PURE MATHEMATICS -1
9709
(March, June and November series 2020 – 2023 With marking scheme)
INTEGRATION
EXERCISE -1

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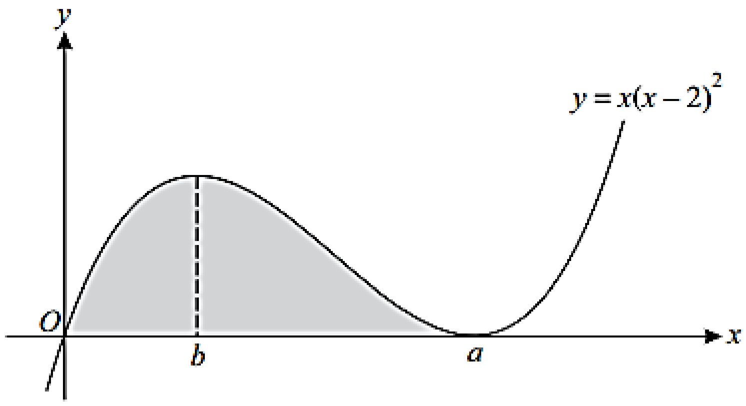
1) SP-2020 _9709_1 Q4

A curve has equation $y = f(x)$. It is given that $f'(x) = \frac{1}{\sqrt{x+6}} + \frac{6}{x^2}$ and that $f(3) = 1$.

Find $f(x)$. [5]

2) SP-2020 _9709_1 Q12



The diagram shows the curve with equation $y = x(x-2)^2$. The minimum point on the curve has coordinates $(a, 0)$ and the x-coordinate of the maximum point is b, where a and b are constants.

(a) State the value of a. [1]

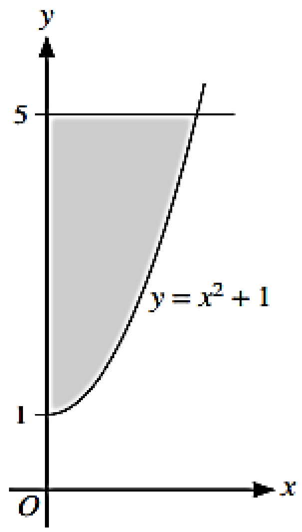
(b) Calculate the value of b. [4]

(c) Find the area of the shaded region. [4]

(d) The gradient, $\frac{dy}{dx}$, of the curve has a minimum value m.

Calculate the value of m. [4]

3) MARCH 2020_9709_12 Q3



The diagram shows part of the curve with equation $y = x^2 + 1$. The shaded region enclosed by the curve, the y-axis and the line $y = 5$ is rotated through 360° about the **y-axis**.

Find the volume obtained. [4]

4) MARCH 2020_9709_12 Q10

The gradient of a curve at the point (x, y) is given by $\frac{dy}{dx} = 2(x+3)^{\frac{1}{2}} - x$. The curve has a stationary point at $(a, 14)$, where a is a positive constant.

(a) Find the value of a. [3]

(b) Determine the nature of the stationary point. [3]

(c) Find the equation of the curve. [4]

5) MARCH 2021_9709_12 Q6(b)

A curve is such that $\frac{dy}{dx} = \frac{6}{(3x-2)^3}$ and $A\,(1, -3)$ lies on the curve. A point is moving along the curve and at A the y-coordinate of the point is increasing at 3 units per second.

(b) Find the equation of the curve. [4]

6) MARCH 2021_9709_12 Q11(a)

The diagram shows the curve with equation $y = 9(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}})$. The curve crosses the x-axis at the point A.

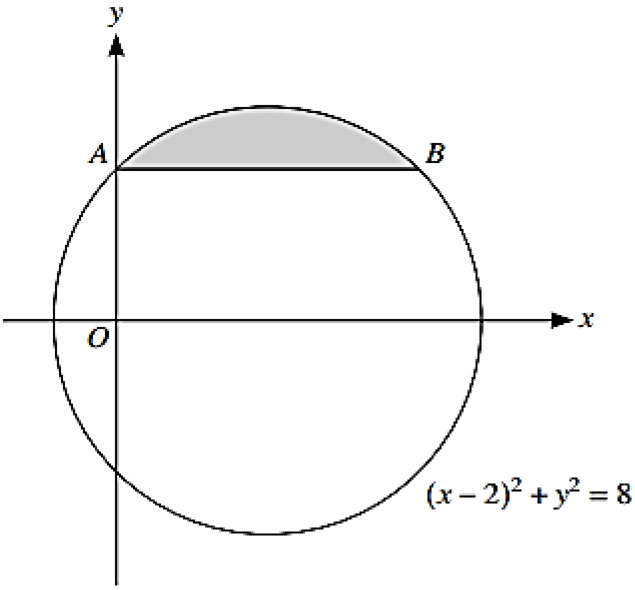
(a) Find the x-coordinate of A. [2]

7) MARCH 2022_9709_12 Q1

A curve with equation $y = f(x)$ is such that $f'(x) = 2x^{-\frac{1}{3}} - x^{\frac{1}{3}}$. It is given that $f(8) = 5$.

Find $f(x)$. [4]

8) MARCH 2022_9709_12 Q8

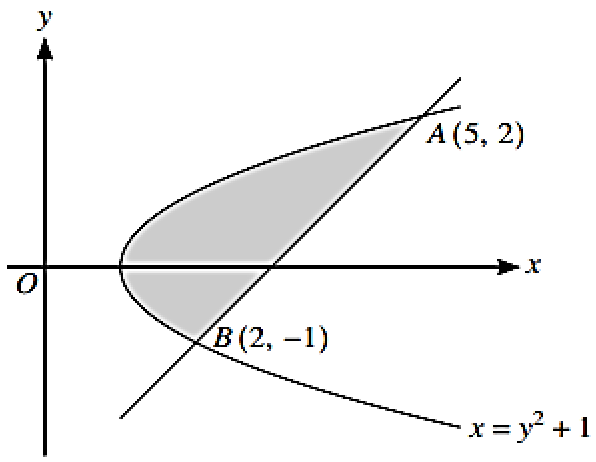


The diagram shows the circle with equation $(x-2)^2 + y^2 = 8$. The chord AB of the circle intersects the positive y-axis at A and is parallel to the x-axis.

(a) Find, by calculation, the coordinates of A and B. [3]

(b) Find the volume of revolution when the shaded segment, bounded by the circle and the chord AB, is rotated through 360° about the x-axis. [5]

9) MARCH 2023_9709_12 Q11

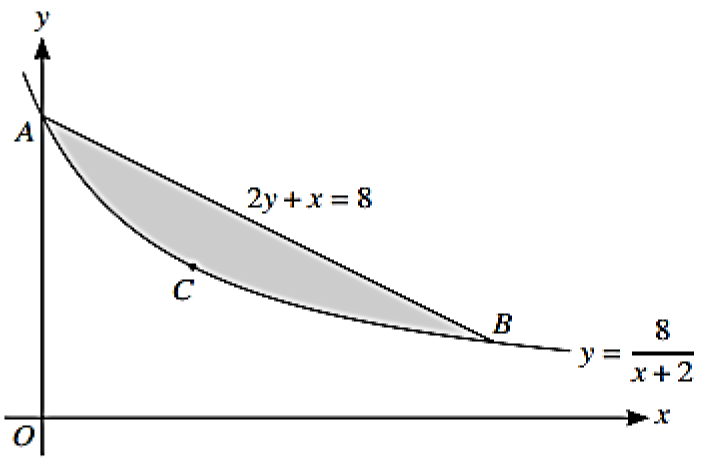


The diagram shows the curve with equation $x = y^2 + 1$. The points $A\,(5, 2)$ and $B\,(2, -1)$ lie on the curve.

(a) Find an equation of the line AB. [2]

(b) Find the volume of revolution when the region between the curve and the line AB is rotated through 360° about the **y-axis**. [9]

10) JUNE 2020_9709_11 Q11

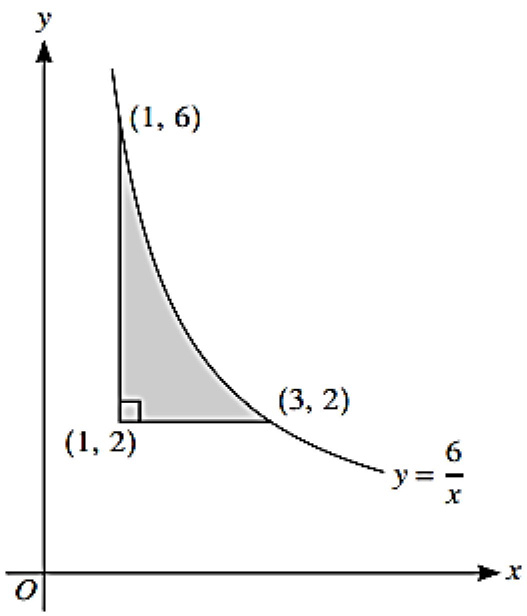


The diagram shows part of the curve $y = \dfrac{8}{x+2}$ and the line $2y + x = 8$, intersecting at points A and B. The point C lies on the curve and the tangent to the curve at C is parallel to AB.

(a) Find, by calculation, the coordinates of A, B and C. [6]

(b) Find the volume generated when the shaded region, bounded by the curve and the line, is rotated through 360° about the x-axis. [6]

11) JUNE 2020_9709_12 Q8



The diagram shows part of the curve $y = \frac{6}{x}$. The points (1, 6) and (3, 2) lie on the curve. The shaded region is bounded by the curve and the lines $y = 2$ and $x = 1$.

(a) Find the volume generated when the shaded region is rotated through 360° about the **y-axis**. [5]

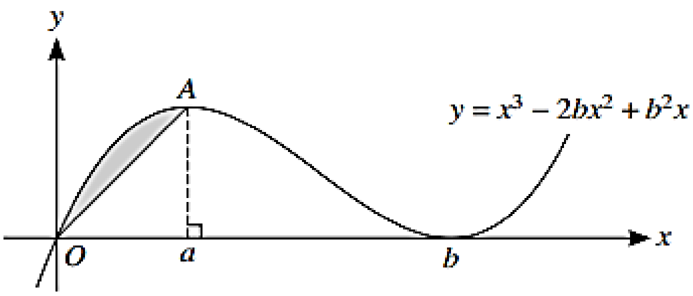
(b) The tangent to the curve at a point X is parallel to the line $y + 2x = 0$. Show that X lies on the line $y = 2x$. [3]

12) JUNE 2020_9709_13 Q2

The equation of a curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$. It is given that the point (4, 7) lies on the curve.

Find the equation of the curve. [4]

13) JUNE 2020_9709_13 Q11



The diagram shows part of the curve with equation $y = x^3 - 2bx^2 + b^2x$ and the line OA, where A is the maximum point on the curve. The x-coordinate of A is a and the curve has a minimum point at $(b, 0)$, where a and b are positive constants.

(a) Show that $b = 3a$. [4]

(b) Show that the area of the shaded region between the line and the curve is ka^4, where k is a fraction to be found. [7]

14) JUNE 2021_9709_11 Q1

The equation of a curve is such that $\frac{dy}{dx} = \frac{3}{x^4} + 32x^3$. It is given that the curve passes through the point $(\frac{1}{2}, 4)$.

