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Pure Maths 1

Trigonometry

Exercise 1. Solution (Revision)

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Trigonometry - Revision

Example 1(a) Show that the equation $1+\sin x\tan x = 5\cos x$ can be expressed as, $6\cos^2 x - \cos x - 1 = 0$ ---[3]

(b) Hence solve the equation $1+\sin x\tan x = 5\cos x$ for $0 \le x \le 180°$

[SP-20/12/Q7] ---[3]

Solution(a): $1+\sin x\tan x = 5\cos x$

or $1+\sin x \cdot \frac{\sin x}{\cos x} = 5\cos x$

or $\cos x + \sin^2 x = 5\cos^2 x$

or $\cos x + 1 - \cos^2 x = 5\cos^2 x$

$\Rightarrow 6\cos^2 x - \cos x - 1 = 0$ ✓

(b) To solve $1+\sin x\tan x = 5\cos x$ for $0 \le x \le 180°$

Using Part(i) $\Rightarrow 6\cos^2 x - \cos x - 1 = 0$

or $(3\cos x + 1)(2\cos x - 1) = 0$

or $\cos x = -\frac{1}{3}$ or $\cos x = \frac{1}{2}$

or $\cos x = -\cos 70.5°$ | $\Rightarrow x = 60°$

$x = (180 - 70.5)$

$x = \underline{109.5°}$ ✓ or $\underline{60°}$ ✓

Example 2: Solve the equation: $\frac{\tan\theta + 3\sin\theta + 2}{\tan\theta - 3\sin\theta + 1} = 2$ for $0° \le \theta \le 90°$ ---[5]

[M-20/12/Q5]

Solution: $\frac{\tan\theta + 3\sin\theta + 2}{\tan\theta - 3\sin\theta + 1} = 2$

$\Rightarrow \tan\theta + 3\sin\theta + 2 = 2\tan\theta - 6\sin\theta + 2$

$\Rightarrow \tan\theta - 9\sin\theta = 0$

$\Rightarrow \frac{\sin\theta}{\cos\theta} - 9\sin\theta = 0$

$\Rightarrow \sin\theta - 9\sin\theta\cos\theta = 0$

$\Rightarrow \sin\theta(1 - 9\cos\theta) = 0$

$\Rightarrow \sin\theta = 0$ or $\cos\theta = \frac{1}{9}$

$\Rightarrow \theta = 0°$ or $\theta = \cos^{-1}\left(\frac{1}{9}\right) = 83.6° \Rightarrow \underline{\theta = 0° \text{ or } 83.6°}$

Example 3 (a) Solve the equation $3\tan^2 x - 5\tan x - 2 = 0$ for $0 \le x \le 180°$ --- [4]

(b) Find the set of values of k for which the equation $3\tan^2 x - 5\tan x + k = 0$ has no solution. -- [2]

(c) For the equation $3\tan^2 x - 5\tan x + k = 0$, state the value of k for which there are three solutions in the interval $0 \le x \le 180°$, and find these solutions. [M-20/12/Q11] --[3]

Solution:

(a) $3\tan^2 x - 5\tan x - 2 = 0 \quad 0 \le x \le 180$

or $(\tan x - 2)(3\tan x + 1) = 0$

$\Rightarrow \tan x = -\frac{1}{3}$ or $\tan x = 2$

$\tan x = -\tan 18.43$ or $x = \tan^{-1} 2$

$x = (180 - 18.43)$ | $x = 63.4°$

$x = 161.6°$ or $x = 63.4°$

(b) $3\tan^2 x - 5\tan x + k = 0$

for no solution $b^2 - 4ac < 0$

$\Rightarrow 25 - 4 \times 3 \times k < 0$

$\Rightarrow k > \frac{25}{12}$

(c) for $k = 0$

$3\tan^2 x - 5\tan x = 0$

$\tan x(3\tan x - 5) = 0$

$\tan x = 0$ or $\tan x = 5/3$

$x = 0, 180, 59.0°$

4. Solve the equation: $\frac{\tan\theta + 2\sin\theta}{\tan\theta - 2\sin\theta} = 3$, for $0 < \theta < 180°$ ---[4]

[M-21/12/Q3]

<u>Solution:</u> $\frac{\tan\theta + 2\sin\theta}{\tan\theta - 2\sin\theta} = 3$

$\Rightarrow \tan\theta + 2\sin\theta = 3(\tan\theta - 2\sin\theta)$

$\Rightarrow 8\sin\theta = 2\tan\theta$

$\Rightarrow 8\sin\theta = 2\frac{\sin\theta}{\cos\theta}$

$\Rightarrow 8\sin\theta\cos\theta - 2\sin\theta = 0$

$\Rightarrow 2\sin\theta(4\cos\theta - 1) = 0$

$\Rightarrow \sin\theta = 0$; $\cos\theta = \frac{1}{4}$

$\Rightarrow \theta = 0°\times, 180°\times$; $\theta = \cos^{-1}\frac{1}{4}$

(as $0 < \theta < 180°$) $\theta = \underline{75.5°}$

5 (a) Show that: $\frac{\sin\theta+2\cos\theta}{\cos\theta-2\sin\theta}-\frac{\sin\theta-2\cos\theta}{\cos\theta+2\sin\theta}\equiv\frac{4}{5\cos^2\theta-4}$ ---[4]

(b) Hence solve the equation:
$\frac{\sin\theta+2\cos\theta}{\cos\theta-2\sin\theta}-\frac{\sin\theta-2\cos\theta}{\cos\theta+2\sin\theta}=5$ for $0^\circ<\theta<180^\circ$ ---[3]

[M-22/12/Q7]

Solution (a) L.H.S $\frac{\sin\theta+2\cos\theta}{\cos\theta-2\sin\theta}-\frac{\sin\theta-2\cos\theta}{\cos\theta+2\sin\theta}$

$$=\frac{(\sin\theta+2\cos\theta)(\cos\theta+2\sin\theta)-(\sin\theta-2\cos\theta)(\cos\theta-2\sin\theta)}{(\cos\theta-2\sin\theta)(\cos\theta+2\sin\theta)}$$

$$=\frac{5\sin\theta\cos\theta+2\cos^2\theta+2\sin^2\theta-(5\sin\theta\cos\theta-2\sin^2\theta-2\cos^2\theta)}{\cos^2\theta-4\sin^2\theta}$$

$$=\frac{4(\cos^2\theta+\sin^2\theta)}{\cos^2\theta-4(1-\cos^2\theta)}=\frac{4}{5\cos^2\theta-4}=R.H.S$$

To Solve:

(b) $\frac{\sin\theta+2\cos\theta}{\cos\theta-2\sin\theta}-\frac{\sin\theta-2\cos\theta}{\cos\theta+2\sin\theta}=5$; $0^\circ<\theta<180^\circ$

$$\Rightarrow \frac{4}{5\cos^2\theta-4}=5$$

$$\Rightarrow 25\cos^2\theta-20=4$$

$$\Rightarrow \cos^2\theta=24$$

$$\cos^2\theta=\frac{24}{25}$$

$\cos\theta=+\sqrt{\frac{24}{25}}$; $\cos\theta=-\sqrt{24/25}$

$=0.9798$; $=-0.9798$

$=\cos 11.5^\circ$; $\cos\theta=-\cos 11.5^\circ$

$\theta=11.5^\circ$; $\theta=180-11.5^\circ$

$\theta=\underline{11.5^\circ}$ or $\theta=\underline{168.5^\circ}$ ✓

6(a) By first obtaining a quadratic equation in $\cos\theta$,
Solve the equation: $\tan\theta \cdot \sin\theta = 1$ for $0° < \theta < 360°$ --- [5]

(b) Show that $\dfrac{\tan\theta}{\sin\theta} - \dfrac{\sin\theta}{\tan\theta} = \tan\theta \sin\theta$ --- [3]

M-23 | 12 | Q7

Solution (a) $\tan\theta \cdot \sin\theta = 1$ for $0 < \theta < 360°$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} \cdot \sin\theta = 1$

$\Rightarrow \sin^2\theta = \cos\theta$

$\Rightarrow (1-\cos^2\theta) = \cos\theta$

$\Rightarrow \cos^2 + \cos\theta - 1 = 0$

$\Rightarrow \cos\theta = \dfrac{-1 \pm \sqrt{5}}{2} = 0.618$ or $\cancel{-1.618}$

$\theta = \cos^{-1} 0.618$

$\theta = 51.8°$ or $360 - 51.8$

$\theta = 51.8°$ ✓ or $\theta = 308.20°$ ✓

(b) Consider (L.H.S) $\dfrac{\tan\theta}{\sin\theta} - \dfrac{\sin\theta}{\tan\theta}$

