PURE MATHEMATICS - 1

9709

(March, June and November series 2020 – 2023 With marking scheme)

<u>Quadratics</u>

Exercise - 1

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1.

The equation of a line is $y = mx + c$, where m and c are constants, and the equation of a curve is $xy = 16$.

(a) Given that the line is a tangent to the curve, express m in terms of c. [3]

(b) Given instead that $m = -4$, find the set of values of c for which the line intersects the curve at two distinct points. [3]

QUESTION – 5: QP_S20_11

2.

The equation of a curve is $y = 2x^2 + kx + k - 1$, where k is a constant.

(a) Given that the line $y = 2x + 3$ is a tangent to the curve, find the value of k. [3]

It is now given that $k = 2$.

(b) Express the equation of the curve in the form $y = 2(x + a)^2 + b$, where a and b are constants, and hence state the coordinates of the vertex of the curve. [3]

QUESTION – 6: QP_S20_12

3.

Find the set of values of m for which the line with equation $y = mx + 1$ and the curve with equation $y = 3x^2 + 2x + 4$ intersect at two distinct points. [4]

QUESTION – 1: QP_S20_13

4.

Find the set of values of m for which the line with equation $y = mx - 3$ and the curve with equation $y = 2x^2 + 5$ do not meet. [3]

QUESTION – 1: QP_W20_11

5.

(a) Express $x^2 + 6x + 5$ in the form $(x + a)^2 + b$, where a and b are constants. [2]

(b) The curve with equation $y = x^2$ is transformed to the curve with equation $y = x^2 + 6x + 5$.

Describe fully the transformation(s) involved. [2]

QUESTION – 1: QP_W20_13

6.

A curve has equation $y = 3x^2 - 4x + 4$ and a straight line has equation $y = mx + m - 1$, where m is a constant.

Find the set of values of m for which the curve and the line have two distinct points of intersection. [5]

QUESTION – 4: QP_W20_13

7.

By using a suitable substitution, solve the equation

$$(2x - 3)^2 - \frac{4}{(2x - 3)^2} - 3 = 0.$$ [4]

QUESTION – 2: QP_M21_12

8.

A line has equation $y = 3x + k$ and a curve has equation $y = x^2 + kx + 6$, where k is a constant.

Find the set of values of k for which the line and curve have two distinct points of intersection. [5]

QUESTION – 4: QP_M21_12

9.

The equation of a curve is $y = (2k - 3)x^2 - kx - (k - 2)$, where k is a constant. The line $y = 3x - 4$ is a tangent to the curve.

Find the value of k. [5]

QUESTION – 6: QP_S21_11

10.

(a) Express $16x^2 - 24x + 10$ in the form $(4x + a)^2 + b$. [2]

(b) It is given that the equation $16x^2 - 24x + 10 = k$, where k is a constant, has exactly one root.

Find the value of this root. [2]

QUESTION – 1: QP_S21_12

11.

A line with equation $y = mx - 6$ is a tangent to the curve with equation $y = x^2 - 4x + 3$.

Find the possible values of the constant m, and the corresponding coordinates of the points at which the line touches the curve. [6]

QUESTION – 3: QP_S21_13

12.

A curve has equation $y = kx^2 + 2x - k$ and a line has equation $y = kx - 2$, where k is a constant.

Find the set of values of k for which the curve and line do not intersect. [5]

QUESTION – 2: QP_W21_11

13.

(a) Express $5y^2 - 30y + 50$ in the form $5(y + a)^2 + b$, where a and b are constants. [2]

QUESTION – 3(a): QP_W21_13

14.

A curve has equation $y = x^2 + 2cx + 4$ and a straight line has equation $y = 4x + c$, where c is a constant.

Find the set of values of c for which the curve and line intersect at two distinct points. [5]

QUESTION – 2: QP_M22_12

15.

(a) Express $2x^2 - 8x + 14$ in the form $2[(x - a)^2 + b]$. [2]

QUESTION – 5(a): QP_M22_12

16.

(a) Solve the equation $6\sqrt{y} + \dfrac{2}{\sqrt{y}} - 7 = 0$. [4]

QUESTION – 5(a): QP_S22_13

17.

