PURE MATHEMATICS -1

9709

(March, June and November series 2020 – 2023 With marking scheme)

SERIES

EXERCISE -1

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1) SP-2020 _9709_1 Q3

An arithmetic progression has first term 7. The nth term is 84 and the $(3n)$th term is 245.

Find the value of n. [4]

2) MARCH 2020_9709_12 Q8

A woman's basic salary for her first year with a particular company is $30 000 and at the end of the year she also gets a bonus of $600.

(a) For her first year, express her bonus as a percentage of her basic salary. [1]

At the end of each complete year, the woman's basic salary will increase by 3% and her bonus will increase by $100.

(b) Express the bonus she will be paid at the end of her 24th year as a percentage of the basic salary paid during that year. [5]

3) MARCH 2021_9709_12 Q9

The first term of a progression is $\cos\theta$, where $0 < \theta < \frac{1}{2}\pi$.

(a) For the case where the progression is geometric, the sum to infinity is $\frac{1}{\cos\theta}$.

(i) Show that the second term is $\cos\theta\sin^2\theta$. [3]

(ii) Find the sum of the first 12 terms when $\theta = \frac{1}{3}\pi$, giving your answer correct to 4 significant figures. [2]

(b) For the case where the progression is arithmetic, the first two terms are again $\cos\theta$ and $\cos\theta\sin^2\theta$ respectively.

Find the 85th term when $\theta = \frac{1}{3}\pi$. [4]

4) MARCH 2022_9709_12 Q4

The first term of a geometric progression and the first term of an arithmetic progression are both equal to a.

The third term of the geometric progression is equal to the second term of the arithmetic progression.

The fifth term of the geometric progression is equal to the sixth term of the arithmetic progression.

Given that the terms are all positive and not all equal, find the sum of the first twenty terms of the arithmetic progression in terms of a. [6]

5) MARCH 2023_9709_12 Q4

The circumference round the trunk of a large tree is measured and found to be 5.00 m. After one year the circumference is measured again and found to be 5.02 m.

(a) Given that the circumferences at yearly intervals form an arithmetic progression, find the circumference 20 years after the first measurement. [2]

(b) Given instead that the circumferences at yearly intervals form a geometric progression, find the circumference 20 years after the first measurement. [3]

6) JUNE 2020_9709_11 Q1

The sum of the first nine terms of an arithmetic progression is 117. The sum of the next four terms is 91.

Find the first term and the common difference of the progression. [4]

7) JUNE 2020_9709_11 Q3

Each year the selling price of a diamond necklace increases by 5% of the price the year before. The selling price of the necklace in the year 2000 was \$36 000.

(a) Write down an expression for the selling price of the necklace n years later and hence find the selling price in 2008. [3]

(b) The company that makes the necklace only sells one each year. Find the total amount of money obtained in the ten-year period starting in the year 2000. [2]

8) JUNE 2020_9709_12 Q4

The nth term of an arithmetic progression is $\frac{1}{2}(3n - 15)$.

Find the value of n for which the sum of the first n terms is 84. [5]

9) JUNE 2020_9709_13 Q8

The first term of a progression is $\sin^2\theta$, where $0 < \theta < \frac{1}{2}\pi$. The second term of the progression is $\sin^2\theta\cos^2\theta$.

(a) Given that the progression is geometric, find the sum to infinity. [3]

It is now given instead that the progression is arithmetic.

(b) (i) Find the common difference of the progression in terms of $\sin\theta$. [3]

(ii) Find the sum of the first 16 terms when $\theta = \frac{1}{3}\pi$. [3]

10) JUNE 2021_9709_11 Q2

The sum of the first 20 terms of an arithmetic progression is 405 and the sum of the first 40 terms is 1410.

Find the 60th term of the progression. [5]

11) JUNE 2021_9709_11 Q5

The fifth, sixth and seventh terms of a geometric progression are $8k$, -12 and $2k$ respectively.

Given that k is negative, find the sum to infinity of the progression. [4]

12) JUNE 2021_9709_12 Q8

The first, second and third terms of an arithmetic progression are a, $\frac{3}{2}a$ and b respectively, where a and b are positive constants. The first, second and third terms of a geometric progression are a, 18 and $b + 3$ respectively.

(a) Find the values of a and b. [5]

(b) Find the sum of the first 20 terms of the arithmetic progression. [3]

13) JUNE 2021_9709_13 Q9

(a) A geometric progression is such that the second term is equal to 24% of the sum to infinity.

Find the possible values of the common ratio. [3]

(b) An arithmetic progression P has first term a and common difference d. An arithmetic progression Q has first term $2(a+1)$ and common difference $(d+1)$. It is given that

$$\frac{\text{5th term of } P}{\text{12th term of } Q} = \frac{1}{3} \quad \text{and} \quad \frac{\text{Sum of first 5 terms of } P}{\text{Sum of first 5 terms of } Q} = \frac{2}{3}.$$

Find the value of a and the value of d. [6]

14) JUNE 2022_9709_11 Q2

The thirteenth term of an arithmetic progression is 12 and the sum of the first 30 terms is -15.

