_9709_13 Q7(a)

The curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}'(x) = \dfrac{-3}{(x+2)^4}$.

(a) The tangent at a point on the curve where $x = a$ has gradient $-\frac{16}{27}$.

Find the possible values of a. **[4]**

MARKING SCHEME

1) SP-2020 _9709_1 Q1

1(a)	Gradient of $CE = 2.5$	1	B1	
	Gradient of $DE = 2.1$	1	B1	
		2		
1(b)	$f'(2) = 2$	1	B1	Accept reasonable conclusion following *their* gradient

2) SP-2020 _9709_1 Q8

(a)	$-12(3-2x)^{-2} \times -2$	2	B1B1	B1 for $-12(3-2x)^{-2}$, B1 for -2
(b)	$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = 0.4 \div 0.15$	1	M1	OE; chain rule used correctly
	$\frac{24}{(3-2x)^2} = \frac{8}{3}$	1	M1	Equates their $\frac{dy}{dx}$ with their $\frac{8}{3}$ or $\frac{3}{8}$ and method seen for solution of quadratic equation
	$x = 0$ or 3	2	A1A1	
		4		

3) MARCH 2020_9709_12 Q1

$f'(x) = \left[-(3x+2)^{-2}\right] \times [3] + [2x]$	B2, 1, 0	
< 0 hence decreasing	B1	Dependent on at least B1 for $f'(x)$ and must include < 0 or '(always) neg'
	3	

4) MARCH 2020_9709_12 Q4

$\frac{dy}{dx} = 2x - 2$	B1	
$\frac{dy}{dx} = \frac{4}{6}$	B1	OE, SOI
$their(2x-2) = their\frac{4}{6}$	M1	LHS and RHS must be *their* $\frac{dy}{dx}$ expression and value
$x = \frac{4}{3}$ oe	A1	
	4	

5) MARCH 2020_9709_12 Q10(a)(b)

(a)	$2(a+3)^{\frac{1}{2}} - a = 0$	M1	SOI. Set $\frac{dy}{dx} = 0$ when $x = a$. Can be implied by an answer in terms of a
	$4(a+3) = a^2 \to a^2 - 4a - 12 = 0$	M1	Take a to RHS and square. Form 3-term quadratic
	$(a-6)(a+2) \to a = 6$	A1	Must show factors, or formula or completing square. Ignore $a = -2$ **SC** If a is never used maximum of M1A1 for $x = 6$,with visible solution
		3	
(b)	$\frac{d^2y}{dx^2} = (x+3)^{-\frac{1}{2}} - 1$	B1	
	Sub *their* $a \to \frac{d^2y}{dx^2} = \frac{1}{3} - 1 = -\frac{2}{3}$ $(or < 0) \to$ MAX	M1A1	A mark only if completely correct If the second differential is not $-\frac{2}{3}$ correct conclusion must be drawn to award the M1
		3	

6) MARCH 2021_9709_12 Q6a

(a)	At $x=1$, $\frac{dy}{dx}=6$	B1	
	$\frac{dx}{dt}=\left(\frac{dx}{dy}\times\frac{dy}{dt}\right)=\frac{1}{6}\times 3=\frac{1}{2}$	M1 A1	Chain rule used correctly. Allow alternative and minimal notation.
		3	

7) MARCH 2021_9709_12 Q11

(a)	$9\left(x^{-\frac{1}{2}}-4x^{-\frac{3}{2}}\right)=0$ leading to $9x^{-\frac{3}{2}}(x-4)=0$	M1	OE. Set y to zero and attempt to solve.
	$x=4$ only	A1	From use of a correct method.
		2	
(b)	$\frac{dy}{dx}=9\left(-\frac{1}{2}x^{-\frac{3}{2}}+6x^{-\frac{5}{2}}\right)$	B2, 1, 0	B2; all 3 terms correct: 9, $-\frac{1}{2}x^{-\frac{3}{2}}$ and $6x^{-\frac{5}{2}}$ B1; 2 of the 3 terms correct
	At $x=4$ gradient $=9\left(-\frac{1}{16}+\frac{6}{32}\right)=\frac{9}{8}$	M1	Using *their* $x=4$ in *their* differentiated expression and attempt to find equation of the tangent.
	Equation is $y=\frac{9}{8}(x-4)$	A1	or $y=\frac{9x}{8}-\frac{9}{2}$ OE
		4	
(c)	$9x^{-\frac{5}{2}}\left(-\frac{1}{2}x+6\right)=0$	M1	Set *their* $\frac{dy}{dx}$ to zero and an attempt to solve.
	$x=12$	A1	Condone $(\pm)12$ from use of a correct method.
		2	
(d)	$\int 9\left(x^{-\frac{1}{2}}-4x^{-\frac{3}{2}}\right)dx=9\left(\frac{x^{\frac{1}{2}}}{\frac{1}{2}}-\frac{4x^{-\frac{1}{2}}}{-\frac{1}{2}}\right)$	B2, 1, 0	B2; all 3 terms correct: 9, $\frac{x^{\frac{1}{2}}}{\frac{1}{2}}$, $\frac{-4x^{-\frac{1}{2}}}{-\frac{1}{2}}$ B1; 2 of the 3 terms correct
	$9\left[\left(6+\frac{8}{3}\right)-(4+4)\right]$	M1	Apply limits *their* 4 → 9 to an integrated expression with no consideration of other areas.
	6	A1	Use of π scores A0
		4	

8) MARCH 2022_9709_12 Q11

a)	$\frac{dy}{dx}=\left\{-k(3x-k)^{-2}\right\}\{\times 3\}\{+3\}$	B2, 1, 0	
	$\frac{-3k}{(3x-k)^2}+3=0$ leading to $(3)(3x-k)^2=(3)k$ leading to $3x-k=[\pm]\sqrt{k}$	M1	Set $\frac{dy}{dx}=0$ and remove the denominator
	$x=\frac{k\pm\sqrt{k}}{3}$	A1	OE
		4	
b)	$a=\frac{4\pm\sqrt{4}}{3}$ leading to $a=2$	B1	Substitute $x=a$ when $k=4$. Allow $x=2$.
	$f''(x)=f'\left[-12(3x-4)^{-2}+3\right]=72(3x-4)^{-3}$	B1	Allow $18k(3x-k)^{-3}$
	>0 (or 9) when $x=2\rightarrow$ minimum	B1 FT	FT on *their* $x=2$, providing their $x\geqslant\frac{3}{2}$ and $f''(x)$ is correct