$= \dfrac{\frac{\sin\theta}{\cos\theta}}{\sin\theta} - \dfrac{\sin\theta}{\sin\theta/\cos\theta}$

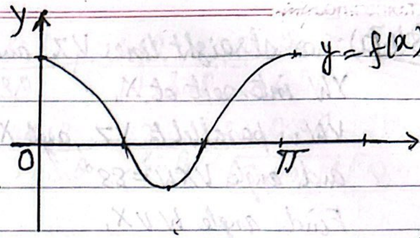
$= \dfrac{\sin\theta}{\sin\theta\cos\theta} - \dfrac{\sin\theta\cos\theta}{\sin\theta}$

$= \dfrac{1}{\cos\theta} - \cos\theta$

$= \dfrac{1-\cos^2\theta}{\cos\theta} = \dfrac{\sin^2\theta}{\cos\theta} = \dfrac{\sin\theta}{\cos\theta} \cdot \sin\theta$

$= \tan\theta \cdot \sin\theta$ ✓ = RHS

Example 7: The diagram shows the graph of $y=f(x)$, where $f(x)=\frac{3}{2}\cos 2x+\frac{1}{2}$ for $0\leqslant x\leqslant \pi$



(a) State the range of f. --[2]

A function g is such that $g(x)=f(x)+k$, where k is a positive constant. The x-axis is a tangent to the curve $y=g(x)$.

(b) State the value of k and hence describe fully the transformation that maps the curve $y=f(x)$ on to $y=g(x)$ --[2]

(c) State the equation of the curve which is the reflection of $y=f(x)$ in the x-axis. Give your answer in the form $y=a\cos 2x+b$, where a and b are constants. --[1]

[S-20/11/Q4]

Solution:

(a) $-1\leqslant f(x)\leqslant 2$

(b) $k=1$

Translation by 1 unit upwards parallel to the y-axis.

(c) $y=-\frac{3}{2}\cos 2x-\frac{1}{2}$ ✓

<u>Example 8</u> (a) Prove the identity $\dfrac{1+\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{1+\sin\theta}=\dfrac{2}{\cos\theta}$ ---[3]

(b) Hence solve the equation $\dfrac{1+\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{1+\sin\theta}=\dfrac{3}{\sin\theta}$ for $0\le\theta\le 2\pi$ --[3]

$\boxed{S\text{-}20/11/Q7}$

Solution:

(a) L.H.S $\dfrac{1+\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{1+\sin\theta}$

$$=\frac{(1+\sin\theta)^2+\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$=\frac{1+2\sin\theta+\sin^2\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$=\frac{2\cancel{(1+\sin\theta)}}{\cos\theta\cancel{(1+\sin\theta)}} \quad [\because \sin^2\theta+\cos^2\theta=1]$$

$$=\frac{2}{\cos\theta}=\text{R.H.S} \checkmark$$

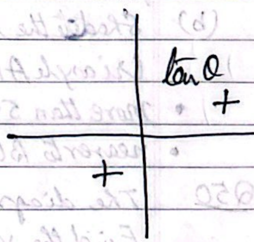
(b) To Solve $\dfrac{1+\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{1+\sin\theta}=\dfrac{3}{\sin\theta}$ $\quad 0\le\theta\le 2\pi$

Using Part (a) $\Rightarrow$ $\dfrac{2}{\cos\theta}=\dfrac{3}{\sin\theta}$

$$\Rightarrow \tan\theta = 3/2$$
$$= \tan 0.983$$

$\therefore \theta = 0.983$ or $0.983+\pi$

$\theta = 0.983$ or 4.12 ✓



Example-9 (a) Express the equation $3\cos\theta = 8\tan\theta$ as a quadratic equation in $\sin\theta$. ---[3]

(b) Hence find the acute angle, in degrees, for which $3\cos\theta = 8\tan\theta$ --[2]

[S-20/12/Q 2]

Solution (a) $3\cos\theta = 8\tan\theta$

$\Rightarrow 3\cos\theta = 8\dfrac{\sin\theta}{\cos\theta} \Rightarrow 3\cos^2\theta = 8\sin\theta$

$\Rightarrow 3(1-\sin^2\theta) = 8\sin\theta$

$\Rightarrow 3\sin^2\theta + 8\sin\theta - 3 = 0$

(b) To solve $3\cos\theta = 8\tan\theta$, $0<\theta<90$

Using Part (a) $\Rightarrow$ $3\sin^2\theta + 8\sin\theta - 3 = 0$

$(3\sin\theta - 1)(\sin\theta + 3) = 0$

$\Rightarrow \sin\theta = \frac{1}{3}$ or $\sin\theta = -3$ $(-1 \leq \sin\theta \leq 1)$

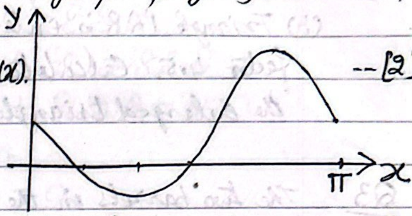
$\Rightarrow \theta = 19.5^\circ$

Example 10: Functions f and g are such that:

$$f(x) = 2 - 3\sin 2x \quad \text{for } 0 \le x \le \pi,$$
$$g(x) = -2f(x) \quad \text{for } 0 \le x \le 2\pi$$

(a) State the range of f and g. ---[3]

The diagram below shows the graph of $y = f(x)$



(b) Sketch on this diagram, the graph of $y = f(x)$. --[2]

The function h is such that.

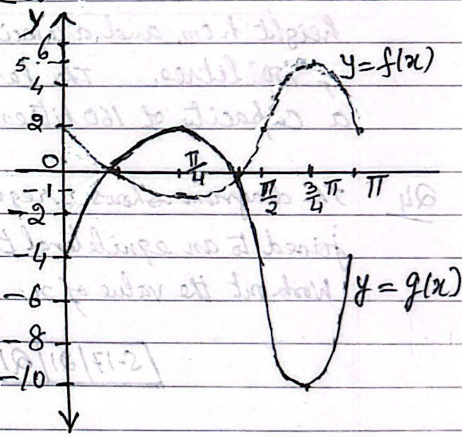
$h(x) = g(x+\pi)$ for $-\pi \le x \le 0$

(c) Describe fully a sequence of transformation that maps the curve $y = f(x)$ on to $y = h(x)$ --[3]

[S-20/12/Q 9]

Solution (a) $-1 \le f(x) \le 5$, $-10 \le g(x) \le 2$

(b)



(c) Reflection in x-axis

Stretch by factor 2 in the y-direction

Translation by $-\pi$ in the x-direction.

or translation by $\begin{pmatrix} -\pi \\ 0 \end{pmatrix}$

Example 11 (a) Show that $\frac{\tan\theta}{1+\cos\theta} + \frac{\tan\theta}{1-\cos\theta} = \frac{2}{\sin\theta\cos\theta}$ ---[4]

(b) Hence solve the equation:

$$\frac{\tan\theta}{1+\cos\theta} + \frac{\tan\theta}{1-\cos\theta} = \frac{6}{\tan\theta} \text{ for } 0^\circ < \theta < 180^\circ$$ --[4]

[S-20/13/Q7]

Solution:

(a) L.H.S. $\frac{\tan\theta}{1+\cos\theta} + \frac{\tan\theta}{1-\cos\theta} = \frac{\tan\theta(1-\cos\theta) + \tan(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$$= \frac{2\tan\theta}{1-\cos^2\theta}$$

$$= \frac{2\tan\theta}{\sin^2\theta}$$

$$= 2\frac{\sin\theta}{\cos\theta} \times \frac{1}{\sin^2\theta}$$

$$= \frac{2}{\sin\theta\cos\theta} = R.H.S$$

(b) Solve $\frac{\tan\theta}{1+\cos\theta} + \frac{\tan\theta}{1-\cos\theta} = \frac{6}{\tan\theta}$ for $0 < \theta < 180^\circ$