Find the sum of the first 50 terms of the progression. [5]

15) JUNE 2022_9709_12 Q2

The second and third terms of a geometric progression are 10 and 8 respectively.

Find the sum to infinity. [4]

16) JUNE 2022_9709_12 Q4

The first, second and third terms of an arithmetic progression are k, $6k$ and $k+6$ respectively.

(a) Find the value of the constant k. [2]

(b) Find the sum of the first 30 terms of the progression. [3]

17) JUNE 2022_9709_13 Q3

An arithmetic progression has first term 4 and common difference d. The sum of the first n terms of the progression is 5863.

(a) Show that $(n-1)d = \frac{11726}{n} - 8$. [1]

(b) Given that the nth term is 139, find the values of n and d, giving the value of d as a fraction. [4]

18) JUNE 2023_9709_11 Q6

The first three terms of an arithmetic progression are $\frac{p^2}{6}$, $2p-6$ and p.

(a) Given that the common difference of the progression is not zero, find the value of p. [3]

(b) Using this value, find the sum to infinity of the geometric progression with first two terms $\frac{p^2}{6}$ and $2p-6$. [2]

19) JUNE 2023_9709_12 Q 9

The second term of a geometric progression is 16 and the sum to infinity is 100.

(a) Find the two possible values of the first term. [4]

(b) Show that the nth term of one of the two possible geometric progressions is equal to 4^{n-2} multiplied by the nth term of the other geometric progression. [4]

20) JUNE 2023_9709_13 Q8

A progression has first term a and second term $\frac{a^2}{a+2}$, where a is a positive constant.

(a) For the case where the progression is geometric and the sum to infinity is 264, find the value of a. [5]

(b) For the case where the progression is arithmetic and $a = 6$, determine the least value of n required for the sum of the first n terms to be less than -480. [5]

21) OCT 2020_9709_11 Q8

A geometric progression has first term a, common ratio r and sum to infinity S. A second geometric progression has first term a, common ratio R and sum to infinity $2S$.

(a) Show that $r = 2R - 1$. [3]

It is now given that the 3rd term of the first progression is equal to the 2nd term of the second progression.

(b) Express S in terms of a. [4]

22) OCT 2020_9709_12 Q2

The first, second and third terms of a geometric progression are $2p + 6$, $-2p$ and $p + 2$ respectively, where p is positive.

Find the sum to infinity of the progression. [5]

23) OCT2020_9709_12 Q4

The sum, S_n, of the first n terms of an arithmetic progression is given by

$$S_n = n^2 + 4n.$$

The kth term in the progression is greater than 200.

Find the smallest possible value of k. [5]

24) OCT 2020_9709_13 Q7

The first and second terms of an arithmetic progression are $\frac{1}{\cos^2\theta}$ and $-\frac{\tan^2\theta}{\cos^2\theta}$, respectively, where $0 < \theta < \frac{1}{2}\pi$.

(a) Show that the common difference is $-\frac{1}{\cos^4\theta}$. [4]

(b) Find the exact value of the 13th term when $\theta = \frac{1}{6}\pi$. [3]

25) OCT 2021_9709_11 Q4

The first term of an arithmetic progression is a and the common difference is -4. The first term of a geometric progression is $5a$ and the common ratio is $-\frac{1}{4}$. The sum to infinity of the geometric progression is equal to the sum of the first eight terms of the arithmetic progression.

(a) Find the value of a. [4]

The kth term of the arithmetic progression is zero.

(b) Find the value of k. [2]

26) OCT 2021_9709_12 Q5

The first, third and fifth terms of an arithmetic progression are $2\cos x$, $-6\sqrt{3}\sin x$ and $10\cos x$ respectively, where $\frac{1}{2}\pi < x < \pi$.

(a) Find the exact value of x. [3]

(b) Hence find the exact sum of the first 25 terms of the progression. [3]

27) OCT 2021_9709_12 Q6

The second term of a geometric progression is 54 and the sum to infinity of the progression is 243. The common ratio is greater than $\frac{1}{2}$.

Find the tenth term, giving your answer in exact form. [5]

28) OCT 2021_9709_13 Q4

The first term of an arithmetic progression is 84 and the common difference is -3.

(a) Find the smallest value of n for which the nth term is negative. [2]

It is given that the sum of the first $2k$ terms of this progression is equal to the sum of the first k terms.

(b) Find the value of k. [3]

29) OCT 2022_9709_11 Q7

A tool for putting fence posts into the ground is called a 'post-rammer'. The distances in millimetres that the post sinks into the ground on each impact of the post-rammer follow a geometric progression. The first three impacts cause the post to sink into the ground by 50 mm, 40 mm and 32 mm respectively.

(a) Verify that the 9th impact is the first in which the post sinks less than 10 mm into the ground. [3]

(b) Find, to the nearest millimetre, the total depth of the post in the ground after 20 impacts. [2]

(c) Find the greatest total depth in the ground which could theoretically be achieved. [2]

30) OCT 2022_9709_12 Q2

The first, second and third terms of an arithmetic progression are a, $2a$ and a^2 respectively, where a is a positive constant.

