PURE MATHEMATICS - 1

9709

(March, June and November series 2020 – 2023 With marking scheme)

Functions

Exercise - 1

HARSHIL MITTAL (A LEVEL)

KOTHARI INTERNATIONAL SCHOOL

NOIDA

1.

Functions f and g are defined by

$$f : x \mapsto 3x + 2, \quad x \in \mathbb{R},$$

$$g : x \mapsto 4x - 12, \quad x \in \mathbb{R}.$$

Solve the equation $f^{-1}(x) = gf(x)$. [4]

Question 2: 9709/01/SP/20

2.

(a) The curve $y = x^2 + 3x + 4$ is translated by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

Find and simplify the equation of the translated curve. [2]

(b) The graph of $y = f(x)$ is transformed to the graph of $y = 3f(-x)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [3]

Question 5: 9709/01/SP/20

3.

The graph of $y = f(x)$ is transformed to the graph of $y = 1 + f(\frac{1}{2}x)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [4]

Question 2: 9709/12/FM/20

4.

(a) Express $2x^2 + 12x + 11$ in the form $2(x + a)^2 + b$, where a and b are constants. [2]

The function f is defined by $f(x) = 2x^2 + 12x + 11$ for $x \leqslant -4$.

(b) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

The function g is defined by $g(x) = 2x - 3$ for $x \leqslant k$.

(c) For the case where $k = -1$, solve the equation $fg(x) = 193$. [2]

(d) State the largest value of k possible for the composition fg to be defined. [1]

Question 9: 9709/12/FM/20

5.

Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto \tfrac{1}{2}x - a,$$

$$g : x \mapsto 3x + b,$$

where a and b are constants.

(a) Given that $gg(2) = 10$ and $f^{-1}(2) = 14$, find the values of a and b. [4]

(b) Using these values of a and b, find an expression for $gf(x)$ in the form $cx + d$, where c and d are constants. [2]

Question 6: 9709/11/MJ/20

6.

The function f is defined for $x \in \mathbb{R}$ by

$$f : x \mapsto a - 2x,$$

where a is a constant.

(a) Express $ff(x)$ and $f^{-1}(x)$ in terms of a and x. [4]

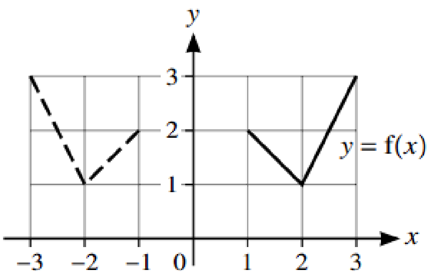
(b) Given that $ff(x) = f^{-1}(x)$, find x in terms of a. [2]

Question 5: 9709/12/MJ/20

7.

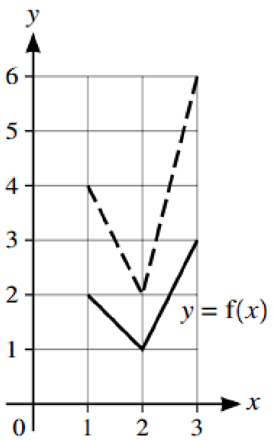
In each of parts **(a)**, **(b)** and **(c)**, the graph shown with solid lines has equation $y = f(x)$. The graph shown with broken lines is a transformation of $y = f(x)$.

(a)



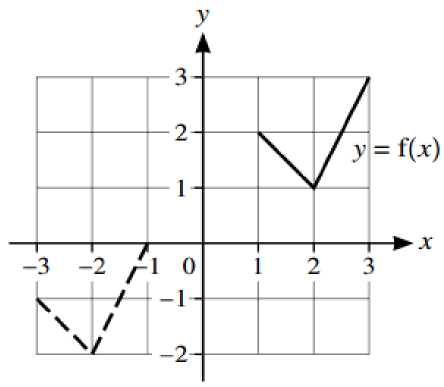
State, in terms of f, the equation of the graph shown with broken lines. [1]

(b)



State, in terms of f, the equation of the graph shown with broken lines. [1]

(c)



State, in terms of f, the equation of the graph shown with broken lines. [2]

Question 3: 9709/13/MJ/20

8.

The functions f and g are defined by

$$f(x) = x^2 + 3 \quad \text{for } x > 0,$$
$$g(x) = 2x + 1 \quad \text{for } x > -\tfrac{1}{2}.$$

(a) Find an expression for $fg(x)$. [1]

(b) Find an expression for $(fg)^{-1}(x)$ and state the domain of $(fg)^{-1}$. [4]

(c) Solve the equation $fg(x) - 3 = gf(x)$. [4]

Question 11: 9709/11/ON/20

9.

Functions f and g are defined by

$$f(x) = 4x - 2, \quad \text{for } x \in \mathbb{R},$$
$$g(x) = \frac{4}{x+1}, \quad \text{for } x \in \mathbb{R},\ x \neq -1.$$

(a) Find the value of $fg(7)$. [1]

(b) Find the values of x for which $f^{-1}(x) = g^{-1}(x)$. [5]

Question 5: 9709/12/ON/20

10.

The function f is defined by $f(x) = \dfrac{2x}{3x-1}$ for $x > \frac{1}{3}$.

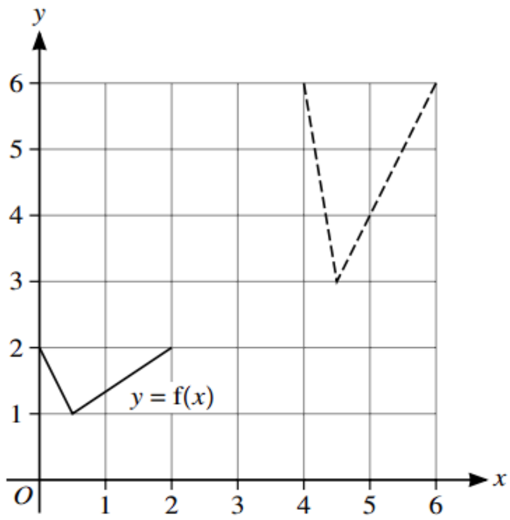
(a) Find an expression for $f^{-1}(x)$. [3]

(b) Show that $\dfrac{2}{3} + \dfrac{2}{3(3x-1)}$ can be expressed as $\dfrac{2x}{3x-1}$. [2]

(c) State the range of f. [1]

Question 6: 9709/13/ON/20

11.



In the diagram, the graph of $y = \mathrm{f}(x)$ is shown with solid lines. The graph shown with broken lines is a transformation of $y = \mathrm{f}(x)$.

(a) Describe fully the two single transformations of $y = \mathrm{f}(x)$ that have been combined to give the resulting transformation. [4]

(b) State in terms of y, f and x, the equation of the graph shown with broken lines. [2]

Question 5: 9709/12/FM/21

12.

Functions f and g are defined as follows:

$$\mathrm{f} : x \mapsto x^2 + 2x + 3 \text{ for } x \leqslant -1,$$
$$\mathrm{g} : x \mapsto 2x + 1 \text{ for } x \geqslant -1.$$

(a) Express $\mathrm{f}(x)$ in the form $(x + a)^2 + b$ and state the range of f. [3]

(b) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

(c) Solve the equation $\mathrm{gf}(x) = 13$. [3]

Question 7: 9709/12/FM/21

13.

Functions f and g are defined as follows:

$$\mathrm{f}(x) = (x - 2)^2 - 4 \text{ for } x \geqslant 2,$$
$$\mathrm{g}(x) = ax + 2 \text{ for } x \in \mathbb{R},$$

where a is a constant.

(a) State the range of f. [1]

(b) Find $f^{-1}(x)$. [2]

(c) Given that $a = -\frac{5}{3}$, solve the equation $f(x) = g(x)$. [3]

Question 9: 9709/11/MJ/21

14.

The function f is defined by $f(x) = 2x^2 + 3$ for $x \geqslant 0$.

(a) Find and simplify an expression for $ff(x)$. [2]

(b) Solve the equation $ff(x) = 34x^2 + 19$. [4]

Question 5: 9709/12/MJ/21

15.

Functions f and g are defined as follows:

$$f : x \mapsto x^2 - 1 \text{ for } x < 0,$$

$$g : x \mapsto \frac{1}{2x+1} \text{ for } x < -\tfrac{1}{2}.$$

(a) Solve the equation $fg(x) = 3$. [4]

(b) Find an expression for $(fg)^{-1}(x)$. [3]

Question 8: 9709/13/MJ/21

16.

(a) Express $-3x^2 + 12x + 2$ in the form $-3(x-a)^2 + b$, where a and b are constants. [2]

The one-one function f is defined by $f : x \mapsto -3x^2 + 12x + 2$ for $x \leqslant k$.

(b) State the largest possible value of the constant k. [1]

It is now given that $k = -1$.

(c) State the range of f. [1]

(d) Find an expression for $f^{-1}(x)$. [3]

The result of translating the graph of $y = f(x)$ by $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$ is the graph of $y = g(x)$.

(e) Express $g(x)$ in the form $px^2 + qx + r$, where p, q and r are constants. [3]

Question 8: 9709/11/ON/21

17.