Using Part (a) $\Rightarrow \frac{2}{\sin\theta\cos\theta} = \frac{6}{\tan\theta}$

$$\Rightarrow \frac{2}{\sin\theta\cos\theta} = \frac{6\cos\theta}{\sin\theta}$$

$$\Rightarrow \cos^2\theta = \frac{1}{3}$$

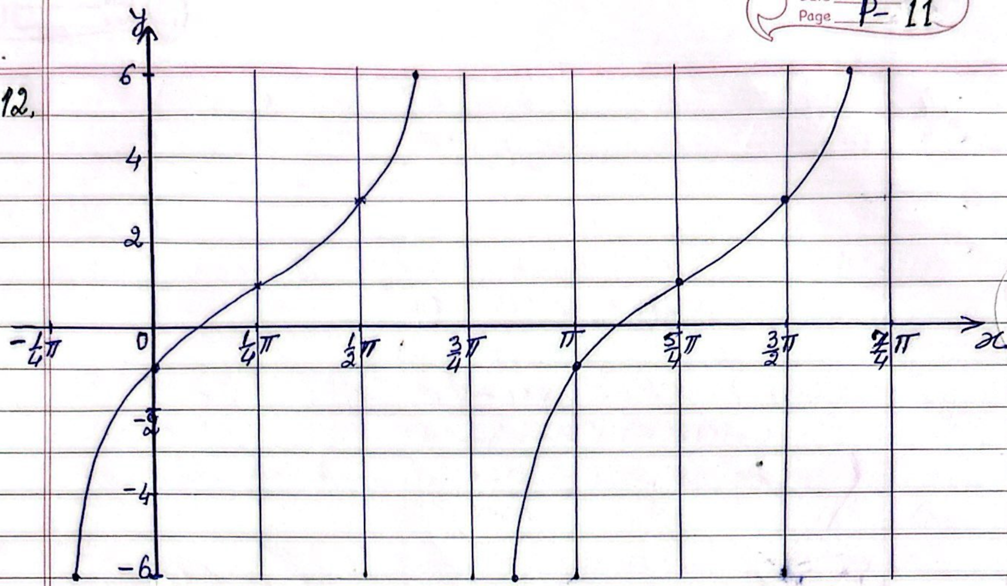
$$\cos\theta = \frac{1}{\sqrt{3}} \text{ or } \cos\theta = -\frac{1}{\sqrt{3}}$$

$(\theta = \cos^{-1}\frac{1}{\sqrt{3}})$ $\quad \theta = 54.7^\circ$ or $\cos\theta = -\cos 54.7^\circ$

$$\theta = 54.7^\circ \text{ or } \theta = 180 - 54.7^\circ$$

$$\theta = 54.7^\circ \text{ or } \theta = 125.3^\circ$$

12.



The diagram shows part of the graph of $y = a\tan(x-b)+c$

Given that $0<b<\pi$, state the values of the constants a, b and c.

[S-21/11/Q4] --[3]

Solution: $a = 2$, $b = \frac{\pi}{4}$ and $c = 1$.

13.(a) Prove the identity $\frac{1-2\sin^2\theta}{1-\sin^2\theta} = 1-\tan^2\theta$ --[2]

(b) Hence solve the equation $\frac{1-2\sin^2\theta}{1-\sin^2\theta} = 2\tan^4\theta$ for $0^\circ \le \theta \le 180^\circ$ --[3]

[S-21/11/Q7]

Solution:
(a) L.H.S. $\frac{1-2\sin^2\theta}{1-\sin^2\theta} = \frac{1-2\sin^2\theta}{\cos^2\theta}$

$= \frac{1}{\cos^2\theta} - \frac{2\sin^2\theta}{\cos^2\theta}$

$= \sec^2\theta - 2\tan^2\theta$

$= 1+\tan^2\theta - 2\tan^2\theta$

$= 1-\tan^2\theta$ ✓

(b) To Solve $\frac{1-2\sin^2\theta}{1-\sin^2\theta} = 2\tan^4\theta$ using part (a)

$\Rightarrow 1-\tan^2\theta = 2\tan^4\theta$

$\Rightarrow 2\tan^4\theta + \tan^2\theta - 1 = 0$

$(2\tan^2\theta - 1)(\tan^2\theta+1) = 0$

$\tan^2\theta = \frac{1}{2}$ or $\tan^2\theta = -1$ ×

$\tan\theta = \pm 0.7071$

$\tan\theta = 0.7071$ or $\tan\theta = -0.7071$

$\theta = 35.3^\circ$ or $180 - 35.3$

$\theta = 35.3^\circ$ ✓ or 144.7° ✓

14.(a) Prove the identity: $\dfrac{1+\sin x}{1-\sin x} - \dfrac{1-\sin x}{1+\sin x} \equiv \dfrac{4\tan x}{\cos x}$ ---- [4].

(b) Hence solve the equation: $\dfrac{1+\sin x}{1-\sin x} - \dfrac{1-\sin x}{1+\sin x} = 8\tan x$; $0 \leq x \leq \frac{\pi}{2}$ ---[3]

[S-21/12/Q10]

Solution(a) L.H.S $\dfrac{1+\sin x}{1-\sin x} - \dfrac{1-\sin x}{1+\sin x}$

$= \dfrac{(1+\sin x)^2 - (1-\sin x)^2}{(1-\sin x)(1+\sin x)}$

$= \dfrac{1+\sin^2 x + 2\sin x - (1+\sin^2 x - 2\sin x)}{1-\sin^2 x}$

$= \dfrac{4\sin x}{\cos^2 x} = \dfrac{4\sin x}{\cos x} \times \dfrac{1}{\cos x}$

$= \dfrac{4\tan x}{\cos x} = R.H.S$ ✓

(b) To solve;
$\dfrac{1+\sin x}{1-\sin x} - \dfrac{1-\sin x}{1+\sin x} = 8\tan x$ $\quad 0 \leq x \leq \frac{\pi}{2}$

using Part (a)

$\Rightarrow \dfrac{4\tan x}{\cos x} = 8\tan x$

$\Rightarrow 4\tan x = 8\tan x \cdot \cos x$ $\quad (\cos x \neq 0)$

$\Rightarrow 8\tan x\cos x - 4\tan x = 0$

$4\tan x(2\cos x - 1) = 0$

$\Rightarrow \tan x = 0$ or $\cos x = \frac{1}{2}$

$\Rightarrow x = 0$ ✓ or $x = \frac{\pi}{3}$ ✓ for $0 \leq x \leq \frac{\pi}{2}$

15 (a) Show that the equation: $\dfrac{\tan x + \sin x}{\tan x - \sin x} = k$
where k is a constant, may be expressed as: $\dfrac{1+\cos x}{1-\cos x} = k$ --- [2]

(b) Hence express $\cos x$ in terms of k. ---[2]

(c) Hence solve the equation: $\dfrac{\tan x + \sin x}{\tan x - \sin x} = 4$ for $-\pi < x < \pi$ ---[2]

[S-21/13/Q4]

Solution(a). $\dfrac{\tan x + \sin x}{\tan x - \sin x} = k$

$\Rightarrow \dfrac{\frac{\sin x}{\cos x} + \sin x}{\frac{\sin x}{\cos x} - \sin x} = k$

$\Rightarrow \dfrac{\sin x + \sin x \cdot \cos x}{\sin x - \sin x\cos x} = k$

$\Rightarrow \dfrac{\cancel{\sin x}(1+\cos x)}{\cancel{\sin x}(1-\cos x)} = k$

$\Rightarrow \dfrac{1+\cos x}{1-\cos x} = k$ ✓

(b) $\dfrac{1+\cos x}{1-\cos x} = k \Rightarrow 1+\cos x = k(1-\cos x)$

$\Rightarrow \cos x(k+1) = k-1$

$\Rightarrow \cos x = \dfrac{k-1}{k+1}$

(c) To solve; $\dfrac{\tan x + \sin x}{\tan x - \sin x} = 4$; $-\pi < x < \pi$

using part (a)

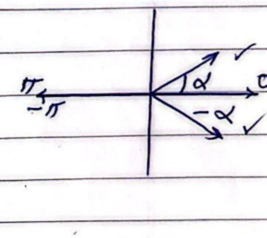
$\Rightarrow \dfrac{1-\cos x}{1+\cos x} = 4 = k$ $\quad [k=4]$

$\Rightarrow$

$\Rightarrow \cos x = \dfrac{4-1}{4+1} = \dfrac{3}{5}$ [Using Part (b)]

$x = \cos^{-1}\frac{3}{5}, -\cos^{-1}\frac{3}{5}$

$x = 0.927; -0.927$ ✓



16(a) Prove the identity $\frac{\sin^3\theta}{\sin\theta - 1} - \frac{\sin^2\theta}{1+\sin\theta} = -\tan^2\theta(1+\sin^2\theta)$ ---[4]

(b) Hence solve the equation: $\frac{\sin^3\theta}{\sin\theta-1} - \frac{\sin^2\theta}{1+\sin\theta} = \tan^2\theta(1-\sin^2\theta)$

for $0 < \theta < 2\pi$ ---[2]

[S-22/11/Q4]

Solution: (a) $\frac{\sin^3\theta}{\sin\theta-1} - \frac{\sin^2\theta}{1+\sin\theta}$

$= \frac{\sin^3\theta(1+\sin\theta) - \sin^2\theta(\sin\theta-1)}{(\sin\theta-1)(\sin\theta+1)}$