Find the sum of the first 50 terms of the progression. [5]

31) OCT 2022_9709_12 Q4

A geometric progression is such that the third term is 1764 and the sum of the second and third terms is 3444.

Find the 50th term. [4]

32) OCT 2022_9709_13 Q9

The first term of a geometric progression is 216 and the fourth term is 64.

(a) Find the sum to infinity of the progression. [3]

The second term of the geometric progression is equal to the second term of an arithmetic progression.

The third term of the geometric progression is equal to the fifth term of the same arithmetic progression.

(b) Find the sum of the first 21 terms of the arithmetic progression. **[6]**

MARKING SCHEME

1) SP-2020 _9709_1 Q3

$7+(n-1)d=84$ and/or $7+(3n-1)d=245$	1	B1	
$(n-1)d=77$, $(3n-1)d=238$ SOI **OR** $2nd=161$ explicitly stated	1	B1	
$\frac{n-1}{3n-1}=\frac{77}{238}$	1	M1	(must be from the correct u_n formula) **OR** other attempt to eliminate d e.g. substitute $d=\frac{161}{2n}$. (If n is eliminated d must be found)
$n=23$ ($d=\frac{77}{22}=3.5$)	1	A1	
	4		

2) MARCH 2020_9709_12 Q8

(a)	2%	B1
		1
(b)	Bonus $=600+23\times100=2900$	B1
	Salary $=30000\times1.03^{23}$	M1
	$=59207.60$	A1
	$\frac{\textit{their } 2900}{\textit{their } 59200}$	M1
	4.9(0)%	A1
		5

3) MARCH 2021_9709_12 Q9

(a)(i)	$\frac{\cos\theta}{1-r}=\frac{1}{\cos\theta}$	B1
	$1-r=\cos^2\theta$ leading to $r=1-\cos^2\theta$	M1
	$r=\sin^2\theta$ leading to 2nd term $=\cos\theta\sin^2\theta$	A1
		3
(a)(ii)	$S_{12}=\frac{\cos\left(\frac{\pi}{3}\right)\left[1-\left(\sin^2\left(\frac{\pi}{3}\right)\right)^{12}\right]}{1-\sin^2\left(\frac{\pi}{3}\right)}=\frac{0.5\left[1-(0.75)^{12}\right]}{1-0.75}$	M1
	1.937	A1
		2
(b)	$[d=]\cos\theta\sin^2\theta-\cos\theta$	M1
	$-\frac{1}{8}$	A1
	[85th term =] $\frac{1}{2}+84\times-\frac{1}{8}$	M1
	-10	A1
		4

4) MARCH 2022_9709_12 Q4

$ar^2 = a + d$	B1
$ar^4 = a + 5d$	B1
$a^2r^4 = a(a+5d)$ leading to $a^2 + 5ad = (a+d)^2$	*M1
$[3ad - d^2 = 0$ leading to$]$ $d = 3a$ OR $[r = 2$ leading to$]$ $d = 3a$	A1
$S_{20} = \frac{20}{2}[2a + 19 \times 3a]$	DM1
$590a$	A1
	6

5) MARCH 2023_9709_12 Q4

a)	$5.00 + 20 \times 0.02$ or $5.02 + 19 \times 0.02$	M1
	5.40	A1
		2
b)	$r = \frac{5.02}{5} = 1.004$ or $\frac{251}{250}$	B1
	$5.00 \times (\textit{their}\,1.004)^{20}$ or $5.02 \times (\textit{their}\,1.004)^{19}$	M1
	5.42	A1
		3

6) JUNE 2020_9709_11 Q1

$117 = \frac{9}{2}(2a + 8d)$	B1
Either $91 = S_4$ with 'a' as $a + 4d$ **or** $117 + 91 = S_{13}$ (**M1** for overall approach. **M1** for S_n)	M1M1
Simultaneous Equations → $a = 7$, $d = 1.5$	A1
	4

7) JUNE 2020_9709_11 Q3

(a)	$36 000 × $(1.05)^r$ (**B1** for $r = 1.05$. **M1** method for rth term)	B1M1
	$53 200 after 8 years.	A1
		3
(b)	$S_{10} = 36000\frac{(1.05^{10} - 1)}{(1.05 - 1)}$	M1
	$453 000	A1
		2

8) JUNE 2020_9709_12 Q4

1st term is −6, 2nd term is −4.5 (M1 for using kth terms to find both a and d)	M1
$\rightarrow a = -6, d = 1.5$	A1 A1
$S_n = 84 \rightarrow 3n^2 - 27n - 336 = 0$	M1
Solution $n = 16$	A1
	5

9) JUNE 2020_9709_13 Q8

(a)	$r = \cos^2\theta$ SOI	M1
	$S_\infty = \dfrac{\sin^2\theta}{1-\cos^2\theta}$	M1
	1	A1
		3
)(i)	$d = \sin^2\theta\cos^2\theta - \sin^2\theta$	M1
	$\sin^2\theta(\cos^2\theta - 1)$	M1
	$-\sin^4\theta$	A1
		3

