# PURE MATHEMATICS -1

# 9709

(March, June and November series 2020 – 2023 With marking scheme)

# Trigonometry

# EXERCISE -1

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1) SP-2020 _9709_1 Q7

**(a)** Show that the equation $1 + \sin x \tan x = 5\cos x$ can be expressed as

$$6\cos^2 x - \cos x - 1 = 0.$$ [3]

**(b)** Hence solve the equation $1 + \sin x \tan x = 5\cos x$ for $0° \leqslant x \leqslant 180°$. [3]

2) MARCH 2020_9709_12 Q5

Solve the equation

$$\frac{\tan\theta + 3\sin\theta + 2}{\tan\theta - 3\sin\theta + 1} = 2$$

for $0° \leqslant \theta \leqslant 90°$. [5]

3) MARCH 2020_9709_12 Q11

**(a)** Solve the equation $3\tan^2 x - 5\tan x - 2 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

**(b)** Find the set of values of $k$ for which the equation $3\tan^2 x - 5\tan x + k = 0$ has no solutions. [2]

**(c)** For the equation $3\tan^2 x - 5\tan x + k = 0$, state the value of $k$ for which there are three solutions in the interval $0° \leqslant x \leqslant 180°$, and find these solutions. [3]

4) MARCH 2021_9709_12 Q3

Solve the equation $\dfrac{\tan\theta + 2\sin\theta}{\tan\theta - 2\sin\theta} = 3$ for $0° < \theta < 180°$. [4]

5) MARCH 2022_9709_12 Q7

**(a)** Show that $\dfrac{\sin\theta + 2\cos\theta}{\cos\theta - 2\sin\theta} - \dfrac{\sin\theta - 2\cos\theta}{\cos\theta + 2\sin\theta} \equiv \dfrac{4}{5\cos^2\theta - 4}$. [4]

**(b)** Hence solve the equation $\dfrac{\sin\theta + 2\cos\theta}{\cos\theta - 2\sin\theta} - \dfrac{\sin\theta - 2\cos\theta}{\cos\theta + 2\sin\theta} = 5$ for $0° < \theta < 180°$. [3]

6) MARCH 2023_9709_12 Q7

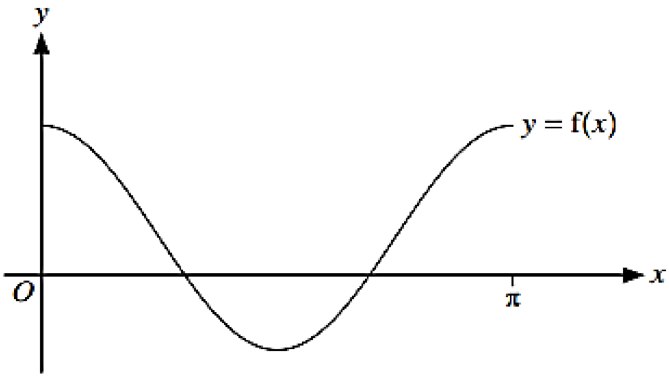
**(a)** By first obtaining a quadratic equation in $\cos\theta$, solve the equation

$$\tan\theta\sin\theta = 1$$

for $0° < \theta < 360°$. [5]

**(b)** Show that $\dfrac{\tan\theta}{\sin\theta} - \dfrac{\sin\theta}{\tan\theta} \equiv \tan\theta\sin\theta$. [3]

7) JUNE 2020_9709_11 Q4




The diagram shows the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \frac{3}{2}\cos 2x + \frac{1}{2}$ for $0 \leqslant x \leqslant \pi$.

**(a)** State the range of f. [2]

A function g is such that $\mathrm{g}(x) = \mathrm{f}(x) + k$, where $k$ is a positive constant. The $x$-axis is a tangent to the curve $y = \mathrm{g}(x)$.

**(b)** State the value of $k$ and hence describe fully the transformation that maps the curve $y = \mathrm{f}(x)$ on to $y = \mathrm{g}(x)$. [2]

**(c)** State the equation of the curve which is the reflection of $y = \mathrm{f}(x)$ in the $x$-axis. Give your answer in the form $y = a\cos 2x + b$, where $a$ and $b$ are constants. [1]

8) JUNE 2020_9709_11 Q7

**(a)** Prove the identity $\dfrac{1+\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1+\sin\theta} \equiv \dfrac{2}{\cos\theta}$. [3]

**(b)** Hence solve the equation $\dfrac{1+\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1+\sin\theta} = \dfrac{3}{\sin\theta}$, for $0 \leqslant \theta \leqslant 2\pi$. [3]

9) JUNE 2020_9709_12 Q2

**(a)** Express the equation $3\cos\theta = 8\tan\theta$ as a quadratic equation in $\sin\theta$. [3]

**(b)** Hence find the acute angle, in degrees, for which $3\cos\theta = 8\tan\theta$. [2]

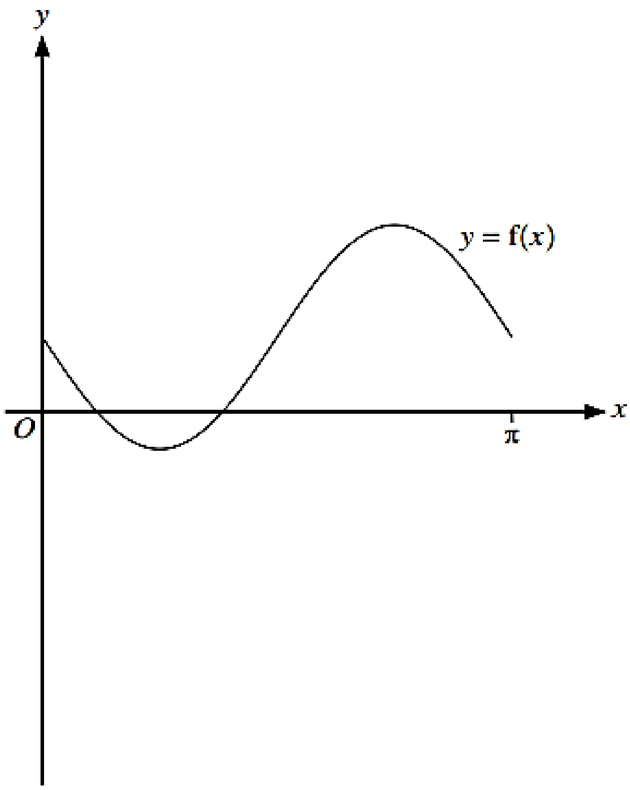
10) JUNE 2020_9709_12 Q9

Functions f and g are such that

$$\mathrm{f}(x) = 2 - 3\sin 2x \quad \text{for } 0 \leqslant x \leqslant \pi,$$
$$\mathrm{g}(x) = -2\mathrm{f}(x) \quad \text{for } 0 \leqslant x \leqslant \pi.$$

**(a)** State the ranges of f and g. [3]

The diagram below shows the graph of $y = f(x)$.