$= \frac{\cancel{\sin^3\theta} + \sin^4\theta - \cancel{\sin^3\theta} + \sin^2\theta}{\sin^2\theta - 1}$

$= \frac{\sin^2\theta(\sin^2\theta+1)}{-(1-\sin^2\theta)}$

$= -\frac{\sin^2\theta}{\cos^2\theta}\cdot(1+\sin^2\theta)$

$= -\tan^2\theta(1+\sin^2\theta)$ - (1)

(b) Solve $0<\theta<2\pi$

$\frac{\sin^3\theta}{\sin\theta-1} - \frac{\sin^2\theta}{1+\sin\theta} = \tan^2\theta(1-\sin^2\theta)$ using part (a)

$\Rightarrow -\tan^2\theta(1+\sin^2\theta) = \tan^2\theta(1-\sin^2\theta)$

$\Rightarrow -\tan^2\theta - \cancel{\tan^2\theta\sin^2\theta} = \tan^2\theta - \cancel{\sin^2\theta\tan^2\theta}$

$\Rightarrow 2\tan^2\theta = 0 \Rightarrow \tan\theta = 0$

$\Rightarrow \theta = 0^{\times}, \pi, 2\pi^{\times}$

(as $0<\theta<2\pi$)

$\theta = \pi$ ✓

17.(a) The curve $y = \sin x$ is transformed to curve, $y = 4\sin(\frac{1}{2}x - 30^\circ)$ describe fully a sequence of transformations that have been combined, make clear the order in which the transformation are applied. ---[5]

(b) Find the exact solution of the equation:

$4\sin(\frac{1}{2}x - 30^\circ) = 2\sqrt{2}$ for $0^\circ \le x \le 360^\circ$ ---[3]

[S-22/11/Q8]

Solution (a) (i) Translation $\begin{pmatrix} 30^\circ \\ 0 \end{pmatrix}$

(ii) Strech (factor 2) in x-direction

(iii) Strech factor 4 in y-direction.

(b) Solve $0 \le x \le 360^\circ$

$4\sin(\frac{1}{2}x - 30^\circ) = 2\sqrt{2}$

$\Rightarrow \sin(\frac{1}{2}x - 30^\circ) = \frac{2\sqrt{2}}{4} = \frac{1}{\sqrt{2}}$

$\Rightarrow (\frac{1}{2}x - 30^\circ) = \sin^{-1}\frac{1}{\sqrt{2}}$

$= 45^\circ$ or $180 - 45^\circ$

$\left\{ \begin{array}{l} 0 \le x \le 360^\circ \\ 0 \le \frac{x}{2} \le 180^\circ \\ -30 \le \frac{x}{2} - 30^\circ \le 150^\circ \end{array} \right.$

$\frac{1}{2}x - 30^\circ = 45^\circ$ or 135°

$\frac{1}{2}x = 75^\circ$ or 165°

$x = 150^\circ$ ✓ or 330° ✓

18. The function f is given by $f(x) = 4\cos^4 x + \cos^2 x - k$ for $0 \leq x \leq 2\pi$, where k is a constant.
(a) Given that $k = 3$, find the exact solution of the equation $f(x) = 0$ ---[5]
(b) Use the quadratic formula to show that, when $k > 5$, the equation $f(x) = 0$ has no solution. ---[5]

[S-22 | 12 | Q 11]

Solution: $f(x) = 4\cos^4 x + \cos^2 x - k$

(a) for $k = 3$, $f(x) = 0$ $0 \leq x \leq 2\pi$

$f(x) = 4(\cos^2 x)^2 + \cos^2 x - 3 = 0$

$(4\cos^2 x - 3)(\cos^2 x + 1) = 0$

$\cos^2 x = 3/4$; $\cos^2 x = -1^{\times}$

$\cos x = \pm \frac{\sqrt{3}}{2} = \pm \cos\frac{\pi}{6}$

$x = \frac{\pi}{6}, \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$= \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11}{6}\pi$ ✓

(b) $f(x) = 0$ for $k > 5$

$4\cos^4 x + \cos^2 x - k = 0$

$4(\cos^2 x)^2 + \cos^2 x - k = 0$

$\cos^2 x = \frac{-1 \pm \sqrt{1 + 16k}}{8}$

$\cos^2 x = \frac{-1 + \sqrt{1+16k}}{8}$; $-\frac{(1 + \sqrt{1+16k})}{8}$

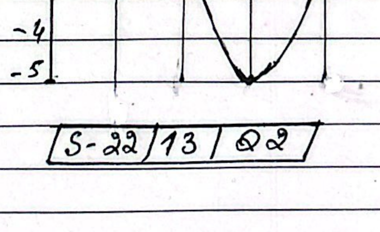
$= \frac{-1 + \sqrt{1+16k}}{8} > 1$ for $k > 5$, $\sqrt{1+16k} > 9$

and $-\left(\frac{1 + \sqrt{1+16k}}{8}\right) < -1$

but $0 \leq \cos^2 x \leq 1$

$\therefore$ Equation has no solution for $k > 5$

19. The diagram shows part of the curve, with equation $y = p\sin(q\theta) + r$, where p, q and r are constants.
(a) State the values of p, q and r. ---[3]



[S-22 | 13 | Q 2]

Solution: $p = 3$ ✓

$q = 1/2$ ✓

$r = -2$ ✓

20(a) Solve the equation $6\sqrt{y} + \frac{2}{\sqrt{y}} - 7 = 0$ ---[4]

(b) Hence solve the equation: $6\sqrt{\tan x} + \frac{2}{\sqrt{\tan x}} - 7 = 0; \quad 0 \leq x \leq 360^\circ$ ---[3]

S-22/13/Q5

Solution (a) $6\sqrt{y} + \frac{2}{\sqrt{y}} - 7 = 0$

$\Rightarrow 6y - 7\sqrt{y} + 2 = 0$

$6y - 4\sqrt{y} - 3\sqrt{y} + 2 = 0$

$2\sqrt{y}(3\sqrt{y} - 2) - 1(3\sqrt{y} - 2) = 0$

$(3\sqrt{y} - 2)(2\sqrt{y} - 1) = 0$

$\Rightarrow \sqrt{y} = 2/3$ or $\sqrt{y} = \frac{1}{2}$

$y = 4/9$ or $y = 1/4$

(b) Solve; $6\sqrt{\tan x} + \frac{2}{\sqrt{\tan x}} - 7 = 0 \quad 0 \leq x \leq 360^\circ$

Using part (a)

$\tan x = 4/9$ or $\tan x = \frac{1}{4}$

$= \tan 24^\circ$ or $\tan x = 14^\circ$

$\therefore x = 24^\circ, 180 + 24^\circ$ or $14^\circ, 180 + 14^\circ$

$x = 14^\circ, 24^\circ, 194^\circ, 204^\circ$

α +

$180+\alpha$ +

21 Solve the equation: $4\sin\theta + \tan\theta = 0$ for $0° < \theta < 180°$ ...[3]

S-23 | 11 | Q1

Solution: $4\sin\theta + \tan\theta = 0$

$\Rightarrow 4\sin\theta + \frac{\sin\theta}{\cos\theta} = 0$

$\Rightarrow \frac{4\sin\theta\cos\theta + \sin\theta}{\cos\theta} = 0$

$\Rightarrow \sin\theta(4\cos\theta + 1) = 0$

$\Rightarrow \sin\theta = 0$ or $\cos\theta = -\frac{1}{4}$

$\alpha = \cos^{-1}\frac{1}{4} = 75.5$

$\Rightarrow \theta = 0^{\times}, 180^{\times}$ $(0 < \theta < 180°)$

$= -\cos\alpha$

$0 < \theta < 180°$

$\theta = 180 - \alpha$

$= 180 - 75.5$

$= \underline{104.5°}$ ✓

22. A curve has equation $y = 2 + 3\sin\frac{1}{2}x$ for $0 \leq x \leq 4\pi$

(a) State the least and greatest value of y. ---[2]

(b) Sketch the curve. ---[2]

(c) State the number of solutions of the equation: ---[1]

$$2 + 3\sin\frac{1}{2}x = 5 - 2x, \quad 0 \leq x \leq 4\pi$$

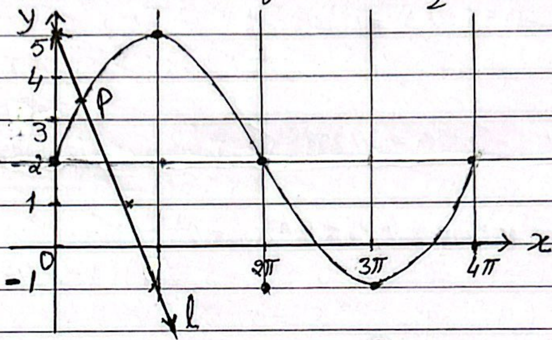
S-23/11/Q7

Solution: Curve: $y = 2 + 3\sin\frac{1}{2}x$ for $0 \leq x \leq 4\pi$

(a) Greatest value at $x = \pi$, $y = 2 + 3\sin\frac{\pi}{2} = 2 + 3\times 1 = \underline{5}$ ✓

Least Value at $x = 3\pi$, $y = 2 + 3\sin\frac{3\pi}{2} = 2 + 3\times(-1) = \underline{-1}$ ✓

(b)



Curve: $y = 2 + 3\sin\frac{1}{2}x$ --- ① $0 \leq x \leq 4\pi$

x	0	π	2π	3π	4π
y	2	5	2	-1	2

(c) Solve; $2 + 3\sin\frac{1}{2}x = 5 - 2x$ --- ②

$\pi = 3.1$

Consider line; $y = 5 - 2x$ --- ③

x	0	2	3
y	5	1	-1

line ③ and curve ① intersect at only one point P.