The function f is defined by $f(x) = 2x^2 - 16x + 23$ for $x < 3$.

(a) Express $f(x)$ in the form $2(x + a)^2 + b$. [2]

QUESTION – 6(a): QP_S22_13

18.

Solve the equation $3x + 2 = \dfrac{2}{x - 1}$. [3]

QUESTION – 1: QP_W22_11

19.

(a) Find the set of values of k for which the equation $8x^2 + kx + 2 = 0$ has no real roots. [2]

QUESTION – 3(a): QP_W22_12

20.

The equation of a curve is $y = 4x^2 + 20x + 6$.

(a) Express the equation in the form $y = a(x + b)^2 + c$, where a, b and c are constants. [3]

(b) Hence solve the equation $4x^2 + 20x + 6 = 45$. [3]

QUESTION – 6: QP_W22_12

21.

(a) Find the coordinates of the minimum point of the curve $y = \frac{9}{4}x^2 - 12x + 18$. [3]

QUESTION – 11(a): QP_W22_12

22.

A line has equation $y = 3x - 2k$ and a curve has equation $y = x^2 - kx + 2$, where k is a constant.

Show that the line and the curve meet for all values of k. [4]

QUESTION – 1: QP_M23_12

23.

The line with equation $y = kx - k$, where k is a positive constant, is a tangent to the curve with equation $y = -\dfrac{1}{2x}$.

Find, in either order, the value of k and the coordinates of the point where the tangent meets the curve. [5]

QUESTION – 5: QP_S23_11

24.

(a) Express $4x^2 - 24x + p$ in the form $a(x + b)^2 + c$, where a and b are integers and c is to be given in terms of the constant p. [2]

(b) Hence or otherwise find the set of values of p for which the equation $4x^2 - 24x + p = 0$ has no real roots. [1]

QUESTION – 3: QP_S23_12

25.

Solve the equation $8x^6 + 215x^3 - 27 = 0$. [3]

QUESTION – 4: QP_S23_12

26.

The function f is defined for $x \in \mathbb{R}$ by $f(x) = x^2 - 6x + c$, where c is a constant. It is given that $f(x) > 2$ for all values of x.

Find the set of possible values of c. [4]

QUESTION – 2: QP_S23_13

MARK SCHEME

1.

Answer	Marks
$x(mx+c) = 16 \rightarrow mx^2 + cx - 16 = 0$	**B1**
Use of $b^2 - 4ac = c^2 + 64m$	**M1**
Sets to 0 $\rightarrow m = \frac{-c^2}{64}$	**A1**
	3
$x(-4x+c) = 16$ Use of $b^2 - 4ac \rightarrow c^2 - 256$	**M1**
$c > 16$ and $c < -16$	**A1 A1**
	3

2.

Answer	Marks
$2x^2 + kx + k - 1 = 2x + 3 \rightarrow 2x^2 + (k-2)x + k - 4 = 0$	**M1**
Use of $b^2 - 4ac = 0 \rightarrow (k-2)^2 = 8(k-4)$	**M1**
$k = 6$	**A1**
	3
$2x^2 + 2x + 1 = 2\left(x + \frac{1}{2}\right)^2 + 1 - \frac{1}{2}$ $a = \frac{1}{2}, b = \frac{1}{2}$	**B1 B1**
vertex $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (**FT** on a and b values)	**B1FT**
	3

3.

Answer	Marks
$3x^2 + 2x + 4 = mx + 1 \rightarrow 3x^2 + x(2-m) + 3\ (=0)$	**B1**
$(2-m)^2 - 36$ SOI	**M1**
$(m+4)(m-8)$ (>/= 0) **or** $2 - m$ >/= 6 and $2 - m$ </= -6 OE	**A1**
$m < -4$, $m > 8$ WWW	**A1**
Alternative method for question 1	
$\frac{dy}{dx} = 6x + 2 \rightarrow m = 6x + 2 \rightarrow 3x^2 + 2x + 4 = (6x+2)x + 1$	**M1**
$x = \pm 1$	**A1**
$m = \pm 6 + 2 \rightarrow m = 8$ or -4	**A1**
$m < -4$, $m > 8$ WWW	**A1**
	4

4.