The graph of $y = f(x)$ is transformed to the graph of $y = f(2x) - 3$.

(a) Describe fully the two single transformations that have been combined to give the resulting transformation. [3]

The point $P(5, 6)$ lies on the transformed curve $y = f(2x) - 3$.

(b) State the coordinates of the corresponding point on the original curve $y = f(x)$. [2]

Question 2: 9709/12/ON/21

18.

The function f is defined as follows:

$$f(x) = \frac{x+3}{x-1} \text{ for } x > 1.$$

(a) Find the value of ff(5). [2]

(b) Find an expression for $f^{-1}(x)$. [3]

Question 3: 9709/12/ON/21

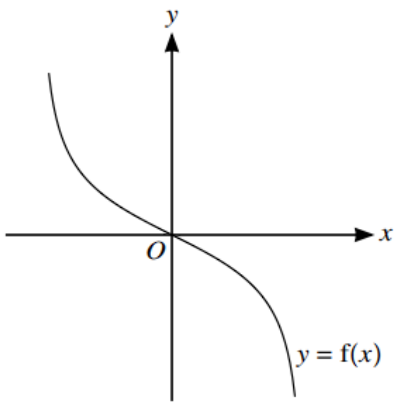
19.

The graph of $y = f(x)$ is transformed to the graph of $y = 3 - f(x)$.

Describe fully, in the correct order, the two transformations that have been combined. [4]

Question 1: 9709/13/ON/21

20.



The diagram shows the graph of $y = f(x)$.

(a) On this diagram sketch the graph of $y = f^{-1}(x)$. [1]

It is now given that $f(x) = -\frac{x}{\sqrt{4-x^2}}$ where $-2 < x < 2$.

(b) Find an expression for $f^{-1}(x)$. [4]

The function g is defined by $g(x) = 2x$ for $-a < x < a$, where a is a constant.

(c) State the maximum possible value of a for which fg can be formed. [1]

(d) Assuming that fg can be formed, find and simplify an expression for $fg(x)$. [2]

Question 6: 9709/13/ON/21

21.

(a) Express $2x^2 - 8x + 14$ in the form $2[(x-a)^2 + b]$. [2]

The functions f and g are defined by

$$f(x) = x^2 \quad \text{for } x \in \mathbb{R},$$
$$g(x) = 2x^2 - 8x + 14 \quad \text{for } x \in \mathbb{R}.$$

(b) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ onto the graph of $y = g(x)$, making clear the order in which the transformations are applied. [4]

Question 5: 9709/12/FM/22

22.

Functions f, g and h are defined as follows:

$$f : x \mapsto x - 4x^{\frac{1}{2}} + 1 \quad \text{for } x \geqslant 0,$$
$$g : x \mapsto mx^2 + n \quad \text{for } x \geqslant -2, \text{ where } m \text{ and } n \text{ are constants,}$$
$$h : x \mapsto x^{\frac{1}{2}} - 2 \quad \text{for } x \geqslant 0.$$

(a) Solve the equation $f(x) = 0$, giving your solutions in the form $x = a + b\sqrt{c}$, where a, b and c are integers. [4]

(b) Given that $f(x) \equiv gh(x)$, find the values of m and n. [4]

Question 9: 9709/12/FM/22

23.

The function f is defined as follows:

$$f(x) = \frac{x^2 - 4}{x^2 + 4} \quad \text{for } x > 2.$$

(a) Find an expression for $f^{-1}(x)$. [3]

(b) Show that $1 - \dfrac{8}{x^2 + 4}$ can be expressed as $\dfrac{x^2 - 4}{x^2 + 4}$ and hence state the range of f. [4]

(c) Explain why the composite function ff cannot be formed. [1]

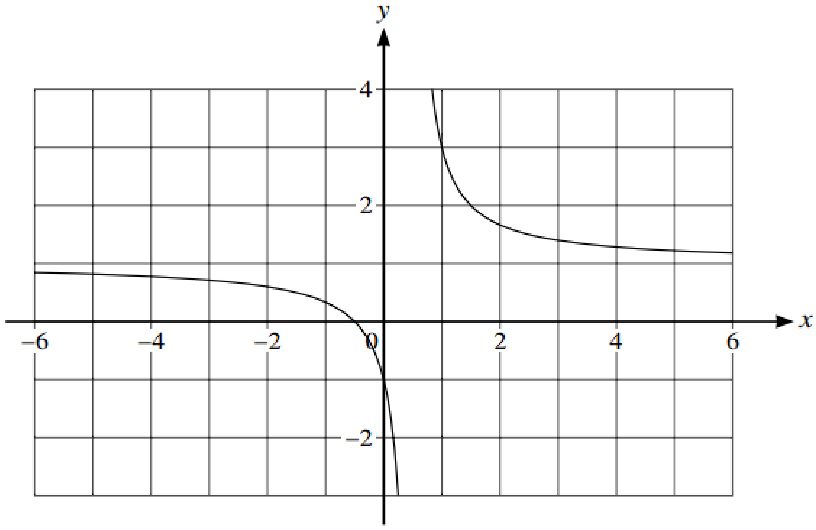
Question 6: 9709/11/MJ/22

24.

Functions f and g are defined as follows:

$$f(x) = \frac{2x+1}{2x-1} \quad \text{for } x \neq \tfrac{1}{2},$$
$$g(x) = x^2 + 4 \quad \text{for } x \in \mathbb{R}.$$

(a)



The diagram shows part of the graph of $y = f(x)$.

State the domain of f^{-1}. [1]

(b) Find an expression for $f^{-1}(x)$. [3]

(c) Find $gf^{-1}(3)$. [2]

(d) Explain why $g^{-1}(x)$ cannot be found. [1]

(e) Show that $1 + \frac{2}{2x-1}$ can be expressed as $\frac{2x+1}{2x-1}$. Hence find the area of the triangle enclosed by the tangent to the curve $y = f(x)$ at the point where $x = 1$ and the x- and y-axes. [6]

Question 10: 9709/12/MJ/22

25.

The function f is defined by $f(x) = 2x^2 - 16x + 23$ for $x < 3$.

(a) Express $f(x)$ in the form $2(x + a)^2 + b$. [2]

(b) Find the range of f. [1]

(c) Find an expression for $f^{-1}(x)$. [3]

The function g is defined by $g(x) = 2x + 4$ for $x < -1$.

(d) Find and simplify an expression for $fg(x)$. [2]

Question 6: 9709/13/MJ/22

26.

The function f is defined by $f(x) = 2 - \frac{3}{4x - p}$ for $x > \frac{p}{4}$, where p is a constant.

(a) Find $f'(x)$ and hence determine whether f is an increasing function, a decreasing function or neither. [3]

(b) Express $f^{-1}(x)$ in the form $\frac{p}{a} - \frac{b}{cx - d}$, where a, b, c and d are integers. [4]

(c) Hence state the value of p for which $f^{-1}(x) \equiv f(x)$. [1]

Question 8: 9709/11/ON/22

27.

Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 4x + 9,$$
$$g(x) = 2x^2 + 4x + 12.$$

(a) Express $f(x)$ in the form $(x - a)^2 + b$. [1]

(b) Express $g(x)$ in the form $2[(x + c)^2 + d]$. [2]

(c) Express $g(x)$ in the form $kf(x + h)$, where k and h are integers. [1]

(d) Describe fully the two transformations that have been combined to transform the graph of $y = f(x)$ to the graph of $y = g(x)$. [4]

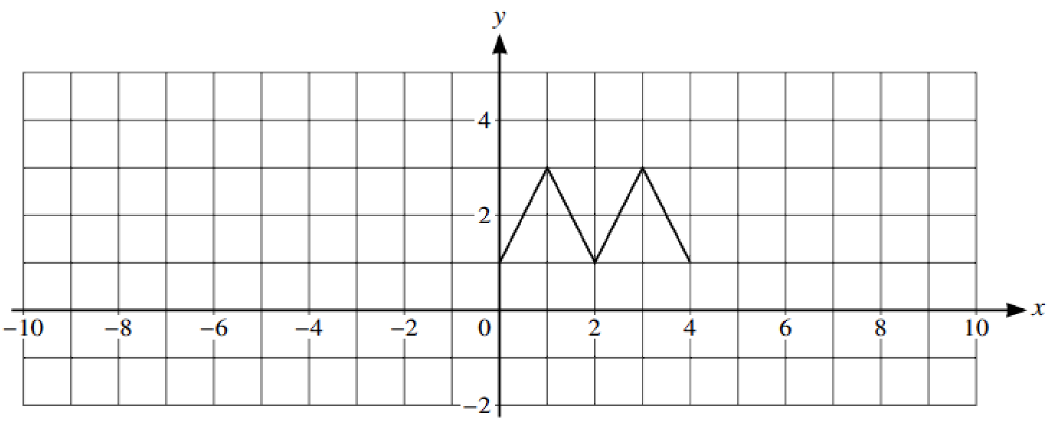
Question 9: 9709/11/ON/22

28.

The graph with equation $y = f(x)$ is transformed to the graph with equation $y = g(x)$ by a stretch in the x-direction with factor 0.5, followed by a translation of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

(a) The diagram below shows the graph of $y = f(x)$.