Number of solution of equation ② $= \underline{1}$ ✓

23.(a) By first expanding $(\cos\theta+\sin\theta)^2$, find the three solutions of the
(i) equation: $(\cos\theta+\sin\theta)^2=1$. for $0\leq\theta\leq\pi$ ...[3]
(ii) Hence verify that the only solutions of the equation $\cos\theta+\sin\theta=1$ for $0\leq\theta\leq\pi$ are 0 and $\frac{1}{2}\pi$. ...[2]
(b) Prove the identity: $\dfrac{\sin\theta}{\cos\theta+\sin\theta}+\dfrac{1-\cos\theta}{\cos\theta-\sin\theta}\equiv\dfrac{\cos\theta+\sin\theta-1}{1-2\sin^2\theta}$ ... [3]
(c) Using the results of (a)(ii) and (b), solve the equation:
$\dfrac{\sin\theta}{\cos\theta+\sin\theta}+\dfrac{1-\cos\theta}{\cos\theta-\sin\theta}=2(\cos\theta+\sin\theta-1)$, for $0\leq\theta\leq\pi$ .... [3]
S-23/12/Q7

Solution (a)(i) solve: $(\cos\theta+\sin\theta)^2=1$ for $0\leq\theta\leq\pi \longrightarrow 0\leq 2\theta\leq 2\pi$

$\Rightarrow \cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta=1$

$\rightarrow 1+\sin2\theta=1\Rightarrow\sin2\theta=0$

$\Rightarrow 2\theta=0,\pi,2\pi\Rightarrow\theta=0,\frac{\pi}{2},\pi$ ✓

(ii) $\cos\theta+\sin\theta=1$

for $\theta=0\Rightarrow\cos0+\sin0=1\Rightarrow 1+0=1$ True ✓

$\theta=\frac{\pi}{2}\Rightarrow\cos\frac{\pi}{2}+\sin\frac{\pi}{2}=1\Rightarrow 0+1=1$ True ✓

at $\theta=\pi\Rightarrow\cos\pi+\sin\pi=1\Rightarrow -1+0=1$ false.

$\therefore$ Only solutions are '0 and $\frac{\pi}{2}$'

(b) To Prove: L.H.S

$\dfrac{\sin\theta}{\cos\theta+\sin\theta}+\dfrac{1-\cos\theta}{\cos\theta-\sin\theta}$

$=\dfrac{\sin\theta(\cos\theta-\sin\theta)+(\cos\theta+\sin\theta)(1-\cos\theta)}{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}$

$=\dfrac{\cancel{\sin\theta\cos\theta}-\sin^2\theta+\cos\theta-\cos^2\theta+\sin\theta-\cancel{\sin\theta\cos\theta}}{\cos^2\theta-\sin^2\theta}$

$=\dfrac{\cos\theta+\sin\theta-(\sin^2\theta+\cos^2\theta)}{(1-\sin^2\theta)-\sin^2\theta}$

$=\dfrac{\cos\theta+\sin\theta-1}{1-2\sin^2\theta}$ = R.H.S ✓

(c) To Solve:

$\dfrac{\sin\theta}{\cos\theta+\sin\theta}+\dfrac{1-\cos\theta}{\cos\theta-\sin\theta}=2(\cos\theta+\sin\theta-1)$

from part (b)

$\Rightarrow\dfrac{(\cos\theta+\sin\theta-1)}{1-2\sin^2\theta}=2(\cos\theta+\sin\theta-1)$

$\Rightarrow (\cos\theta+\sin\theta-1)=0$ — ①

$\Rightarrow$ or $1=2(1-2\sin^2\theta)$ — ②

$\Rightarrow 1=2-4\sin^2\theta$

$\longrightarrow \sin^2\theta=1\Rightarrow\sin^2\theta=\frac{1}{4}$

$\Rightarrow\sin\theta=\pm\frac{1}{4}$

$\sin\theta=\frac{1}{2}$, $\sin\theta=-\frac{1}{2}$ ×

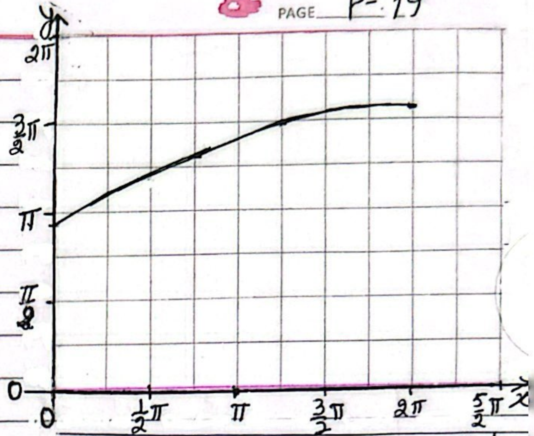
for ② $\theta=\frac{\pi}{6},\frac{5\pi}{6}$ -- ② $\quad 0\leq\theta\leq\pi$

for ① $\theta=0,\frac{\pi}{2}$ -- ①

$\therefore\ \theta=0,\frac{\pi}{6},\frac{\pi}{2},\frac{5}{6}\pi$ ✓

24. The diagram shows the graph of $y = f(x)$ where the function f is defined by:

$f(x) = 3 + 2\sin\frac{1}{4}x$ for $0 \leq x \leq 2\pi$



(a) On the diagram, sketch the graph of $y = f^{-1}(x)$. ---[2]

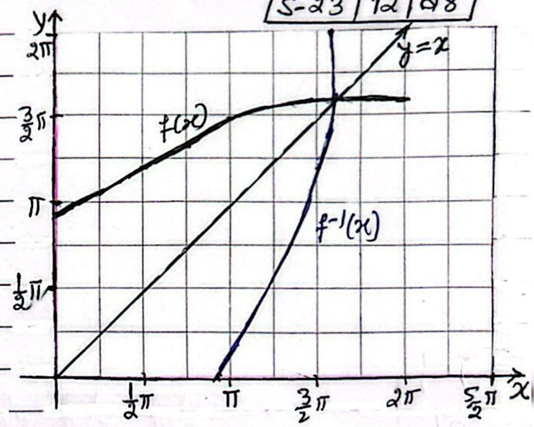
(b) Find an expression for $f^{-1}(x)$ --[2]

(d) Describe fully a sequence of three transformations which can be combined to transform the graph of $y = \sin x$ for $0 \leq x \leq \frac{1}{2}\pi$ to the graph of $y = f(x)$, making clear the order in which the transformation are applied. ---[6]

S-23 | 12 | Q8

Solution: (a) Draw a line $y = x$

Graph of $f^{-1}(x)$ is the reflection of the graph of $f(x)$ in line $y = x$.



(b) $f(x)$:

$y = 3 + 2\sin x/4$

Interchange x and y.

$\Rightarrow x = 3 + 2\sin y/4$

$\Rightarrow \sin y/4 = \frac{x-3}{2}$

$\Rightarrow y = 4\sin^{-1}\frac{x-3}{2}$

Hence $f^{-1}(x) = 4\sin^{-1}\left(\frac{x-3}{2}\right)$ ✓

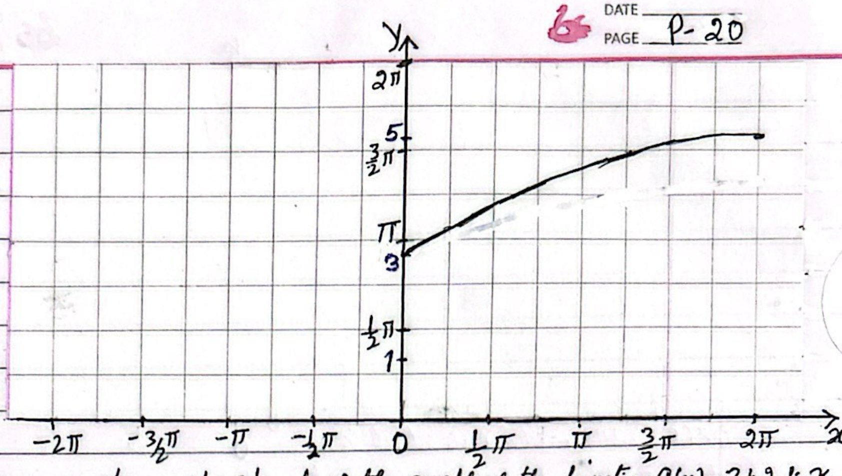
(d) Stretch (factor 4) in x-direction.