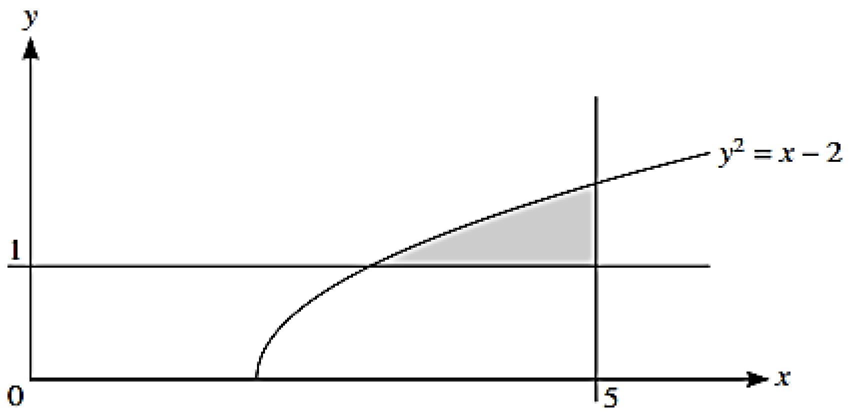
Find the equation of the curve. [4]

15) JUNE 2021_9709_11 Q11(a)

The equation of a curve is $y = 2\sqrt{3x+4} - x$.

(a) Find the equation of the normal to the curve at the point (4, 4), giving your answer in the form $y = mx + c$. [5]

16) JUNE 2021_9709_12 Q9



The diagram shows part of the curve with equation $y^2 = x - 2$ and the lines $x = 5$ and $y = 1$. The shaded region enclosed by the curve and the lines is rotated through 360° about the x-axis.

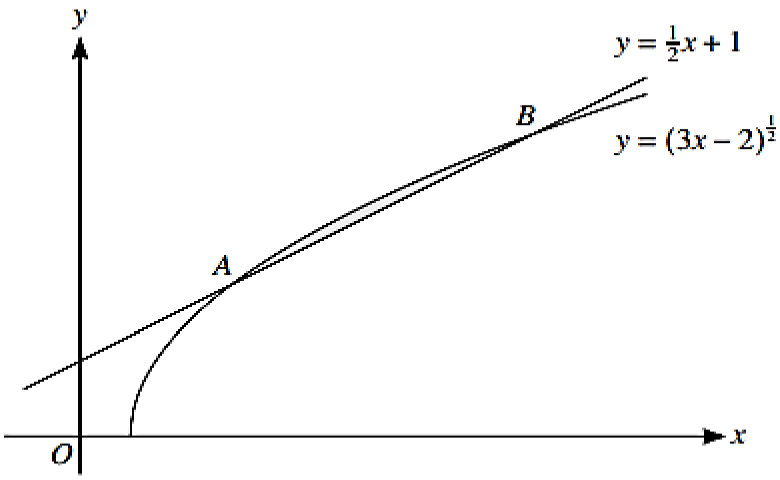
Find the volume obtained. [6]

17) JUNE 2021_9709_13 Q1

A curve with equation $y = f(x)$ is such that $f'(x) = 6x^2 - \frac{8}{x^2}$. It is given that the curve passes through the point (2, 7).

Find $f(x)$. [3]

18) JUNE 2022_9709_11 Q7



The diagram shows the curve with equation $y = (3x - 2)^{\frac{1}{2}}$ and the line $y = \frac{1}{2}x + 1$. The curve and the line intersect at points A and B.

(a) Find the coordinates of A and B. [4]

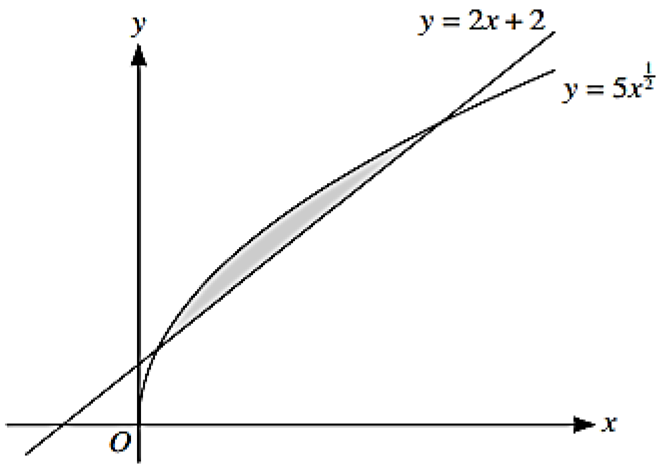
(b) Hence find the area of the region enclosed between the curve and the line. [5]

19) JUNE 2022_9709_12 Q3

The equation of a curve is such that $\frac{dy}{dx} = 3(4x - 7)^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$. It is given that the curve passes through the point $(4, \frac{5}{2})$.

Find the equation of the curve. [4]

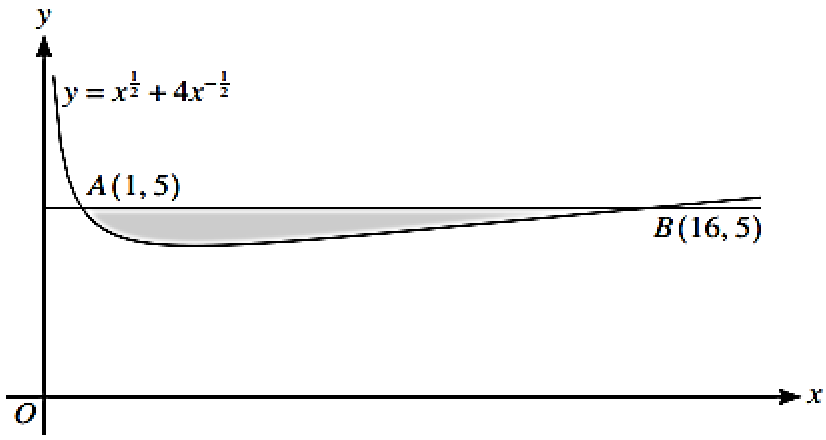
20) JUNE 2022_9709_12 Q6



The diagram shows the curve with equation $y = 5x^{\frac{1}{2}}$ and the line with equation $y = 2x + 2$.

Find the exact area of the shaded region which is bounded by the line and the curve. [5]

21) JUNE 2022_9709_13 Q8



The diagram shows the curve with equation $y = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$. The line $y = 5$ intersects the curve at the points $A\,(1, 5)$ and $B\,(16, 5)$.

(a) Find the equation of the tangent to the curve at the point A. [4]

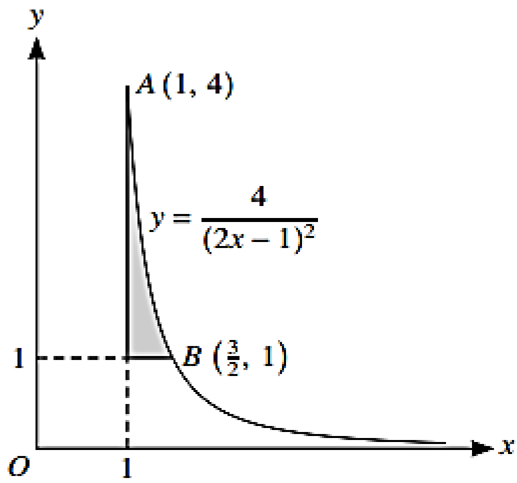
(b) Calculate the area of the shaded region. [4]

22) JUNE 2022_9709_13 Q10(a)

The function f is defined by $f(x) = (4x + 2)^{-2}$ for $x > -\frac{1}{2}$.

(a) Find $\int_1^{\infty} f(x)\,dx$. [4]

23) JUNE 2023_9709_11 Q10(a)



The diagram shows part of the curve with equation $y = \frac{4}{(2x-1)^2}$ and parts of the lines $x = 1$ and $y = 1$. The curve passes through the points $A\,(1, 4)$ and $B, (\frac{3}{2}, 1)$.

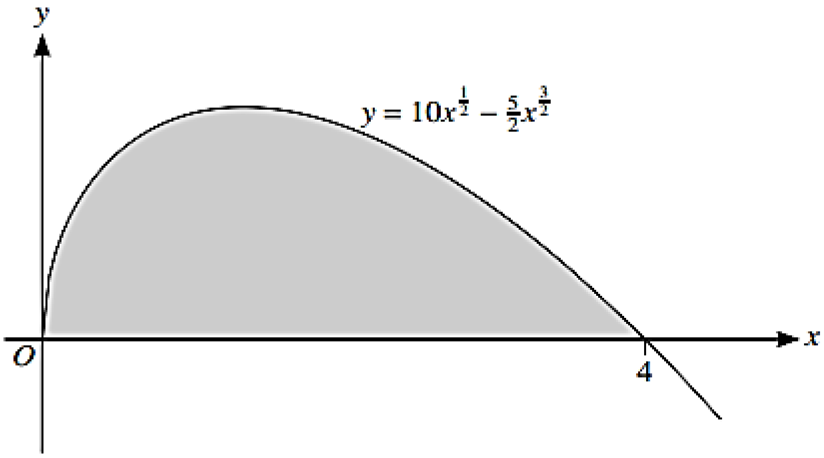
(a) Find the exact volume generated when the shaded region is rotated through 360° about the x-axis. [5]

24) JUNE 2023_9709_12 Q1

The equation of a curve is such that $\frac{dy}{dx} = \frac{4}{(x-3)^3}$ for $x > 3$. The curve passes through the point (4, 5).

Find the equation of the curve. [3]

25) JUNE 2023_9709_12 Q5



The diagram shows the curve with equation $y = 10x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}}$ for $x > 0$. The curve meets the x-axis at the points (0, 0) and (4, 0).

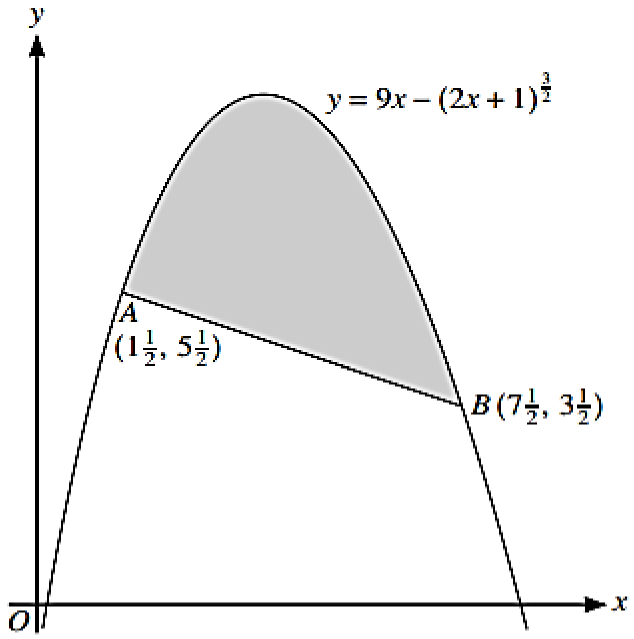
Find the area of the shaded region. [4]

26) JUNE 2023_9709_13 Q9(a)

A curve which passes through (0, 3) has equation $y = f(x)$. It is given that $f'(x) = 1 - \frac{2}{(x-1)^3}$.