On the diagram sketch the graph of $y = g(x)$. [3]



(b) Find an expression for $g(x)$ in terms of $f(x)$. [2]

Question 5: 9709/12/ON/22

29.

Functions f and g are defined by

$$f(x) = x + \frac{1}{x} \quad \text{for } x > 0,$$
$$g(x) = ax + 1 \quad \text{for } x \in \mathbb{R},$$

where a is a constant.

(a) Find an expression for $gf(x)$. [1]

(b) Given that $gf(2) = 11$, find the value of a. [2]

(c) Given that the graph of $y = f(x)$ has a minimum point when $x = 1$, explain whether or not f has an inverse. [1]

It is given instead that $a = 5$.

(d) Find and simplify an expression for $g^{-1}f(x)$. [3]

(e) Explain why the composite function fg cannot be formed. [1]

Question 9: 9709/12/ON/22

30.

The function f is defined by $f(x) = -2x^2 - 8x - 13$ for $x < -3$.

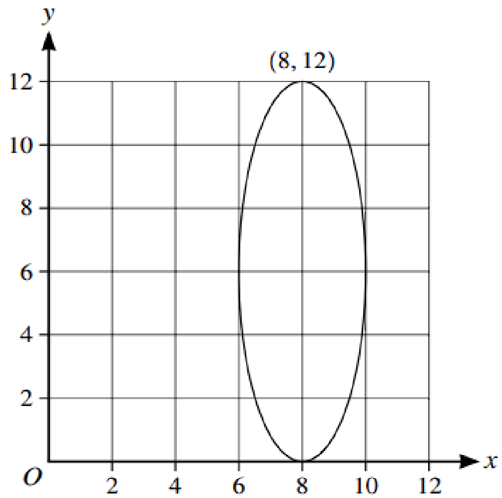
(a) Express $f(x)$ in the form $-2(x + a)^2 + b$, where a and b are integers. [2]

(b) Find the range of f. [1]

(c) Find an expression for $f^{-1}(x)$. [3]

Question 2: 9709/13/ON/22

31.



The diagram shows a curve which has a maximum point at $(8, 12)$ and a minimum point at $(8, 0)$. The curve is the result of applying a combination of two transformations to a circle. The first transformation applied is a translation of $\begin{pmatrix} 7 \\ -3 \end{pmatrix}$. The second transformation applied is a stretch in the y-direction.

(a) State the scale factor of the stretch. [1]

(b) State the radius of the original circle. [1]

(c) State the coordinates of the centre of the circle after the translation has been completed but before the stretch is applied. [2]

(d) State the coordinates of the centre of the original circle. [2]

Question 5: 9709/13/ON/22

32.

A function f is defined by $f(x) = x^2 - 2x + 5$ for $x \in \mathbb{R}$. A sequence of transformations is applied in the following order to the graph of $y = f(x)$ to give the graph of $y = g(x)$.

Stretch parallel to the x-axis with scale factor $\frac{1}{2}$

Reflection in the y-axis

Stretch parallel to the y-axis with scale factor 3

Find $g(x)$, giving your answer in the form $ax^2 + bx + c$, where a, b and c are constants. [4]

Question 2: 9709/13/ON/22

33.

The function f is defined by $f(x) = -3x^2 + 2$ for $x \leqslant -1$.

(a) State the range of f. [1]

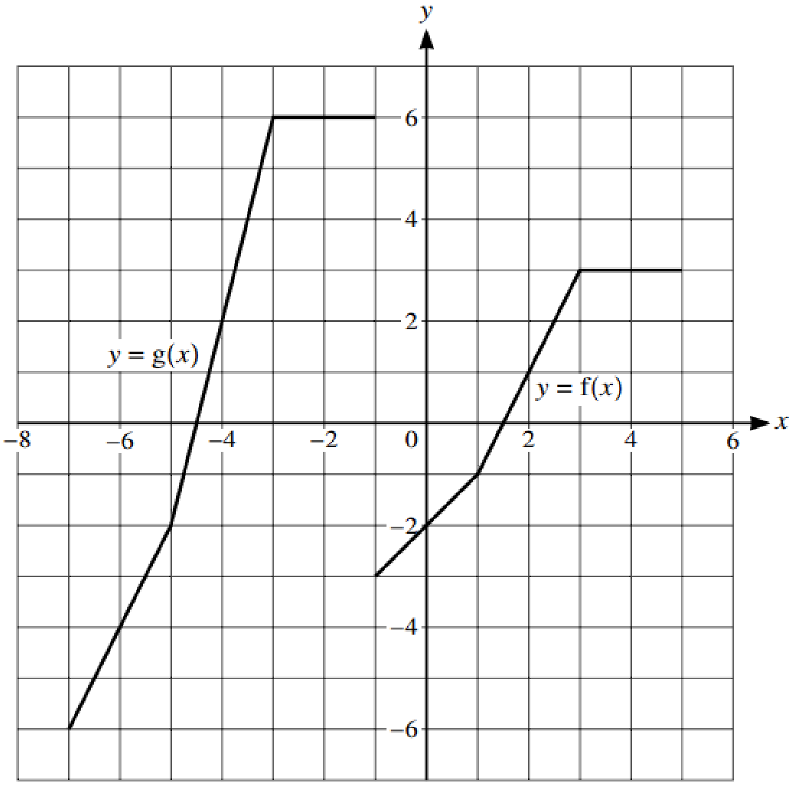
(b) Find an expression for $f^{-1}(x)$. [3]

The function g is defined by $g(x) = -x^2 - 1$ for $x \leqslant -1$.

(c) Solve the equation $fg(x) - gf(x) + 8 = 0$. [5]

Question 9: 9709/13/ON/22

34.



The diagram shows graphs with equations $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$.

Describe fully a sequence of two transformations which transforms the graph of $y = \mathrm{f}(x)$ to $y = \mathrm{g}(x)$. [4]

Question 3: 9709/11/MJ/23

35.

The functions f and g are defined as follows, where a and b are constants.

$$\mathrm{f}(x) = 1 + \frac{2a}{x-a} \text{ for } x > a$$

$$\mathrm{g}(x) = bx - 2 \text{ for } x \in \mathbb{R}$$

(a) Given that $\mathrm{f}(7) = \frac{5}{2}$ and $\mathrm{gf}(5) = 4$, find the values of a and b. [4]

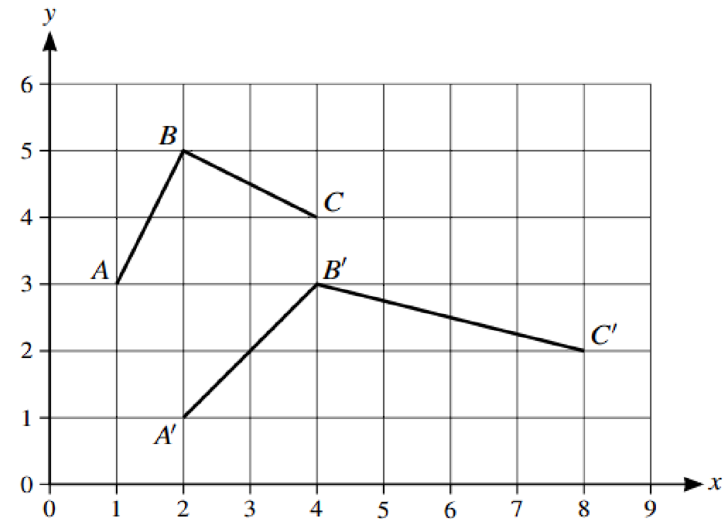
For the rest of this question, you should use the value of a which you found in **(a)**.

(b) Find the domain of f^{-1}. [1]

(c) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

Question 8: 9709/11/MJ/23

36.



The diagram shows the graph of $y = f(x)$, which consists of the two straight lines AB and BC. The lines $A'B'$ and $B'C'$ form the graph of $y = g(x)$, which is the result of applying a sequence of two transformations, in either order, to $y = f(x)$.

State fully the two transformations. [4]

Question 1: 9709/13/MJ/23

37.

The function f is defined, for $x \in \mathbb{R}$, by $f : x \mapsto x^2 + ax + b$, where a and b are constants.

(a) It is given that $a = 6$ and $b = -8$.

Find the range of f. [3]

(b) It is given instead that $a = 5$ and that the roots of the equation $f(x) = 0$ are k and $-2k$, where k is a constant.

Find the values of b and k. [3]

(c) Show that if the equation $f(x + a) = a$ has no real roots then $a^2 < 4(b - a)$. [3]

Question 11: 9709/01/SP/20

38.

The functions f and g are defined by

$$f(x) = x^2 - 4x + 3 \quad \text{for } x > c, \text{ where } c \text{ is a constant,}$$

$$g(x) = \frac{1}{x+1} \quad \text{for } x > -1.$$

(a) Express $f(x)$ in the form $(x - a)^2 + b$. [2]

It is given that f is a one-one function.

(b) State the smallest possible value of c. [1]

It is now given that $c = 5$.