Stretch (factor 2) in y-direction.

Translation $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$

(Part (c) on the next page)

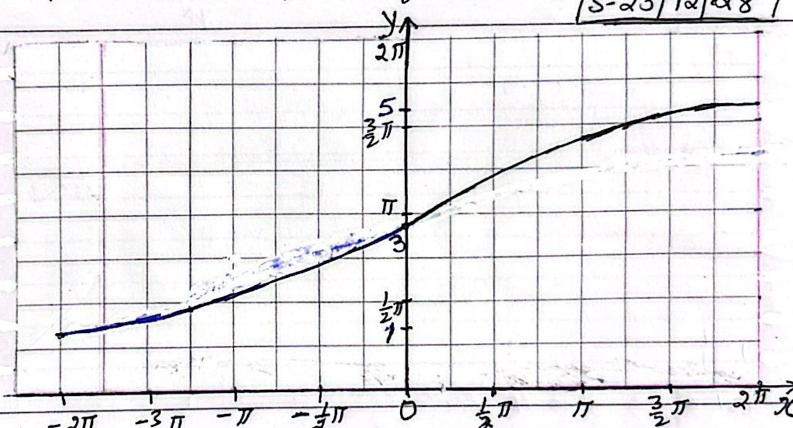
24(c)



The diagram above shows part of the graph of the function $g(x) = 3 + 2\sin\frac{x}{4}$ for $-2\pi \leq x \leq 2\pi$. Complete the graph of $g(x)$ on the diagram above and hence explain whether the function g has an inverse. --- [2]

[S-23/12/Q8]

Solution:



Yes $g(x)$ has an inverse, because the graph is always increasing the given interval. (or it is one-one).

Note
$(\sin(-\theta) = -\sin\theta)$

$g(2\pi) = 3 + 2\sin\frac{\pi}{2} = 3 + 2 = 5$

$g(-2\pi) = 3 + 2\sin(-\frac{\pi}{2}) = 3 - 2 = 1$

25. (a) Show that the equation, $3\tan^2 x - 3\sin^2 x - 4 = 0$, may be expressed in the form $a\cos^4 x + b\cos^2 x + c = 0$, where a, b, c are constants. ...[3]

(b) Hence solve the equation $3\tan^2 x - 3\sin^2 x - 4 = 0$ for $0^\circ \leq x \leq 180^\circ$ ...[4]

S-23 | 13 | Q4

Solution: $3\tan^2 x - 3\sin^2 x - 4 = 0$

(a) $\Rightarrow 3\dfrac{\sin^2 x}{\cos^2 x} - 3\sin^2 x - 4 = 0$

$\Rightarrow 3\sin^2 x - 3\sin^2 x \cdot \cos^2 x - 4\cos^2 x = 0$

$\Rightarrow 3(1-\cos^2 x) - 3\cos^2 x(1-\cos^2 x) - 4\cos^2 x = 0$

$\Rightarrow 3 - 3\cos^2 x - 3\cos^2 x + 3\cos^4 x - 4\cos^2 x = 0$

$\Rightarrow \underline{3\cos x - 10\cos^2 x + 3 = 0}$ ✓

(b) Solve: $3\tan^2 x - 3\sin^2 x - 4 = 0$ for $0 \leq x \leq 180^\circ$

$\Rightarrow 3\cos^4 x - 10\cos^2 x + 3 = 0$ (Using part (a))

$\Rightarrow 3(\cos^2 x)^2 - 10\cos^2 x + 3 = 0$

$\Rightarrow (3\cos^2 x - 1)(\cos^2 x - 3) = 0$

$\Rightarrow \cos^2 x = \dfrac{1}{3}$ or $\cos^2 x = 3^{\times}$ $(\because 0 \leq \cos^2 x \leq 1)$

Let $\cos^2 x = y$

$3y^2 - 10y + 3 = 0$

$3y^2 - 9y - y + 3 = 0$

$3y(y-3) - 1(y-3) = 0$

$(y-3)(3y-1) = 0$

$\Rightarrow \cos x = \dfrac{1}{\sqrt{3}}$ or $\cos x = -\dfrac{1}{\sqrt{3}}$

$= \cos\alpha$ or $\cos x = -\cos\alpha$

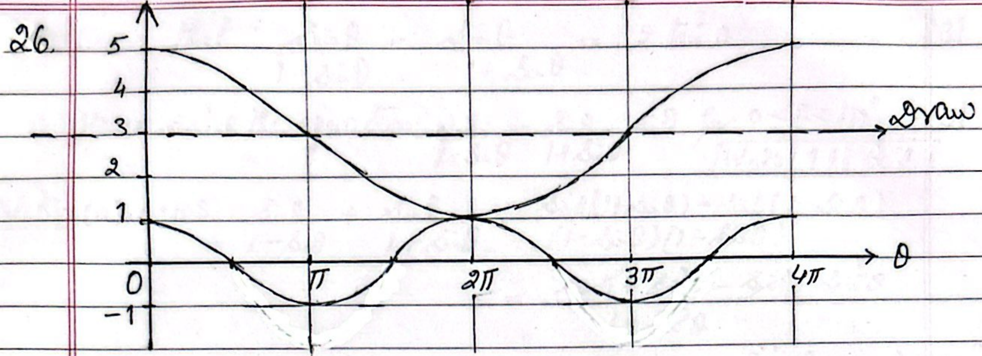
$\cos^{-1}\dfrac{1}{\sqrt{3}} = 54.7^\circ = \alpha$

$x = \alpha$ or $x = 180 - \alpha$

$x = 54.7$ or $x = 180 - 54.7$

$\underline{x = 54.7^\circ \text{ or } 125.3^\circ}$

26.



In the diagram, the lower curve has equation $y = \cos\theta$. The upper curve shows the result of applying a combination of transformations to $y = \cos\theta$.
Find in terms of cosine function, the equation of the upper curve. [W-20 / 11 / Q4] --- [3]

Solution: $y = a\cos(b\theta - h) + k$ [If the graph of $y = \cos\theta$ is applied the following transformations]

Strech along Y axis = amplitude = a $= 2$ ✓

Stretch along X-axis = Period = $\frac{2\pi}{b}$ $= 4\pi \Rightarrow b = \frac{1}{2}$ ✓

Translation along X-axis = $h = 0$

Translation along Y-axis = $k = 3$

∴ The equation of the upper curve is;

$$y = 2\cos\left(\tfrac{1}{2}\theta\right) + 3$$ ✓

27(a) Show that $\dfrac{\sin\theta}{1-\sin\theta} - \dfrac{\sin\theta}{1+\sin\theta} = 2\tan^2\theta$ --- [3]

(b) Hence solve the equation: $\dfrac{\sin\theta}{1-\sin\theta} - \dfrac{\sin\theta}{1+\sin\theta} = 8$ for $0 < \theta < 180^\circ$ --- [3]

$\boxed{W\text{-}20 / 11 / Q7}$

Solution (a) L.H.S. $\dfrac{\sin\theta}{1-\sin\theta} + \dfrac{\sin\theta}{1+\sin\theta} = \dfrac{\sin\theta(1+\sin\theta) - \sin\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$

$= \dfrac{\cancel{\sin\theta} + \sin^2\theta - \cancel{\sin\theta} - \sin^2\theta}{1-\sin^2\theta}$

$= \dfrac{2\sin^2\theta}{\cos^2\theta}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= 2\tan^2\theta = R.H.S$ ✓

(b) To Solve. $\dfrac{\sin\theta}{1-\sin\theta} - \dfrac{\sin\theta}{1+\sin\theta} = 8$ $0 < \theta < 180^\circ$

$\Rightarrow 2\tan^2\theta = 8$ [using Part (a) for L.H.S]