10) JUNE 2021_9709_11 Q2

$10(2a + 19d) = 405$	B1	
$20(2a + 39d) = 1410$	B1	
Solving simultaneously two equations obtained from using the correct sum formulae [$a = 6, d = 1.5$]	M1	Reach $a =$ or $d =$
Using the correct formula for 60th term with their a and d	M1	
60th term = 94.5	A1	OE, e.g. $\dfrac{189}{2}$
	5	

11) JUNE 2021_9709_11 Q5

$(-12)^2 = 8k \times 2k$	M1	Forming an equation in k
$k = -3$	A1	
Using correct formula for S_∞ [$r = 0.5, a = -384$]	M1	With $-1 < r < 1$
$S_\infty = -768$	A1	
Alternative method for Question 5		
$r^2 = \dfrac{2k}{8k}$	M1	
$r = [\pm]0.5$	A1	
Using correct formula for S_∞ [$r = 0.5, a = -384$]	M1	$-1 < r < 1$
$S_\infty = -768$	A1	
	4	

12) JUNE 2021_9709_12 Q8

(a)	$\left(a+b=2\times\frac{3}{2}a\right)\Rightarrow b=2a$	B1	SOI
	$18^2 = a(b+3)$ OE or 2 correct statements about r from the GP, e.g. $r=\frac{18}{a}$ and b + 3 = 18r or $r^2=\frac{b+3}{a}$	B1	SOI
	$324 = a(2a+3) \Rightarrow 2a^2+3a-324[=0]$ or $b^2+3b-648[=0]$ or $6r^2-r-12[=0]$ or $4d^2+3d-162[=0]$	M1	Using the correct connection between AP and GP to form a 3-term quadratic with all terms on one side.
	$(a-12)(2a+27)[=0]$ or $(b-24)(b+27)[=0]$ or $(2r-3)(3r+4)[=0]$ or $(d-6)(4d+27)[=0]$	M1	Solving *their* 3-term quadratic by factorisation, formula or completing the square to obtain answers for *a*, *b*, *r* or *d*.
	$a = 12,\ b = 24$	A1	WWW. Condone extra 'solution' $a=-13.5, b=-27$ only.
		5	
(b)	Common difference $d = 6$	B1 FT	SOI. FT *their* $\frac{a}{2}$
	$S_{20}=\frac{20}{2}(2\times12+19\times6)$	M1	Using correct sum formula with *their a*, *their* calculated *d* and 20.
	1380	A1	
		3	

13) JUNE 2021_9709_13 Q9

(a)	$ar=\frac{24}{100}\times\frac{a}{1-r}$	M1	Form an equation using a numerical form of the percentage and correct formula for u_2 and S_∞
	$100r^2-100r+24[=0]$	A1	OE. All 3 terms on one side of an equation.
	$(20r-8)(5r-3)[=0]\rightarrow\ r=\frac{2}{5},\frac{3}{5}$	A1	Dependent on factors or formula seen from their quadratic.
		3	

(b)	$3\times\{(a+4d)\}=\{(2(a+1)+11(d+1))\}$	*M1	SOI Attempt to cross multiply with contents of at least one { } correct
	Simplifies to $a+d=13$	A1	
	$\left[\frac{5}{2}\right]\times3\{(2a+4d)\}=\left[\frac{5}{2}\right]\times2\{(4(a+1)+4(d+1))\}$	*M1	SOI Attempt to cross multiply with contents of at least one { } correct
	Simplifies to $-a+2d=8$	A1	
	Solve 2 linear equations simultaneously	DM1	Elimination or substitution expected
	$d=7,\ a=6$	A1	SC B1 for a=6, d=7 without complete working
		6	

14) JUNE 2022_9709_11 Q2

$a+12d=12$	B1	For correct equation.
$\frac{30}{2}(2a+(30-1)d)=-15$	B1	For correct equation in a and d. If using $\frac{n}{2}(a+l)$, must replace l with an expression involving a and d.
$a=72,\ d=-5$	B1	Both values correct SOI.
$S_{50}=\frac{50}{2}(2(\textit{their } a)+49(\textit{their } d))$	M1	Using sum formula with *their a* and *d* values obtained via a valid method.
$S_{50}=-2525$	A1	
	5	

15) JUNE 2022_9709_12 Q2

$r=0.8$	B1	OE
$a=12.5$	B1	OE
$S_\infty=12.5\div(1-0.8)$	M1	Using $\frac{a}{1-r}$ with '*their a*' and '*their r*' but $\lvert r\rvert$ must be <1.
$S_\infty=\frac{125}{2}, 62\frac{1}{2}$ or 62.5	A1	$\frac{12\frac{1}{2}}{\frac{1}{5}}$ or similar **does not** get A1.
	4	

16) JUNE 2022_9709_12 Q4

4(a)	$2\times6k=k+k+6$ or $6k-k=k+6-6k$ or $2d=6$ leading to $d=3,\ \therefore 6k-3=k$	B1	OE A correct equation in k only. Can be implied by correct final answer.
	$k=\frac{6}{10}$ or 0.6	B1	OE
		2	
(b)	$d=3$	B1	Correct value of d can be implied by a correct final answer. Working may be seen in part (a) but must be used in (b).
	$S_{30}=\frac{30}{2}(2\times\text{'}\textit{their } k\text{'}+29\times\text{'}\textit{their } d\text{'})$	M1	It needs to be clear that the candidate is using a correct sum formula. There is no requirement to check the candidates working for d but it must be clearly identified.
	$S_{30}=1323$	A1	ISW if corrected to 1320.
		3	