(c) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

(d) Find an expression for $gf(x)$ and state the range of gf. [3]

Question 9: 9709/13/MJ/20

39.

(a) The graph of $y = f(x)$ is transformed to the graph of $y = 2f(x - 1)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [3]

(b) The curve $y = \sin 2x - 5x$ is reflected in the y-axis and then stretched by scale factor $\frac{1}{3}$ in the x-direction.

Write down the equation of the transformed curve. [2]

Question 2: 9709/12/MJ/21

40.

Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 2x + 5,$$
$$g(x) = x^2 + 4x + 13.$$

(a) By first expressing each of $f(x)$ and $g(x)$ in completed square form, express $g(x)$ in the form $f(x + p) + q$, where p and q are constants. [4]

(b) Describe fully the transformation which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. [2]

Question 6: 9709/13/MJ/21

41.

(a) The curve with equation $y = x^2 + 2x - 5$ is translated by $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

Find the equation of the translated curve, giving your answer in the form $y = ax^2 + bx + c$. [3]

(b) The curve with equation $y = x^2 + 2x - 5$ is transformed to a curve with equation $y = 4x^2 + 4x - 5$.

Describe fully the single transformation that has been applied. [2]

Question 4: 9709/13/MJ/22

42.

The function f is defined by $f(x) = 2 - \dfrac{5}{x + 2}$ for $x > -2$.

(a) State the range of f. [1]

(b) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

The function g is defined by $g(x) = x + 3$ for $x > 0$.

(c) Obtain an expression for $fg(x)$ giving your answer in the form $\dfrac{ax + b}{cx + d}$, where a, b, c and d are integers. [3]

Question 7: 9709/13/MJ/23

MARK SCHEME

1.

Answer	Marks	Partial Marks	Guidance
$f^{-1}(x) = \frac{x-2}{3}$	1	B1	
$gf(x) = 4(3x+2) - 12$	1	B1	
Equate $f^{-1}(x)$ and $gf(x)$ expressions, $x = \frac{2}{7}$	2	M1A1	
	4		

2.

Answer	Marks	Partial Marks	Guidance
$y = (x-2)^2 + 3(x-2) + 4 = x^2 - x + 2$	2	M1A1	
Reflection [in] y axis	1	B1	In either order
Stretch factor 3 in y direction	2	B1B1	B1 for stretch, B1 for factor 3 in y direction
	3		

3.

Answer	Marks	Guidance
[Stretch] [factor 2, x direction (or y-axis invariant)]	*B1 DB1	
[Translation or Shift] [1 unit in y direction] **or** [Translation/Shift] $\left[\begin{pmatrix}0\\1\end{pmatrix}\right]$	B1B1	Accept transformations in either order. Allow (0, 1) for the vector
	4	

4.

Answer	Marks	Guidance
$\left[2(x+3)^2\right]\,[-7]$	B1B1	Stating $a=3, b=-7$ gets B1B1
	2	
$y=2(x+3)^2-7 \to 2(x+3)^2=y+7 \to (x+3)^2=\frac{y+7}{2}$	M1	First 2 operations correct. Condone sign error or with x/y interchange
$x+3=(\pm)\sqrt{\frac{y+7}{2}} \to x=(\pm)\sqrt{\frac{y+7}{2}}-3 \to f^{-1}(x)=-\sqrt{\frac{x+7}{2}}-3$	A1FT	FT on *their* a and b. Allow $y = \ldots$
Domain: $x \geqslant -5$ **or** $\geqslant -5$ **or** $[-5, \infty)$	B1	Do not accept $y=\ldots,\ f(x)=\ldots, f^{-1}(x)=\ldots$
	3	
$fg(x) = 8x^2 - 7$	B1FT	SOI. FT on *their* -7 from part **(a)**
$8x^2-7=193 \to x^2=25 \to x=-5$ only	B1	
Alternative method for question 9(c)		
$g(x) = f^{-1}(193) \to 2x-3=-\sqrt{100}-3$	M1	FT on *their* $f^{-1}(x)$
$x=-5$ only	A1	
	2	
(Largest k is) $-\frac{1}{2}$	B1	Accept $-\frac{1}{2}$ or $k \leqslant -\frac{1}{2}$
	1	

5.

Answer	Marks
$3(3x+b)+b=9x+4b \rightarrow 10 = 18 + 4b$	M1
$b=-2$	A1
Either $f(14) = 2$ **or** $f^{-1}(x) = 2(x + a)$ etc.	M1
$a = 5$	A1
	4
$gf(x) = 3\left(\frac{1}{2}x-5\right)-2$	M1
$gf(x) = \frac{3}{2}x-17$	A1
	2

6.

Answer	Marks
$ff(x) = a-2(a-2x)$	M1
$ff(x) = 4x-a$	A1
$f^{-1}(x) = \frac{a-x}{2}$	M1 A1
	4
$4x-a=\frac{a-x}{2} \rightarrow 9x = 3a$	M1
$x=\frac{a}{3}$	A1
	2

7.

Answer	Marks
$(y)=f(-x)$	B1
	1
$(y)=2f(x)$	B1
	1
$(y)=f(x+4)-3$	B1 B1
	2

8.

Answer	Marks	Guidance
$\text{fg}(x) = (2x+1)^2 + 3$	B1	OE
	1	
$y = (2x+1)^2 + 3 \rightarrow 2x+1 = (\pm)\sqrt{y-3}$	M1	1st two operations. Allow one sign error or x/y interchanged
$x = (\pm)\frac{1}{2}\left(\sqrt{y-3} - 1\right)$	M1	OE 2nd two operations. Allow one sign error or x/y interchanged
$\left(\text{fg}^{-1}(x) =\right) \frac{1}{2}\left(\sqrt{x-3} - 1\right)$ for $(x) > 3$	A1 B1	Allow $(3, \infty)$
	4	
$\text{gf}(x) = 2\left(x^2+3\right)+1$	B1	SOI
$(2x+1)^2 + 3 - 3 = 2\left(x^2+3\right)+1 \rightarrow 2x^2 + 4x - 6\ (=0)$	*M1	Express as 3-term quadratic
$(2)(x+3)(x-1)\ (=0)$	DM1	Or quadratic formula or completing the square
$x = 1$	A1	
	4	

9.

Answer	Marks	Guidance
0	B1	
	1	
$(\text{f}^{-1}(x)) = \frac{x+2}{4}$, $(\text{g}^{-1}(x)) = \frac{4-x}{x}$ or $\frac{4}{x} - 1$	B1 B1	OE. Sight of correct inverses.
$x^2 + 6x - 16\ (= 0)$	B1	Equating inverses and simplifying.
$(x+8)$ and $(x-2)$	M1	Correct attempt at solution of *their* 3-term quadratic-factorising, completing the square or use of formula.
$(x =)\ 2$ or -8	A1	Do not accept answers obtained with no method shown.
	5	

10.

Answer	Marks	Guidance
$y = \frac{2x}{3x-1} \rightarrow 3xy - y = 2x \rightarrow 3xy - 2x = y$ (or $-y = 2x - 3xy$)	*M1	For 1st two operations. Condone a sign error
$x(3y-2) = y \rightarrow x = \frac{y}{3y-2}$ (or $x = \frac{-y}{2-3y}$)	DM1	For 2nd two operations. Condone a sign error
$\left(\text{f}^{-1}(x)\right) = \frac{x}{3x-2}$	A1	Allow $\left(\text{f}^{-1}(x)\right) = \frac{-x}{2-3x}$
	3	
$\left[\frac{2(3x-1)+2}{3(3x-1)}\right] = \left[\frac{6x}{3(3x-1)} = \frac{2x}{3x-1}\right]$	B1 B1	AG, WWW First B1 is for a correct single unsimplified fraction. An intermediate step needs to be shown. Equivalent methods accepted.
	2	

Answer	Marks	Guidance
$(\text{f}(x)) > \frac{2}{3}$	B1	Allow $(y) > \frac{2}{3}$. Do not allow $x > \frac{2}{3}$
	1	

11.

Answer	Marks	Guidance
(Stretch) (factor 3 in y direction **or** parallel to the y-axis)	**B1 B1**	
(Translation) $\begin{pmatrix}4\\0\end{pmatrix}$	**B1 B1**	Allow Translation 4 (units) in x direction. N.B. Transformations can be given in either order.
	4	
$[y =]\ 3\text{f}(x-4)$	**B1 B1**	B1 for 3 , B1 for $(x-4)$ with no extra terms.
	2	

12.

Answer	Marks	Guidance
$[\text{f}(x)=](x+1)^2+2$	**B1 B1**	Accept $a=1, b=2$.
Range [of f is (y)] $\geqslant 2$	**B1FT**	OE. Do not allow $x \geqslant 2$, FT on *their b*.
	3	
$y=(x+1)^2+2$ leading to $x=[\pm]\sqrt{y-2}-1$	**M1**	Or by using the formula. Allow one sign error.
$\text{f}^{-1}(x)=-\sqrt{x-2}-1$	**A1**	
	2	

Answer	Marks	Guidance
$2(x^2+2x+3)+1=13$	**B1**	Or using a correct completed square form of f(x).
$2x^2+4x-6[=0]$ leading to $(2)(x-1)(x+3)[=0]$	**B1**	Or $x=1, x=-3$ using formula or completing square. Must reach 2 solutions.
$x=-3$ **only**	**B1**	
	3	

13.