Answer	Marks	Guidance
$2x^2+5=mx-3 \rightarrow 2x^2-mx+8\ (=0)$	B1	Form 3-term quadratic
m^2-64	M1	Find b^2-4ac .
$-8<m<8$	A1	Accept (-8, 8) and equality included
	3	

5.

Answer	Marks	Guidance
$\left[(x+3)^2\right]$ $[-4]$	B1 B1	
	2	
[Translation or shift] $\begin{pmatrix}-3\\-4\end{pmatrix}$	B1 B1 FT	Accept [translation/shift] $\begin{pmatrix}-\textit{their } a\\ \textit{their } b\end{pmatrix}$ OR translation –3 units in x-direction and (translation) –4 units in y-direction.
	2	

6.

Answer	Marks	Guidance
$3x^2-4x+4=mx+m-1 \rightarrow 3x^2-(4+m)x+(5-m)\ (=0)$	M1	3-term quadratic
$b^2-4ac=(4+m)^2-4\times3\times(5-m)$	M1	Find b^2-4ac for *their* quadratic
$m^2+20m-44$	A1	
$(m+22)(m-2)$	A1	Or use of formula or completing square. This step must be seen
$m>2$, $m<-22$	A1	Allow $x>2$, $x<-22$
	5	

7.

Answer	Marks	Guidance
$u=2x-3$ leading to $u^4-3u^2-4\ [=0]$	M1	Or $u=(2x-3)^2$ leading to $u^2-3u-4\ [=0]$
$(u^2-4)(u^2+1)\ [=0]$	M1	Or $(u-4)(u+1)\ [=0]$
$2x-3=[\pm]2$	A1	
$x=\frac{1}{2}, \frac{5}{2}$ **only**	A1	
	4	

8.

Answer	Marks	Guidance
$x^2+kx+6=3x+k$ leading to $x^2+x(k-3)+(6-k)$ [=0]	M1	Eliminate y and form 3-term quadratic.
$(k-3)^2-4(6-k)[>0]$	M1	OE. Apply b^2-4ac .
$k^2-2k-15[>0]$	A1	Form 3-term quadratic.
$(k+3)(k-5)[>0]$	A1	Or $k=-3,\ 5$ from use of formula or completing square.
$k<-3,\quad k>5$	A1 FT	Or any correct alternative notation, do not allow $\leqslant, \geqslant$. FT for *their* outside regions.
	5	

9.

Answer	Marks	Guidance
$(2k-3)x^2-kx-(k-2)=3x-4$	*M1	Equating curve and line
$(2k-3)x^2-(k+3)x-(k-6)[=0]$	DM1	Forming a 3-term quadratic
$(k+3)^2+4(2k-3)(k-6)[=0]$	DM1	Use of discriminant (dependent on **both** previous M marks)
$9k^2-54k+81[=0]$ [leading to $k^2-6k+9=0$]	M1	Simplifying and solving *their* 3-term quadratic in k
$k=3$	A1	
Alternative method for Question 6		
$(2k-3)x^2-kx-(k-2)=3x-4$	*M1	Equating curve and line
$2(2k-3)x-k=3 \Rightarrow x=\dfrac{k+3}{4k-6}$ or $k=\dfrac{3+6x}{4x-1}$	DM1	Differentiating and solving for x or k
Either $(2k-3)\left(\dfrac{k+3}{4k-6}\right)^2-k\left(\dfrac{k+3}{4k-6}\right)-(k-2)=3\left(\dfrac{k+3}{4k-6}\right)-4$ **Or** $4x\left(\dfrac{3x^2+3x-6}{2x^2-x-1}\right)-6x-\left(\dfrac{3x^2+3x-6}{2x^2-x-1}\right)=3$	DM1	Substituting *their* x into equation or *their* $k=\dfrac{3x^2+3x-6}{2x^2-x-1}$ or $k=\dfrac{3x+6}{2x+1}$ into derivative equation (dependent on **both** previous M marks)
$9k^2-54k+81[=0]$ [leading to $k^2-6k+9=0$]	M1	Simplifying and solving *their* 3-term quadratic in k (or solving for x)
$k=3$	A1	
		SC If M0, B1 for differentiating, equating to 3 and solving for x or k
	5	

10.