(c)	Substitute $k=-1$ leading to $g'(x)=\frac{3}{(3x+1)^2}+3$	M1	Condone one error.
	$g'(x)>0$ or $g'(x)$ always positive, hence g is an increasing function	A1	WWW. A0 if the conclusion depends on substitution of values into $g'(x)$.
	Alternative method for question 11(c)		
	$x=\frac{k\pm\sqrt{k}}{3}$ when $k=-1$ has no solutions, so g is increasing or decreasing	M1	Allow the statement 'no turning points' for increasing or decreasing
	Show g'(x) is positive for any value of x, hence g is an increasing function	A1	Or show $g(b)>g(a)$ for $b>a \rightarrow$ g, hence g is an increasing function
		2	

9) MARCH 2023_9709_12 Q3

$\frac{dy}{dx}=\left\{\frac{1}{60}(3x+1)\times 2\right\}\times\{3\}$	B1 B1	May see $\frac{1}{60}(18x+6)$.
$\frac{1}{10}(3x+1)=1$	M1	Equate *their* $\frac{dy}{dx}$ to 1.
$x=3$	A1	
	4	

10) MARCH 2023_9709_12 Q10

a)	$-\frac{3}{2}=\frac{1}{2}+k$ leading to $k=-2$	B1	AG Need to see $4^{-\frac{1}{2}}$ evaluated as $\frac{1}{4^{\frac{1}{2}}}$ or better.
		1	
b)	$[y]=2x^{\frac{1}{2}}-2x \quad [+c]$	M1 A1	Allow $\frac{x^{\frac{1}{2}}}{1/2}-2x$.
	$-1=4-8+c$	M1	Substitute $x=4$, $y=-1$ (c present) Expect $c=3$.
	$y=2x^{\frac{1}{2}}-2x+3$ or $y=2\sqrt{x}-2x+3$	A1	Allow if $f(x)=$ or $y=$ anywhere in the solution.
		4	
c)	$x^{-1/2}-2=0$	M1	Set *their* $\frac{dy}{dx}$ to zero.
	$x=\frac{1}{4}$	A1	If $\left(\frac{1}{2}\right)^2=\pm\frac{1}{4}$ max of M1A1 if $\left(\frac{1}{4},3\frac{1}{2}\right)$ seen.
	(¼, 3½)	A1	
		3	
d)	$\frac{d^2y}{dx^2}=-\frac{1}{2}x^{-\frac{3}{2}}$	B1	
	<0 (or -4) hence Maximum	DB1	WWW Ignore extra solutions from $x=-\frac{1}{4}$.
		2	

11) JUNE 2020_9709_11 Q9

(a)	$\frac{dy}{dx} = 3(3-2x)^2 \times -2 + 24 = -6(3-2x)^2 + 24$ (**B1** without ×−2. **B1** for ×−2)	**B1B1**
	$\frac{d^2y}{dx^2} = -12(3-2x)\times -2 = 24(3-2x)$ (**B1FT** from $\frac{dy}{dx}$ without − 2)	**B1FT** **B1**
		4
(b)	$\frac{dy}{dx} = 0$ when $6(3-2x)^2 = 24 \to 3-2x = \pm 2$	**M1**
	$x = ½, y = 20$ or $x = 2½, y = 52$ (**A1** for both x values or a correct pair)	**A1A1**
		3
(c)	If $x = ½$, $\frac{d^2y}{dx^2} = 48$ Minimum	**B1FT**
	If $x = 2½$, $\frac{d^2y}{dx^2} = -48$ Maximum	**B1FT**
		2

12) JUNE 2020_9709_12 Q3

(a)	Volume after 30 s = 18000 $\quad \frac{4}{3}\pi r^3 = 18000$	**M1**
	$r = 16.3$ cm	**A1**
		2
(b)	$\frac{dV}{dr} = 4\pi r^2$	**B1**
	$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt} = \frac{600}{4\pi r^2}$	**M1**
	$\frac{dr}{dt} = 0.181$ cm per second	**A1**
		3

13) JUNE 2020_9709_12 Q10

(a)	$\frac{dy}{dx} = 54 - 6(2x-7)^2$	**B2,1**
	$\frac{d^2y}{dx^2} = -24(2x-7)$ (**FT** only for omission of '×2 ' from the bracket)	**B2,1 FT**
		4
(b)	$\frac{dy}{dx} = 0 \to (2x-7)^2 = 9$	**M1**
	$x = 5, y = 243$ or $x = 2, y = 135$	**A1 A1**
		3
(c)	$x = 5$ $\frac{d^2y}{dx^2} = -72 \to$ Maximum (**FT** only for omission of '×2 ' from the bracket)	**B1FT**
	$x = 2$ $\frac{d^2y}{dx^2} = 72 \to$ Minimum (**FT** only for omission of '×2 ' from the bracket)	**B1FT**
		2

14) JUNE 2020_9709_13 Q6

(a)	$\frac{dy}{dx}=\left[\frac{1}{2}(5x-1)^{-1/2}\right]\times[5]$	B1 B1
	Use $\frac{dy}{dt}=2\times\left(their\frac{dy}{dx}\text{when } x=1\right)$	M1
	$\frac{5}{2}$	A1
		4
(b)	$2\times their\frac{5}{2}(5x-1)^{-1/2}=\frac{5}{8}$ oe	M1
	$(5x-1)^{1/2}=8$	A1
	$x=13$	A1
		3

15) JUNE 2020_9709_13 Q11(a)