(a) Find the equation of the curve. [4]

27) JUNE 2023_9709_13 Q10(c)



The diagram shows the points $A\,(1\frac{1}{2}, 5\frac{1}{2})$ and $B\,(7\frac{1}{2}, 3\frac{1}{2})$ lying on the curve with equation $y = 9x - (2x+1)^{\frac{3}{2}}$.

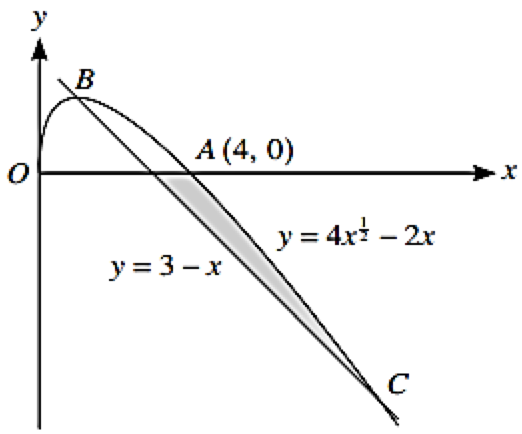
(c) Find the area of the shaded region. [5]

28) OCT 2020_9709_11 Q2

The equation of a curve is such that $\frac{dy}{dx} = \frac{1}{(x-3)^2} + x$. It is given that the curve passes through the point (2, 7).

Find the equation of the curve. [4]

29) OCT 2020_9709_11 Q12



The diagram shows a curve with equation $y = 4x^{\frac{1}{2}} - 2x$ for $x \geqslant 0$, and a straight line with equation $y = 3 - x$. The curve crosses the x-axis at $A\,(4, 0)$ and crosses the straight line at B and C.

(a) Find, by calculation, the x-coordinates of B and C. [4]

(b) Show that B is a stationary point on the curve. [2]

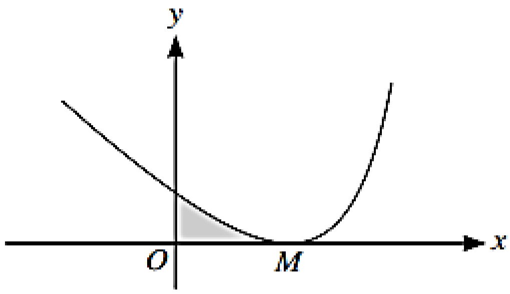
(c) Find the area of the shaded region. [6]

30) OCT 2020_9709_12 Q7(b)

The point (4, 7) lies on the curve $y = f(x)$ and it is given that $f'(x) = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$.

(b) Find the equation of the curve. [4]

31) OCT2020_9709_12 Q10



The diagram shows part of the curve $y = \dfrac{2}{(3-2x)^2} - x$ and its minimum point M, which lies on the x-axis.

(a) Find expressions for $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}$ and $\int y\,dx$. [6]

(b) Find, by calculation, the x-coordinate of M. [2]

(c) Find the area of the shaded region bounded by the curve and the coordinate axes. [2]

32) OCT 2020_9709_13 Q2

The function f is defined by $f(x) = \dfrac{2}{(x+2)^2}$ for $x > -2$.

(a) Find $\int_1^{\infty} f(x)\,dx$. [3]

(b) The equation of a curve is such that $\dfrac{dy}{dx} = f(x)$. It is given that the point $(-1, -1)$ lies on the curve.

Find the equation of the curve. [2]

33) OCT 2020_9709_13 Q10(b)

A curve has equation $y = \dfrac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \dfrac{1}{k^2}$ where $x > 0$ and k is a positive constant.

(b) It is given instead that $\displaystyle\int_{\frac{1}{4}k^2}^{k^2} \left(\frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2}\right) dx = \frac{13}{12}$.

Find the value of k. [5]

34) OCT 2021_9709_11 Q9

A curve has equation $y = f(x)$, and it is given that $f'(x) = 2x^2 - 7 - \frac{4}{x^2}$.

(a) Given that $f(1) = -\frac{1}{3}$, find $f(x)$. [4]

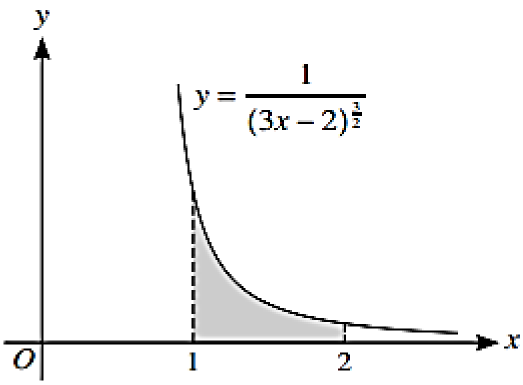
(b) Find the coordinates of the stationary points on the curve. [5]

(c) Find $f''(x)$. [1]

(d) Hence, or otherwise, determine the nature of each of the stationary points. [2]

35) OCT 2021_9709_11 Q10(a)(b)

(a) Find $\int_1^{\infty} \frac{1}{(3x-2)^{\frac{3}{2}}}\,dx$. [4]



The diagram shows the curve with equation $y = \frac{1}{(3x-2)^{\frac{3}{2}}}$. The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$. The shaded region is rotated through 360° about the x-axis.

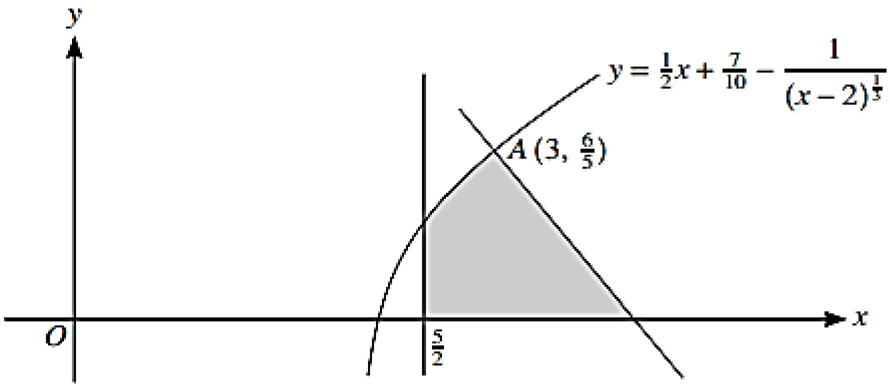
(b) Find the volume of revolution. [4]

36) OCT 2021_9709_12 Q4

A curve is such that $\frac{dy}{dx} = \frac{8}{(3x+2)^2}$. The curve passes through the point $(2, 5\frac{2}{3})$.

Find the equation of the curve. [4]

37) OCT 2021_9709_12 Q11

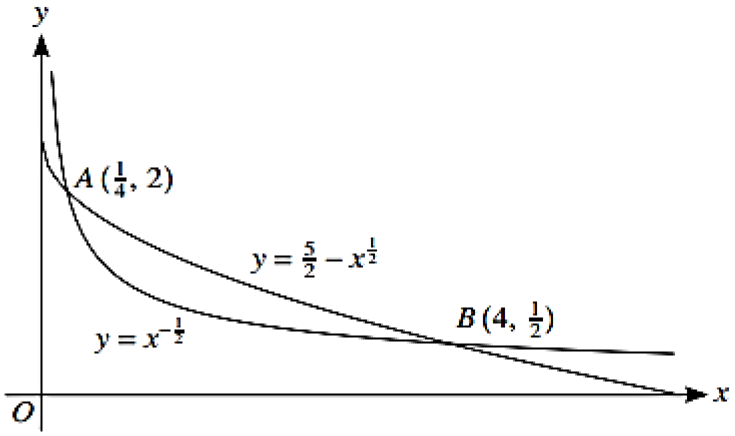


The diagram shows the line $x = \frac{5}{2}$, part of the curve $y = \frac{1}{2}x + \frac{7}{10} - \dfrac{1}{(x-2)^{\frac{1}{3}}}$ and the normal to the curve at the point $A\ \left(3, \frac{6}{5}\right)$.

(a) Find the x-coordinate of the point where the normal to the curve meets the x-axis. [5]

(b) Find the area of the shaded region, giving your answer correct to 2 decimal places. [6]

38) OCT 2021_9709_13 Q8(a)



The diagram shows the curves with equations $y = x^{-\frac{1}{2}}$ and $y = \frac{5}{2} - x^{\frac{1}{2}}$. The curves intersect at the points $A\left(\frac{1}{4}, 2\right)$ and $B\left(4, \frac{1}{2}\right)$.

(a) Find the area of the region between the two curves. [6]

39) OCT 2021_9709_13 Q10

A curve has equation $y = \mathrm{f}(x)$ and it is given that

$$\mathrm{f}'(x) = \left(\tfrac{1}{2}x + k\right)^{-2} - (1+k)^{-2},$$

where k is a constant. The curve has a minimum point at $x = 2$.

(a) Find $\mathrm{f}''(x)$ in terms of k and x, and hence find the set of possible values of k. [3]

It is now given that $k = -3$ and the minimum point is at $(2, 3\frac{1}{2})$.

(b) Find $f(x)$. [4]

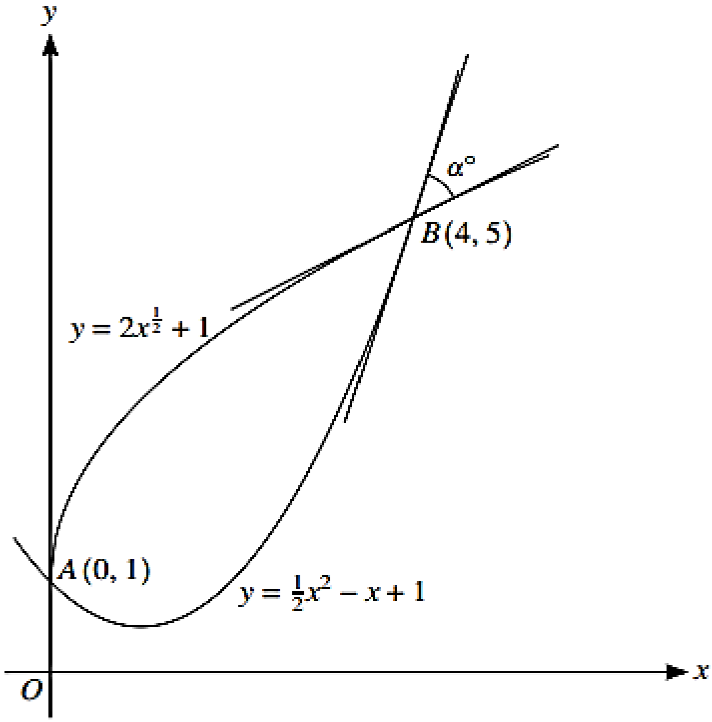
(c) Find the coordinates of the other stationary point and determine its nature. [4]

40) OCT 2022_9709_11 Q2(b)

The equation of a curve is such that $\frac{dy}{dx} = 12(\frac{1}{2}x - 1)^{-4}$. It is given that the curve passes through the point $P(6, 4)$.

(b) Find the equation of the curve. [4]

41) OCT 2022_9709_11 Q10



Curves with equations $y = 2x^{\frac{1}{2}} + 1$ and $y = \frac{1}{2}x^2 - x + 1$ intersect at $A(0, 1)$ and $B(4, 5)$, as shown in the diagram.