Answer	Marks	Guidance
$(4x-3)^2$ or $(4x+(-3))^2$ or $a=-3$	B1	$k(4x-3)^2$ where $k \neq 1$ scores B0 but mark final answer, allow recovery.
$+1$ or $b=1$	B1	
	2	
[For one root] $k=1$ or '*their b*'	B1 FT	Either by inspection or solving or from $24^2 - 4 \times 16 \times (10-k) = 0$ WWW
[Root or $x=$]$\frac{3}{4}$ or 0.75	B1	**SC B2** for correct final answer WWW.
	2	

11.

Answer	Marks	Guidance
$x^2-4x+3=mx-6$ leading to $x^2-x(4+m)+9$	***M1**	Equating and gathering terms. May be implied on the next line.
b^2-4ac leading to $(4+m)^2-4\times9$	**DM1**	SOI. Use of the discriminant with *their* a, b and c
$4+m=\pm6$ or $(m-2)(m+10)=0$ leading to $m=2$ or -10	**A1**	Must come from $b^2-4ac=0$ SOI
Substitute both *their* m values into *their* equation in line 1	**DM1**	
$m=2$ leading to $x=3$; $m=-10$ leading to $x=-3$	**A1**	
(3, 0), (–3, 24)	**A1**	Accept 'when $x=3$, $y=0$; when $x=-3$, $y=24$' If final A0A0 scored, **SC B1** for one point correct WWW
Alternative method for Question 3		
$\frac{dy}{dx}=2x-4 \rightarrow 2x-4=m$	***M1**	
$x^2-4x+3=(2x-4)x-6$	**DM1**	
$x^2-4x+3=2x^2-4x-6 \rightarrow 9=x^2 \rightarrow x=\pm3$	**A1**	
$y=0,\ 24$ or (3, 0), (–3, 24)	**A1**	
Substitute both *their* x values into *their* equation in line 1	**DM1**	Or substitute both *their* (x,y) into $y=mx-6$
When $x=3$, $m=2$; when $x=-3$, $m=-10$	**A1**	If A0, DM1, A0 scored, **SC B1** for one point correct WWW
	6	

12.

Answer	Marks	Guidance
$kx^2+2x-k=kx-2$ leading to $kx^2+(-k+2)x-k+2\ [=0]$	*M1	Eliminate y and form 3-term quadratic. Allow 1 error.
$(-k+2)^2-4k(-k+2)$	DM1	Apply b^2-4ac ; allow 1 error but a, b and c must be correct for *their* quadratic.
$5k^2-12k+4$ or $(-k+2)(-k+2-4k)$	A1	May be shown in quadratic formula.
$(-k+2)(-5k+2)$	DM1	Solving a 3-term quadratic in k (all terms on one side) by factorising, use of formula or completing the square. Factors must expand to give *their* coefficient of k^2.
$\frac{2}{5}<k<2$	A1	WWW, accept two separate correct inequalities. If M0 for solving quadratic, **SC B1** can be awarded for correct final answer.
	5	

13.

Answer	Marks	Guidance
$\{5(y-3)^2\}$ $\{+5\}$	B1 B1	Accept $a=-3$, $b=5$
	2	

14.

Answer	Marks	Guidance
$x^2+2cx+4=4x+c$ leading to $x^2+2cx-4x+4-c\ [=0]$	*M1	Equate ys and move terms to one side of equation.
$b^2-4ac=(2c-4)^2-4(4-c)$	DM1	Use of discriminant with *their* correct coefficients.
$[4c^2-16c+16-16+4c=]\ 4c^2-12c$	A1	
$b^2-4ac>0$ leading to $(4)c(c-3)>0$	M1	Correctly apply '>0' considering both regions.
$c<0,\ c>3$	A1	Must be in terms of c. **SC B1** instead of M1A1 for $c \leqslant 0$, $c \geqslant 3$
	5	

15.