Answer	Marks	Guidance
Range of f is $\text{f}(x) \geqslant -4$	**B1**	Allow y, f or 'range' or $[-4,\infty)$
	1	
$y=(x-2)^2-4 \Rightarrow (x-2)^2=y+4 \Rightarrow x-2=+\sqrt{(y+4)}$ or $\pm\sqrt{(y+4)}$	**M1**	May swap variables here
$[\text{f}^{-1}(x)]=\sqrt{(x+4)}+2$	**A1**	
	2	
$(x-2)^2-4=-\frac{5}{3}x+2 \Rightarrow x^2-4x+4-4=-\frac{5}{3}x+2$ $[\Rightarrow x^2-\frac{7}{3}x-2=0]$	**M1**	Equating and simplifying to a 3-term quadratic
$(3x+2)(x-3)[=0]$ or $\frac{7\pm\sqrt{7^2-4(3)(-6)}}{6}$ OE	**M1**	Solving quadratic
$x=3$ only	**A1**	
	3	

Answer	Marks	Guidance
$f^{-1}(12) = 6$	**M1**	Substitute 12 into *their* $f^{-1}(x)$ and evaluate
$g(f^{-1}(12)) = 6a + 2$	**M1**	Substitute *their* '6' into $g(x)$
$g(g(f^{-1}(12))) = a(6a + 2) + 2 = 62$	**M1**	Substitute the result into $g(x)$ and = 62
$6a^2 + 2a - 60$ [= 0]	**M1**	Forming and solving a 3-term quadratic
$a = -\frac{10}{3}$ or 3	**A1**	
Alternative method for Question 9(d)		
$g(f^{-1}(x)) = a\left(\sqrt{x+4}+2\right)+2$ or $gg(x) = a(ax+2)+2$	**M1**	Substitute *their* $f^{-1}(x)$ or $g(x)$ into $g(x)$
$g(g(f^{-1}(x))) = a\left(a\left(\sqrt{x+4}+2\right)+2\right)+2$	**M1**	Substitute the result into $g(x)$
$g(g(f^{-1}(12))) = a(6a + 2) + 2 = 62$	**M1**	Substitute 12 and = 62
$6a^2 + 2a - 60$ [= 0]	**M1**	Forming and solving a 3-term quadratic
$a = -\frac{10}{3}$ or 3	**A1**	
	5	

14.

Answer	Marks	Guidance
$ff(x) = 2(2x^2 + 3)^2 + 3$	M1	Condone = 0.
$8x^4 + 24x^2 + 21$	A1	ISW if correct answer seen. Condone = 0.
	2	
$8x^4 + 24x^2 + 21 = 34x^2 + 19 \Rightarrow 8x^4 + 24x^2 - 34x^2 + 21 - 19$ [= 0]	M1	Equating $34x^3 + 19$ to *their* 3-term ff(x) and collect all terms on one side condone ± sign errors.
$8x^4 - 10x^2 + 2$[= 0]	A1	
$[2](x^2 - 1)(4x^2 - 1)$	M1	Attempt to solve 3-term quartic or 3-term quadratic by factorisation, formula or completing the square or factor theorem.
$\left[x^2 = 1 \text{ or } \frac{1}{4} \text{ leading to}\right] x = 1 \text{ or } x = \frac{1}{2}$	A1	If factorising, factors must expand to give $8x^4$ or $4x^4$ 4 or *their* ax^4 otherwise M0A0 due to calculator use. Condone ± 1, $\pm\frac{1}{2}$ but not $\sqrt{\frac{1}{4}}$ or $\sqrt{1}$.
	4	

15.

Answer	Marks	Guidance
$[\text{fg}(x) =]1/(2x+1)^2 - 1$	B1	SOI
$1/(2x+1)^2 - 1 = 3$ leading to $4(2x+1)^2 = 1$ or $\frac{1}{(2x+1)} = [\pm]2$ or $16x^2 + 16x + 3 = 0$	M1	Setting $\text{fg}(x) = 3$ and reaching a stage before $2x+1 = \pm½$ or reaching a 3 term quadratic in x
$2x+1 = \pm½$ or $2x+1 = -½$ or $(4x+1)(4x+3)[=0]$	A1	Or formula or completing square on quadratic
$x = -\frac{3}{4}$ only	A1	
Alternative method for Question 8(a)		
$x^2 - 1 = 3$	M1	
$\text{g}(x) = -2$	A1	
$\frac{1}{(2x+1)} = -2$	M1	
$x = -\frac{3}{4}$ only	A1	
	4	

Answer	Marks	Guidance
$y = \frac{1}{(2x+1)^2} - 1$ leading to $(2x+1)^2 = \frac{1}{y+1}$ leading to $2x+1 = [\pm]\frac{1}{\sqrt{y+1}}$	*M1	Obtain $2x+1$ or $2y+1$ as the subject
$x = [\pm]\frac{1}{2\sqrt{y+1}} - \frac{1}{2}$	DM1	Make x (or y) the subject
$-\frac{1}{2\sqrt{x+1}} - \frac{1}{2}$	A1	OE e.g. $-\frac{\sqrt{x+1}}{2x+2} - \frac{1}{2}$, $-\left(\sqrt{\frac{-x}{4x+4} + \frac{1}{4}} + \frac{1}{2}\right)$
	3	

16.

Answer	Marks	Guidance
$\{-3(x-2)^2\}$ $\{+14\}$	B1 B1	B1 for each correct term; condone $a = 2$, $b = 14$.
	2	
$[k =]\ 2$	B1	Allow $[x] \leqslant 2$.
	1	

Answer	Marks	Guidance
[Range is] [y] $\leqslant -13$	B1	Allow [f(x)] $\leqslant -13$, [f] $\leqslant -13$ but NOT $x \leqslant -13$.
	1	
$y = -3(x-2)^2 + 14$ leading to $(x-2)^2 = \frac{14-y}{3}$	M1	Allow $\frac{y-14}{-3}$. Allow 1 error in rearrangement if x, y on opposite sides.
$x = 2(\pm)\sqrt{\frac{14-y}{3}}$	A1	Allow $\frac{y-14}{-3}$.
$[f^{-1}(x)] = 2 - \sqrt{\frac{14-x}{3}}$	A1	OE. Allow $\frac{x-14}{-3}$. Must be x on RHS; must be negative square root only.
Alternative method for question 8(d)		
$x = -3(y-2)^2 + 14$ leading to $(y-2)^2 = \frac{14-x}{3}$	M1	Allow $\frac{x-14}{-3}$. Allow 1 error in rearrangement if x, y on opposite sides.
$= 2(\pm)\sqrt{\frac{14-x}{3}}$	A1	Allow $\frac{x-14}{-3}$.
$[f^{-1}(x)] = 2 - \sqrt{\frac{14-x}{3}}$	A1	OE. Allow $\frac{x-14}{-3}$. Must be x on RHS; must be negative square root only.
	3	

Answer	Marks	Guidance
[g(x) =] $\{-3(x+3-2)^2\} + \{14+1\}$	B2, 1, 0	OR $\{-3(x+3)^2\} + \{12(x+3)\} + \{3\}$
$g(x) = -3x^2 - 6x + 12$	B1	
	3	

17.

Answer	Marks	Guidance
Stretch with [scale factor] either ± 2 or $\pm\frac{1}{2}$	B1	
Scale factor $\frac{1}{2}$ in the x-direction	B1	
Translation $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$ or translation of 3 units in negative y-direction	B1	
	3	
(10 , 9)	B1 B1	B1 for each correct co-ordinate.
	2	

18.

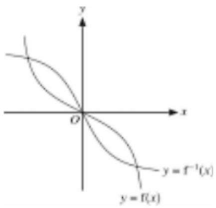
Answer	Marks	Guidance
$f(5) = [2]$ and $f(\textit{their}\ 2) = [5]$ OR $ff(5) = \left[\frac{2+3}{2-1}\right]$ OR $\dfrac{\frac{x+3}{x-1}+3}{\frac{x+3}{x-1}-1}$ and an attempt to substitute x =5.	M1	Clear evidence of applying f twice with x = 5.
5	A1	
	2	

Answer	Marks	Guidance
$\frac{x+3}{x-1}=y \Rightarrow x+3=xy-y$ OR $\frac{y+3}{y-1}=x \Rightarrow y+3=xy-x$	*M1	Setting f(*x*) = *y* or swapping *x* and *y*, clearing of fractions and expanding brackets. Allow ± sign errors.
$xy-x=y+3 \Rightarrow x=\frac{y+3}{y-1}$ OE OR $y+3=xy-x \Rightarrow y=\left[\frac{x+3}{x-1}\right]$ OE	DM1	Finding *x* or *y* = . Allow ± sign errors.
$[f^{-1}(x) \text{or } y]=\frac{x+3}{x-1}$	A1	OE e.g. $1+\frac{4}{x-1}$ etc. Must be a function of *x*, cannot be *x* = .
	3	

19.