$\frac{dy}{dx}=3x^2-4bx+b^2$	B1
$3x^2-4bx+b^2=0\to(3x-b)(x-b)\ (=0)$	M1
$x=\frac{b}{3}$ or b	A1
$a=\frac{b}{3}\ \to\ b=3a$ AG	A1
Alternative method for question 11(a)	
$\frac{dy}{dx}=3x^2-4bx+b^2$	B1
Sub $b=3a$ & obtain $\frac{dy}{dx}=0$ when $x=a$ and when $x=3a$	M1
$\frac{d^2y}{dx^2}=6x-12a$	A1
<0 Max at $x=a$ and >0 Min at $x=3a$. Hence $b=3a$ AG	A1
	4

16) JUNE 2021_9709_11 Q11

(a)	$\frac{dy}{dx}=3(3x+4)^{-0.5}-1$	B1 B1	B1 All correct with 1 error, B2 if all correct
	Gradient of tangent $=-\frac{1}{4}$ and Gradient of normal = 4	*M1	Substituting $x=4$ into a differentiated expression and using $m_1m_2=-1$
	Equation of line is $(y-4)=4(x-4)$ or evaluate c	DM1	With (4, 4) and *their* gradient of normal
	So $y=4x-12$	A1	
		5	
(b)	$3(3x+4)^{-0.5}-1=0$	M1	Setting *their* $\frac{dy}{dx}=0$
	Solving as far as $x=$	M1	Where $\frac{dy}{dx}$ contains $a(bx+c)^{-0.5}$ a, b, c any values
	$x=\frac{5}{3}$, $y=2\left(3\times\frac{5}{3}+4\right)^{0.5}-\frac{5}{3}=\frac{13}{3}$	A1	

(c)	$\frac{d^2y}{dx^2} = -\frac{9}{2}(3x+4)^{-1.5}$	M1	Differentiating *their* $\frac{dy}{dx}$ OR checking $\frac{dy}{dx}$ to find +ve and -ve either side of their $x = \frac{5}{3}$
	At $x = \frac{5}{3}$ $\frac{d^2y}{dx^2}$ is negative so the point is a maximum	A1	
		2	
(d)	Area $= \left[\int 2(3x+4)^{0.5} - x\,dx =\right] \frac{4}{9}(3x+4)^{1.5} - \frac{1}{2}x^2$	B1 B1	B1 for each correct term (unsimplified)
	$\left(\frac{4}{9}(16)^{1.5} - \frac{1}{2}(4)^2\right) - \frac{4}{9}(4)^{1.5} = \frac{256}{9} - 8 - \frac{32}{9}$	M1	Substituting limits 0 and 4 into an expression obtained by integrating y
	$16\frac{8}{9}$	A1	Or $\frac{152}{9}$
		4	

17) JUNE 2021_9709_12 Q11

(a)	At stationary point $\frac{dy}{dx} = 0$ so $6(3\times2-5)^3 - k\times2^2 = 0$	M1	Setting given $\frac{dy}{dx} = 0$ and substituting $x = 2$ into it.
	$[k =]\frac{3}{2}$	A1	OE
		2	
(b)	$[y =]\frac{6}{4\times3}(3x-5)^4 - \frac{1}{3}kx^3$ [$+c$].	*M1 A1FT	Integrating (increase of power by 1 in at least one term) given $\frac{dy}{dx}$. Expect $\frac{1}{2}(3x-5)^4 - \frac{1}{2}x^3$. FT *their* non zero k.
	$-\frac{7}{2} = \frac{1}{2}(3\times2-5)^4 - \frac{1}{3}\times\frac{3}{2}\times2^3 + c$ [leading to $-3.5 + c = -3.5$]	DM1	Using (2,-3.5) in an integrated expression. $+c$ needed. Substitution needs to be seen, simply stating $c = 0$ is DM0.
	$y = \frac{1}{2}(3x-5)^4 - \frac{1}{2}x^3$	A1	$y =$ or f(x)= must be seen somewhere in solution.
(b)	**Alternative method for Question 11(b)**		
	$[y =]\frac{81}{2}x^4 - \frac{541}{2}x^3 + 675x^2 - 750x(+c)$ or $-270x^3 - k\frac{x^3}{3}$	*M1 A1 FT	From $\frac{dy}{dx} = 162x^3 - 810x^2 - kx^2 - 1350x - 750$. FT *their* k
	$-\frac{7}{2} = \frac{81}{2}\times2^4 - \frac{541}{2}\times2^3 + 675\times2^2 - 750\times2 + c$	DM1	Using (2, -3.5) in an integrated expression. $+c$ needed
	$y = \frac{81}{2}x^4 - \frac{541}{2}x^3 + 675x^2 - 750x + \frac{625}{2}$	A1	$y =$ or f(x)= must be seen somewhere in solution.
		4	
(c)	$[3\times]\left[18(3x-5)^2\right][-2kx]$	B2,1,0 FT	FT *their* k. Square brackets indicate each required component. B2 for fully correct, B1 for one error or one missing component, B0 for 2 or more errors.
	Alternative method for Question 11(c)		
	$486x^2 - 1623x + 1350$ or $-1620x - 2kx$	B2,1,0 FT	FT *their* k. B2 for fully correct, B1 for one error, B0 for 2 or more errors.
		2	
(d)	[At $x = 2$] $\left[\frac{d^2y}{dx^2} =\right] 54(3\times2-5)^2 - 4k$ or 48	M1	OE. Substituting $x = 2$ into *their* second differential or other valid method.
	[> 0] Minimum	A1	WWW
		2	