(a) Find the area of the region between the two curves. [5]

The acute angle between the two tangents at B is denoted by $\alpha°$, and the scales on the axes are the same.

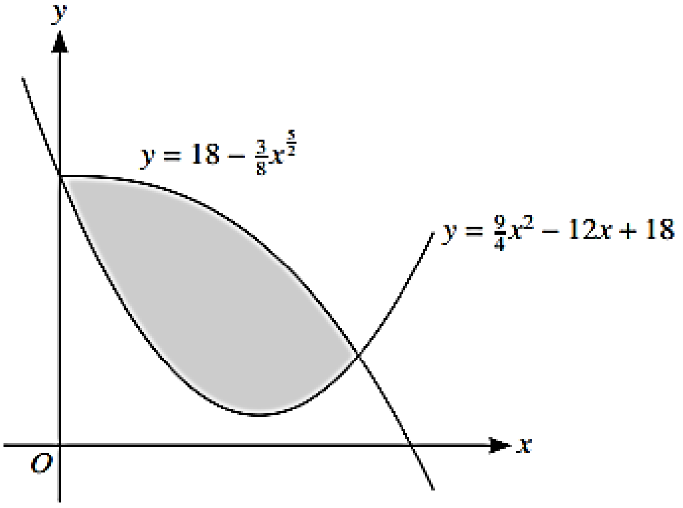
(b) Find α. [5]

42) OCT 2022_9709_12 Q8(a)

The equation of a curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$. The curve passes through the point (3, 5).

(a) Find the equation of the curve. [4]

43) OCT 2022_9709_12 Q11(b)



The diagram shows the curves with equations $y = \frac{9}{4}x^2 - 12x + 18$ and $y = 18 - \frac{3}{8}x^{\frac{5}{2}}$. The curves intersect at the points (0, 18) and (4, 6).

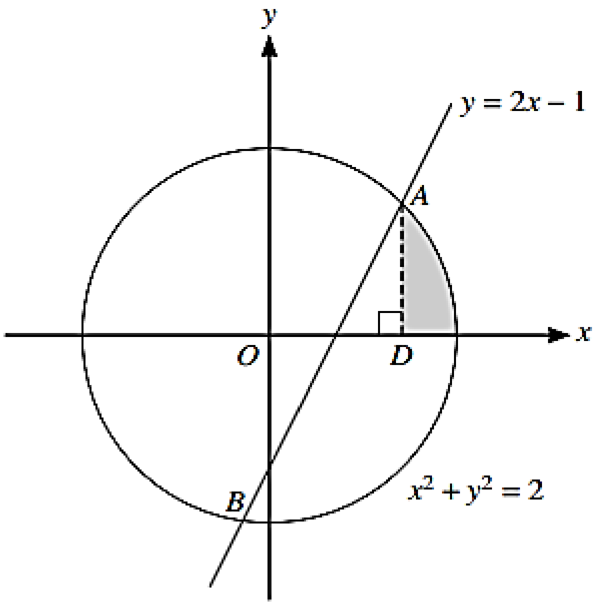
(b) Find the area of the shaded region. [5]

44) OCT 2022_9709_13 Q7(b)

The curve $y = f(x)$ is such that $f'(x) = \frac{-3}{(x+2)^4}$.

(b) Find $f(x)$ given that the curve passes through the point $(-1, 5)$. [3]

45) OCT 2022_9709_13 Q10



The diagram shows the circle $x^2 + y^2 = 2$ and the straight line $y = 2x - 1$ intersecting at the points A and B. The point D on the x-axis is such that AD is perpendicular to the x-axis.

(a) Find the coordinates of A. [4]

(b) Find the volume of revolution when the shaded region is rotated through 360° about the x-axis. Give your answer in the form $\frac{\pi}{a}(b\sqrt{c}-d)$, where a, b, c and d are integers. [4]

(c) Find an exact expression for the perimeter of the shaded region. [2]

MARKING SCHEME

1) SP-2020 _9709_1 Q4

Attempt integration	1	M1	
$f(x) = 2(x+6)^{\frac{1}{2}} - \frac{6}{x}(+c)$	2	A1A1	Accept unsimplified terms, A1 for each term
$2(3) - \frac{6}{3} + c = 1$	1	M1	Substitute $x = 3, y = 1$. c must be present
$[c = -3]$ $f(x) = 2(x+6)^{\frac{1}{2}} - \frac{6}{x} - 3$	1	A1	
	5		

2) SP-2020 _9709_1 Q12

(a)	$a = 2$	1	B1	
(b)	$y = x^3 - 4x^2 + 4x$	1	B1	
	$\frac{dy}{dx} = 3x^2 - 8x + 4$	2	B2FT	FT B1 for $3x^2$, B1 for $-8x + 4$
	$(x-2)(3x-2) = 0 \rightarrow b = \frac{2}{3}$	1	B1	Dependent on method seen for solving quadratic equation
		4		
(c)	Area $= \int y\,dx = \left[\frac{x^4}{4} - \frac{4x^3}{3} + 2x^2\right]$	2	B2	B1 for $\frac{x^4}{4}$, B1 for $\frac{4x^3}{3} + 2x^2$
	$4 - \frac{32}{3} + 8$	1	M1	Apply limits $0 \rightarrow 2$
	$\frac{4}{3}$	1	A1	Unsupported answer receives 0 marks
		4		
(d)	$\frac{d^2y}{dx^2} = 6x - 8 = 0, x = \frac{4}{3}$	2	M1*A1	Attempt 2nd derivative and set = 0
	When $x = \frac{4}{3}, \frac{dy}{dx}$ (or m) $= -\frac{4}{3}$	2	DM1A1	
		4		

3) MARCH 2020_9709_12 Q3

$(\pi)\int(y-1)dy$	*M1	SOI Attempt to integrate x^2 or $(y-1)$
$(\pi)\left[\frac{y^2}{2} - y\right]$	A1	
$(\pi)\left[\left(\frac{25}{2} - 5\right) - \left(\frac{1}{2} - 1\right)\right]$	DM1	Apply limits $1 \rightarrow 5$ to an integrated expression
8π or AWRT 25.1	A1	
	4	

4) MARCH 2020_9709_12 Q10

a)	$2(a+3)^{\frac{1}{2}}-a=0$	M1	SOI. Set $\frac{dy}{dx}=0$ when $x=a$. Can be implied by an answer in terms of a
	$4(a+3)=a^2 \to a^2-4a-12=0$	M1	Take a to RHS and square. Form 3-term quadratic
	$(a-6)(a+2) \to a=6$	A1	Must show factors, or formula or completing square. Ignore $a=-2$ SC If a is never used maximum of M1A1 for $x=6$,with visible solution
		3	
b)	$\frac{d^2y}{dx^2}=(x+3)^{-\frac{1}{2}}-1$	B1	
	Sub *their* $a \to \frac{d^2y}{dx^2}=\frac{1}{3}-1=-\frac{2}{3}$ $(or<0) \to$ MAX	M1A1	A mark only if completely correct If the second differential is not $-\frac{2}{3}$ correct conclusion must be drawn to award the M1
		3	
c)	$(y=)\frac{2(x+3)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{1}{2}x^2$ $(+c)$	B1B1	
	Sub $x=$ *their* a and $y=14$ $\to 14=\frac{4}{3}(9)^{\frac{3}{2}}-18+c$	M1	Substitute into an integrated expression. c must be present. Expect $c=-4$
	$y=\frac{4}{3}(x+3)^{\frac{3}{2}}-\frac{1}{2}x^2-4$	A1	Allow $f(x)=$....
		4	

5) MARCH 2021_9709_12 Q6(b)

b)	$[y=]\left(\frac{6(3x-2)^{-2}}{-2}\right)\div(3)$ $[+c]$	B1 B1	
	$-3=-1+c$	M1	Substitute $x=1$, $y=-3$. c must be present.
	$y=-(3x-2)^{-2}-2$	A1	OE. Allow f(x)=
		4	

6) MARCH 2021_9709_12 Q11(a)

a)	$9\left(x^{-\frac{1}{2}}-4x^{-\frac{3}{2}}\right)=0$ leading to $9x^{-\frac{3}{2}}(x-4)=0$	M1	OE. Set y to zero and attempt to solve.
	$x=4$ only	A1	From use of a correct method.
		2	

7) MARCH 2022_9709_12 Q1

$[f(x)=]\frac{2x^{\frac{2}{3}}}{\frac{2}{3}}-\frac{x^{\frac{4}{3}}}{\frac{4}{3}}$ $[+c]$	B1 B1	$\frac{2}{3}$ and $\frac{4}{3}$ may be seen as sums of 1 and a fraction.
$5=12-12+c$	M1	Substituting (8,5) into an integral.
$[f(x)=]3x^{\frac{2}{3}}-\frac{3}{4}x^{\frac{4}{3}}+5$	A1	Fractions in the denominators scores A0.
	4	

8) MARCH 2022_9709_12 Q8

a)	$(-2)^2+y^2=8$ leading to $y=2$ leading to $A=(0,2)$	B1	
	Substitute $y=\mathit{their}\,2$ into circle leading to $(x-2)^2+4=8$	M1	Expect $x=4$.
	$B=(4, 2)$	A1	
		3	
b)	Attempt to find $[\pi]\int\left(8-(x-2)^2\right)dx$	*M1	
	$[\pi]\left[8x-\frac{(x-2)^3}{3}\right]$ or $[\pi]\left[8x-\left(\frac{x^3}{3}-2x^2+4x\right)\right]$	A1	
	$[\pi]\left(32-\frac{16}{3}\right)$ or $[\pi]\left[32-\left(\frac{64}{3}-32+16\right)\right]$	DM1	Apply limits $0\rightarrow$ *their* 4.
	Volume of cylinder $=\pi\times2^2\times4=16\pi$	B1 FT	OR from $\pi\int2^2dx$ with *their* limits from **(a)**. FT on *their* A and B
	[Volume of revolution $=26\frac{2}{3}\pi-16\pi=$] $10\frac{2}{3}\pi$	A1	Accept 33.5
		5	

9) MARCH 2023_9709_12 Q11

a)	Gradient of $AB=\frac{2-(-1)}{5-2}$	M1	Expect 1, must be from $\Delta y/\Delta x$.
	Equation of AB is $y-2=1(x-5)$ or $y+1=1(x-2)$	A1	OE. Expect $y=x-3$.
		2	
b)	$[\pi]\int x^2dy=[\pi]\int(y^2+1)^2dy=[\pi]\int(y^4+2y^2+1)dy$	M1	For curve: Attempt to square y^2+1 and attempt integration. Subtracting curve equation from line equation before squaring is M0. Integration before squaring M0.
	$[\pi]\left(\frac{y^5}{5}+\frac{2y^3}{3}+y\right)$	A2, 1, 0	
	$[\pi]\int(y+3)^2dy=[\pi]\int(y^2+6y+9)dy$	M1	For line: Attempt to square *their* $y+3$ and attempt integration.
	$[\pi]\left(\frac{y^3}{3}+3y^2+9y\right)$ or $[\pi]\left(\frac{(y+3)^3}{3}\right)$	A2, 1, 0	Not available for incorrect line equations.
	$[\pi]\left\{\frac{8}{3}+12+18-\left(-\frac{1}{3}+3-9\right)\right\}$ or $[\pi]\left\{\frac{32}{5}+\frac{16}{3}+2-\left(-\frac{1}{5}-\frac{2}{3}-1\right)\right\}$	DM1	Apply limits $-1\rightarrow2$ to either integral providing they have been awarded M1. Expect $15\frac{3}{5}$ $[\pi]$ and/or $39[\pi]$. Some evidence of substitution of both -1 and 2 must be seen. Dependent on at least one of the first 2 M1 marks.
	Volume $=[\pi](39-15\frac{3}{5})$	DM1	Appropriate subtraction. Dependent on at least one of the first 2 M1 marks.
	$=23\frac{2}{5}\pi$ or $\frac{117}{5}\pi$ or awrt 73.5[1327]	A1	
		9	