Answer	Marks	Guidance
$2[\{(x-2)^2\}\ \{+3\}]$	B1 B1	B1 for $a=2$, B1 for $b=3$. $2(x-2)^2+6$ gains B1B0
	2	

16.

Answer	Marks	Guidance
$6y+2-7y^{1/2}$ [= 0]	*M1	OE Rearrange to a 3-term quadratic.
$\left(2y^{\frac{1}{2}}-1\right)\left(3y^{\frac{1}{2}}-2\right)$ [= 0] or e.g. $(2u-1)(3u-2)$ [= 0]	DM1	Or use of formula or completing the square.
$[y^{1/2}=]\frac{1}{2},\frac{2}{3}$	A1	Answers only **SC B1** if DM1 not scored.
$[y=]\frac{1}{4},\frac{4}{9}$	A1	Answers only **SC B1** if DM1 not scored.
	4	

17.

Answer	Marks	Guidance
$\left\{2(x-4)^2\right\}$ $\{-9\}$	B1 B1	OE When *a* and *b* stated give priority to marking algebraic expression.
	2	

18.

Answer	Marks	Guidance
$(3x+2)(x-1)=2 \Rightarrow 3x^2-x-4$ $[=0]$	M1	OE Multiply by denominator and obtain a quadratic.
$(3x-4)(x+1)[=0]$	M1	Solve by factorising, formula or completing the square.
$[x=]$ $-1, \frac{4}{3}$	A1	Allow 1.33 If M1 M0, **SC B1** possible for two correct answers.
	3	

19.

Answer	Marks	Guidance
$k^2-4\times 8\times 2$ [< 0]	M1	Use of b^2-4ac but not just in the quadratic formula.
$-8<k<8$ or $-8<k$, $k<8$ or $\lvert k\rvert<8$ or $(-8, 8)$	A1	Condone '$-8<k$ or $k<8$', '$-8<k$ and $k<8$' but not $\sqrt{64}$.
	2	

20.

Answer	Marks	Guidance
$\left(Their\ 4\left(x+\frac{5}{2}\right)^2-19\right)=45\ \left[\Rightarrow\left(x+\frac{5}{2}\right)^2=16\right]$	*M1	Equate their quadratic completed square form from **6(a)** to 45 or re-start and use completing the square.
Solve as far as $x=$	DM1	Any valid method leading to two answers.
$[x=]\frac{3}{2},-\frac{13}{2}$	A1	**SC:** If M0 or M1 DM0 awarded, B1 available for correct final answers.
	3	
Quadratic **curve** that is the right way up **(**must be seen either side of stationary point**)**	B1	No axes required, ignore any axes even if incorrect.
Stationary point stated using any valid method or correctly labelled on their diagram.	B1 FT B1 FT	FT *their* values from **6(a)** as long as *their* expression is of the form $p(qx+r)^2+s$. Expect $\left(-\frac{5}{2},-19\right)$. Condone if stated correctly but plotted incorrectly.
	3	

21.

Answer	Marks	Guidance
$\left[\frac{dy}{dx}=\right]\frac{9}{2}x-12\ [=0]$ or $[y=]\ \frac{9}{4}\left\{\left(x-\frac{8}{3}\right)^2+\frac{8}{9}\right\}$ or $\frac{9}{4}\left(x-\frac{8}{3}\right)^2+2$	B1	OE Either $\frac{dy}{dx}$ or a correct expression in completed square form. Allow unsimplified.
$x=\frac{24}{9}$	B1	OE Condone 2.67 AWRT.
$y=2$	B1	CAO Note: $x=\frac{-b}{2a}=\frac{8}{3}$ Bl; substitute $\frac{8}{3}$ for x in y = B1; y=2 B1.
	3	

22.

Answer	Marks	Guidance
$x^2-kx+2=3x-2k$ leading to $x^2-x(k+3)+(2+2k)\ [=0]$	M1	3-term quadratic, may be implied in the discriminant.
$b^2-4ac=(k+3)^2-8(1+k)$ (ignore '= 0' at this stage)	DM1	Cannot just be seen in the quadratic formula.
$=(k-1)^2$ accept $(k-1)(k-1)$	A1	Or use of calculus to show minimum of zero at $k=1$ or sketch of $f(k)=k^2-2k+1$.
$\geqslant 0$ Hence will meet for all values of k	A1	Clear conclusion.
	4	

23.