18) JUNE 2021_9709_13 Q2

$\left[f'(x) =\right] \left((2x-1)^{1/2}\right) \times \left(\frac{1}{3} \times 2 \times \frac{3}{2}\right) (-2)$	B2, 1, 0	Expect $(2x-1)^{1/2} - 2$
$(2x-1)^{1/2} - 2 \leqslant 0 \;\rightarrow\; 2x-1 \leqslant 4$ or $2x-1 < 4$	M1	SOI. Rearranging and then squaring, must have power of ½ not present Allow '=0' at this stage but do not allow '≥ 0' or '> 0' If '–2' missed then must see $\leqslant$ or $<$ for the M1
Value [of a] is 2½ or $a = 2½$	A1	WWW, OE e.g. $\frac{5}{2}$, 2.5 Do not allow from '=0' unless some reference to negative gradient.
	4	

19) JUNE 2021_9709_13 Q11

(a)	$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} - \frac{1}{2}k^2 x^{-3/2}$	B1 B1	Allow any correct unsimplified form
	$\frac{1}{2}x^{-1/2} - \frac{1}{2}k^2 x^{-3/2} = 0$ leading to $\frac{1}{2}x^{-1/2} = \frac{1}{2}k^2 x^{-3/2}$	M1	OE. Set to zero and one correct algebraic step towards the solutions. $\frac{dy}{dx}$ must only have 2 terms.
	$(k^2, 2k)$	A1	
		4	
(b)	When $x = 4k^2$, $\frac{dy}{dx} = \left[\frac{1}{4k} - \frac{1}{16k} = \right] \frac{3}{16k}$	B1	OE
	$y = \left[2k + k^2 \times \frac{1}{2k}\right] = \frac{5k}{2}$	B1	OE. Accept $2k + \frac{k}{2}$
	Equation of tangent is $y - \frac{5k}{2} = \frac{3}{16k}(x - 4k^2)$ or $y = mx + c \rightarrow \frac{5k}{2} = \frac{3}{16k}(4k^2) + c$	M1	Use of line equation with *their* gradient and $(4k^2, their\ y)$,
	When $x = 0$, $y = \left[\frac{5k}{2} - \frac{3k}{4} = \right] \frac{7k}{4}$ or from $y = mx + c$, $c = \frac{7k}{4}$	A1	OE
		4	
(c)	$\int \left(x^{\frac{1}{2}} + k^2 x^{-\frac{1}{2}}\right) dx = \frac{2x^{\frac{3}{2}}}{3} + 2k^2 x^{\frac{1}{2}}$	B1	Any unsimplified form
	$\left(\frac{16k^3}{3} + 4k^3\right) - \left(\frac{9k^3}{4} + 3k^3\right)$	M1	Apply limits $\frac{9}{4}k^2 \rightarrow 4k^2$ to an integration of y. M0 if volume attempted.
	$\frac{49k^3}{12}$	A1	OE. Accept $4.08k^3$
		3	

20) JUNE 2022_9709_11 Q10

(a)	$\frac{d^2y}{dx^2}=6(-1)^2-\frac{4}{(-1)^3}>0\ \therefore$ minimum or $\frac{d^2y}{dx^2}=10\ \therefore$ minimum	B1	Sub $x=-1$ into $\frac{d^2y}{dx^2}$, correct conclusion. WWW
		1	
(b)	$\frac{dy}{dx}=2x^3+\frac{2}{x^2}[+c]$	*M1	Integrating $\frac{d^2y}{dx^2}$ (at least one term correct).
	$0=-2+2+c$ leading to $c=[0]$	DM1	Substituting $x=-1, \frac{dy}{dx}=0$ (need to see) to evaluate c. DM0 if simply state $c=0$ or omit $+c$.
	$y=\frac{1}{2}x^4-\frac{2}{x}+(\textit{their } c)x+k$	A1 FT	Integrated. FT *their* non-zero value of c if DM1 awarded.
	$\frac{9}{2}=\frac{1}{2}+2+k$ leading to $k=[2]$	DM1	Substituting $x=-1, y=\frac{9}{2}$ to evaluate k (dep on *M1).
	$y=\frac{1}{2}x^4-\frac{2}{x}+2$	A1	OE e.g. $2x^{-1}$ or $\frac{4}{2}$. A0 (wrong process) if c not evaluated but correct answer obtained.
)(c)	$\frac{dy}{dx}=2x^3+\frac{2}{x^2}=0$	M1	*Their* $\frac{dy}{dx}=0$.
	Leading to $x^5=-1$	M1	Reaching equation of the form $x^5=a$.
	So only stationary point is when $x=-1$	A1	$x=-1$ and stating e.g. 'only' or 'no other solutions.
		3	
(d)	At $x=1$, $\frac{dy}{dx}=[4]$	*M1	Substituting $x=1$ into *their* $\frac{dy}{dx}$.
	$\frac{dx}{dt}=\frac{dx}{dy}\times\frac{dy}{dt}=\frac{1}{4}\times 5$	DM1	OE Using chain rule correctly SOI.
	$\frac{5}{4}$	A1	OE e.g. 1.25.
		3	

21) JUNE 2022_9709_12 Q9

(a)	$\frac{dy}{dx}=\{3\}+\left\{-4\times\frac{1}{2}(3x+1)^{-\frac{1}{2}}\times 3\right\}\left[=3-6(3x+1)^{-\frac{1}{2}}\right]$	B1 B1	Correct differentiation of $3x+1$ and no other terms and correct differentiation of $-4(3x+1)^{\frac{1}{2}}$. Accept unsimplified.
	$\left[\frac{d^2y}{dx^2}=\right]-\frac{1}{2}\times-6(3x+1)^{-\frac{3}{2}}\times 3\ [=9(3x+1)^{-\frac{3}{2}}]$	B1	WWW. Accept unsimplified. Do not award if $\frac{dy}{dx}$ is incorrect.
		3	
(b)	$\frac{dy}{dx}=0$ leading to $3-6(3x+1)^{-\frac{1}{2}}=0$	M1	Setting *their* $\frac{dy}{dx}=0$.
	$(3x+1)^{\frac{1}{2}}=2 \Rightarrow 3x+1=4$ leading to $x=1$	A1	CAO – do not ISW for a second answer.
	$y=-4$ [coordinates $(1,-4)$]	A1	Condone inclusion of second value from a second answer.
	$\frac{d^2y}{dx^2}=9(3\times1+1)^{-\frac{3}{2}}=\frac{9}{8}$ or >0 so minimum	A1	Some evidence of substitution needed but $\frac{d^2y}{dx^2}$. Do not award if $\frac{d^2y}{dx^2}$ is incorrect or wrongly evaluated. Accept correct consideration of gradients either side of $x=1$.