10) JUNE 2020_9709_11 Q11

(a)	Simultaneous equations $\dfrac{8}{x+2} = 4 - \frac{1}{2}x$	M1
	$x = 0$ or $x = 6 \rightarrow A\ (0, 4)$ and $B\ (6, 1)$	B1A1
	At C $\dfrac{-8}{(x+2)^2} = -\dfrac{1}{2} \rightarrow C\ (2, 2)$ (B1 for the differentiation. M1 for equating and solving)	B1 M1A1
		6
(b)	Volume under line $= \pi\int\left(-\frac{1}{2}x+4\right)^2 dx = \pi\left[\dfrac{x^3}{12} - 2x^2 + 16x\right] = (42\pi)$ (M1 for volume formula. A2,1 for integration)	M1 A2,1
	Volume under curve $= \pi\int\left(\dfrac{8}{x+2}\right)^2 dx = \pi\left[\dfrac{-64}{x+2}\right] = (24\pi)$	A1
	Subtracts and uses 0 to 6 $\rightarrow 18\pi$	M1A1

11) JUNE 2020_9709_12 Q8

(a)	Volume $= \pi\int x^2 dy = \pi\int \dfrac{36}{y^2} dy$	*M1
	$= \pi\left[\dfrac{-36}{y}\right]$	A1
	Uses limits 2 to 6 correctly $\rightarrow (12\pi)$	DM1
	Vol of cylinder $= \pi.1^2.4$ or $\int 1^2.dy$ $= [y]$ from 2 to 6	M1
	Vol $= 12\pi - 4\pi = 8\pi$	A1
		5
(b)	$\dfrac{dy}{dx} = \dfrac{-6}{x^2}$	B1
	$\dfrac{-6}{x^2} = -2 \rightarrow x = \sqrt{3}$	M1
	$y = \dfrac{6}{\sqrt{3}} = 2\sqrt{3}$ Lies on $y = 2x$	A1
		3

12) JUNE 2020_9709_13 Q2

$(y) = \dfrac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \dfrac{3x^{\frac{1}{2}}}{\frac{1}{2}}\ (+c)$	B1 B1
$7 = 16 - 12 + c$ (M1 for subsituting $x = 4$, $y = 7$ into *their* integrated expansion)	M1
$y = 2x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + 3$	A1
	4

13) JUNE 2020_9709_13 Q11

(a)	$\frac{dy}{dx} = 3x^2 - 4bx + b^2$	B1
	$3x^2 - 4bx + b^2 = 0 \rightarrow (3x-b)(x-b)\ (=0)$	M1
	$x = \frac{b}{3}$ or b	A1
	$a = \frac{b}{3} \rightarrow b = 3a$ AG	A1
	Alternative method for question 11(a)	
	$\frac{dy}{dx} = 3x^2 - 4bx + b^2$	B1
	Sub $b = 3a$ & obtain $\frac{dy}{dx} = 0$ when $x = a$ and when $x = 3a$	M1
	$\frac{d^2y}{dx^2} = 6x - 12a$	A1
	< 0 Max at $x = a$ and > 0 Min at $x = 3a$. Hence $b = 3a$ AG	A1
		4
(b)	Area under curve $= \int (x^3 - 6ax^2 + 9a^2x)dx$	M1
	$\frac{x^4}{4} - 2ax^3 + \frac{9a^2x^2}{2}$	B2,1,0
	$\frac{a^4}{4} - 2a^4 + \frac{9a^4}{2}\ \left(= \frac{11a^4}{4}\right)$ (M1 for applying limits $0 \rightarrow a$)	M1
	When $x = a$, $y = a^3 - 6a^3 + 9a^3 = 4a^3$	B1
	Area under line $= \frac{1}{2}a \times$ *their* $4a^3$	M1
	Shaded area $= \frac{11a^4}{4} - 2a^4 = \frac{3}{4}a^4$	A1
		7

14) JUNE 2021_9709_11 Q1

$[y=] - \frac{1}{x^3} + 8x^4\ [+c]$	B1 B1	OE. Accept unsimplified.
$4 = -8 + \frac{1}{2} + c$	M1	Substituting $\left(\frac{1}{2}, 4\right)$ into an integrated expression
$y = -\frac{1}{x^3} + 8x^4 + \frac{23}{2}$	A1	OE. Accept $-x^{-3}$; must be 8; $y =$ must be seen in working.
	4	

15) JUNE 2021_9709_11 Q11(a)

(a)	$\frac{dy}{dx} = 3(3x+4)^{-0.5} - 1$	B1 B1	B1 All correct with 1 error, B2 if all correct
	Gradient of tangent $= -\frac{1}{4}$ and Gradient of normal $= 4$	*M1	Substituting $x = 4$ into a differentiated expression and using $m_1m_2 = -1$
	Equation of line is $(y-4) = 4(x-4)$ or evaluate c	DM1	With (4, 4) and *their* gradient of normal
	So $y = 4x - 12$	A1	
		5	

16) JUNE 2021_9709_12 Q9

Curve intersects $y = 1$ at (3, 1)	B1	**Throughout Question 9: 1 < *their* 3 < 5** Sight of $x = 3$
Volume $= [\pi]\int(x-2)[\mathrm{d}x]$	M1	M1 for showing the intention to integrate $(x-2)$. Condone missing π or using 2π.
$[\pi]\left[\frac{1}{2}x^2 - 2x\right]$ or $[\pi]\left[\frac{1}{2}(x-2)^2\right]$	A1	Correct integral. Condone missing π or using 2π.
$=[\pi]\left[\left(\frac{5^2}{2}-2\times5\right)-\left(\frac{their\,3^2}{2}-2\times their\,3\right)\right]$ $=[\pi]\left[\frac{5}{2}+\frac{3}{2}\right]$ as a minimum requirement for *their* values	M1	Correct use of '*their* 3' and 5 in an integrated expression. Condone missing π or using 2π. Condone +c. Can be obtained by integrating and substituting between 5 and 2 and then 3 and 2 then subtracting.
Volume of cylinder $= \pi\times1^2\times(5 - their\,3)[=2\pi]$	B1 FT	Or by integrating 1 to obtain x (condone y if 5 and *their* 3 used).
[Volume of solid $= 4\pi - 2\pi =$]2π or 6.28	A1	AWRT
Alternative method for Question 9		
Curve intersects $y = 1$ at (3, 1)	B1	Sight of $x = 3$
Volume of solid $= \pi\int(x-2)-1[\mathrm{d}x]$	M1 B1	M1 for showing the intention to integrate $(x-2)$ B1 for correct integration of -1. Condone missing π or 2π for M1 but not for B1.
$[\pi]\left[\frac{1}{2}x^2 - 3x\right]$ or $[\pi]\left[\frac{1}{2}(x-3)^2\right]$	A1	Correct integral, allow as two integrals. Condone missing π or using 2π.
$=[\pi]\left[\left(\frac{5^2}{2}-3\times5\right)-\left(\frac{their\,3^2}{2}-3\times their\,3\right)\right]$	M1	Correct use of '*their* 3' and 5 in an integrated expression. Condone missing π or using 2π. Condone +c. Can be obtained by integrating and substituting between 5 and 2 and then 3 and 2 then subtracting.
[Volume of solid $= 4\pi - 2\pi =$]2π or 6.28	A1	AWRT
	6	

17) JUNE 2021_9709_13 Q1

$[\mathrm{f}(x)=]\ 2x^3 + \frac{8}{x}\ [+c]$	**B1**	Allow any correct form
$7 = 16 + 4 + c$	**M1**	Substitute f(2) = 7 into an integral. c must be present. Expect $c = -13$
$\mathrm{f}(x) = 2x^3 + \frac{8}{x} - 13$	**A1**	Allow $y =$, $\mathrm{f}(x)$ or y can appear earlier in answer
	3	

18) JUNE 2022_9709_11 Q7

(a)	$(3x-2)^{\frac{1}{2}} = \frac{1}{2}x+1 \Rightarrow 3x-2=\left(\frac{1}{2}x+1\right)^2 = \frac{1}{4}x^2+x+1$	**M1**	Equating curve and line, attempt to square; $\frac{1}{4}x^2+1$ M0
	$\Rightarrow \frac{1}{4}x^2-2x+3[=0]\left[\Rightarrow x^2-8x+12=0\right]\Rightarrow(x-6)(x-2)[=0]$	**M1**	Forming and solving a 3TQ by factorisation, formula or completing the square – see guidance.
	(2, 2) and (6, 4)	**A1 A1**	A1 for each point, or A1 A0 for two correct x-values. If M0 for solving, **SC B2** possible: B1 for each point or B1 B0 for two correct x-values.
		4	

(b)	$\text{Area} = \pm\int_{[2]}^{[6]}\left((3x-2)^{\frac{1}{2}} - \left(\frac{1}{2}x+1\right)\right)[\text{d}x]$	*M1	For intention to integrate and subtract (M0 if squared).
	$\pm\left[\frac{2}{9}(3x-2)^{\frac{3}{2}} - \left(\frac{1}{4}x^2+x\right)\right]_2^6$	B1 B1	B1 for each bracket integrated correctly (in any form).
	$\pm\left(\left[\frac{2}{9}(16)^{\frac{3}{2}} - \left(\frac{1}{4}\times 36+6\right)\right] - \left[\frac{2}{9}(4)^{\frac{3}{2}} - \left(\frac{1}{4}\times 4+2\right)\right]\right)$	DM1	±(F(*their* 6) – F(*their* 2)) with *their* integral. Allow 1 sign error.
	$\frac{4}{9}$	A1	AWRT 0.444. **SC1 B1** for $\frac{4}{9}$ if *M1 B1 B1 DM0. **SC2 B1** for $\frac{4}{9}$ if *M1 B0 B0 DM0, provided limits stated.
	Alternative method for question 7(b)		
	$\text{Area} = \pm\int_{[2]}^{[6]}(3x-2)^{\frac{1}{2}}\ [\text{d}x]$ – area of trapezium (or triangle + rectangle)	*M1	For intention to integrate and subtract (M0 if squared).
	$\pm\left[\frac{2}{9}(3x-2)^{\frac{3}{2}}\right]_2^6 - 4\left(\frac{2+4}{2}\right)$ or $\pm\left[\frac{2}{9}(3x-2)^{\frac{3}{2}}\right]_2^6 - \left(\frac{2+4}{2}+(2\times 4)\right)$	B1 B1 FT	B1 for bracket integrated correctly (in any form). B1 FT for using correct formula with *their* values.
	$\pm\left(\left(\frac{2}{9}(16)^{\frac{3}{2}} - \frac{2}{9}(4)^{\frac{3}{2}}\right) - 12\right)$	DM1	±F(*their* 6) – F(*their* 2)) using *their* integral. Allow 1 sign error.
b)	$\frac{4}{9}$	A1	AWRT 0.444. **SC1 B1** for $\frac{4}{9}$ if *M1 B1 B1 DM0. **SC2 B1** for $\frac{4}{9}$ if *M1 B0 B0 DM0, provided limits stated.
		5	