17) JUNE 2022_9709_13 Q3

(a)	$\frac{n}{2}\left[8+(n-1)d\right]=5863$ leading to $n\left[8+(n-1)d\right]=11726$ leading to $(n-1)d=\frac{11726}{n}-8$	B1	Must show a useful intermediate step. WWW AG.
		1	
(b)	$4+(n-1)d=139$ leading to $\frac{11726}{n}-8=135$	*M1	OE Use of correct u_n formula with expression from (a) or S_n formula to eliminate *d*.
	$n=\frac{11726}{143}=82$	A1	
	$81d=\frac{11726}{82}-8$	DM1	Substitute *their n* into a correct u_n or S_n formula
	$d=\frac{5}{3}$	A1	Accept $\frac{138}{81}$ OE fraction only If M0 DM0 scored them SC B1 B1 for correct *n* and *d* values only.
		4	

18) JUNE 2023_9709_11 Q6

(a)	$2(2p-6)=p+\frac{p^2}{6}\Rightarrow\frac{p^2}{6}-3p+12[=0]$ OR $(2p-6)-\frac{p^2}{6}=p-(2p-6)\Rightarrow\frac{p^2}{6}-3p+12[=0]$ OR $\frac{1}{6}d^2+d[=0]$	*M1	Correct method leading to formation of a 3-term quadratic in p (all terms on one side) or 2-term quadratic in d. OE e.g. $p^2-18p+72[=0]$, $\frac{1}{2}p^2-9p+36[=0]$.
	$p^2-18p+72[=0]\Rightarrow(p-6)(p-12)[=0]$ or $\frac{18\pm\sqrt{(-18)^2-4(1)(72)}}{2}$ OR $d\left(\frac{1}{6}d+1\right)[=0]\Rightarrow d=-6$	DM1	Solve a 3-term quadratic in p by factorisation, formula or completing the square or solve a 2-term quadratic in d by factorisation.
	$p=12$ only	A1	Since $p=6$ gives $d=0$. If *M1 DM0 then $p=12$ only, award SC B1, max 2/3 marks. A0 XP if error in either factor and $p=12$ only. $p=12$ only by trial and improvement 3/3.
		3	
(b)	For GP $r=\left[\frac{2p-6}{\frac{p^2}{6}}\right]=\frac{18}{24}\left[=\frac{3}{4}\right]$	B1	OE SOI.
	Sum to infinity $=\frac{24}{1-\frac{3}{4}}=96$	B1 FT	FT *their* value of p if used correctly to find r (B0 if 'p' used) provided $\lvert r\rvert<1$. e.g. $p=18\Rightarrow[S_\infty=]\frac{54}{1-\frac{5}{9}}=121.5$.
		2	

19) JUNE 2023_9709_12 Q 9

a)	$\left[ar=16, \frac{a}{1-r}=100\right]$ leading to $a=\frac{16}{r}$ and $a=100(1-r)$	B1	Rearranging two algebraic statements to give $a=$. These can be implied by a correct equation in one variable.
	$100(1-r)r=16$ leading to $100r^2-100r+16[=0]$	*M1	Using their two expressions and rearranging to get a 3-term quadratic expression with all of the terms on one side. Condone sign errors only.
	$(5r-4)(5r-1)=0$ OR $\frac{25\pm\sqrt{25^2-4.25.4}}{2.25}$ leading to $r=\left[\frac{4}{5} \text{ or } \frac{1}{5}\right]$	DM1	Condone $(5r-4)(5r-1)$ following $100r^2-100r+16$.
	$a=20, a=80$	A1	SC: if DM0 scored SCB1 is available for sight of 20 and 80.
	Alternative Method for Question 9(a)		
	$\left[ar=16, \frac{a}{1-r}=100\right]$ leading to $r=\frac{16}{a}$ and $r=\frac{100-a}{100}$	B1	Rearranging two algebraic statements to give $r=$. These can be implied by a correct equation in one variable.
	$1600=100a-a^2$ leading to $a^2-100a+1600[=0]$	*M1	Using their two expressions and rearranging to get a 3-term quadratic expression with all of the terms on one side. Condone sign errors and 160 instead of 1600 only.
	$(a-20)(a-80)=0$ OR $\frac{100\pm\sqrt{100^2-4.1600}}{2}$	DM1	
	$a=20, a=80$	A1	SC: if DM0 scored SCB1 is available for sight of 20 and 80.
		4	
(b)	$r=\frac{4}{5}, \frac{1}{5}$	B1	OE SOI
	$[u_n=]$*their* $20\times$ *their* $\left(\frac{4}{5}\right)^{n-1}$ $[v_n=]$*their* $80\times$ *their* $\left(\frac{1}{5}\right)^{n-1}$	B1FT	2 expressions for the nth term FT *their* values from part **(a)** if $\lvert r\rvert$ less than 1.
	Method 1 for final 2 marks		
	$20\times\left(\frac{1}{5}\right)^{n-1}\times 4^{n-1}$	M1	Correctly separating the numerator and denominator of *their* $\left(\frac{4}{5}\right)^{n-1}$ or one correct step towards the solution eg $u_n=80\times\frac{4^{n-2}}{5^{n-1}}$.
	$u_n=\frac{1}{4}\times 80\times\left(\frac{1}{5}\right)^{n-1}\times 4^{n-1}=4^{n-2}\times 80\times\left(\frac{1}{5}\right)^{n-1}=4^{n-2}\times v_n$	A1	AG Given result clearly shown
	Method 2 for final 2 marks		
	$\frac{20\times 0.8^{n-1}}{80\times 0.2^{n-1}}=\frac{1}{4}\times 4^{n-1}$	M1	Dividing two nth terms of the correct format and simplifying their terms in r.
	$=4^{-1}\times 4^{n-1}=4^{n-2}$	A1	AG
		4	