22) JUNE 2022_9709_13 Q8a

(a)	$\left[\frac{dy}{dx}=\right]\frac{1}{2}x^{-1/2}-2x^{-3/2}$	B1 B1	Allow unsimplified versions.
	At $x=1$, $\frac{dy}{dx}=\frac{1}{2}-2=-\frac{3}{2}$	M1	Substitute $x=1$ into a differentiated y.
	Equation of tangent is $y-5=-\frac{3}{2}(x-1)$	A1	WWW Or $y=-\frac{3}{2}x+\frac{13}{2}$.

23) JUNE 2022_9709_13 Q10b

(b)	$\frac{dy}{dx}=\left\{-2(4x+2)^{-3}\right\}\ \{\times 4\}$	B1 B1	Allow unsimplified forms.
	Recognise $\frac{dy}{dx}=-1$	B1	SOI
	their $\frac{-8}{(4x+2)^3}=$ *their* -1	M1	Must be numerical. Must be some attempt to solve *their* equation and $\frac{dy}{dx}\neq 0$.
	(0, ¼)	A1 A1	Accept $x=0$, $y=$ ¼. $y=$ ¼ must be from $x=0$ not $x=-1$.
		6	

24) JUNE 2023_9709_11 Q9

(a)	$\frac{dV}{dh}=\frac{4}{3}\times 3(25+h)^2$ [$=4900$ when $h=10$]	B1	Correct expression for $\frac{dV}{dh}$.
	$\frac{dV}{dh}\times\frac{dh}{dt}=\frac{dV}{dt}\Rightarrow$ *their* "$4(25+10)^2$"$\times\frac{dh}{dt}=500\Rightarrow\frac{dh}{dt}=\left[\frac{500}{4900}\right]$	M1	Use chain rule correctly to find a numerical expression for $\frac{dh}{dt}$. Accept e.g. $\frac{500}{2500+2000+400}$.
	$\frac{dh}{dt}=0.102\left[\text{cms}^{-1}\right]$	A1	AWRT OE e.g. $\frac{5}{49}$ ISW.
		3	
(b)	$\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}\Rightarrow 500=$ *their* "$4(25+h)^2$"$\times 0.075$	*M1	SOI Use chain rule correctly to form equation in h.
	$\left[(25+h)^2=\frac{5000}{3}\right]\Rightarrow h=[15.8248\ldots]$	DM1	Solve quadratic to find h. Exact value of h is $\sqrt{\frac{5000}{3}}-25$ or $\frac{50\sqrt{6}}{3}-25$ $h+25=40.82\ldots$
	$V=69900\text{ cm}^3$	A1	AWRT ISW Look for 698(88.5).
		3	

25) JUNE 2023_9709_11 Q10b

b)	$\left[\frac{dy}{dx}=\right]\left\{-8(2x-1)^{-3}\right\}\{\times 2\}$	B2, 1, 0	OE B1 for each correct element in {}.
	At B gradient $=-2$	B1	
	Eqn of tangent $y-1=$ *their* "-2"$\left(x-\frac{3}{2}\right)$ OR Eqn of normal $y-1=$ *their* "$\frac{1}{2}$"$\left(x-\frac{3}{2}\right)$	M1	SOI Following differentiation OE e.g. $y=-2x+4$ or $y=\frac{1}{2}x+\frac{1}{4}$. (Must have $m_N=-\frac{1}{m_T}$ for M1).
	Tangent crosses x-axis at 2 or normal crosses x-axis at $-\frac{1}{2}$	A1	SOI For at least one intercept correct or correct integration.
	Area $=\frac{5}{4}$	A1	From intercepts: $\frac{1}{2}\times\frac{5}{2}\times 1=\frac{5}{4}$ or $1+\frac{1}{4}=\frac{5}{4}$, from lengths: $\frac{1}{2}\times\sqrt{5}\times\frac{\sqrt{5}}{2}=\frac{5}{4}$ or by integration.
		6	

26) JUNE 2023_9709_11 Q11

(a)	$6a^2-30a+6a=0$ $[\Rightarrow 6a(a-4)=0]$	B1	Sub $x=a$ into $\frac{dy}{dx}=0$. May see $a^2-5a+a=0$.
	$a=4$ only	B1	
		2	
(b)	$\frac{d^2y}{dx^2}=12x-30$ or correct values of $\frac{dy}{dx}$ either side of $x=4$	M1	Differentiate $\frac{dy}{dx}$ (mult. by power or dec. power by 1) M0 if no values of $\frac{dy}{dx}$, only signs.
	At $x=4, \frac{d^2y}{dx^2}>0$ $\therefore$ minimum or $\frac{d^2y}{dx^2}=18$ $\therefore$ minimum or concludes minimum from $\frac{dy}{dx}$ values	A1	WWW A0 XP if $a=4$ obtained incorrectly in (a) Must see 'minimum'. If M0, SC **B1** for 'minimum' from $\frac{dy}{dx}$ sign diagram.
		2	
(c)	$[y=]$ $\frac{6}{3}x^3-\frac{30}{2}x^2+6(\textit{their } a)x[+c]$	B1 FT	Expect $2x^3-15x^2+24x[+c]$. B1 poss. even if uses 'a' – no value in (a) – max 1/3.
	$-15=2(\textit{their } \text{"4"})^3-15(\textit{their } \text{"4"})^2+6(\textit{their } \text{"4"})^2+c$	M1	Sub $x=\textit{their}$"4", $y=-15$ into integral (must incl $+c$) Look for $-15=128-240+96+c$ $[\Rightarrow c=1]$.
	$y=2x^3-15x^2+24x+1$	A1	Coefficients must be correct and simplified. Need to see '$y=$' or '$f(x)=$' in the working.
		3	
(d)	$\frac{dy}{dx}=6x^2-30x+6(\textit{their } \text{"4"})[=0]$ If correct, $[6](x-1)(x-4)[=0]$ or $\frac{30\pm\sqrt{(-30)^2-4(6)(24)}}{12}$	M1	OE Forming a 3-term quadratic using the given $\frac{dy}{dx}$ and solving by factorisation, formula or completing the square. Check for working in (b).
	Coordinates $(1,12)$	A1	Allow $x=1, y=12$ (ignore $x=4$ if present). If M0, award SC **B1** for $(1,12)$.
		2	