19) JUNE 2022_9709_12 Q3

$[y=]\left\{\frac{3(4x-7)^{\frac{3}{2}}}{\frac{3}{2}\times 4}\right\} + \left\{-\frac{4}{\frac{1}{2}}x^{\frac{1}{2}}\right\}\left[\Rightarrow \frac{1}{2}(4x-7)^{\frac{3}{2}} - 8x^{\frac{1}{2}}\right][+c]$	B1 B1	Marks can be awarded for correct unsimplified expressions ISW.
$\frac{5}{2} = \frac{1}{2}(9)^{\frac{3}{2}} - 8\times 4^{\frac{1}{2}} + c\quad [\Rightarrow c=5]$	M1	Using $(4, \frac{5}{2})$ in an integrated expression (defined by at least one correct power) including + *c*.
$y = \frac{3}{6}(4x-7)^{\frac{3}{2}} - 8x^{\frac{1}{2}} + 5$.	A1	Condone *c* = 5 as their final line if either *y* = or f(*x*) = seen elsewhere in the solution. Coefficients must not contain unresolved double fractions.
	4	

20) JUNE 2022_9709_12 Q6

Line meets curve when: $2x+2=5x^{\frac{1}{2}}$ leading to $2x-5x^{\frac{1}{2}}+2[=0]$ or $4x^2+8x+4=25x$ leading to $4x^2-17x+4[=0]$ or $x=\frac{y^2}{25}$ leading to $2y^2-25y+50[=0]$	M1	Equating line and curve and rearranging so that terms are all on same side, condone sign errors, and making a valid attempt to solve by factorising, using the formula or completing the square. Factors are:$(2x^{\frac{1}{2}}-1)(x^{\frac{1}{2}}-2)$, $(4x-1)(x-4)$ and $(2y-5)(y-10)$.
$x=\frac{1}{4}, x=4$	A1	**SC:** If M1 not scored, **SC B1** available for correct answers, could just be seen as limits.
Area = $\int 5x^{\frac{1}{2}}-(2x+2)\mathrm{d}x=\int 5x^{\frac{1}{2}}-2x-2\ \mathrm{d}x$	*M1	Intention to integrate and subtract areas. Condone missing brackets and/or subtraction wrong way around.
$=\left[\frac{10}{3}x^{\frac{3}{2}}-x^2-2x\right]_{\frac{1}{4}}^{4}=\left(\left(\frac{10}{3}\times8-16-8\right)-\left(\frac{10}{3}\times\frac{1}{8}-\frac{1}{16}-\frac{1}{2}\right)\right)$	DM1	Integrating($kx^{\frac{3}{2}}$ seen) and substituting '*their* points of intersection' (but limits need to be found, not assumed to be 0 and something else).
$\frac{45}{16}$ or $2\frac{13}{16}$ or 2.8125	A1	OE exact answer. Condone $-\frac{45}{16}$ if corrected to $\frac{45}{16}$. A0 for inclusion of π. **SC:** If *M1 DM0 scored, **SC B1** available for correct answer.

21) JUNE 2022_9709_13 Q8

a)	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\right]\frac{1}{2}x^{-1/2}-2x^{-3/2}$	B1 B1	Allow unsimplified versions.
	At $x=1$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}-2=-\frac{3}{2}$	M1	Substitute $x=1$ into a differentiated y.
	Equation of tangent is $y-5=-\frac{3}{2}(x-1)$	A1	WWW Or $y=-\frac{3}{2}x+\frac{13}{2}$.
		4	
(b)	$\frac{x^{3/2}}{3/2}+8x^{1/2}$	B1	OE Integrate to find area under curve, allow unsimplified versions.
	$\left[\left(\frac{128}{3}+32\right)-\left(\frac{2}{3}+8\right)\right]$	M1	Apply limits $1 \to 16$ to an integrated expression.
	Area under line = $15\times5=75$	B1	Or by $\int_1^{16}5\mathrm{d}x$.
	Required area = $75-66=9$	A1	
		4	

22) JUNE 2022_9709_13 Q10(a)

a)	$\left\{\frac{(4x+2)^{-1}}{-1}\right\}\{\div4\}$ or eg $\left\{\frac{1}{16}\right\}\left\{-(x+0.5)^{-1}\right\}$ or $\frac{-1}{(16x+8)}$	B1 B1	OE If more than one function of x present then B0 B0.
	$0-(-1/24)$	M1	Apply limits to an integral, ∞ must be used correctly.
	1/24	A1	Allow 0.0417 AWRT.
		4	

23) JUNE 2023_9709_11 Q10(a)

(a)	$[\pi]\int\frac{16}{(2x-1)^4}[dx]=[\pi]\int 16(2x-1)^{-4}[dx]=[\pi]\left(-\frac{16}{3\times2\times(2x-1)^3}\right)$	*M1	Integrate y^2 (power incr. by 1 or div by *their* new power). M0 if more than 1 error or $-\frac{16}{6}x(2x-1)^{-3}$.
	$[\pi]\left(-\frac{16}{3\times2\times(2x-1)^3}\right)$	A1	OE e.g. $\left(-\frac{8}{3}(2x-1)^{-3}\right)$.
	$[\pi]\left(-\frac{16}{6\times8}+\frac{16}{6\times1}\right)$ $\left[=[\pi]\frac{112}{48}=[\pi]\frac{7}{3}\right]$	DM1	Sub correct limits into *their* integral: $F\left(\frac{3}{2}\right)-F(1)$. Must see at least $\left(-\frac{1}{3}+\frac{8}{3}\right)$. Allow 1 sign error. Decimal: 2.33π or 7.33.
	Volume of cylinder $\left[=\pi\times1^2\times\frac{1}{2}\right]=\frac{1}{2}\pi$ OR $[\pi]\int_1^{1.5}1[dx]=\frac{1}{2}\pi$	B1	$\frac{1}{2}\pi$ or $\pm\pi\left(\frac{3}{2}-1\right)$ seen.
	Volume of revolution $\left[=\frac{7}{3}\pi-\frac{1}{2}\pi\right]=\frac{11}{6}\pi$	A1	A0 for 5.76 (not exact). If DM0 for insufficient substitution, or B0, SC **B1** for $\frac{11}{6}\pi$.
		5	

24) JUNE 2023_9709_12 Q1

$[y]=\frac{4}{-2}(x-3)^{-3+1}$ or $\frac{4}{-2(x-3)^2}[+c]$	B1	OE Allow $\frac{4}{-3+1}$ and $-3+1$ for the power.
$5=\frac{4}{-2}(4-3)^{-2}+c$ or $5=\frac{4}{-2(4-3)^2}+c$ leading to $c=$	M1	Correct use of (4, 5) to find c in an integrated expression (defined by the correct power and no extra x's or terms).
$y=\frac{-2}{(x-3)^2}+7$ or $y=-2(x-3)^{-2}+7$	A1	OE $-\frac{4}{2}$ must be simplified to -2. Condone c = 7 as their final line as long as either y or f(x) = is seen elsewhere. Do not ISW if the result is of the form y = mx+c.
	3	

25) JUNE 2023_9709_12 Q5

$\left[\int\left(10x^{\frac{1}{2}}-\frac{5}{2}x^{\frac{3}{2}}\right)=\right]\left\{\frac{10}{\frac{3}{2}}x^{\frac{3}{2}}\right\}\left\{-\frac{5}{2\times\frac{5}{2}}x^{\frac{5}{2}}\right\}$ $\left[=\frac{20}{3}x^{\frac{3}{2}}-x^{\frac{5}{2}}\right]$	B1 B1	B1 for contents of each { } then ISW.
$=\left(their\frac{20}{3}\times8-32\right)[-0]$	M1	Using limit(s) correctly in an integrated expression (defined by one correct power). Minimum acceptable working is their $\left(\frac{160}{3}-32\right)$.
[Area of shaded region =] $\frac{64}{3}$, $21\frac{1}{3}$ or 21.3[333…]	A1	Condone the presence of π for the first 3 marks. Condone using the limits the wrong way around for the M mark and if -21.3 is corrected to 21.3 allow the A mark. **SC**: if M0 scored **SCB1** is available for correct final answer If $\int\left(10x^{\frac{1}{2}}-\frac{5}{2}x^{\frac{3}{2}}\right)=21.3$ and no integration seen B1 only.
	4	

26) JUNE 2023_9709_13 Q9(a)

(a)	$[y=]\{x\}\{+(x-1)^{-2}\}$ $[+c]$	B1 B1	May be unsimplified.
	Sub $x=0$, $y=3$ leading to 3 = 0 + 1 + c	M1	Substitution into an integral, expect $c=2$.
	$y=x+(x-1)^{-2}+2$ or $f(x)=x+(x-1)^{-2}+2$	A1	$\frac{-2}{(-2)(x-1)^3}$ or $\frac{-2(x-1)^{-2}}{-2}$ must be simplified.
		4	

27) JUNE 2023_9709_13 Q10(c)

(c)	$\left\{\frac{9x^2}{2}\right\}+\left\{\frac{-(2x+1)^{\frac{5}{2}}}{\frac{5}{2}\times 2}\right\}$	B1 B1	Integrating y with respect to x.
	$\left\{\frac{9}{2}7.5^2-\frac{1}{5}(2\times 7.5+1)^{2.5}\right\}-\left\{\frac{9}{2}1.5^2-\frac{1}{5}(2\times 1.5+1)^{2.5}\right\}$ or $\left(\frac{9}{2}\times\frac{225}{4}-\frac{1024}{5}\right)-\left(\frac{81}{8}-\frac{32}{5}\right)$ or $\frac{1933}{40}-\frac{149}{40}$ or $48.325-3.725$	M1	OE Apply limits 1½ to 7½ to an integral. Working must be seen. Expect 44.6 .
	$\frac{1}{2}\left(5\frac{1}{2}+3\frac{1}{2}\right)\times 6$ or $\int_{\frac{3}{2}}^{\frac{15}{2}}(\frac{-1}{3}x+6)\mathrm{d}x=$ $\left(\frac{-1}{6}\times\left(\frac{15}{2}\right)^2+6\times\frac{15}{2}\right)-\left(\frac{-1}{6}\times\left(\frac{3}{2}\right)^2+6\times\frac{3}{2}\right)$ or $\frac{285}{8}-\frac{69}{8}$ $[=27]$	B1	SOI Area of trapezium. May be seen combined with the area under the curve integral.
	[Shaded area = 44.6 – 27 =] 17.6	A1	**SC B1** if no substitution of the limits seen.
		5	

28) OCT 2020_9709_11 Q2

$(y=)\left[-(x-3)^{-1}\right]\left[+\frac{1}{2}x^2\right]$ $(+c)$	B1 B1	
$7=1+2+c$	M1	Substitute $x = 2$, $y = 7$ into an integrated expansion (c present). Expect $c = 4$
$y=-(x-3)^{-1}+\frac{1}{2}x^2$ $+4$	A1	OE
	4	