**(b)** Sketch, on this diagram, the graph of $y = g(x)$. [2]

The function h is such that

$$h(x) = g(x + \pi) \quad \text{for } -\pi \leqslant x \leqslant 0.$$

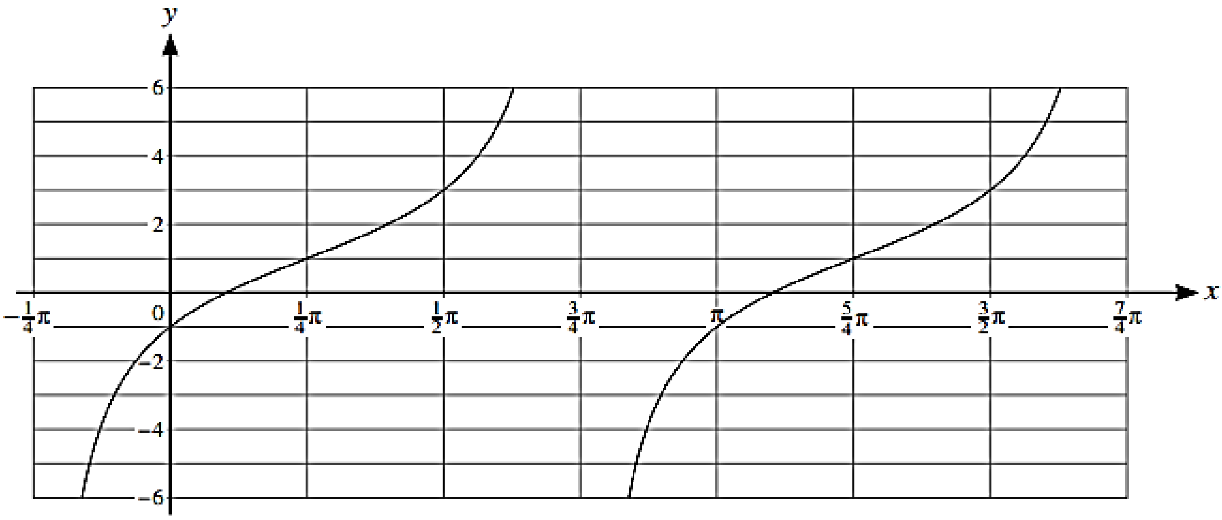
**(c)** Describe fully a sequence of transformations that maps the curve $y = f(x)$ on to $y = h(x)$. [3]

11) JUNE 2020_9709_13 Q7

**(a)** Show that $\dfrac{\tan\theta}{1+\cos\theta} + \dfrac{\tan\theta}{1-\cos\theta} \equiv \dfrac{2}{\sin\theta\cos\theta}$. [4]

**(b)** Hence solve the equation $\dfrac{\tan\theta}{1+\cos\theta} + \dfrac{\tan\theta}{1-\cos\theta} = \dfrac{6}{\tan\theta}$ for $0° < \theta < 180°$. [4]

12) JUNE 2021_9709_11 Q4




The diagram shows part of the graph of $y = a\tan(x - b) + c$.

Given that $0 < b < \pi$, state the values of the constants $a$, $b$ and $c$. [3]

13) JUNE 2021_9709_11 Q7

**(a)** Prove the identity $\dfrac{1 - 2\sin^2\theta}{1 - \sin^2\theta} \equiv 1 - \tan^2\theta$. [2]

**(b)** Hence solve the equation $\dfrac{1 - 2\sin^2\theta}{1 - \sin^2\theta} = 2\tan^4\theta$ for $0° \leqslant \theta \leqslant 180°$. [3]

14) JUNE 2021_9709_12 Q10

**(a)** Prove the identity $\dfrac{1 + \sin x}{1 - \sin x} - \dfrac{1 - \sin x}{1 + \sin x} \equiv \dfrac{4\tan x}{\cos x}$. [4]

**(b)** Hence solve the equation $\dfrac{1 + \sin x}{1 - \sin x} - \dfrac{1 - \sin x}{1 + \sin x} = 8\tan x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. [3]

15) JUNE 2021_9709_13 Q4

**(a)** Show that the equation

$$\frac{\tan x + \sin x}{\tan x - \sin x} = k,$$

where $k$ is a constant, may be expressed as

$$\frac{1 + \cos x}{1 - \cos x} = k.$$ [2]

**(b)** Hence express $\cos x$ in terms of $k$. [2]

**(c)** Hence solve the equation $\dfrac{\tan x + \sin x}{\tan x - \sin x} = 4$ for $-\pi < x < \pi$. [2]

16) JUNE 2022_9709_11 Q4

**(a)** Prove the identity $\dfrac{\sin^3\theta}{\sin\theta - 1} - \dfrac{\sin^2\theta}{1 + \sin\theta} \equiv -\tan^2\theta(1 + \sin^2\theta)$. [4]

**(b)** Hence solve the equation

$$\frac{\sin^3\theta}{\sin\theta - 1} - \frac{\sin^2\theta}{1+\sin\theta} = \tan^2\theta(1-\sin^2\theta)$$

for $0 < \theta < 2\pi$. [2]

17) JUNE 2022_9709_11 Q8

**(a)** The curve $y = \sin x$ is transformed to the curve $y = 4\sin(\frac{1}{2}x - 30°)$.

Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations are applied. [5]

**(b)** Find the exact solutions of the equation $4\sin(\frac{1}{2}x - 30°) = 2\sqrt{2}$ for $0° \leqslant x \leqslant 360°$. [3]

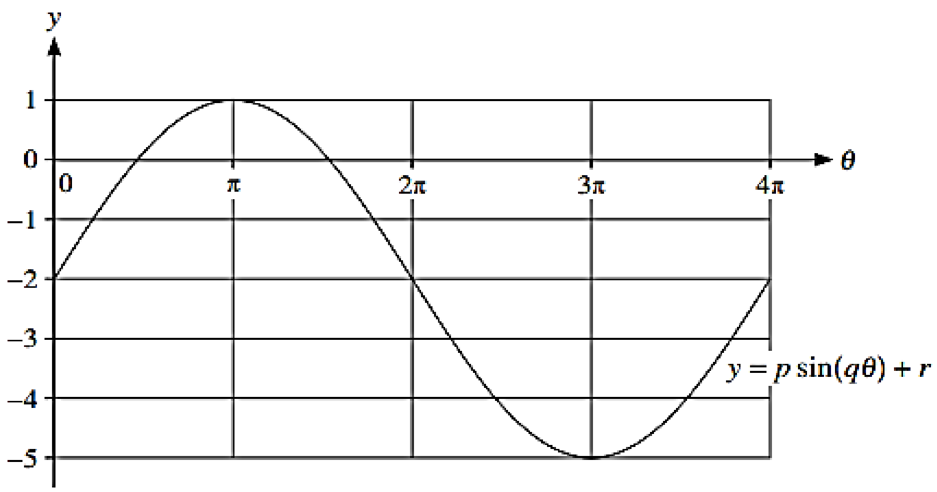
18) JUNE 2022_9709_12 Q11

The function f is given by $f(x) = 4\cos^4 x + \cos^2 x - k$ for $0 \leqslant x \leqslant 2\pi$, where $k$ is a constant.