27) JUNE 2023_9709_12 Q 11

(a)	$\frac{dy}{dx}=\left\{k\frac{1}{2}(4x+1)^{-\frac{1}{2}}\right\}\{\times 4\}\{-1\}$	B 2,1,0	OE e.g. $2k(4x+1)^{-\frac{1}{2}}-1$ B2 Three correct unsimplified { } and no others. B1 Two correct { } or three correct { } and an additional term e.g. +5. B0 More than one error.
		2	
(b)	$2k(4x+1)^{-\frac{1}{2}}-1=0$ leading to $(4x+1)^{\frac{1}{2}}=2k$ or $\frac{2k}{(4x+1)^{\frac{1}{2}}}=1$	M1	OE Equating their $\frac{dy}{dx}$ of the form $ak(4x+1)^{-\frac{1}{2}}-1$ where $a=2$ or 0.5, to 0 and dealing with the negative power correctly including k not multiplied by $(4x+1)^{\frac{1}{2}}$.
	$x=\frac{4k^2-1}{4}$	A1	CAO OE simplified expression ISW.
		2	
(c)	$2\times 10.5(4x+1)^{-\frac{1}{2}}-1=2$	M1	Putting k= 10.5 into their $\frac{dy}{dx}$ and equating to 2.
	$7=(4x+1)^{\frac{1}{2}}$ leading to $4x+1=49$ leading to $x=12$	A1	If M1 earned SCB1 available for $x=\frac{33}{64}$ from $a=\frac{1}{2}$.
	$y=[10.5\sqrt{4x+1}-x+5=]66.5$ [leading to (12, 66.5)]	A1	
	$y-66.5=-\frac{1}{2}(x-12)$	A1	OE
		4	

28) JUNE 2023_9709_13 Q9(b)(c)

b)	[Gradient of tangent =] $f'(0) = 3$	B1	
	Equation of tangent is $y - 3 = \textit{their}$ gradient at $x = 0(x - 0)$	M1*	Expect $y = 3x + 3$, normal gets M0.
	Intersection given by $3x + 3 = x + (x-1)^{-2} + 2$	DM1	FT *their* equation from part (a).
	$2x + 1 = \frac{1}{(x-1)^2} \rightarrow (2x+1)(x-1)^2 - 1 = 0$ or solve equation before given form reached and show solution ($x = 3/2$) satisfies given result	A1	WWW AG
		4	
c)	Substitute $x = \frac{3}{2}$ leading to $(2x+1)(x-1)^2 - 1$ leading to $4 \times ¼ - 1 = 0$. Hence $x = \frac{3}{2}$ If shown in (b) must be referenced here (in part (c))	B1	Evaluation of each bracket must be shown. Allow $\left(\frac{1}{2}\right)^2$ for second bracket. Solution of $(2x+1)(x-1)^2 - 1 = 0$ is acceptable.
	When $x = \frac{3}{2}$ $y = 7½$	B1	
		2	

29) JUNE 2023_9709_13 Q10(a)(b)

(a)	$\left[\frac{dy}{dx} =\right]\{9\} + \left\{-\frac{3}{2}(2x+1)^{1/2} \times 2\right\}$	B1, B1	Including '+c' makes the second term B0.
	$9 - 3(2x+1)^{1/2} = 0$ leading to $2x + 1 = 9$	M1	Set differential to zero and solve by squaring SOI. Beware $9^2 - 3^2(2x+1) = 0$ M0A0. $2x + 1 = \sqrt{3}$ *or* $2x + 1 = \pm 9$ get M0.
	Max point = (4, 9)	A1	WWW y = 9 must come from original equation.
		4	
(b)	When x = 1½, shows substitution or $\frac{dy}{dx} = 3$	M1	Substituting x = 1½ into their $\frac{dy}{dx}$.
	Gradient of *AB* is $\frac{5½ - 3½}{1½ - 7½}\left[= \frac{-1}{3}\right]$	M1	Substituting into a correct expression for m_{AB}.
	$-\frac{1}{3} x 3 = -1$. [Hence *AB* is the normal]	A1	
	Alternative method for Question 10(b)		
	When x = 1½ $\frac{dy}{dx} = 3$,[perpendicular gradient is -1/3]	M1	
	Perpendicular through A has equation $y = \frac{-x}{3} + 6$ which contains B(7.5,3.5) leading to AB is a normal to the curve at A	M1 A1	
		3	

30) OCT 2020_9709_11 Q3

(Derivative =) $4\pi r^2$ ($\rightarrow$ 400π)	B1	SOI Award this mark for $\frac{dr}{dV}$
$\frac{50}{\textit{their}\text{ derivative}}$	M1	Can be in terms of r
$\frac{1}{8\pi}$ or 0.0398	A1	AWRT
	3	

31) OCT 2020_9709_11 Q6

$\frac{dy}{dx}=\left[\frac{1}{2}(25-x^2)^{-1/2}\right]\times[-2x]$	B1 B1	
$\frac{-x}{(25-x^2)^{1/2}}=\frac{4}{3}\to\frac{x^2}{25-x^2}=\frac{16}{9}$	M1	Set = $\frac{4}{3}$ and square both sides
$16(25-x^2)=9x^2\to 25x^2=400\to x=(\pm)4$	A1	
When $x=-4, y=5\to(-4, 5)$	A1	
	5	