29) OCT 2020_9709_11 Q12

(a)	$4x^{\frac{1}{2}}-2x=3-x\ \rightarrow x-4x^{\frac{1}{2}}+3(=0)$	*M1	3-term quadratic. Can be expressed as e.g. u^2-4u+3 (=0)
	$\left(x^{\frac{1}{2}}-1\right)\left(x^{\frac{1}{2}}-3\right)(=0)$ or $(u-1)(u-3)(=0)$	DM1	Or quadratic formula or completing square
	$x^{\frac{1}{2}}=1,\ 3$	A1	SOI
	$x=1,\ 9$	A1	
	Alternative method for question 12(a)		
	$\left(4x^{\frac{1}{2}}\right)^2=(3+x)^2$	*M1	Isolate $x^{\frac{1}{2}}$
	$16x=9+6x+x^2\ \rightarrow\ x^2-10x+9\ (=0)$	A1	3-term quadratic
	$(x-1)(x-9)\ (=0)$	DM1	Or formula or completing square on a quadratic obtained by a correct method
	$x=1,\ 9$	A1	
		4	
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=2x^{1/2}-2$	*B1	
	$\frac{\mathrm{d}y}{\mathrm{d}x}$ or $2x^{1/2}-2=0$ when $x=1$ hence B is a stationary point	DB1	
		2	

(c)	Area of correct triangle = $\frac{1}{2}$ (9 – 3) × 6	M1	or $\int_3^9 (3-x)(\mathrm{d}x) = \left[3x - \frac{1}{2}x^2\right] \to -18$
	$\int(4x^{\frac{1}{2}} - 2x)(\mathrm{d}x) = \left[\frac{4x^{\frac{3}{2}}}{\frac{3}{2}} - x^2\right]$	B1 B1	
	$(72-81) - \left(\frac{64}{3} - 16\right)$	M1	Apply limits 4 → *their* 9 to an integrated expression
	$-14\frac{1}{3}$	A1	OE
	Shaded region = $18 - 14\frac{1}{3} = 3\frac{2}{3}$	A1	OE
		6	

30) OCT 2020_9709_12 Q7(b)

(b)	$\frac{6x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{4x^{-\frac{1}{2}}}{-\frac{1}{2}} (+c)$	B1 B1	B1 for each unsimplified integral.
	Uses (4, 7) leading to $c = (-21)$	M1	Uses (4, 7) to find a c value
	y or $\mathrm{f}(x) = 12x^{\frac{1}{2}} + 8x^{-\frac{1}{2}} - 21$ or $12\sqrt{x} + \frac{8}{\sqrt{x}} - 21$	A1	Need to see y or f(x) = somewhere in *their* solution and 12 and 8

31) OCT2020_9709_12 Q10

(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = [8] \times \left[(3-2x)^{-3}\right] + [-1]$ $\left(= \frac{8}{(3-2x)^3} - 1\right)$	B2, 1, 0	B2 for all three elements correct, B1 for two elements correct, B0 for only one or no elements correct.
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -3 \times 8 \times (3-2x)^{-4} \times (-2)$ $\left(= \frac{48}{(3-2x)^4}\right)$	B1 FT	FT providing *their* bracket is to a negative power
	$\int y\mathrm{d}x = [(3-2x)^{-1}]\,[2 \div (-1 \times -2)]\,[-\frac{1}{2}x^2]\,(+c)$ $\left(= \frac{1}{3-2x} - \frac{1}{2}x^2 + c\right)$	B1 B1 B1	Simplification not needed, B1 for each correct element
		6	
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \to (3-2x)^3 = 8 \to 3 - 2x = \mathrm{k} \to x =$	M1	Setting *their* 2-term differential to 0 and attempts to solve as far as $x =$
	$\frac{1}{2}$	A1	
	Alternative method for question 10(b)		
	$y = 0 \to \frac{2}{(3-2x)^2} - x = 0 \to (x-2)(2x-1)^2 = 0 \to x =$	M1	Setting y to 0 and attempts to solve a cubic as far as $x =$ (3 factors needed)
	$\frac{1}{2}$	A1	
		2	
(c)	Area under curve = *their* $\left[\frac{1}{3-2\times\left(\frac{1}{2}\right)} - \frac{\left(\frac{1}{2}\right)^2}{2}\right] - \left[\frac{1}{3-2\times0} - 0\right]$	M1	Using *their* integral, *their* positive x limit from **part (b)** and 0 correctly.
	$\frac{1}{24}$	A1	
		2	

32) OCT 2020_9709_13 Q2

(a)	$\frac{-2}{x+2}$	B1	Integrate f(x). Accept $-2(x+2)^{-1}$. Can be unsimplified.
	$0-\left(-\frac{2}{3}\right) = \frac{2}{3}$	M1 A1	Apply limit(s) to an integrated expansion. CAO for A1
		3	
(b)	$-1=-2+c$	M1	Substitute $x=-1, y=-1$ into *their* integrated expression (*c* present)
	$y=\frac{-2}{x+2}+1$	A1	Accept $y=-2(x+2)^{-1}+1$. –2 must be resolved.
		2	

33) OCT 2020_9709_13 Q10(b)

(b)	$\int \frac{1}{k}x^{1/2}+x^{-1/2}+\frac{1}{k^2} = \left[\frac{2x^{3/2}}{3k}\right]+\left[2x^{1/2}\right]+\left[\frac{x}{k^2}\right]$	B2, 1, 0	OE
	$\left(\frac{2k^2}{3}+2k+1\right)-\left(\frac{k^2}{12}+k+\frac{1}{4}\right)$	M1	Apply limits $\frac{k^2}{4}\to k^2$ to an integrated expression. Expect $\frac{7}{12}k^2+k+\frac{3}{4}$
	$\frac{7}{12}k^2+k+\frac{3}{4}=\frac{13}{12}$	M1	Equate to $\frac{13}{12}$ and simplify to quadratic. OE, expect $7k^2+12k-4\ (=0)$
	$k=\frac{2}{7}$ only (or 0.286)	A1	Dependent on $(7k-2)(k+2)\ (=0)$ or formula or completing square.
		5	

34) OCT 2021_9709_11 Q9

(a)	$f(x)=\frac{2}{3}x^3-7x+4x^{-1}\ [+c]$	B2, 1, 0	Allow terms on different lines; allow unsimplified.
	$-\frac{1}{3}=\frac{2}{3}-7+4+c$ leading to $c=[2]$	M1	Substitute f(1) = $-\frac{1}{3}$ into an integrated expression and evaluate *c*.
	$f(x)=\frac{2}{3}x^3-7x+4x^{-1}+2$	A1	OE.
		4	
(b)	$2x^4-7x^2-4\ [=0]$	M1	Forms 3-term quadratic in x^2 with all terms on one side. Accept use of substitution e.g. $2y^2-7y-4[=0]$.
	$(2x^2+1)(x^2-4)\ [=0]$	M1	Attempt factors or use formula or complete the square. Allow ± sign errors. Factors must expand to give *their* coefficient of x^2 or e.g. *y*. Must be quartic equation. Accept use of substitution e.g. $(2y+1)(y-4)$.
	$x=[\pm]2$	A1	If M0 for solving quadratic, SC **B1** can be awarded for $[\pm]2$.
	$\left[\frac{2}{3}(2)^3-7(2)+\frac{4}{2}+2 \text{ leading to}\right]\left(2, -\frac{14}{3}\right)$ $\left[\frac{2}{3}(-2)^3-7(-2)+\frac{4}{-2}+2 \text{ leading to}\right]\left(-2, \frac{26}{3}\right)$	B1 B1	B1 B1 for correct coordinates clearly paired; B1 for each correct point; B1 B0 if additional point.
		5	
(c)	$f''(x)=4x+8x^{-3}$	B1	OE
		1	

(d)	$f''(2)=9>0$ MINIMUM at $x=$ *their* 2	B1 FT	FT on *their* $x = [\pm]2$ provided $f''(x)$ is correct. Must have correct value of $f''(x)$ if $x=2$.
	$f''(-2)=-9<0$ MAXIMUM at $x=$ *their* -2	B1 FT	FT on *their* $x = [\pm]2$ provided $f''(x)$ is correct. Must have correct value of $f''(x)$ if $x=-2$. **Special case:** If values not shown and B0B0 scored, **SC B1** for $f''(2)>0$ MIN and $f''(-2)<0$ MAX
	Alternative method for question 9(d)		
	Evaluate $f'(x)$ for x-values either side of 2 and –2	M1	FT on *their* $x = [\pm]2$
	MINIMUM at $x=$ *their* 2, MAXIMUM at $x=$ *their* 2	A1 FT	FT on *their* $x = [\pm]2$. Must have correct values of $f'(x)$ if shown. **Special case:** If values not shown and M0A0 scored **SC B1** $f'(2)$ –/0/+ MIN and $f'(-2)$ +/0/– MAX
	Alternative method for question 9(d)		
	Justify maximum and minimum using correct sketch graph	B1 B1	Need correct coordinates in **(b)** for this method.
		2	

35) OCT 2021_9709_11 Q10(a)(b)

(a)	$\left\{\frac{(3x-2)^{-\frac{1}{2}}}{-1/2}\right\} \div \{3\}$	B2, 1, 0	Attempt to integrate
	$-\frac{2}{3}[0-1]$	M1	M1 for applying limits $1 \to \infty$ to an integrated expression (either correct power or dividing by their power).
	$\frac{2}{3}$	A1	
		4	
(b)	$[\pi]\int y^2 dx = [\pi]\int (3x-2)^{-3} dx = [\pi]\frac{(3x-2)^{-2}}{-2\times 3}$	*M1 A1	M1 for attempt to integrate y^2 (power increases); allow 1 error. A1 for correct result in any form.
	$[\pi]\left[-\frac{1}{6}\right]\left[\frac{1}{16}-1\right]$	DM1	Apply limits 1 and 2 to an integrated expression and subtract correctly; allow 1 error.
	$\frac{5\pi}{32}$	A1	OE
		4	

36) OCT 2021_9709_12 Q4

$y=-\frac{\frac{8}{3}}{(3x+2)}[+c]$	*B1	For $(3x+2)^{-1}$
	DB1	For $-\frac{8}{3}$
$5\frac{2}{3}=-\frac{\frac{8}{3}}{(3\times 2+2)}+c$	M1	Substituting $\left(2,\ 5\frac{2}{3}\right)$ into *their* integrated expression – defined by power = –1, or dividing by their power. + c needed
$y=-\frac{8}{3(3x+2)}+6$	A1	OE e.g. $y=-\frac{8}{3}(3x+2)^{-1}+6$
	4	