Answer	Marks	Guidance
{Reflection} {[in the] *x*-axis} or {Stretch of scale factor -1} {parallel to y-axis}	*B1 DB1	{} indicate how the B1 marks should be awarded throughout.
Then {Translation} $\left\{\begin{pmatrix}0\\3\end{pmatrix}\right\}$	B1 B1	Or Translation 3 units in the positive *y*-direction. **N.B.** If order reversed a maximum of 3 out of 4 marks awarded.
Alternative method for question 1		
{Translation} $\left\{\begin{pmatrix}0\\-3\end{pmatrix}\right\}$	B1 B1	Or Translation 3 units in the negative *y*-direction.
Then {Reflection} {in the *x*-axis} or {Stretch of scale factor -1} {parallel to *y*-axis}	*B1 DB1	**N.B.** If order reversed a maximum of 3 out of 4 marks awarded.
	4	

20.

Answer	Marks	Guidance
	B1	A reflection of the given curve in *y* = *x* (the line *y* = *x* can be implied by position of curve).
	1	

Answer	Marks	Guidance
$y=\frac{-x}{\sqrt{4-x^2}}$ leading to $x^2=y^2(4-x^2)$	*M1	Squaring and clearing the fraction. Condone one error in squaring $-x$ or y
$x^2(1+y^2)=4y^2$	DM1	OE. Factorisation of the new subject with order of operations correct. Condone sign errors.
$x=(\pm)\frac{2y}{\sqrt{1+y^2}}$	DM1	$x=(\pm)\sqrt{\left(\frac{4y^2}{(1+y^2)}\right)}$ OE is acceptable for this mark. Isolating the new subject. Order of operations correct. Condone sign errors.
$f^{-1}(x)=\frac{-2x}{\sqrt{1+x^2}}$	A1	Selecting the correct square root. Must not have fractions in numerator or denominator.
	4	
1 or $a=1$	B1	Do not allow $x=1$ or $-1<x<1$
	1	
$[\text{fg}(x)=\text{f}(2x)=]\ \frac{-2x}{\sqrt{4-4x^2}}$	B1	Allow $\frac{-2x}{\sqrt{4-(2x)^2}}$ or any correct unsimplified form.
$\text{fg}(x)=\frac{-x}{\sqrt{1-x^2}}$ or $\frac{-x}{1-x^2}\sqrt{1-x^2}$ or $\frac{x}{x^2-1}\sqrt{1-x^2}$	B1	Result of cancelling 2 in numerator and denominator.
	2	

21.

Answer	Marks	Guidance
$2[\{(x-2)^2\}\ \{+3\}]$	B1 B1	B1 for $a=2$, B1 for $b=3$. $2(x-2)^2+6$ gains B1B0
	2	
{Translation} $\begin{pmatrix}\{2\}\\\{3\}\end{pmatrix}$ OR {Stretch} {y direction} {factor 2}	B2,1,0	B2 for fully correct, B1 with two elements correct. {} indicates different elements.
{Stretch} {y direction} {factor 2} OR {Translation} $\begin{pmatrix}\{2\}\\\{6\}\end{pmatrix}$	B2,1,0	B2 for fully correct, B1 with two elements correct. {} indicates different elements.
	4	

22.

Answer	Marks	Guidance
$\left[x^{\frac{1}{2}}=\right]\frac{4\pm\sqrt{16-4}}{2}=2\pm\sqrt{3}$	M1 A1	OE. Answer must come from formula or completing square. If M0A0 scored then **SC B1** for $2\pm\sqrt{3}$ only.
$[x=](2\pm\sqrt{3})^2$	M1	Attempt to square *their* $2\pm\sqrt{3}$
$7+4\sqrt{3}$, $7-4\sqrt{3}$	A1	Accept $7\pm4\sqrt{3}$ or $a=7, b=\pm4, c=3$ **SC B1** instead of second M1A1 for correct final answer only.
Alternative method for question 9(a)		
$-4x^{\frac{1}{2}}+1=0$ leading to $(x+1)^2=16x$ leading to $x^2-14x+1=0$	*M1 A1	OE
$x=\frac{14\pm\sqrt{196-4}}{2}$	DM1	Attempt to solve for x
$7+4\sqrt{3}$, $7-4\sqrt{3}$	A1	**SC B1** instead of second M1A1 for correct final answer only.
	4	
$[\text{gh}(x)=]\ m\left(x^{\frac{1}{2}}-2\right)^2+n$	M1	SOI
$[\text{gh}(x)=]\ m\left(x-4x^{\frac{1}{2}}+4\right)+n\ \equiv\ x-4x^{\frac{1}{2}}+1$	A1	SOI
$m=1,\ n=-3$	A1 A1	WWW
	4	

23.

Answer	Marks	Guidance
$y=\frac{x^2-4}{x^2+4}$ leading to $(x^2+4)y=(x^2-4)$ leading to $x^2y+4y=x^2-4$	*M1	For clearing denominator and expanding brackets. If swap variables first, look for $y^2x+4x=y^2-4$.
$x^2y-x^2=-4y-4$ leading to $x^2(1-y)=4y+4$ leading to $x^2=...$	DM1	For making x^2 the subject. If swap variables first, look for $y^2(1-x)=4x+4\Rightarrow y^2=\ldots$
$x^2=\frac{4y+4}{1-y}$ leading to $x=\sqrt{\frac{4y+4}{1-y}}$ leading to $[\text{f}^{-1}(x)]=\sqrt{\frac{4x+4}{1-x}}$	A1	OE e.g. $\sqrt{\frac{-4x-4}{x-1}}$ without ± in final answer.
Alternative method for Q6(a)		
$x=\frac{y^2-4}{y^2+4}$ leading to $x=1-\frac{8}{y^2+4}$ leading to $x-1=\frac{-8}{y^2+4}$	*M1	For division and reaching $x-1=...$ (or $y-1=...$)
$y^2+4=\frac{-8}{x-1}$ leading to $y^2=\frac{-8}{x-1}-4$	DM1	For making y^2 (or x^2) the subject.
$[y=][\text{f}^{-1}(x)]=\sqrt{\frac{-8}{x-1}-4}$	A1	OE without ± in final answer.
	3	

Answer	Marks	Guidance
$1-\frac{8}{x^2+4}=\frac{x^2+4}{x^2+4}-\frac{8}{x^2+4}\left[=\frac{x^2+4-8}{x^2+4}\right]=\frac{x^2-4}{x^2+4}$	**M1 A1**	Using common denominator or division to reach 1. Remainder –8. WWW
$0<f(x)<1$	**B1 B1**	B1 for each correct inequality. B0 if contradictory statement seen. Accept $f(x)>0,\ f(x)<1;\ 1>f(x)>0;\ (0,1)$ **SC B1** for $0 \leqslant f(x) \leqslant 1$.
	4	
Because the range of f does not include the whole of the domain of f (or any of it)	**B1**	Accept an answer that includes an example outside the domain of f, e.g. $f(4)=\frac{12}{20}$. Must refer to the domain or > 2. Need not explicitly use the term 'domain' but must not refer just to the range.
	1	

24.

Answer	Marks	Guidance
$x \neq 1$ or $x<1,\ x>1$ or $(-\infty,1),(1,\infty)$ $[x \in \mathbb{R}]$	**B1**	Must be x not $f^{-1}(x)$ or y. Do not accept $1<x<1$.
	1	
$y=\frac{2x+1}{2x-1}$ leading to $(2x-1)y=2x+1$ leading to $2xy-y=2x+1$	***M1**	Setting $y =$, removing fraction and expanding brackets.
$2xy-2x=y+1$ leading to $2x(y-1)=y+1$ leading to $x=\frac{y+1}{2(y-1)}$	**DM1**	Reorganising to get $x =$. Condone ± sign errors only.
$[f^{-1}(x)]=\frac{x+1}{2(x-1)},\ \frac{x+1}{x-1}\times\frac{1}{2}$ or $\frac{1}{x-1}+\frac{1}{2}$	**A1**	OE. Must be in terms of x. Do not allow $\frac{x+1}{x-1}\div 2$.
	3	
$(\textit{their}\ f^{-1}(3))$ leading to $(\textit{their}\ f^{-1}(3))^2+4$ $[f^{-1}(3)=1, 1+4=]$	**M1**	Correct order of operations and substitution of $x = 3$ needed.
5	**A1**	
	2	
Sight of 'not one to one' or 'many to one' or 'one to many'	**B1**	Any reason mentioning 2 values, or + and —, such as: square root gives 2 values or horizontal line test crosses curve twice or 2 values because of turning point or 2 values because it is a quadratic.
	1	

Answer	Marks	Guidance
$f(x)=1+\frac{2}{2x-1}=\frac{2x-1}{2x-1}+\frac{2}{2x-1}=\frac{2x+1}{2x-1}$	**B1**	AG Do not condone equating expressions and verification.
$f'(x)=-4(2x-1)^{-2}$ or $2(2x-1)^{-1}+\{-(2x+1)2(2x-1)^{-2}\}$ or $\frac{(2x-1)2-2(2x+1)}{(2x-1)^2}$	***M1**	For $k(2x-1)^{-2}$ and no other terms or correct use of the product or quotient rule then ISW.
Gradient $m=-4$	**A1**	Differentiation must have clearly taken place.
Equation of tangent is $y-3=-4(x-1)$ $[\Rightarrow y=-4x+7]$	**DM1**	Using (1, 3) in the equation of a line with *their* gradient.
Crosses axes at $\left(\frac{7}{4},0\right)$ and $(0,7)$	**A1 FT**	SOI from *their* straight line or by integration from 0 to '*their* 7/4'.
[Area =] $\frac{49}{8}$	**A1**	OE e.g. 6.13 AWRT. If M0 A0 DM0, **SC B2** available for correct answer.
	6	

25.