$\tan^2\theta = 4$

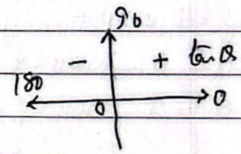
$\tan\theta = \pm 2$

$\tan\theta = 2$ or $\tan\theta = -2$

$= \tan 63.4^\circ$ or $\tan\theta = -\tan 63.4$

$\therefore \theta = 63.4^\circ$ or $(180 - 63.4)^\circ$

$\theta = \underline{63.4^\circ}$ or $\underline{116.6^\circ}$ ✓



28(a) Prove the identity: $\left(\dfrac{1}{\cos x} - \tan x\right)\left(\dfrac{1}{\sin x} + 1\right) = \dfrac{1}{\tan x}$ --- [4]

(b) Hence solve: $\left(\dfrac{1}{\cos x} - \tan x\right)\left(\dfrac{1}{\sin x} + 1\right) = 2\tan^2 x$ for $0 \leq x \leq 180^\circ$ --- [2]

$\boxed{W\text{-}20 / 12 / Q6}$

Solution: (a) L.H.S. $\left(\dfrac{1}{\cos x} - \tan x\right)\left(\dfrac{1}{\sin x} + 1\right)$

$= \left(\dfrac{1}{\cos x} - \dfrac{\sin x}{\cos x}\right)\left(\dfrac{1+\sin x}{\sin x}\right)$

$= \left(\dfrac{1-\sin x}{\cos x}\right)\left(\dfrac{1+\sin x}{\sin x}\right)$

$= \dfrac{(1-\sin^2 x)}{\sin x \cdot \cos x}$

$= \dfrac{\cos^2 x}{\sin x \cdot \cos x}$

$= \dfrac{1}{\sin x/\cos x} = \dfrac{1}{\tan x} = \underline{R.H.S}$ ✓

(b) Solve $\left(\dfrac{1}{\cos x} - \tan x\right)\left(\dfrac{1}{\sin x} + 1\right) = 2\tan^2 x$

$\Rightarrow \dfrac{1}{\tan x} = 2\tan^2 x$ [$\because$ Part (a)]

$\Rightarrow \tan^3 x = \dfrac{1}{2}$

$\Rightarrow \tan x = 0.7937$

$\Rightarrow x = \underline{38.4^\circ}$ ✓

29. A curve has equation $y = 3\cos 2x + 2$ for $0 \leq x \leq \pi$

(a) State the greatest and the least value of y. ---[2]

(b) Sketch the graph of $y = 3\cos 2x + 2$ for $0 \leq x \leq \pi$ ---[2]

(c) By considering the straight line $y = kx$, where k is a constant, state the number of solutions of the equation $3\cos 2x + 2 = kx$ for $0 \leq x \leq \pi$, in each of the following cases;

(i) $k = -3$ ---[1]

(ii) $k = 1$ ---[1]

(iii) $k = 3$ ---[1]

Functions f, g, and h are defined for $x \in R$ by:

$f(x) = 3\cos 2x + 2$
$g(x) = f(2x) + 4$
$h(x) = 2f(x + \frac{1}{2}\pi)$

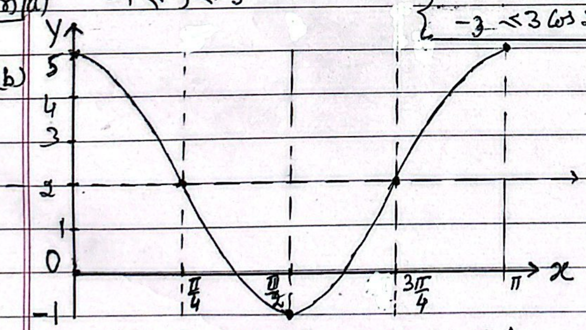
(d) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ onto $y = g(x)$ --[2]

(e) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ onto $y = f(x)$ onto $y = h(x)$ $\boxed{W\text{-}20|12|Q11}$ --[2]

Solution (a) $-1 \leq y \leq 5$

$\left\{ \begin{array}{l} -1 \leq \cos 2x \leq 1 \\ -3 \leq 3\cos 2x \leq 3 \Rightarrow -1 \leq 3\cos 2x + 2 \leq 5 \end{array} \right.$ ✓

(b)



Graph of
$y = 3\cos 2x + 2$

c. (i) 0 solution (ii) 2 solution (iii) 1 solution

(d) stretch by (scalar factor) $\frac{1}{2}$, parallel to x-axis, and Translation $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$.

(e) Translation of ______ $\begin{pmatrix} -\frac{\pi}{2} \\ 0 \end{pmatrix}$ and stretch (scalar factor 2) parallel to y-axis (or vertically).

30. Solve the equation, $3\tan^2\theta + 1 = \dfrac{2}{\tan^2\theta}$ for $0 \leq \theta \leq 180°$ --[5]

[W-20/13/Q3]

Solution: $3\tan^2\theta + 1 = \dfrac{2}{\tan^2\theta}$

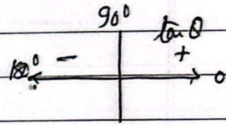
$\Rightarrow 3\tan^4\theta + \tan^2\theta - 2 = 0$

$\Rightarrow (3\tan^2\theta - 2)(\tan^2\theta + 1) = 0$

$\Rightarrow \tan^2\theta = 2/3$ or $\tan^2\theta = -1^{\times}$

$\Rightarrow \tan\theta = \pm\sqrt{2/3}$

$\Rightarrow \tan\theta = \sqrt{2/3}$ or $\tan\theta = -\sqrt{2/3}$



$= \tan 39.2$ or $\tan\theta = -\tan 39.2$

$\therefore \theta = 39.2°, (180 - 39.2)°$

$\underline{\theta = 39.2°}, \underline{140.8°}$ ✓

31. Solve, by factorising, the equation: for $0 \leq \theta \leq 180°$

$$6\cos\theta\tan\theta - 3\cos\theta + 4\tan\theta - 2 = 0$$

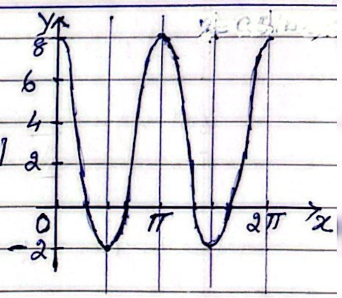
----[4] [W-21/11/Q3]

Solution:

$$6\cos\theta\tan\theta - 3\cos\theta + 4\tan\theta - 2 = 0$$
$$3\cos\theta(2\tan\theta - 1) + 2(2\tan\theta - 1) = 0$$
$$(2\tan\theta - 1)(3\cos\theta + 2) = 0$$
$$\tan\theta = \frac{1}{2} \quad \text{or} \quad \cos\theta = -\frac{2}{3}$$
$$\theta = \tan^{-1}\frac{1}{2}, \quad \cos\theta = -\cos$$
$$\theta = 26.6 \quad \cos\theta = -\cos 48.2°$$
$$\theta = 26.6° \quad \theta = 180 - 48.2° = 131.8° \Rightarrow \theta = 26.6° ; \theta = 131.8°$$

32 The diagram shows part of the graph of:

$$y = a\cos(bx) + c$$



(a) Find the values of positive integers a, b, c. --[3]

(b) For these values of a, b and c, determine the number of solutions in the interval $0 < x < 2\pi$, for each of the following equations:

(i) $a\cos(bx) + c = \frac{6}{\pi}x$ (ii) $a\cos(bx) + c = 6 - \frac{6}{\pi}x$

---[2]

[W-21/11/Q5]

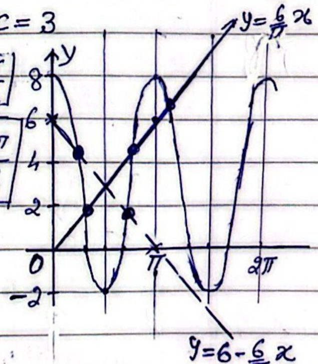
Solution (a) $a = 5$ (amplitude), $b = 2$ (Period $= \frac{2\pi}{2} = \pi$), $c = 3$

(b) (i) 3 (Draw a line $y = \frac{6}{\pi}x$)

x	0	π	2π
y	0	6	12

(ii) 2 (Draw a line $y = 6 - \frac{6}{\pi}x$)

x	0	π	2π
y	6	0	-6



33. Solve the equation: $2\cos\theta = 7 - \frac{3}{\cos\theta}$ for $-90^\circ < \theta < 90^\circ$ ---[4]
[W-21/12/Q1]

<u>Solution:</u> $2\cos\theta = 7 - \frac{3}{\cos\theta}$ for $-90^\circ < \theta < 90^\circ$

$\Rightarrow 2\cos^2\theta - 7\cos\theta + 3 = 0$

$(2\cos\theta - 1)(\cos\theta - 3) = 0 \Rightarrow \cos\theta = \frac{1}{2}$ ✓ or $\cos\theta = 3$ ✗ $(-1 \leq \cos\theta \leq 1)$