**(a)** Given that $k = 3$, find the exact solutions of the equation $f(x) = 0$. [5]

**(b)** Use the quadratic formula to show that, when $k > 5$, the equation $f(x) = 0$ has no solutions. [5]

19) JUNE 2022_9709_13 Q2




The diagram shows part of the curve with equation $y = p\sin(q\theta) + r$, where $p$, $q$ and $r$ are constants.

**(a)** State the value of $p$. [1]

**(b)** State the value of $q$. [1]

**(c)** State the value of $r$. [1]

20) JUNE 2022_9709_13 Q5

**(a)** Solve the equation $6\sqrt{y} + \dfrac{2}{\sqrt{y}} - 7 = 0$. [4]

**(b)** Hence solve the equation $6\sqrt{\tan x} + \dfrac{2}{\sqrt{\tan x}} - 7 = 0$ for $0° \leqslant x \leqslant 360°$. [3]

21) JUNE 2023_9709_11 Q1

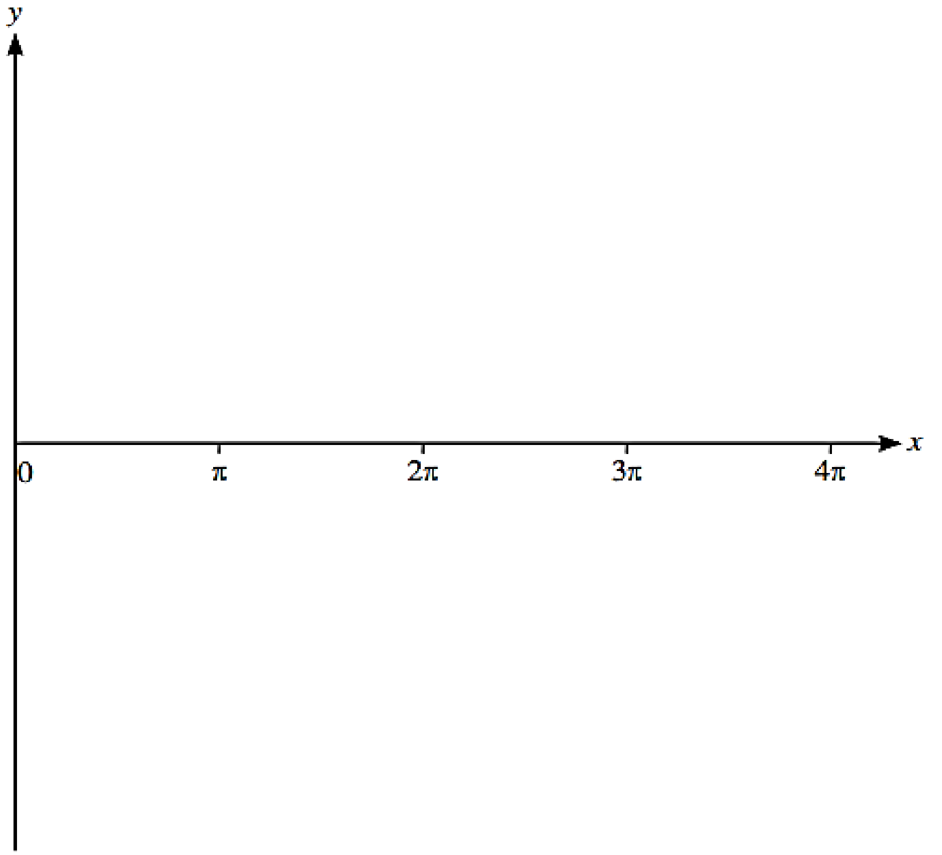
Solve the equation $4\sin\theta + \tan\theta = 0$ for $0° < \theta < 180°$. [3]

22) JUNE 2023_9709_11 Q7

A curve has equation $y = 2 + 3\sin\frac{1}{2}x$ for $0 \leqslant x \leqslant 4\pi$.

**(a)** State greatest and least values of $y$. **[2]**

**(b)** Sketch the curve. **[2]**




**(c)** State the number of solutions of the equation

$$2 + 3\sin\frac{1}{2}x = 5 - 2x$$

for $0 \leqslant x \leqslant 4\pi$. **[1]**

23) JUNE 2023_9709_12 Q7

**(a)** **(i)** By first expanding $(\cos\theta + \sin\theta)^2$, find the three solutions of the equation

$$(\cos\theta + \sin\theta)^2 = 1$$

for $0 \leqslant \theta \leqslant \pi$. **[3]**

**(ii)** Hence verify that the only solutions of the equation $\cos\theta + \sin\theta = 1$ for $0 \leqslant \theta \leqslant \pi$ are 0 and $\frac{1}{2}\pi$. **[2]**

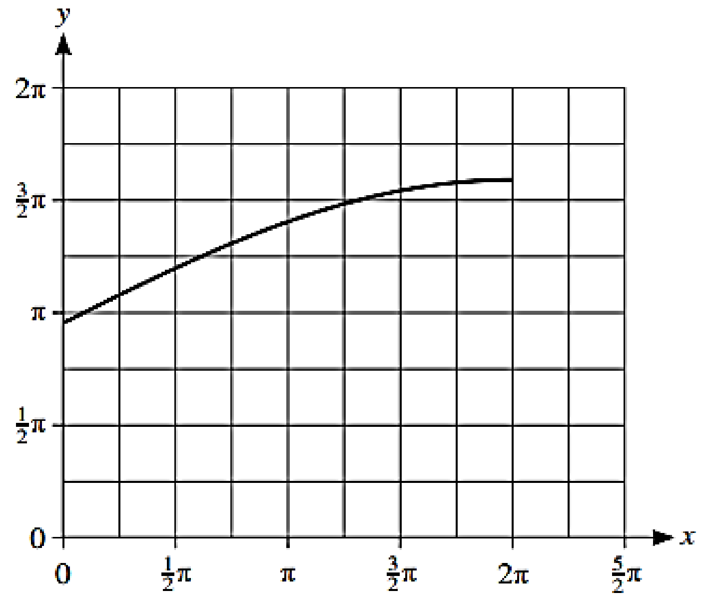
**(b)** Prove the identity $\dfrac{\sin\theta}{\cos\theta + \sin\theta} + \dfrac{1 - \cos\theta}{\cos\theta - \sin\theta} \equiv \dfrac{\cos\theta + \sin\theta - 1}{1 - 2\sin^2\theta}$. **[3]**

**(c)** Using the results of **(a)(ii)** and **(b)**, solve the equation

$$\frac{\sin\theta}{\cos\theta + \sin\theta} + \frac{1 - \cos\theta}{\cos\theta - \sin\theta} = 2(\cos\theta + \sin\theta - 1)$$

for $0 \leqslant \theta \leqslant \pi$. [3]