Answer	Marks	Guidance
$kx - k = -\frac{1}{2x} \Rightarrow 2kx^2 - 2kx + 1[=0]$ OR quadratic in y: $x = \frac{y+k}{k} \Rightarrow y = -\frac{1}{2\left(\frac{y+k}{k}\right)} \Rightarrow 2y^2 + 2ky + k = 0$	*M1	OE e.g. $kx^2 - kx + \frac{1}{2}[=0]$, $x^2 - x + \frac{1}{2k}[=0]$ Equate line and curve to form 3-term quadratic (all terms on one side).
$b^2 - 4ac[=0] \Rightarrow ([-]2k)^2 - 4(2k)(1)[=0]$ or $4k^2 - 8k\ [=0 \Rightarrow 4k(k-2)=0]$ OR using equation in y: $(2k)^2 - 4(2)(k) = 0$	DM1	Use discriminant correctly with their a,b,c not in quadratic formula. DM0 if x still present. May see $k^2 - 4(k)\left(\frac{1}{2}\right) = 0$ or $1 - 4\left(\frac{1}{2k}\right) = 0$.
$k = 2$ only	A1	If DM0 then $k = 2$, award A0 XP then B0 B0 Allow A1 even if divides by k to solve. If $k = 0$ also present but uses $k = 2$, award A1.
$4x^2 - 4x + 1 = 0\left[\Rightarrow (2x-1)^2 = 0\right] \Rightarrow x = \frac{1}{2}$	B1	
$y = 2 \times \frac{1}{2} - 2 = -1$	B1	

24.

Answer	Marks	Guidance
$4(x-3)^2$ seen or $a = 4$ and $b = -3$	B1	OE Award marks for the correct expression or their values a, b and c. Condone $4(x-3)+p-36 = 0$ and $4\left(\frac{p}{4}-9\right)$.
$-36+p$ or $p-36$ seen or $c = p-36$	B1	
	2	
$p-36>0$ leading to $p>36$ or $24^2 - 4\times 4p\langle 0 \Rightarrow p\rangle 36$ or $36<p$	B1	Allow $(36,\infty)$ or $36<p<\infty$. Consider final answer only.
	1	

25.

Answer	Marks	Guidance
$[8x^6 + 215x^3 - 27 = 0]$ leading to $(8x^3 - 1)(x^3 + 27)[=0]$ **OR** $\frac{-215 \pm \sqrt{215^2 - 4.8.-27}}{2.8}$ or $\frac{-215 \pm \sqrt{47089}}{2.8}$	M1	OE If a substitution is used then the correct coefficients must be retained. Condone substitution of $x = x^3$.
$\frac{1}{8}$, -27	A1	Both correct values seen. **SC**: if M0 scored **SC B1** is available for sight of $\frac{1}{8}$ and -27 OE
$\frac{1}{2}$ or $0.5, -3$	A1	**SC**: if M0SCB1 scored then **SCB1** is available for the correct answers and no others. Do not ISW if answers given as a range.
	3	

26.

Answer	Marks	Guidance
$x^2-6x+c>2$ leading to $(x-3)^2-9+c>2$	M1 A1	M1 for completion of the square with an equation or in equality with the '2'.
$c>11-(x-3)^2$ and $(x-3)^2\geq 0$	M1	SOI
$c>11$	A1	
Alternative Method 1		
$\frac{dy}{dx}=2x-6=0$	M1	M1 for differentiating and setting $\frac{dy}{dx}=0$.
$x=3$	A1	
When $x=3$, $y=9-18+c$	M1	
$[-9+c>2]$ $c>11$	A1	
Alternative Method 2		
$x^2-6x+c>2$ leading to $x^2-6x+c-2[>0]$ then use of $'b^2-4ac'$	M1	
$36-4(1)(c-2)<0$	M1 A1	OE Must be correct inequality for M1.
$c>11$	A1	
	4	