$\underline{\theta = 60^\circ \text{ or } -60^\circ}$ ✓ $(-90^\circ < \theta < 90^\circ)$

34(a) Show that the equation, $\frac{\tan x + \cos x}{\tan x - \cos x} = k$, where k is a constant, can be expressed as; $(k+1)\sin^2 x + (k-1)x - (k+1) = 0$ ---[4]

(b) Hence solve the equation $\frac{\tan x + \cos x}{\tan x - \cos x} = 4$ for $0 \leq x \leq 360^\circ$ ---[4]
[W-21/13/Q7]

<u>Solution</u> (a) $\frac{\tan x + \cos x}{\tan x - \cos x} = k \Rightarrow \frac{\frac{\sin x}{\cos x} + \cos x}{\frac{\sin x}{\cos x} - \cos x} = k$

$\Rightarrow \frac{\sin x + \cos^2 x}{\sin x - \cos^2 x} = k$

$\Rightarrow \sin x + (1 - \sin^2 x) = k[\sin x - (1 - \sin^2 x)]$

$\Rightarrow (k+1)\sin^2 x + (k-1)\sin x - (k+1) = 0$ ✓

(b) To solve $\frac{\tan x + \cos x}{\tan x - \cos x} = 4$ for $0 \leq x \leq 360^\circ$

using part (a) and put $k = 4$ (part b)

$\Rightarrow (4+1)\sin^2 x + (4-1)\sin x - (4+1) = 0$

$\Rightarrow 5\sin^2 x + 3\sin x - 5 = 0$

$\sin x = \frac{-3 \pm \sqrt{109}}{10} = \frac{-3 \pm 10.44}{10} = .744 ; -1.344$ ✗

$\therefore \sin x = 0.744$
$= \sin 48.1^\circ$

$0 \leq x \leq 360$

$(180-\alpha) \mid \alpha$ ✓

$x = 48.1, \ 180 - 48.1$

$\underline{x = 48.1^\circ ; 131.9^\circ}$ ✓

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35(a) Show that the equation: $\dfrac{1}{\sin\theta+\cos\theta}+\dfrac{1}{\sin\theta-\cos\theta}=1$

may be expressed in the form $a\sin^2\theta+b\sin\theta+c=0$, where a, b and c are constants to be found. ---[3]

(b) Hence solve the equation. $\dfrac{1}{\sin\theta+\cos\theta}+\dfrac{1}{\sin\theta-\cos\theta}=1$ for $0^\circ\leq\theta\leq360^\circ$ ---[3]

W-22 | 11 | Q6

Solution:

(a) $\dfrac{1}{\sin\theta+\cos\theta}+\dfrac{1}{\sin\theta-\cos\theta}=1$

$\dfrac{\sin\theta-\cancel{\cos\theta}+\sin\theta+\cancel{\cos\theta}}{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}=1$

$\Rightarrow 2\sin\theta=\sin^2\theta-\cos^2\theta$

$2\sin\theta=\sin^2\theta-(1-\sin^2\theta)$

$\Rightarrow 2\sin^2\theta-2\sin\theta-1=0$

(b) Solve; $\dfrac{1}{\sin\theta+\cos\theta}+\dfrac{1}{\sin\theta-\cos\theta}=1$

Using part (a)

$\Rightarrow 2\sin^2\theta-2\sin\theta-1=0$

$\sin\theta=\dfrac{2\pm\sqrt{(-2)^2-4\times2\times(-1)}}{2\times2}$

$=\dfrac{2\pm\sqrt{12}}{2}=\dfrac{1\pm\sqrt{3}}{2}$

$\sin\theta=-0.366$ or $\sin\theta=\dfrac{1+\sqrt{3}}{2}$ ✗ >1

$=-\sin\alpha$, $\alpha=\sin^{-1}0.366=21.5^\circ$

$\theta=180+\alpha,\ 360-\alpha$

$\theta=180+21.5^\circ$ or $360-21.5$

$=\underline{201.5^\circ}$ or $\underline{338.5^\circ}$ ✓

-ve | -ve
$180+\alpha$ | $360-\alpha$

36. Solve the equation: $8\cos^2\theta - 10\cos\theta + 2 = 0$ for $0^\circ \leq \theta \leq 180^\circ$ ---[3]

$\boxed{W\text{-}22/12/3(b)}$

Solution: $8\cos^2\theta - 10\cos\theta + 2 = 0$

$2(4\cos\theta - 1)(\cos\theta - 1) = 0$

$\Rightarrow \cos\theta = \frac{1}{4}, \cos\theta = 1$

$\cos\theta = \cos^{-1}\frac{1}{4}$ or $\theta = \cos^{-1}1$, $0 \leq \theta \leq 180$

$\theta = 75.5^\circ$ or $\theta = 0^\circ$

$\therefore \theta = 0^\circ$ or 75.5° ✓

37.(a) Prove the identity: $\frac{\sin\theta}{\sin\theta + \cos\theta} + \frac{\cos\theta}{\sin\theta - \cos\theta} \equiv \frac{\tan^2\theta + 1}{\tan^2\theta - 1}$ --[3]

(b) Hence find the exact solution of the equation: $\frac{\sin\theta}{\sin\theta + \cos\theta} + \frac{\cos\theta}{\sin\theta - \cos\theta} = 2$ for $0 \leq \theta \leq \pi$, ---[4]

$\boxed{W\text{-}22/12/Q7}$

Solution: (a) L.H.S $\frac{\sin\theta}{\sin\theta + \cos\theta} + \frac{\cos\theta}{\sin\theta - \cos\theta}$

$= \frac{\sin\theta(\sin\theta - \cos\theta) + \cos\theta(\sin\theta + \cos\theta)}{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta - \cos^2\theta}$

$= \frac{\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta} - \frac{\cos^2\theta}{\cos^2\theta}}$ (Divide N^r and D^r by $\cos^2\theta$.

$= \frac{\tan^2\theta + 1}{\tan^2\theta - 1}$ ✓ R.H.S.

(b) Solve; $\frac{\sin\theta}{\sin\theta + \cos\theta} + \frac{\cos\theta}{\sin\theta - \cos\theta} = 2$

$0 \leq \theta \leq \pi$

using part (a)

$\frac{\tan^2\theta + 1}{\tan^2\theta - 1} = 2$

$\Rightarrow \tan^2\theta + 1 = 2(\tan^2\theta - 1)$

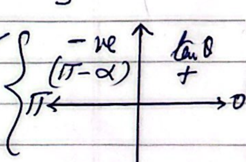
$\Rightarrow \tan^2\theta = 3$

$\tan\theta = \pm\sqrt{3}$

$\tan\theta = \sqrt{3}$, $\tan\theta = -\sqrt{3} = -\tan\pi/3$

$\theta = \frac{\pi}{3}$, $\theta = \pi - \frac{\pi}{3}$

$\theta = \frac{\pi}{3}$ or $\frac{2\pi}{3}$ ✓



38 Solve the equation: $8\sin^2\theta + 6\cos\theta + 1 = 0$ for $0° < \theta < 180°$ ---[3]

[W-22 | 13 | Q 1]

Solution: $8\sin^2\theta + 6\cos\theta + 1 = 0$

$\Rightarrow 8(1-\cos^2\theta) + 6\cos\theta + 1 = 0$

$\Rightarrow 8\cos^2\theta - 6\cos\theta - 9 = 0$

$(4\cos\theta + 3)(2\cos\theta - 3) = 0$

$\Rightarrow \cos\theta = -\frac{3}{4}$ or $\cos\theta = \frac{3}{2} > 1$

$\cos\theta = -0.75$ $\quad 0° < \theta < 180°$

$= -\cos 41.4$

$\therefore \theta = 180 - 41.4$

$\theta = 138.6°$ ✓

39. It is given that $\alpha = \cos^{-1}(8/17)$; Find, without using the trigonometric functions on your calculator, the exact value of $\frac{1}{\sin\alpha} + \frac{1}{\tan\alpha}$. ---[5]

[W-22 | 13 | Q6]

Solution: Given $\alpha = \cos^{-1}(8/17) \Rightarrow \cos\alpha = \frac{8}{17}$

$\Rightarrow \sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\left(\frac{8}{17}\right)^2} = \frac{15}{17}$ ✓

$\therefore \tan\alpha = \sin\alpha / \cos\alpha = {}^{15}/_{17} \Big/ \frac{8}{17} = \frac{15}{8}$ ✓

$\therefore \frac{1}{\sin\alpha} + \frac{1}{\tan\alpha} = \frac{1}{15/17} + \frac{1}{\frac{15}{8}}$

$= \frac{17}{15} + \frac{8}{15}$

$= \frac{25}{15}$

$= \frac{5}{3}$ ✓