24) JUNE 2023_9709_12 Q8

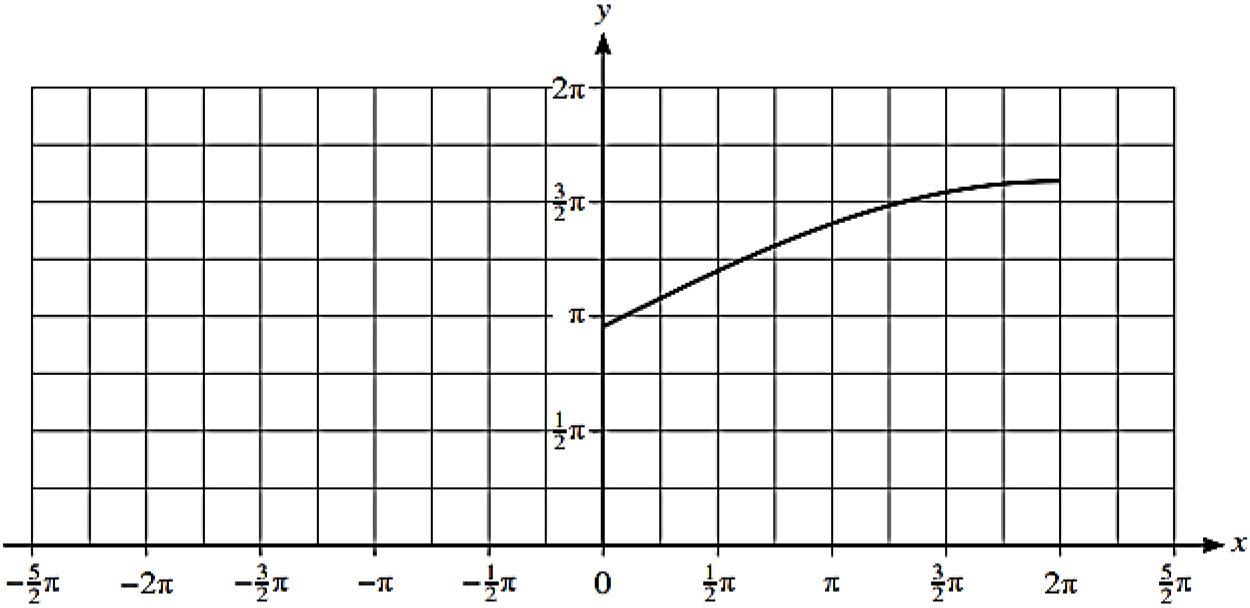



The diagram shows the graph of $y = \mathrm{f}(x)$ where the function f is defined by

$$\mathrm{f}(x) = 3 + 2\sin\tfrac{1}{4}x \text{ for } 0 \leqslant x \leqslant 2\pi.$$

**(a)** On the diagram above, sketch the graph of $y = \mathrm{f}^{-1}(x)$. [2]

**(b)** Find an expression for $\mathrm{f}^{-1}(x)$. [2]




The diagram above shows part of the graph of the function $g(x) = 3 + 2\sin\frac{1}{4}x$ for $-2\pi \leqslant x \leqslant 2\pi$.

Complete the sketch of the graph of $g(x)$ on the diagram above and hence explain whether the function g has an inverse. [2]

**(d)** Describe fully a sequence of three transformations which can be combined to transform the graph of $y = \sin x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$ to the graph of $y = f(x)$, making clear the order in which the transformations are applied. [6]

25) JUNE 2023_9709_13 Q4

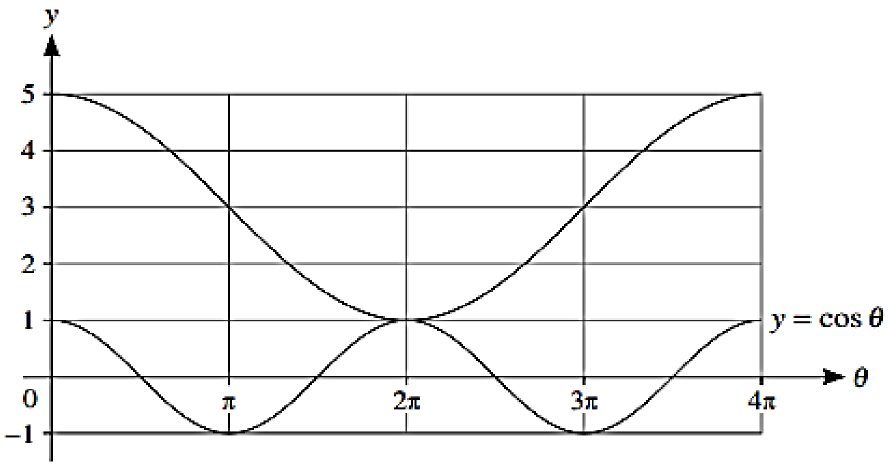
**(a)** Show that the equation

$$3\tan^2 x - 3\sin^2 x - 4 = 0$$

may be expressed in the form $a\cos^4 x + b\cos^2 x + c = 0$, where $a$, $b$ and $c$ are constants to be found. [3]

**(b)** Hence solve the equation $3\tan^2 x - 3\sin^2 x - 4 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

26) OCT 2020_9709_11 Q4




In the diagram, the lower curve has equation $y = \cos\theta$. The upper curve shows the result of applying a combination of transformations to $y = \cos\theta$.

Find, in terms of a cosine function, the equation of the upper curve. [3]

27) OCT 2020_9709_11 Q7

**(a)** Show that $\dfrac{\sin\theta}{1-\sin\theta} - \dfrac{\sin\theta}{1+\sin\theta} \equiv 2\tan^2\theta$. [3]

**(b)** Hence solve the equation $\dfrac{\sin\theta}{1-\sin\theta} - \dfrac{\sin\theta}{1+\sin\theta} = 8$, for $0° < \theta < 180°$. [3]

28) OCT2020_9709_12 Q6

**(a)** Prove the identity $\left(\dfrac{1}{\cos x} - \tan x\right)\left(\dfrac{1}{\sin x} + 1\right) \equiv \dfrac{1}{\tan x}$. [4]

**(b)** Hence solve the equation $\left(\dfrac{1}{\cos x} - \tan x\right)\left(\dfrac{1}{\sin x} + 1\right) = 2\tan^2 x$ for $0° \leqslant x \leqslant 180°$. [2]

29) OCT2020_9709_12 Q11

A curve has equation $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$.

**(a)** State the greatest and least values of $y$. [2]

**(b)** Sketch the graph of $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$. [2]

**(c)** By considering the straight line $y = kx$, where $k$ is a constant, state the number of solutions of the equation $3\cos 2x + 2 = kx$ for $0 \leqslant x \leqslant \pi$ in each of the following cases.