20) JUNE 2023_9709_13 Q8

(a)	$r=\frac{a}{a+2}$	B1	OE SOI
	$\frac{a}{1-\frac{a}{a+2}}=264$	M1	Use of S∞ formula.
	$\frac{a(a+2)}{a+2-a}=264$ leading to $\frac{a(a+2)}{2}=264$ leading to $a^2+2a-528\ [=0]$	M1*	Process to a 3 term quadratic or a 3 term cubic. May contain terms on LHS and RHS.
	$(a-22)(a+24)\ [=0]$	DM1	Attempt to solve.
	$a=22$ (only)	A1	22 without working **SC DB1** (dep on 2nd M1).
		5	
(b)	$d=\frac{6^2}{6+2}-6=-\frac{3}{2}$	B1	
	$\frac{n}{2}\left\{12+(n-1)\left(\frac{-3}{2}\right)\right\}[<]-480$	M1*	Forming an inequation with *their* numerical *d*. May use an equality.
	$[3]\left(n^2-9n-640\right)\ [>0]$	A1	OE May contain terms on LHS and RHS.
	$[n=]\ \frac{9\pm\sqrt{81+2560}}{2}$	DM1	OE. Expect 30.19 . Working for solution must be shown.
	31 only	A1	Must come from a correct first inequality (or an equality). 31 no working **SC DB1** (dep on correct quadratic and correct inequality/equality).
		5	

21) OCT 2020_9709_11 Q8

a)	$S=\frac{a}{1-r}$, $2S=\frac{a}{1-R}$	B1	SOI at least one correct
	$\frac{2a}{1-r}=\frac{a}{1-R}$	M1	SOI
	$2-2R=1-r\ \rightarrow\ r=2R-1$	A1	AG
		3	

b)	$ar^2 = aR \to (a)(2R-1)^2 = R(a)$	*M1	
	$4R^2 - 5R + 1\ (=0) \to (4R-1)(R-1)\ (=0)$	DM1	Allow use of formula or completing square.
	$R = \frac{1}{4}$	A1	Allow $R = 1$ in addition
	$S = \frac{2a}{3}$	A1	
	Alternative method for question 8(b)		
	$ar^2 = aR \to (a)r^2 = ½(r+1)(a)$	*M1	Eliminating 1 variable
	$2r^2 - r - 1\ (=0) \to (2r+1)(r-1)\ (=0)$	DM1	Allow use of formula or completing square. Must solve a quadratic.
	$r = -\frac{1}{2}$	A1	Allow $r = 1$ in addition
	$S = \frac{2a}{3}$	A1	
		4	

22) OCT 2020_9709_12 Q2

$(-2p)^2 = (2p+6) \times (p+2)$ or $\frac{-2p}{2p+6} = \frac{p+2}{-2p}$	M1	OE. Using "a, b, c then $b^2 = ac$" or $a = 2p+6$, $ar = -2p$ and $ar^2 = p+2$ to form a correct relationship in terms of p only
$(2p^2 - 10p - 12 = 0)\ p = 6$	A1	
$a = 18$ and $r = -⅔$	A1	
$(s_\infty) = \text{their } a \div (1 - \text{their } r)$ $\left(= 18 \div \frac{5}{3}\right)$	M1	Correct formula used with their values for a and r, $\lvert r \rvert < 1$ Both a & r from the same value of p.
$(s_\infty =)10.8$	A1	OE. A0 if an extra solution given
		SC B2 for $s_\infty = \frac{2p+6}{1-\frac{-2p}{2p+6}}$ or $\frac{2p+6}{1-\frac{p+2}{-2p}}$ ignore any subsequent algebraic simplification.
	5	