37) OCT 2021_9709_12 Q11

(a)	$\frac{dy}{dx}=\frac{1}{2}+\frac{1}{3(x-2)^{\frac{4}{3}}}$	B1	OE. Allow unsimplified.
	Attempt at evaluating *their* $\frac{dy}{dx}$ at $x=3$ $\left[\frac{1}{2}+\frac{1}{3(3-2)^{\frac{4}{3}}}=\frac{5}{6}\right]$	*M1	Substituting $x = 3$ into *their* differentiated expression – defined by one of 3 original terms with correct power of x.
	Gradient of normal = $\frac{-1}{their\frac{dy}{dx}}$ $\left[=-\frac{6}{5}\right]$	*DM1	Negative reciprocal of *their* evaluated $\frac{dy}{dx}$.
	Equation of normal $y-\frac{6}{5}=(their \text{ normal gradient})(x-3)$ $\left[y=-\frac{6}{5}x+4.8\Rightarrow 5y=-6x+24\right]$	DM1	Using *their* normal gradient and *A* in the equation of a straight line. Dependent on *M1 and *DM1.
	[When $y = 0$,] $x = 4$	A1	or (4, 0)
		5	
b)	Area under curve = $\int\left(\frac{1}{2}x+\frac{7}{10}-\frac{1}{(x-2)^{\frac{1}{3}}}\right)[dx]$	M1	For intention to integrate the curve (no need for limits). Condone inclusion of π for this mark.
	$\frac{1}{4}x^2+\frac{7}{10}x-\frac{3(x-2)^{\frac{2}{3}}}{2}$	A1	For correct integral. Allow unsimplified. Condone inclusion of π for this mark.
	$\left(\frac{9}{4}+2.1-\frac{3}{2}\right)-\left(\frac{6.25}{4}+1.75-\frac{3\times0.5^{\frac{2}{3}}}{2}\right)$	M1	Clear substitution of 3 and 2.5 into *their* integrated expression (with at least one correct term) and subtracting.
	0.48[24]	A1	If M1A1M0 scored then SC B1 can be awarded for correct answer.
	[Area of triangle =] 0.6	B1	OE
	[Total area =] 1.08	A1	Dependent on the first M1 and WWW.
		6	

38) OCT 2021_9709_13 Q8(a)

a)	$\int\left(\frac{5}{2}-x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)dx$	M1	OR as 2 separate integrals $\int\left(\frac{5}{2}-x^{1/2}\right)dx-\int\left(x^{-1/2}\right)dx$
	$\left\{\frac{5}{2}x-\frac{2}{3}x^{\frac{3}{2}}\right\}\{-\}\left\{2x^{\frac{1}{2}}\right\}$	A1 A1 A1	If two separate integrals with no subtraction SC B1 for each correct integral.
	$\left(10-\frac{16}{3}-4\right)-\left(\frac{5}{8}-\frac{1}{12}-1\right)$	DM1	Substitute limits $\frac{1}{4}\to4$ at least once, must be seen.
	$\frac{9}{8}$ or 1.125	A1	WWW. Cannot be awarded if π appears in any integral.
		6	

39) OCT 2021_9709_13 Q10

(a)	$f''(x) = -\left(\frac{1}{2}x + k\right)^{-3}$	B1	
	$f''(2) > 0 \Rightarrow -(1+k)^{-3} > 0$	M1	Allow for solving *their* $f''(2) > 0$
	$k < -1$	A1	WWW
		3	
(b)	$\left[f(x) = \int\left(\left(\frac{1}{2}x-3\right)^{-2} - (-2)^{-2}\right)dx = \right] \left\{\frac{\left(\frac{1}{2}x-3\right)^{-1}}{-1\times\frac{1}{2}}\right\} \left\{-\frac{x}{4}\right\}$	B1 B1	Allow $-2\left(\frac{1}{2}x+k\right)^{-1}$ OE for 1st B1 and $-(1+k)^{-2}x$ OE for 2nd B1
	$3\frac{1}{2} = 1 - \frac{1}{2} + c$	M1	Substitute $x = 2, y = 3\frac{1}{2}$ into *their* integral with c present.
	$f(x) = \frac{-2}{\left(\frac{1}{2}x-3\right)} - \frac{x}{4} + 3$	A1	OE
		4	
(c)	$\left(\frac{1}{2}x-3\right)^{-2} - (-2)^{-2} = 0$	M1	Substitute $k = -3$ and set to zero.
	leading to $\left(\frac{1}{2}x-3\right)^{2} = 4$ $\left[\frac{1}{2}x - 3 = (\pm)2\right]$ leading to $x = 10$	A1	
	$(10, -\frac{1}{2})$	A1	Or when $x = 10, y = -1 - 2\frac{1}{2} + 3 = -\frac{1}{2}$
	$f''(10)\left[= -(5-3)^{-3} \rightarrow\right] < 0 \rightarrow$ MAXIMUM	A1	WWW
		4	

40) OCT 2022_9709_11 Q2(b)

(b)	$[y=]\left(\frac{12\left(\frac{1}{2}x-1\right)^{-3}}{-3}\right) \div \frac{1}{2}\left[= -8\left(\frac{1}{2}x-1\right)^{-3}\right]$	B2, 1, 0	
	$4 = \frac{12\times\left(\frac{1}{2}\times 6-1\right)^{-3}}{\frac{1}{2}\times -3} + c$ $\left[\Rightarrow 4 = -8\times 2^{-3} + c\right] \Rightarrow c = [5]$	M1	Must have $+c$. Substitute $y = 4, x = 6$ and solve for c in an integrated expression. May be unsimplified.
	$[y=]-8\left(\frac{1}{2}x-1\right)^{-3} + 5$	A1	OE Must see '$y =$' or '$f(x) =$' in the working.
		4	

41) OCT 2022_9709_11 Q10

(a)	$\pm\int(2x^{1/2}+1)-\left(\frac{1}{2}x^2-x+1\right)\,dx$ [$=\pm\int 2x^{1/2}-\frac{1}{2}x^2+x\,dx$]	*M1	
	$\pm\left(\frac{4x^{3/2}}{3}+x-\left(\frac{x^3}{6}-\frac{x^2}{2}+x\right)\right)$ or $\pm\left(\frac{4x^{3/2}}{3}-\frac{x^3}{6}+\frac{x^2}{2}\right)$	B2, 1, 0	OE Coefficients may be unsimplified.
	$\pm\left(\frac{32}{3}-\frac{32}{3}+8\right)$ or $\pm\left(\frac{44}{3}-0-\frac{20}{3}+0\right)$	DM1	± (F(4) – F(0)) using *their* integral(s).
	$=8$	A1	Depends on all previous marks. If *M1 B2 DM0 and limits stated, SC B1 for +8
		5	
(b)	Upper curve: $\frac{dy}{dx}=x^{-\frac{1}{2}}$. Lower curve: $\frac{dy}{dx}=x-1$	M1 A1	Attempt at differentiating one function. A1 if both correct.
	At $x=4$: gradient of upper curve $=\frac{1}{2}$, gradient of lower curve $=3$	M1	Evaluate two gradients using $x=4$.
	$\alpha=\tan^{-1}3-\tan^{-1}\frac{1}{2}$ [$=71.57-26.57$]	M1	Use inverse tan to find angles then subtract. **OR** find equations of both tangents then Pythagoras using a point on each e.g. on axes. **OR** cosine rule using intercepts or proportion.
	[$\alpha=$]45°	A1	AWRT
		5	

42) OCT 2022_9709_12 Q8(a)

(a)	$[y=]\left\{\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}\right\}+\left\{-\frac{3x^{\frac{1}{2}}}{\frac{1}{2}}\right\}[+c]\left[=2x^{\frac{3}{2}}-6x^{\frac{1}{2}}\right]$	B1 B1	Marks can be awarded for correct unsimplified expressions, 1 mark each for contents of { } ISW.
	$5=2\times3^{\frac{3}{2}}-6\times3^{\frac{1}{2}}+c$	M1	Correct use of (3,5) in an integrated expression (defined by at least one correct power) including + c.
	$y=2x^{\frac{3}{2}}-6x^{\frac{1}{2}}+5$	A1	Condone $c=5$ as their final line if either $y=$ or f(x) = seen elsewhere in the solution, but coefficients must not contain unresolved double fractions.
		4	

43) OCT 2022_9709_12 Q11(b)

(b)	[Area =] $\int\left(18-\frac{3}{8}x^{\frac{5}{2}}-\left(\frac{9}{4}x^2-12x+18\right)\right)dx$	M1	Intention to integrate and subtract areas (either way around). Can be two separate functions or combined. Using y^2 scores 0/5 but condone inclusion of π except for the final mark.
	Note: Subtraction not required for these marks. Either separately $\left([18x]-\frac{3x^{\frac{7}{2}}}{8\times\frac{7}{2}}\right)$, $\left(\frac{9x^3}{4\times3}-\frac{12x^2}{2}[+18x]\right)$ Or combined $[18x]-\frac{3x^{\frac{7}{2}}}{8\times\frac{7}{2}}-\frac{9x^3}{4\times3}+\frac{12x^2}{2}[-18x]$	B1,B1	One mark for correct integration of each curve, allow unsimplified. $\left([18x]-\frac{3}{28}x^{\frac{7}{2}}\right)$ $\left(\frac{3}{4}x^3-6x^2[+18x]\right)$ or $[18x]-\frac{3}{28}x^{\frac{7}{2}}-\frac{3}{4}x^3+6x^2[-18x]$ **BUT** condone sign errors that are only due to missing brackets.
	$=\left(-\frac{3}{28}\times4^{\frac{7}{2}}-\frac{3}{4}\times4^3+6\times4^2\right)$ $[-(0)]$	M1	Clear substitution of 4 into at least one integrated expression (defined by at least one correct power) which can be unsimplified.
	$=\frac{240}{7}$ or 34.3 AWRT	A1	SC: If all marks awarded except the final M1, SCB1 is available for the correct final answer.
		5	

44) OCT 2022_9709_13 Q7(b)

(b)	$[f(x)] = \frac{1}{(x+2)^3}$ $[+c]$	B1	Allow unsimplified form and '$y =$'
	$5 = 1 + c$	M1	Sub $x = -1$, $y = 5$ into an integral.
	$[f(x)] = \frac{1}{(x+2)^3} + 4$	A1	Allow '$y =$'
		3	

45) OCT 2022_9709_13 Q10

(a)	$x^2 + (2x-1)^2 - 2[=0] \to 5x^2 - 4x - 1\,[=0]$	*M1 A1	Or $5y^2 + 2y - 7\,[=0]$.
	$(5x+1)(x-1)[=0]$ or $(5y+7)(y-1)[=0]$	DM1	May see factors or formula or completing square.
	$x = 1$, $y = 1$ or (1, 1) only	A1	May be implied on the diagram.
		4	
(b)	$(\pi)\int(2-x^2)dx = (\pi)\left(2x - \frac{x^3}{3}\right)$	*M1 A1	Attempt integration of y^2, allow $\int(2-y^2)dy$.
	$(\pi)\left(2\sqrt{2} - \frac{(\sqrt{2})^3}{3}\right) - \left(2 - \frac{1}{3}\right))$	DM1	Apply limits $1 \to \sqrt{2}$.
	$\frac{\pi}{3}(4\sqrt{2} - 5)$	A1	CAO, allow $\frac{\pi}{3}(2\sqrt{8} - 5)$, must be in given form.
		4	
(c)	Arc length $= \frac{1}{8}(2\pi\sqrt{2})$ or $\frac{\pi\sqrt{2}}{4}$ oe	B1	Must be exact.
	Perimeter $= \sqrt{2} +$ *their* arc length	B1 FT	Must be exact, do not allow inverse trig functions.
		2	