**(i)** $k = -3$ [1]

**(ii)** $k = 1$ [1]

**(iii)** $k = 3$ [1]

Functions f, g and h are defined for $x \in \mathbb{R}$ by

$$f(x) = 3\cos 2x + 2,$$
$$g(x) = f(2x) + 4,$$
$$h(x) = 2f\left(x + \tfrac{1}{2}\pi\right).$$

**(d)** Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = g(x)$. [2]

**(e)** Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = h(x)$. [2]

30) OCT 2020_9709_13 Q3

Solve the equation $3\tan^2\theta + 1 = \dfrac{2}{\tan^2\theta}$ for $0° < \theta < 180°$. [5]

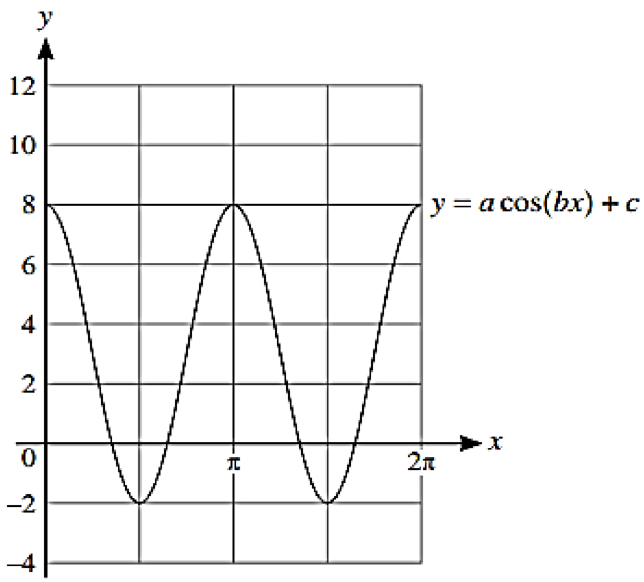
31) OCT 2021_9709_11 Q3

Solve, by factorising, the equation

$$6\cos\theta\tan\theta - 3\cos\theta + 4\tan\theta - 2 = 0,$$

for $0° \leqslant \theta \leqslant 180°$. [4]

32) OCT 2021_9709_11 Q5




The diagram shows part of the graph of $y = a\cos(bx) + c$.

**(a)** Find the values of the positive integers $a$, $b$ and $c$. [3]

**(b)** For these values of $a$, $b$ and $c$, use the given diagram to determine the number of solutions in the interval $0 \leqslant x \leqslant 2\pi$ for each of the following equations.

**(i)** $a\cos(bx) + c = \dfrac{6}{\pi}x$ [1]

**(ii)** $a\cos(bx) + c = 6 - \dfrac{6}{\pi}x$ [1]

33) OCT 2021_9709_12 Q1

Solve the equation $2\cos\theta = 7 - \dfrac{3}{\cos\theta}$ for $-90° < \theta < 90°$. [4]

34) OCT 2021_9709_13 Q7

**(a)** Show that the equation $\dfrac{\tan x + \cos x}{\tan x - \cos x} = k$, where $k$ is a constant, can be expressed as

$$(k+1)\sin^2 x + (k-1)\sin x - (k+1) = 0.$$ [4]

**(b)** Hence solve the equation $\dfrac{\tan x + \cos x}{\tan x - \cos x} = 4$ for $0° \leqslant x \leqslant 360°$. [4]

35) OCT 2022_9709_11 Q6

**(a)** Show that the equation

$$\frac{1}{\sin\theta + \cos\theta} + \frac{1}{\sin\theta - \cos\theta} = 1$$

may be expressed in the form $a\sin^2\theta + b\sin\theta + c = 0$, where $a$, $b$ and $c$ are constants to be found. [3]

**(b)** Hence solve the equation $\dfrac{1}{\sin\theta + \cos\theta} + \dfrac{1}{\sin\theta - \cos\theta} = 1$ for $0° \leqslant \theta \leqslant 360°$. [3]

36) OCT 2022_9709_12 Q3b

Solve the equation $8\cos^2\theta - 10\cos\theta + 2 = 0$ for $0° \leqslant \theta \leqslant 180°$. [3]

37) OCT 2022_9709_12 Q7

**(a)** Prove the identity $\dfrac{\sin\theta}{\sin\theta + \cos\theta} + \dfrac{\cos\theta}{\sin\theta - \cos\theta} \equiv \dfrac{\tan^2\theta + 1}{\tan^2\theta - 1}$. [3]

**(b)** Hence find the exact solutions of the equation $\dfrac{\sin\theta}{\sin\theta + \cos\theta} + \dfrac{\cos\theta}{\sin\theta - \cos\theta} = 2$ for $0 \leqslant \theta \leqslant \pi$. [4]

38) OCT 2022_9709_13 Q1

Solve the equation $8\sin^2\theta + 6\cos\theta + 1 = 0$ for $0° < \theta < 180°$. [3]

39) OCT 2022_9709_13 Q6

It is given that $\alpha = \cos^{-1}\left(\frac{8}{17}\right)$.

Find, without using the trigonometric functions on your calculator, the exact value of $\dfrac{1}{\sin\alpha} + \dfrac{1}{\tan\alpha}$. [5]

**MARKING SCHEME**

1) SP-2020 _9709_1 Q7

| | | | |
|---|---|---|---|
| (a) | Replace $\tan x$ by $\frac{\sin x}{\cos x}$<br>$1+\frac{\sin x^2}{\cos x}=5\cos x$ | 1 | M1 |
| | Replace $\sin x^2$ by $1-\cos x^2$ | 1 | M1 |
| | $6\cos x^2-\cos x-1\ (=0)$ | 1 | A1 |
| | | 3 | |
| (b) | Solution of quadratic $\left[c=-\frac{1}{3}\text{ or }\frac{1}{2}\right]$ | 1 | M1 |
| | $x=60°$ or $109.5°$ | 2 | A1A1 |
| | | 3 | |

2) MARCH 2020_9709_12 Q5

| | |
|---|---|
| $2\tan\theta-6\sin\theta+2=\tan\theta+3\sin\theta+2 \rightarrow \tan\theta-9\sin\theta\ (=0)$ | M1 |
| $\sin\theta-9\sin\theta\cos\theta\ (=0)$ | M1 |
| $\sin\theta(1-9\cos\theta)\ (=0) \rightarrow \sin\theta=0,\quad \cos\theta=\frac{1}{9}$ | M1 |
| $\theta=0$ or $83.6°$ (only answers in the given range) | A1A1 |
| | 5 |

3) MARCH 2020_9709_12 Q11

| | | |
|---|---|---|
| (a) | $(\tan x-2)(3\tan x+1)\ (=0)$. or formula or completing square | M1 |
| | $\tan x=2$ or $-\frac{1}{3}$ | A1 |
| | $x=63.4°$(only value in range) or $161.6°$(only value in range) | B1FT<br>B1FT |
| | | 4 |
| (b) | Apply $b^2-4ac<0$ | M1 |
| | $k>\frac{25}{12}$ | A1 |
| | | 2 |
| (c) | $k=0$ | M1 |
| | $\tan x=0$ or $\frac{5}{3}$ | A1 |
| | $x=0°$ or $180°$ or $59.0°$ | A1 |
| | | 3 |

4) MARCH 2021_9709_12 Q3

| | |
|---|---|
| $\tan\theta + 2\sin\theta = 3\tan\theta - 6\sin\theta$ leading to $2\tan\theta - 8\sin\theta\ [=0]$ | **M1** |
| $2\sin\theta - 8\sin\theta\cos\theta\ (=0)$ leading to $[2]\sin\theta(1-4\cos\theta)\ [=0]$ | **M1** |
| $\cos\theta = \frac{1}{4}$ | **A1** |
| $\theta = 75.5°$ **only** | **A1** |
| | **4** |