23) OCT2020_9709_12 Q4

S_x and S_{x+1}	M1	Using two values of n in the given formula
$a = 5, d = 2$	A1 A1	
$a + (n-1)d > 200 \rightarrow 5 + 2(k-1) > 200$	M1	Correct formula used with their a and d to form an equation or inequality with 200, condone use of n
$(k =) 99$	A1	Condone $\geqslant 99$
Alternative method for question 4		
$\frac{n}{2}(2a + (n-1)d) = n^2 + 4n \rightarrow \left(\frac{d}{2} = 1, a - \frac{1}{2}d = 4\right)$	M1	Equating two correct expressions of S_n and equating coefficients of n and n^2
$d = 2, a = 5$	A1 A1	
$a + (n-1)d > 200 \rightarrow 5 + 2(k-1) > 200$	M1	Correct formula used with their a and d to form an equation or inequality with 200, condone use of n
$(k =) 99$	A1	Condone $\geqslant 99$
Alternative method for question 4		
$sum_k - sum_{k-1} \rightarrow k^2 + 4k - (k-1)^2 - 4(k-1)$	M1 A1	Using given formula with consecutive expressions subtracted. Allow k+1 and k.
$2k + 3 > 200$ or $= 200$	M1 A1	Simplifying to a linear equation or inequality
$(k =) 99$	A1	Condone $\geqslant 99$
	5	

24) OCT 2020_9709_13 Q7

(a)	$(d =) -\frac{\tan^2\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}$	B1	Allow sign error(s). Award only at form (d =)... stage
	$-\frac{\sin^2\theta}{\cos^4\theta} - \frac{1}{\cos^2\theta}$ or $\frac{-\sec^2\theta}{\cos^2\theta}$	M1	Allow sign error(s). Can imply B1
	$\frac{-\sin^2\theta - \cos^2\theta}{\cos^4\theta}$ or $\frac{-\frac{1}{\cos^2\theta}}{\cos^2\theta}$	M1	
	$-\frac{1}{\cos^4\theta}$	A1	AG, WWW
		4	
(b)	$a = \frac{4}{3}$, $d = -\frac{16}{9}$	B1	SOI, both required. Allow $a = \frac{1}{\frac{3}{4}}$, $d = -\frac{1}{\frac{9}{16}}$
	$u_{13} = \frac{1}{\cos^2\theta} - \frac{12}{\cos^4\theta} = \frac{4}{3} + 12\left(\frac{-16}{9}\right)$	M1	Use of correct formula with *their a* and *their d*. The first 2 steps could be reversed
	–20	A1	WWW
		3	

25) OCT 2021_9709_11 Q4

(a)	$\frac{5a}{1-(\pm\frac{1}{4})}$	B1	Use of correct formula for sum to infinity.
	$\frac{8}{2}[2a+7(-4)]$	*M1	Use of correct formula for sum of 8 terms and form equation; allow 1 error.
	$4a = 8a - 112$ leading to $a = [28]$	DM1	Solve equation to reach a value of a.
	$a = 28$	A1	Correct value.
		4	
(b)	$their\ 28 + (k-1)(-4) = 0$	M1	Use of correct method with *their* a.
	$[k=]8$	A1	
		2	

26) OCT 2021_9709_12 Q5

(a)	[(3rd term – 1st term) = (5th term – 3rd term) leading to…] $-6\sqrt{3}\sin x - 2\cos x = 10\cos x + 6\sqrt{3}\sin x$ [leading to $-12\sqrt{3}\sin x = 12\cos x$] OR [(1st term + 5th term) = 2 × 3rd term leading to…] $12\cos x = -12\sqrt{3}\sin x$	*M1	OE. From the given terms, obtain 2 expressions relating to the common difference of the arithmetic progression, attempt to solve them simultaneously and achieve an equation just involving sin$x$ and cos$x$.
	Elimination of sin$x$ and cos$x$ to give an expression in tan$x$ $\left[\tan x = -\frac{1}{\sqrt{3}}\right]$	DM1	For use of $\frac{\sin x}{\cos x} = \tan x$
	$[x=]\frac{5\pi}{6}$ only	A1	CAO. Must be exact.
		3	
(b)	$d = 2\cos x$ or $d = 2\cos(their\ x)$	B1 FT	Or an equivalent expression involving sin$x$ and cos$x$ e.g. $-3\sqrt{3}\sin(their\ x) - \cos(their\ x)\ \left[=-\sqrt{3}\right]$ FT for *their* x from (a) only. If not $\pm\sqrt{3}$, must see unevaluated form.
	$S_{25} = \frac{25}{2}(2\times(2\cos(their\ x)) + (25-1)\times(their\ d))$ $\left[=12.5\left(2\times(-\sqrt{3}) + 24(-\sqrt{3})\right)\right]$	M1	Using the correct sum formula with $\frac{25}{2}$, (25 – 1) and with a replaced by either 2(cos(*their* x)) or $\pm\sqrt{3}$ and d replaced by either 2(cos(*their* x)) or $\pm\sqrt{3}$.
	$-325\sqrt{3}$	A1	Must be exact.
		3	