Answer	Marks	Guidance
$\{2(x-4)^2\}$ $\{-9\}$	B1 B1	OE When *a* and *b* stated give priority to marking algebraic expression.
	2	
$y > -7$	B1	Allow f(*x*) > –7 or (–7, ∞) Don't allow *x* > –7.
	1	
$(x-4)^2 = \frac{y+9}{2}$	M1	2 operations correct. Allow a sign error.
$x = 4\ [\pm]\sqrt{\frac{y+9}{2}}$	M1	2 operations correct. Allow a sign error.
$[\mathrm{f}^{-1}(x)=]4-\sqrt{\frac{x+9}{2}}$	A1 FT	OE FT on *their* answer to **(a)** i.e. $-a-\sqrt{\left(\frac{x-b}{2}\right)}$.
	3	
fg(*x*) = f(2*x* + 4) = $2(2x+4-4)^2-9$	M1	Allow $2(2x+4)^2-16(2x+4)+23$.
$8x^2-9$ only	A1	
	2	

26.

Answer	Marks	Guidance
$\mathrm{f}'(x)=-3(-1)(4)(4x-p)^{-2}\left[=\frac{12}{(4x-p)^2}\right]$	B2, 1, 0	
> 0 Hence increasing function	B1FT	Correct conclusion from *their* $\mathrm{f}'(x)$.
	3	
$y=2-\frac{3}{4x-p}$ $\Rightarrow$ $(y-2)(4x-p)=-3$ **or** $4xy-py=8x-2p-3$	M1	OE Form horizontal equation. Sign errors only, no missing terms. May go directly to $4y=p-\frac{3}{x-2}$ OE M1 M1
$4xy-8x=py-2p-3 \Rightarrow 4x(y-2)=p(y-2)-3$ **or** $4x=-\frac{3}{x-2}+p$	M1	OE Factorise out $[4]x$ or $[4]y$.
$x=\frac{p(y-2)-3}{4(y-2)}\left[\Rightarrow x=\frac{p}{4}-\frac{3}{4y-8}\right]$ **or** $\frac{-\frac{3}{x-2}+p}{4}$	M1	OE Make x (or y) the subject.
$[\mathrm{f}^{-1}(x)=]\frac{p}{4}-\frac{3}{4x-8}$	A1	OE in correct form (must be in terms of *x*).
	4	
$[p=]8$	B1	
	1	

27.

Answer	Marks	Guidance
$(x-2)^2+5$	B1	
	1	
$2\left(\left\{(x+1)^2\right\}+\{5\}\right)$	B2, 1, 0	
	2	
$[\mathrm{g}(x)=]\ 2\mathrm{f}(x+3)$ or $k=2,\ h=3$	B1	In correct form. B0 if contradiction.
	1	
{Translation} $\left\{\begin{pmatrix}-3\\0\end{pmatrix}\right\}$	B2, 1, 0 FT	FT on *their* $x+3$ or $h=3$.
{Stretch} {*y* direction, factor 2}	B2, 1, 0 FT	FT on *their* 2 or $k=2$.
	4	

28.

Answer	Marks	Guidance
Three points at the bottom of their transformed graph plotted at $y=2$	B1	All 5 points of the graph must be connected.
Bottom three points of /\/\ at $x=0$, $x=1$ & $x=2$	B1	Must be this shape.
All correct	B1	Condone extra cycles outside $0\leqslant x\leqslant 2$.
	3	**SC:** If B0 B0 scored, B1 available for /\ in one of correct positions **or** all 5 points correctly plotted and not connected **or** correctly sized shape in the wrong position.
[g(x) =] f(2x) + 1	B1 B1	Award marks for their final answer as follows: f(2x) B1, + 1 B1. Condone $y=$ or $f(x)=$.
	2	

29.

Answer	Marks	Guidance
$a\left(x+\frac{1}{x}\right)+1$	B1	ISW
	1	

Answer	Marks	Guidance
$a\left(2+\frac{1}{2}\right)+1=11$	M1	Substitute $x=2$ into *their* expression from **(a)** and equate to 11. This may be done in 2 stages: $f(2)=2.5, g(2.5)=11$.
$[a=]\,4$	A1	
	2	
No,[because it is] not one-one	B1	Or other suitable explanation that may include one to many or many to one.
	1	
$[g^{-1}(x)]=\frac{x-1}{5}$ WWW	B1	Condone use of a instead of 5.
$[g^{-1}f(x)=]\frac{x+\frac{1}{x}-1}{5}$ OE	M1	Correct combination of their $g^{-1}(x)$ with given $f(x)$ Condone use of a instead of 5.
$\frac{x^2-x+1}{5x}$ or $\frac{1}{5}\left(x+\frac{1}{x}-1\right)$ or $\frac{1}{5}\left(x+x^{-1}-1\right)$ OE ISW	A1	Must not contain unresolved fractions e.g. $\frac{x+x^{-1}-1}{5}$.
	3	
The domain of f does not include the whole of the range of g. **Or** The range of g does not lie in the domain of f.	B1	Accept an answer that includes an example outside the domain of f, e.g. $g(—1)=—4$ but for f, $x>0$.
	1	

30.

Answer	Marks	Guidance
$[f(x)]=\{-2(x+2)^2\}-\{5\}$	B1 B1	
	2	
$[f(x)]<-7$	B1	Allow $y<-7$, <-7, $(-\infty,-7)$ or less than -7 $-\infty\langle f(x)\langle -7, -7\rangle f(x)\rangle -\infty$, $f<-7$
	1	
$y=-2(x+2)^2-5 \rightarrow (x+2)^2=\frac{-(y+5)}{2}$	M1	Operations correct. Allow sign errors. FT *their* quadratic from **(a)**.
$x=[\pm]\sqrt{\frac{-(y+5)}{2}}-2$	M1	Operations correct. Allow sign errors. FT *their* quadratic from **(a)**.
$[f^{-1}(x)]=-2-\sqrt{\frac{-(x+5)}{2}}$ or $-2-\sqrt{-\frac{(x+5)}{2}}$	A1	Allow $[f^{-1}(x)]=-2-\sqrt{\frac{x+5}{-2}}$.
	3	

31.

Answer	Marks	Guidance
3	B1	Ignore any description.
	1	
2	B1	Ignore any description.
	1	
(8, 2)	B1 B1	Ignore any description. Allow vector notation and absence of brackets.
	2	

Answer	Marks	Guidance
(1, 5)	B1 FT B1 FT	FT each coordinate, (*their*8 – 7, *their*2 + 3) Allow vector notation and absence of brackets.
	2	

32.

Answer	Marks	Guidance
Stretch: $(2x)^2-2(2x)+5$ or $(x-1)^2+4$ leading to $(2x-1)^2+4$	M1	Replacing x by $2x$.
Reflection: $(-2x)^2-2(-2x)+5$ or $(-2x-1)^2+4$	M1	Replacing x by $-x$. FT on *their* stretch.
Stretch: $3\{(-2x)^2-2(-2x)+5\}$ or $3\{(-2x-1)^2+4\}$	M1	Multiplying the whole function by 3. FT on *their* (stretch plus reflection).
$12x^2+12x+15$	A1	
	4	

33.

Answer	Marks	Guidance
$[y]\leqslant -1$	B1	Accept f or $f(x)\leqslant -1$, $-\infty<y\leqslant -1$, $(-\infty,-1]$. Do not accept $x\leqslant -1$.
	1	
$y=-3x^2+2$ rearranged to $3x^2=2-y$, leading to $x^2=\dfrac{2-y}{3}$ or $y^2=\dfrac{2-x}{3}$	M1	
$x=[\pm]\sqrt{\dfrac{2-y}{3}}\ \rightarrow\ [f^{-1}(x)]=\{-\}\left\{\sqrt{\dfrac{2-x}{3}}\right\}$	A1 A1	A1 for minus, A1 for $\sqrt{\dfrac{2-x}{3}}$, allow $-\sqrt{\dfrac{x-2}{-3}}$.
	3	
$fg(x)=-3(-x^2-1)^2+2$	M1	SOI expect $-3x^4-6x^2-1$.
$gf(x)=-(-3x^2+2)^2-1$	M1	SOI expect $-9x^4+12x^2-5$.
$fg(x)-gf(x)+8=0$ leading to $6x^4-18x^2+12\ [=0]$	A1	OE
$[6](x^2-1)(x^2-2)\ [=0]$ or formula or completion of the square	M1	Solving a 3-term quadratic equation in x^2 must be seen.
$x=-1$, $-\sqrt{2}$ only these **two** solutions	A1	Allow $-\sqrt{1}$, $-1.41[4]$ Answers only **SC B1**.
	5	

34.