5) MARCH 2022_9709_12 Q7

| | | |
|---|---|---|
| a) | $\frac{(\sin\theta+2\cos\theta)(\cos\theta+2\sin\theta)-(\sin\theta-2\cos\theta)(\cos\theta-2\sin\theta)}{(\cos\theta-2\sin\theta)(\cos\theta+2\sin\theta)}$ | ***M1** |
| | $\frac{5\sin\theta\cos\theta+2\cos^2\theta+2\sin^2\theta-(5\sin\theta\cos\theta-2\sin^2\theta-2\cos^2\theta)}{\cos^2\theta-4\sin^2\theta}$ $=\frac{4(\cos^2\theta+\sin^2\theta)}{\cos^2\theta-4\sin^2\theta}$ | **A1** |
| | $\frac{4}{\cos^2\theta-4(1-\cos^2\theta)}$ | **DM1** |
| | $\frac{4}{5\cos^2\theta-4}$ | **A1** |
| | | **4** |
| b) | $\frac{4}{5\cos^2\theta-4}=5$ leading to $25\cos^2\theta=24$ <br> leading to $\cos\theta=\sqrt{\frac{24}{25}}\left[=(\pm)0.9798\right]$ | **M1** |
| | $\theta=11.5°$ or $168.5°$ | **A1** <br> **A1 FT** |
| | | **3** |

6) MARCH 2023_9709_12 Q7

| | | |
|---|---|---|
| a) | $\tan\theta\sin\theta = 1$ leading to $\sin^2\theta = \cos\theta$ | M1 |
| | $1-\cos^2\theta = \cos\theta$ or $\cos^2\theta + \cos\theta - 1$ $[=0]$ | M1 |
| | $[\cos\theta =]$ $\dfrac{-1\pm\sqrt{5}}{2}$ | M1 |
| | 51.8°, | A1 |
| | 308.2° | A1 FT |
| | | 5 |
| b) | $\dfrac{\tan\theta}{\sin\theta} - \dfrac{\sin\theta}{\tan\theta} = \dfrac{\sin\theta}{\sin\theta\cos\theta} - \dfrac{\sin\theta\cos\theta}{\sin\theta} = \dfrac{1}{\cos\theta} - \cos\theta$ | M1 |
| | $= \dfrac{1-\cos^2\theta}{\cos\theta} = \dfrac{\sin^2\theta}{\cos\theta}$ | M1 |
| | $= \tan\theta\sin\theta$ | A1 |
| | | 3 |

7) JUNE 2020_9709_11 Q4

| | | |
|---|---|---|
| (a) | $-1 \leqslant f(x) \leqslant 2$ | B1 B1 |
| | | 2 |
| (b) | $k = 1$ | B1 |
| | Translation by 1 unit upwards parallel to the y-axis | B1 |
| | | 2 |
| (c) | $y = -\dfrac{3}{2}\cos 2x - \dfrac{1}{2}$ | B1 |

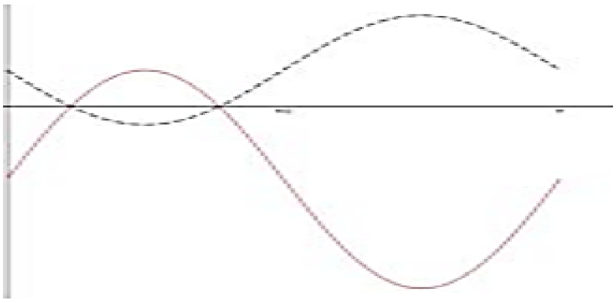
8) JUNE 2020_9709_11 Q7

| | | |
|---|---|---|
| (a) | $\dfrac{(1+\sin\theta)^2 + \cos^2\theta}{\cos\theta(1+\sin\theta)}$ | M1 |
| | Use of $\sin^2\theta + \cos^2\theta = 1 \rightarrow \dfrac{2+2\sin\theta}{\cos\theta(1+\sin\theta)} \rightarrow \dfrac{2}{\cos\theta}$. | M1A1 |
| | | 3 |
| (b) | $\dfrac{2}{\cos\theta} = \dfrac{3}{\sin\theta} \rightarrow \tan\theta = 1.5$ | M1 |
| | $\theta = 0.983$ or $4.12$<br>(FT on second value for 1st value + $\pi$) | A1<br>A1FT |
| | | 3 |

9) JUNE 2020_9709_12 Q2

| | | |
|---|---|---|
| (a) | $3\cos\theta = 8\tan\theta \rightarrow 3\cos\theta = \frac{8\sin\theta}{\cos\theta}$ | M1 |
| | $3(1 - \sin^2\theta) = 8\sin\theta$ | M1 |
| | $3\sin^2\theta + 8\sin\theta - 3 = 0$ | A1 |
| | | 3 |
| (b) | $(3\sin\theta - 1)(\sin\theta + 3) = 0 \rightarrow \sin\theta = \frac{1}{3}$ | M1 |
| | $\theta = 19.5°$ | A1 |
| | | 2 |

10) JUNE 2020_9709_12 Q9

| | | |
|---|---|---|
| (a) | f($x$) from −1 to 5 | B1B1 |
| | g($x$) from −10 to 2<br>(FT from part (a)) | B1FT |
| | | 3 |
| (b) |  | B2, 1 |
| | | 2 |
| (c) | Reflect in $x$-axis | B1 |
| | Stretch by factor 2 in the $y$ direction | B1 |
| | Translation by $-\pi$ in the $x$ direction OR translation by $\begin{pmatrix} 0 \\ -\pi \end{pmatrix}$. | B1 |
| | | 3 |

11) JUNE 2020_9709_13 Q7

| | | |
|---|---|---|
| (a) | $\frac{\tan\theta}{1+\cos\theta}+\frac{\tan\theta}{1-\cos\theta}=\frac{\tan\theta(1-\cos\theta)+\tan\theta(1+\cos\theta)}{1-\cos^2\theta}$ | M1 |
| | $=\frac{2\tan\theta}{\sin^2\theta}$ | M1 |
| | $=\frac{2\sin\theta}{\cos\theta\sin^2\theta}$ | M1 |
| | $=\frac{2}{\sin\theta\cos\theta}$ **AG** | A1 |
| | | 4 |
| (b) | $\frac{2}{\sin\theta\cos\theta}=\frac{6\cos\theta}{\sin\theta}$ | M1 |
| | $\cos^2\theta=\frac{1}{3}\rightarrow\ \cos\theta=(\pm)0.5774$ | A1 |
| | 54.7°, 125.3°<br>(**FT** for 180° – 1st solution) | A1<br>A1FT |
| | | 4 |

12) JUNE 2021_9709_11 Q4

| | |
|---|---|
| $a=2$ | B1 |
| $b=\frac{\pi}{4}$ | B1 |
| $c=1$ | B1 |
| | 3 |

13) JUNE 2021_9709_11 Q7

| | | |
|---|---|---|
| (a) | Reach $\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}$ **or** $\frac{1-\sin^2\theta}{1-\sin^2\theta}-\frac{\sin^2\theta}{\cos^2\theta}$ **or** $\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta}-2\tan^2\theta$<br>**or** $\sec^2\theta-\frac{2\sin^2\theta}{\cos^2\theta}$ **or** $2-\sec^2\theta$ **or** $\frac{\cos 2\theta}{\cos^2\theta}$ | M1 |
| | $1-\tan^2\theta$ | A1 |
| | | 2 |
| (b) | $1-\tan^2\theta=2\tan^4\theta \Rightarrow 2\tan^4\theta+\tan^2\theta-1\ [=0]$ | M1 |
| | $\tan^2\theta=0.5$ or $-1$ leading to $\tan\theta=[\pm]\sqrt{0.5}$ | M1 |
| | $\theta=35.3°$ and $144.7°$ (AWRT) | A1 |
| | | 3 |