32) OCT2020_9709_12 Q7(a)

a)	$f'(4)\left(=\frac{5}{2}\right)$	*M1	Substituting 4 into $f'(x)$
	$\left(\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}\right)\to\left(\frac{dy}{dt}\right)=\frac{5}{2}\times 0.12$	DM1	Multiplies *their* $f'(4)$ by 0.12
	$\left(\frac{dy}{dt}=\right)0.3$	A1	OE
		3	

33) OCT2020_9709_12 Q10(a)(b)

a)	$\left(\frac{dy}{dx}\right)=[8]\times[(3-2x)^{-3}]+[-1]$	$\left(=\frac{8}{(3-2x)^3}-1\right)$	B2, 1, 0	B2 for all three elements correct, B1 for two elements correct, B0 for only one or no elements correct.
	$\frac{d^2y}{dx^2}=-3\times8\times(3-2x)^{-4}\times(-2)$	$\left(=\frac{48}{(3-2x)^4}\right)$	B1 FT	FT providing *their* bracket is to a negative power
	$\int y\,dx=[(3-2x)^{-1}][2\div(-1\times-2)][-\frac{1}{2}x^2](+c)$	$\left(=\frac{1}{3-2x}-\frac{1}{2}x^2+c\right)$	B1 B1 B1	Simplification not needed, B1 for each correct element
			6	
b)	$\frac{dy}{dx}=0\to(3-2x)^3=8\to 3-2x=k\to x=$		M1	Setting *their* 2-term differential to 0 and attempts to solve as far as $x=$
	$\frac{1}{2}$		A1	
	Alternative method for question 10(b)			
	$y=0\to\frac{2}{(3-2x)^2}-x=0\to(x-2)(2x-1)^2=0\to x=$		M1	Setting y to 0 and attempts to solve a cubic as far as $x=$ (3 factors needed)
	$\frac{1}{2}$		A1	
			2	

34) OCT 2020_9709_13 Q8

a)	$\frac{dy}{dx} = [2] \quad [-2(2x+1)^{-2}]$	B1 B1	
	$\frac{d^2y}{dx^2} = 8(2x+1)^{-3}$	B1	
		3	
b)	Set *their* $\frac{dy}{dx} = 0$ and attempt solution	M1	
	$(2x+1)^2 = 1 \to 2x+1 = (\pm)1$ or $4x^2+4x=0 \to (4)x(x+1)=0$	M1	Solving as far as $x =$ ....
	$x = 0$	A1	WWW. Ignore other solution.
	(0, 2)	A1	One solution only. Accept $x = 0, y = 2$ only.
	$\frac{d^2y}{dx^2} > 0$ from a solution $x > -\frac{1}{2}$ hence minimum	B1	Ignore other solution. Condone arithmetic slip in value of $\frac{d^2y}{dx^2}$. *Their* $\frac{d^2y}{dx^2}$ must be of the form $k(2x+1)^{-3}$
		5	

35) OCT 2020_9709_13 Q10(a)

a)	$\frac{dy}{dx} = \left[\frac{x^{-1/2}}{2k}\right] - \left[\frac{x^{-3/2}}{2}\right] + ([0])$	B2, 1, 0	$([0])$ implies that more than 2 terms counts as an error
	Sub $\frac{dy}{dx} = 3$ when $x = \frac{1}{4}$ Expect $3 = \frac{1}{k} - 4$	M1	
	$k = \frac{1}{7}$ (or 0.143)	A1	
		4	

36) OCT 2021_9709_11 Q10(c)

)(c)	$\frac{dy}{dx} = -\frac{3}{2} \times 3(3x-2)^{-\frac{5}{2}}$	M1	M1 for attempt to differentiate (power decreases); allow 1 error.
	At $x = 1$, $\frac{dy}{dx} = -\frac{9}{2}$	*M1	Substitute $x = 1$ into *their* differentiated expression; allow 1 error.
	[Equation of normal is] $y - 1 = \frac{2}{9}(x-1)$ OR evaluates c	DM1	Forms equation of line or evaluates c using (1, 1) and gradient $\frac{-1}{their \frac{dy}{dx}}$.
	At A, $y = \frac{7}{9}$	A1	OE e.g. AWRT 0.778; must clearly identify y-intercept
		4	

37) OCT 2021_9709_12 Q9

(a)	$\left[\frac{dV}{dr}=\right]\ \frac{9}{2}\left(r-\frac{1}{2}\right)^2$	B1	OE. Accept unsimplified.
	$\frac{dr}{dt}=\frac{dr}{dV}\times\frac{dV}{dt}=\frac{1.5}{their\ \frac{dV}{dr}}\ \left[=\frac{1.5}{\frac{9}{2}\left(5.5-\frac{1}{2}\right)^2}=\frac{1.5}{112.5}\right]$	M1	Correct use of chain rule with 1.5, *their* differentiated expression for $\frac{dV}{dr}$ and using $r = 5.5$.
	0.0133 or $\frac{3}{225}$ or $\frac{1}{75}$ [metres per second]	A1	
		3	
(b)	$\frac{dV}{dr}$ or *their* $\frac{dV}{dr}=\frac{1.5}{0.1}$ or 15 OR $0.1=\frac{1.5}{their\ \frac{dV}{dr}}\ \left[=\frac{2\times1.5}{9\left(r-\frac{1}{2}\right)^2}\text{OE}\right]$	B1 FT	Correct statement involving $\frac{dV}{dr}$ or *their* $\frac{dV}{dr}$, 1.5 and 0.1.
	$\left[\frac{9}{2}\left(r-\frac{1}{2}\right)^2=15\Rightarrow\right]\ r=\frac{1}{2}+\sqrt{\frac{10}{3}}$	B1	OE e.g. AWRT 2.3 Can be implied by correct volume.
	[Volume =] 8.13 AWRT	B1	OE e.g. $\frac{-3+5\sqrt{30}}{3}$. CAO.
		3	