27) OCT 2021_9709_12 Q6

$ar = 54$ and $\frac{a \text{ or } their\ a}{1-r} = 243$	B1	SOI
$\frac{54}{r} = 243(1-r)$ leading to $243r^2 - 243r + 54[=0]$ $[9r^2 - 9r + 2 = 0]$ OR $a^2 - 243a + 13122[=0]$	*M1	Forming a 3-term quadratic expression in r or a using *their* 2nd term and S_∞. Allow ± sign errors.
$k(3r-2)(3r-1)[=0]$ OR $(a-81)(a-162)[=0]$	DM1	Solving *their* 3-term quadratic using factorisation, formula or completing the square. If factorising, factors must expand to give ±*their* coefficient of r^2.
$54 \div \left(their\ \frac{2}{3}\right) = a$ OR $54 \div (their\ 81) = r$	DM1	May be implied by final answer.
Tenth term $= \frac{512}{243}$ $\left[\text{OR } 81 \times \left(\frac{2}{3}\right)^9 \text{ OR } 54 \times \left(\frac{2}{3}\right)^8\right]$	A1	OE. Must be exact. **Special case:** If B1M1DM0DM1 scored then SC B1 can be awarded for the correct final answer.
	5	

28) OCT 2021_9709_13 Q4

(a)	$84 - 3(n-1) = 0$	M1	OE, SOI. Allow either = 0 or < 0 (to -3).
	Smallest n is 30	A1	**SC B2** for answer only n = 30 WWW.
		2	
(b)	$\left(\frac{2k}{2}\right)[168 + (2k-1)(-3)] = \left(\frac{k}{2}\right)[168 + (k-1)(-3)]$	M1 A1	M1 for forming an equation using correct formula. A1 for at least one side correct.
	$k = 19$	A1	
		3	

29) OCT 2022_9709_11 Q7

(a)	$r = 0.8$	B1	SOI
	$50 \times (their\ 0.8)^7 = 10.5$	M1	Evaluate 8th or 9th term in GP.
	$50 \times (their\ 0.8)^8 = 8.39$. Hence 9th impact required	A1	**AG** Two terms correct + conclusion (mention of 9th impact or u_9 somewhere in the solution). Statement that one is <10 (and the other >10) is insufficient unless it mentions 9th impact or u_9.
	Alternative method for final two marks: Logarithm method		
	$50 \times (their\ 0.8)^n < 10 \Rightarrow (their\ 0.8)^n < 0.5$ $n \log(their\ 0.8) < \log 0.5$ $n > \frac{\log 0.5}{\log(their\ 0.8)} \Rightarrow [n >]7.2$	M1	
	$n = 8$ hence 9th impact required	A1	**AG** Need conclusion that mentions 9th impact or u_9.
		3	
(b)	$\frac{50\left(1-(their\ 0.8)^{20}\right)}{1 - their\ 0.8}$	M1	OE Use of formula with *their* r SOI.
	$= 247$	A1	Must be to the nearest mm (not 247.1).
		2	
(c)	$\frac{50}{1 - their\ 0.8}$	M1	Use of sum to infinity formula with *their* r SOI. Substituting a value of n into the sum formula M0.
	$= 250$	A1	
		2	

30) OCT 2022_9709_12 Q2

$2a-a=a^2-2a$	B1	OE An unsimplified correct equation in a or d only, e.g. $a^2+a=4a$. Can be implied by correct values for a or d.
$a=3$ or $d=3$	B1	Condone 'extra' solution of $a=0$ or $d=0$.
$a=3$ and $d=3$	B1	SOI
$S_{50}=\frac{50}{2}(2\times their\ a+49\times their\ d)$	M1	May be done using 50th term (=150). Their a and d must be numerical.
3825	A1	ISW **SC B2** for $1275a$ or $1275d$
	5	

31) OCT 2022_9709_12 Q4

$ar^2=1764$ and $ar+ar^2=3444$ or $ar=1680$ or $\frac{a(1-r^3)}{1-r}-a=3444$	B1	Two correct algebraic statements.
Attempt to solve as far as $r=$ or $a=$	M1	Any valid method, e.g. $1764\div1680$ or from $20r^2-41r+21$ OE (condone solving using a calculator).
$r=\frac{1764}{1680}=\frac{21}{20}$ or 1.05 [$a=1600$]	A1	Note: $r=\frac{1764}{3444-1764}$ www implies B1 and M1.
17 500	A1	AWRT e.g. 17 474.1…..
	4	

32) OCT 2022_9709_13 Q9

(a)	$216r^3=64\rightarrow r=2/3$	B1	Allow decimal to 3sf (AWRT).
	$S_\infty=\frac{216}{1-their\frac{2}{3}}=648$ cao	M1 A1	M1 depends on *their* $\lvert r\rvert<1$.
		3	
b)	$216\left(\frac{2}{3}\right)=144\rightarrow 144=a+d$	B1 FT	SOI, may be implied in the use of $96=144+3d$ and finding a. Mis-reads not condoned in **9(b)**.
	$216\left(\frac{2}{3}\right)^2=96\rightarrow 96=a+4d$	B1 FT	SOI, may be implied in the use of $96=144+3d$ and finding a.
	Solve simultaneously	*M1	No working may be seen.
	$d=-16,\quad a=160$	A1	Both required.
	$S_{21}=\frac{21}{2}\{320+20(-16)\}=0$	DM1 A1	Or use of $\frac{21}{2}(a+u_{21})$.
		6	