14) JUNE 2021_9709_12 Q10

| | | |
|---|---|---|
| (a) | $\frac{1+\sin x}{1-\sin x}-\frac{1-\sin x}{1+\sin x}=\frac{(1+\sin x)^2-(1-\sin x)^2}{(1-\sin x)(1+\sin x)}$ | *M1 |
| | $=\frac{1+2\sin x+\sin^2 x-(1-2\sin x+\sin^2 x)}{(1-\sin x)(1+\sin x)}$ | DM1 |
| | $=\frac{4\sin x}{1-\sin^2 x}=\frac{4\sin x}{\cos^2 x}$ | DM1 |
| | $=\frac{4\sin x}{\cos x\cos x}=\frac{4\tan x}{\cos x}$ | A1 |
| | **Alternative method for Question 10(a)** | |
| | $\frac{4\tan x}{\cos x}=\frac{4\sin x}{\cos^2 x}=\frac{4\sin x}{1-\sin^2 x}$ | *M1 |
| | $=\frac{-2}{1+\sin x}+\frac{2}{1-\sin x}$ | DM1 |
| | $=1+\frac{-2}{1+\sin x}+\frac{2}{1-\sin x}-1$ | DM1 |
| | $=-\frac{1-\sin x}{1+\sin x}+\frac{1+\sin x}{1-\sin x}$ | A1 |
| | | **4** |
| (b) | $\cos x=\frac{1}{2}$ | *B1 |
| | $x=\frac{\pi}{3}$ | DB1 |
| | $x=0$ from $\tan x=0$ or $\sin x=0$ | B1 |
| | | **3** |

15) JUNE 2021_9709_13 Q4

| | | |
|---|---|---|
| (a) | $\frac{\tan x+\sin x}{\tan x-\sin x}[=k]$ leading to $\frac{\sin x+\sin x\cos x}{\sin x-\sin x\cos x}[=k]$ or $\frac{\frac{1}{\cos x}+1}{\frac{1}{\cos x}-1}$ [=k] or $\frac{\tan x+\tan x\cos x}{\tan x-\tan x\cos x}$ [=k] | M1 |
| | $\frac{\sin x(1+\cos x)}{\sin x(1-\cos x)}$ or $\frac{\frac{1}{\cos x}+1}{\frac{1}{\cos x}-1}.\frac{\cos x}{\cos x}$ or $\frac{\tan x(1+\cos x)}{\tan x(1-\cos x)}$ leading to $\frac{1+\cos x}{1-\cos x}[=k]$ | A1 |
| | | 2 |
| (b) | $k-k\cos x=1+\cos x$ leading to $k-1=k\cos x+\cos x$ | M1 |
| | $k-1=(k+1)\cos x$ leading to $\cos x=\frac{k-1}{k+1}$ | A1 |
| | | 2 |
| (c) | Obtaining $\cos x$ from *their* **(b)** or **(a)** | M1 |
| | $\pm0.927$ (only solutions in the given range) | A1 |
| | | 2 |

16) JUNE 2022_9709_11 Q4

| | | |
|---|---|---|
| 4(a) | $\frac{\sin^3\theta}{\sin\theta-1}-\frac{\sin^2\theta}{1+\sin\theta}=\frac{\sin^3\theta(1+\sin\theta)}{(\sin\theta-1)(1+\sin\theta)}-\frac{\sin^2\theta(\sin\theta-1)}{(\sin\theta-1)(1+\sin\theta)}$ $\left[=\frac{\sin^3\theta(1+\sin\theta)-\sin^2\theta(\sin\theta-1)}{(\sin\theta-1)(1+\sin\theta)}\right]$ | *M1 |
| | $-\frac{\sin^2\theta+\sin^4\theta}{1-\sin^2\theta}$ | DM1 |
| | $-\frac{\sin^2\theta(1+\sin^2\theta)}{\cos^2\theta}$ | DM1 |
| | $-\tan^2\theta(1+\sin^2\theta)$ | A1 |
| | **Alternative method for Q4(a)** | |
| | $-\tan^2\theta(1+\sin^2\theta)=-\frac{\sin^2\theta(1+\sin^2\theta)}{1-\sin^2\theta}$ | *M1 |
| | $\frac{-\sin^2\theta-\sin^4\theta}{(1-\sin\theta)(1+\sin\theta)}$ | DM1 |
| | $\frac{\sin^2\theta+\sin^3\theta-\sin^3\theta+\sin^4\theta}{(\sin\theta-1)(1+\sin\theta)}=\frac{\sin^3\theta(1+\sin\theta)-\sin^2\theta(\sin\theta-1)}{(\sin\theta-1)(1+\sin\theta)}$ | DM1 |

| | | |
|---|---|---|
| a) | $\frac{\sin^3\theta}{\sin\theta - 1} - \frac{\sin^2\theta}{1+\sin\theta}$ | **A1** |
| | | **4** |
| b) | $-\tan^2\theta(1+\sin^2\theta) = \tan^2\theta(1-\sin^2\theta)$ leading to $[2]\tan^2\theta = 0$ | **M1** |
| | $\tan\theta = 0$ leading to $[\theta =]\pi$ | **A1** |
| | **Alternative method for Q4(b)** | |
| | $-\frac{\sin^2\theta}{\cos^2\theta}(1+\sin^2\theta) = \frac{\sin^2\theta}{\cos^2\theta}(1-\sin^2\theta)$ leading to $-\sin^2\theta - \sin^4\theta = \sin^2\theta - \sin^4\theta$ leading to $[2]\sin^2\theta = 0$ | **M1** |
| | $\sin\theta = 0$ leading to $[\theta =]\pi$ | **A1** |
| | | **2** |

17)

18) JUNE 2022_9709_11 Q8

| | | |
|---|---|---|
| (a) | **EITHER** **(1)**{Translation} $\begin{pmatrix}\{30°\}\\\{0\}\end{pmatrix}$ **OR** **(2)**{Translation} $\begin{pmatrix}\{60°\}\\\{0\}\end{pmatrix}$ | **B2,1,0** |
| | **(3)**{Stretch} {factor 2} {in $x$-direction} | **B2,1,0** |
| | **(4)** Stretch factor 4 in $y$-direction **and** correct order | **B1** |
| | | **5** |
| (b) | $4\sin\left(\frac{1}{2}x - 30°\right) = 2\sqrt{2} \Rightarrow \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)[=45]$ | **M1** |
| | $\frac{1}{2}x - 30 = 45$ or $135 \Rightarrow x = 2(45+30)$ or $x = 2(135+30)$ | **M1** |
| | $x = 150°, x = 330°$ | **A1** |
| | | **3** |