38) OCT 2021_9709_12 Q10

(a)	$[f'(x)=]\ 2x-\frac{k}{x^2}$	B1	
	$f'(2)=0\ \left[2\times2-\frac{k}{2^2}=0\right]\Rightarrow k=\ldots$	M1	Setting *their* 2-term f′(2) = 0, at least one term correct and attempting to solve as far as $k =$.
	$k = 16$	A1	
		3	
(b)	$f''(2)=$ e.g. $2+\frac{2k}{2^3}$	M1	Evaluate a two term f″(2) with at least one term correct. Or other valid method.
	$\left[2+\frac{2k}{2^3}\right]>0\Rightarrow$ minimum or $=6\Rightarrow$ minimum	A1 FT	WWW. FT on positive k value.
		2	
(c)	When $x = 2$, $f(x) = 14$	B1	SOI
	[Range is or y or $f(x)$] $\geqslant$ *their* f(2)	B1 FT	Not $x \geqslant$ *their* f(2)
		2	

39) OCT 2021_9709_13 Q3

3(a)	$\{5(y-3)^2\}\quad\{+5\}$	B1 B1	Accept $a = -3$, $b = 5$
		2	
3(b)	$[f'(x)=]5x^4-30x^2+50$	B1	
	$5(x^2-3)^2+5$ or $b^2<4ac$ and at least one value of $f'(x)>0$	M1	
	> 0 and increasing	A1	WWW
		3	

40) OCT 2021_9709_13 Q8(b)

$\left[\frac{dy}{dx}=\right]-\frac{1}{2}x^{-\frac{3}{2}}$	B1	
When $x=1$, $m=-\frac{1}{2}$	M1	Substitute $x=1$ into a differential.
[Equation of normal is] $y-1=2(x-1)$	M1	Through (1, 1) with gradient $-\frac{1}{m}$ or $\frac{1-p}{1}=2$
[When $x=0$,] $p=-1$	A1	WWW
	4	

41) OCT 2022_9709_11 Q2(a)

$12\left(\frac{1}{2}\times 6-1\right)^{-4}\left[=12(2)^{-4}=\frac{3}{4}\right]$	M1	Substitute $x=6$ into $\frac{dy}{dx}$ SOI by gradient $\frac{3}{4}$ used.
$y-4=\frac{3}{4}(x-6)$ OR evaluates $c=-\frac{1}{2}$	A1	OE e.g. $y=\frac{3}{4}x-\frac{1}{2}$ or evaluates c in $y=\frac{3}{4}x+c$ using (6, 4) and gradient $\frac{3}{4}$. ISW
	2	

42) OCT 2022_9709_11 Q3

$\frac{dy}{dx}=\frac{1}{2}ax^{-\frac{1}{2}}-2$	B2, 1, 0	
$0=\frac{1}{2}a(9)^{-\frac{1}{2}}-2 \Rightarrow \frac{a}{6}-2=0 \Rightarrow a=[12]$	M1	Substitute $x=9$ and $\frac{dy}{dx}=0$ into *their* derivative and solve a linear equation for a.
$[a=]12$	A1	
$\left[y=\textit{their } a\times(9)^{\frac{1}{2}}-18=\right]18$	A1 FT	FT on *their* a.
	5	

43) OCT 2022_9709_11 Q 8(a)

a)	$f'(x)=-3(-1)(4)(4x-p)^{-2}\left[=\frac{12}{(4x-p)^2}\right]$	B2, 1, 0	
	>0 Hence increasing function	B1FT	Correct conclusion from *their* $f'(x)$.
		3	

44) OCT 2022_9709_12 Q8(b)(c)

(b)	$3x^{\frac{1}{2}}-3x^{-\frac{1}{2}}=0$	M1	Setting given differential to 0.
	[x=] 1	A1	CAO WWW Condone extra solution of —1 only if it is rejected.
		2	
(c)	$x>1$ or $x>$ "*their* 8(b)"	B1FT	Allow $\geqslant$
		1	

45) OCT 2022_9709_12 Q11(c)

$\left[\frac{dy}{dx}=\right]\frac{-5\times3}{2\times8}x^{\frac{3}{2}}\left[=-\frac{15}{16}x^{\frac{3}{2}}\right]$	B1	Allow unsimplified.
$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}\Rightarrow\frac{dy}{dt}=-\frac{15}{16}\times8\times2$	M1	Substitute $x=4$ into their $\frac{dy}{dx}$ and multiply by 2.
-15	A1	Accept decreasing [at/by] 15
	3	**Note:** If incorrect curve used, this is not a MR and only M1 mark is available. Expect $(\frac{9(4)}{2}-12)\times2[=12]$

46) OCT 2022_9709_13 Q4

$\left[\frac{dv}{dx}\right]=(9-x)^2$	B1	Allow unsimplified forms. Allow any or no notation
Substitute $x = 4$ into *their* differentiated V,	*M1	Expect 25.
$\frac{dx}{dt}=\frac{1}{\textit{their}\text{ derivative}}\times3.6$ (accept $\frac{dt}{dx}=\frac{\textit{their}\text{ derivative}}{3.6}$)	M1	Correct use of the chain rule, ignore incorrect conversions at this point. Expect 0.144
$=\frac{1}{\textit{their numerical}\text{ derivative}}\times3.6\times\frac{100}{60}$	DM1	Correct use of the conversion factors.
$=\frac{1}{25}\times3.6\times\frac{100}{60}=0.24$	A1	
	5	

47) OCT 2022_9709_13 Q7(a)

$\frac{-3}{(a+2)^4}=-\frac{16}{27}\to$ e.g. $16(a+2)^4=81$	M1	Equate first derivative and $-\frac{16}{27}$ and move term in a (or x) into the numerator.
$\to(a+2)^2=\frac{9}{4}\to a+2=[\pm]\frac{3}{2}$	M1	Solve for $(a+2)$ or $(x+2)$
$a=-\frac{1}{2}$ or $-\frac{7}{2}$	A1 A1	Allow 'x ='
	4	