Answer	Marks	Guidance
{Stretch} {factor 2} {in y-direction}	B2, 1, 0	**B2** for fully correct, **B1** with two elements correct. {} indicates different elements.
{Translation} $\begin{pmatrix}\{-6\}\\ \{0\}\end{pmatrix}$	B2, 1, 0	**B2** for fully correct, **B1** with two elements correct. {} indicates different elements.
	4	Transformations may be in either order.

35.

Answer	Marks	Guidance
$1+\frac{2a}{7-a}=\frac{5}{2}\left[\Rightarrow \frac{2a}{7-a}=\frac{3}{2} \Rightarrow 7a=21\right]\Rightarrow a=\ldots$ OR $1+\frac{2a}{7-a}=\frac{5}{2}\left[\Rightarrow(7-a)+2a=\frac{5}{2}(7-a)\ [\Rightarrow 7a=21\right]\Rightarrow a=\ldots$	M1	OE Substitute $x=7$ then solve for a via legitimate mathematical steps. Condone sign errors only.
$a=3$	A1	If M0, **SC B1** for $a=3$ with no working.
$f(5)=1+\frac{2(their3)}{5-their3}=4[\Rightarrow 4b-2=4]\Rightarrow b=\ldots$ OR $gf(5)=b\left(1+\frac{2(their3)}{5-their3}\right)-2\ [\Rightarrow 4b-2=4]\Rightarrow b=\ldots$	M1	Evaluate f(5), either separately or within gf then solve for b via legitimate mathematical steps. Condone sign errors only. FT *their* a value.
$b=\frac{3}{2}$	A1	OE e.g. $\frac{6}{4}$, 1.5 .
	4	
$x>1$	B1	Accept $(1,\infty)$ or $\{*: *>1\}$ where * is any variable. B0 for $f^{-1}(x)>1$ or $f(x)>1$ or $y>1$.
	1	

Answer	Marks	Guidance
EITHER $x-1=\frac{6}{y-3}\ [\Rightarrow (y-3)(x-1)=6]$ OR $x=1+\frac{6}{y-3} \Rightarrow x(y-3)=(y-3)+6$	*M1	OE $y-1=\frac{6}{x-3} \Rightarrow (x-3)(y-1)=6$. OE $y=1+\frac{6}{x-3} \Rightarrow y(x-3)=(x-3)+6$. Allow *M1 for use of *their* 3 from **(a)**.
$y-3=\frac{6}{x-1}$ or $y(x-1)=3x+3$	DM1	OE $x-3=\frac{6}{y-1}$ or $x(y-1)=3y+3$. Allow DM1 for use of *their* 3 from **(a)**.
$[f^{-1}(x)]=3+\frac{6}{x-1}$	A1	OE Correct answer e.g. $\frac{3x+3}{x-1}$ ISW. Must be in terms of x.
		*M1 DM1 possible for 'a' used, but A0 so max 2/3.
	3	

36.

Answer	Marks	Guidance
{Translation} $\begin{pmatrix}\{0\}\\\{-2\}\end{pmatrix}$	B2, 1, 0	B2 for fully correct, B1 with two elements correct. {} indicates different elements.
{Stretch} {[scale] factor 2} {parallel to *x*-axis}	B2, 1, 0	B2 for fully correct, B1 with two elements correct.
	4	Transformations can be in either order.

37.

(a)	$x^2+6x-8=(x+3)^2-17$ **OR** $2x+6=0\to x=-3\to y=-17$	2	B1B1	B1 for $(x+3)^2$, B1 for -17 **OR** B1 for $x=-3$, B1 for $y=-17$
	Range $f(x)\geqslant -17$	1	B1FT	FT; following through visible method
		3		
(b)	$(x-k)(x+2k)=0\equiv x^2+5x+b=0$	1	M1	Realises the link between roots and the equation
	$k=5$	1	A1	Comparing coefficients of x
	$b=-2k^2=-50$	1	A1	
		3		
(c)	$(x+a)^2+a(x+a)+b=a$	1	M1*	Replaces 'x' by '$x+a$' in 2 terms
	Uses b^2-4ac, $9a^2-4(2a^2+b-a)$	1	DM1	Any use of discriminant
	$a^2<4(b-a)$	1	A1	AG
		3		

38.

(a)	$\left[(x-2)^2\right][-1]$	B1 B1
		2
(b)	Smallest $c = 2$ (**FT** on *their* part **(a)**)	B1FT
		1
(c)	$y=(x-2)^2-1 \rightarrow (x-2)^2 = y+1$	*M1
	$x = 2(\pm)\sqrt{y+1}$	DM1
	$(f^{-1}(x)) = 2+\sqrt{x+1}$ for $x > 8$	A1
		3
(d)	$gf(x) = \frac{1}{(x-2)^2-1+1} = \frac{1}{(x-2)^2}$ OE	B1
	Range of gf is $0 < gf(x) < \frac{1}{9}$	B1 B1
		3

39.

(a)	Translation $\begin{pmatrix}1\\0\end{pmatrix}$	B1	Allow shift and allow by 1 in x-direction or [parallel to/on/in/ along/against] the x-axis or horizontally. 'Translation by 1 to the right' only, scores B0
	Stretch	B1	Stretch. **SC B2** for amplitude doubled.
	Factor 2 in y-direction	B1	With/by **factor** 2 in y-direction or [parallel to/on/in/along/against] the y-axis or vertically or with x axis invariant 'With/by factor 2 upwards' only, scores B0. Accept SF as an abbreviation for scale factor.
		3	**Note:** Transformations can be in either order
(b)	[$-\sin 6x$][$+ 15x$] or [$\sin(-6x)$][$+ 15x$] OE	B1 B1	Accept an unsimplified version. ISW. B1 for each correct component – square brackets indicate each required component.
			If B0, **SC B1** for either $\sin(-2x) + 5x$ or $-\sin(2x) + 5x$ or $\sin 6x - 15x$ or $\sin\left(-\frac{2}{3}x\right)+\frac{5}{3}x$
		2	

40.

(a)	$f(x) = (x-1)^2 + 4$	B1	
	$g(x) = (x+2)^2 + 9$	B1	
	$g(x) = f(x+3)+5$	B1 B1	B1 for each correct element. Accept $p = 3, q = 5$
		4	
(b)	Translation or Shift	B1	
	$\begin{pmatrix}-3\\5\end{pmatrix}$ or acceptable explanation	B1 FT	If given as 2 single translations both must be described correctly e.g. $\begin{pmatrix}-3\\0\end{pmatrix}$ & $\begin{pmatrix}0\\5\end{pmatrix}$ FT from *their* $f(x+p)+q$ *or their* $f(x) \rightarrow g(x)$ Do not accept $\begin{pmatrix}1\\4\end{pmatrix}$ or $\begin{pmatrix}-2\\9\end{pmatrix}$
		2	

41.

(a)	$\{(x+1)^2+2(x+1)-5\}+\{3\}$, or $\{(x+1+1)^2\}+\{-6+3\}$	M1 M1	M1 for dealing with $\begin{pmatrix}-1\\0\end{pmatrix}$ and M1 for dealing with $\begin{pmatrix}0\\3\end{pmatrix}$.
	$[y=]x^2+4x+1$	A1	Answer only given full marks.
		3	
(b)	{Stretch} {x direction or horizontally or y-axis invariant} { factor ½}	B2, 1, 0	Additional transformation B0.
		2	

42.

(a)	$[y]<2$ OR $[f(x)]<2$	B1	OE e.g. $f<2, (-\infty, 2), -\infty<f[x]<2$. Do not accept $x<2$ or $f(x)\leqslant 2$.
		1	
(b)	$y=2-\dfrac{5}{x+2}$ leading to $y(x+2)=2(x+2)-5$ leading to $xy+2y=2x-1$	M1	or $\dfrac{5}{x+2}=2-y$ (allow sign errors).
	$2y+1=2x-xy$ leading to $2y+1=x(2-y)$	DM1	or $\dfrac{5}{2-y}=x+2$ (allow sign errors).
	$x=\dfrac{2y+1}{2-y} \rightarrow f^{-1}(x)=\dfrac{2x+1}{2-x}$	A1	OE or $y=\dfrac{5}{2-x}-2$.
	Domain is $x<2$	B1 FT	FT on the numerical part of *their* range from part **(a),** including $x\neq 2$ not penalized. No FT for $x\in\mathcal{R}, x=k, x\neq k$.
		4	
(c)	$fg(x)=2-\dfrac{5}{x+3+2}$	B1	
	$=\dfrac{2(x+5)-5}{x+5}$ or $\dfrac{2(x+5)}{x+5}-\dfrac{5}{x+5}$	M1	Use of *their* common denominator.
	$=\dfrac{2x+5}{x+5}$	A1	
		3	