19) JUNE 2022_9709_12 Q11

| | | |
|---|---|---|
| 1(a) | $4\cos^4 x + \cos^2 x - 3 = 0 \Rightarrow (4\cos^2 x - 3)(\cos^2 x + 1) = 0$ | **M1** |
| | $\Rightarrow [\cos^2 x =] \frac{3}{4}$ $[\cos^2 x = -1]$ | **A1** |
| | $\Rightarrow \cos x = [\pm]\sqrt{their\,\frac{3}{4}}$ **OE** $\left[= \pm\frac{\sqrt{3}}{2}\right]$ | **M1** |
| | $[x =]\ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ | **A1**<br>**A1 FT** |
| (b) | $\cos^2 x = \frac{-1-\sqrt{1+16k}}{8} < 0$ [$\therefore$ no solutions]. | **B1** |
| | $[\cos^2 x] = \frac{-1 \pm \sqrt{1+16k}}{8}$ | ***M1** |
| | Substituting $k = 5$ and obtain 1 from the formula | **DM1** |
| | $\cos^2 x = 1$ or $\cos^2 x >$ or $\geqslant 1$ | **A1** |
| | Concluding statement having considered both ± cases. ∴ no solutions | **A1** |
| | **Alternative method for question 11(b)** | |
| | $\cos^2 x = \frac{-1-\sqrt{1+16k}}{8} < 0$ [$\therefore$ no solutions]. | **B1** |
| | $[\cos^2 x] = \frac{-1 \pm \sqrt{1+16k}}{8}$ | ***M1** |
| | $\frac{-1+\sqrt{1+16k}}{8} * 1 \Rightarrow -1+\sqrt{1+16k} * 8 \Rightarrow 1+16k * 81$ | **DM1** |
| | $k * 5$ | **A1** |
| | Concluding statement having considered both ± cases. ∴ no solutions | **A1** |
| | | **5** |

20) JUNE 2022_9709_13 Q2

| | | |
|---|---|---|
| (a) | $[p =]\ 3$ | **B1** |
| | | **1** |
| (b) | $[q =]\ \frac{1}{2}$ | **B1** |
| | | **1** |
| (c) | $[r =]\ -2$ | **B1** |
| | | **1** |

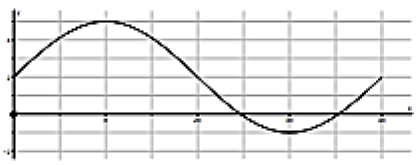
21) JUNE 2022_9709_13 Q5

| | | |
|---|---|---|
| a) | $6y+2-7y^{1/2}$ [= 0] | *M1 |
| | $\left(2y^{\frac{1}{2}}-1\right)\left(3y^{\frac{1}{2}}-2\right)$ [= 0] or e.g. $(2u-1)(3u-2)$ [= 0] | DM1 |
| | $[y^{1/2}=]\frac{1}{2},\frac{2}{3}$ | A1 |
| | $[y=]\frac{1}{4},\frac{4}{9}$ | A1 |
| | | 4 |
| b) | Use of $\tan x$ = *their* $y$ values | M1 |
| | $x=$ 14[.0], 24[.0],<br>$x=$ 194[.0], 204[.0] | A1<br>A1 FT |
| | | 3 |

JUNE 2023_9709_11 Q1

| | |
|---|---|
| $4\sin\theta+\tan\theta=0 \Rightarrow 4\sin\theta+\frac{\sin\theta}{\cos\theta}[=0]$ | M1 |
| $\Rightarrow \sin\theta(4\cos\theta+1)[=0 \Rightarrow \sin\theta=0 \text{ or}]\ \cos\theta=-\frac{1}{4}$ | M1 |
| $\theta=104.5°$ | A1 |
| | 3 |

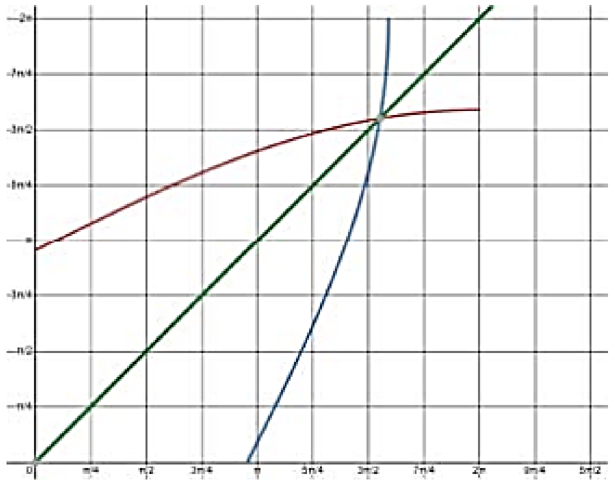
22) JUNE 2023_9709_11 Q7

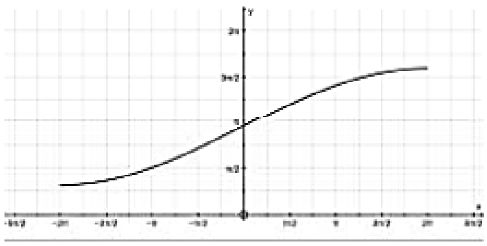
| | | |
|---|---|---|
| (a) | [Greatest =] 5 | B1 |
| | [Least =] –1 | B1 |
| (b) |  | B1<br>B1 FT |
| | | 2 |
| (c) | 1 | B1 |
| | | 1 |

23) JUNE 2023_9709_12 Q7

| | | |
|---|---|---|
| 7(a)(i) | $\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta = 1$ leading to $2\sin\theta\cos\theta = 0$ or $\sin 2\theta = 0$ | *B1 |
| | $[\theta =]\ 0\ ,\dfrac{\pi}{2},\pi$ | DB 2,1,0 |
| | | 3 |
| 7(a)(ii) | $\cos 0 + \sin 0 = [1+0=]1$ and $\cos\dfrac{\pi}{2} + \sin\dfrac{\pi}{2}[=0+1] = 1$ | B1 |
| | $\cos\pi + \sin\pi[=-1+0] = -1$ or $\neq 1$ | B1 |
| | | 2 |
| (b) | $\dfrac{(\cos\theta - \sin\theta)\sin\theta + (\cos\theta + \sin\theta)(1-\cos\theta)}{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}$ | M1 |
| | $= \dfrac{\cos\theta\sin\theta - \sin^2\theta + \cos\theta - \cos^2\theta + \sin\theta - \sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$ | A1 |
| | $= \dfrac{\sin\theta + \cos\theta - \cos^2\theta - \sin^2\theta}{\cos^2\theta - \sin^2\theta} = \dfrac{\cos\theta + \sin\theta - 1}{1 - 2\sin^2\theta}$ | A1 |
| 7(c) | $\dfrac{\cos\theta + \sin\theta - 1}{1 - 2\sin^2\theta} = 2(\cos\theta + \sin\theta - 1)$<br>leading to $1 = 2(1 - 2\sin^2\theta)$ | *M1 |
| | $k\sin^2\theta = 1$ or $3$ leading to $\sin\theta = [\pm]\sqrt{\dfrac{1 \text{ or } 3}{k}}$<br>$\left[4\sin^2\theta = 1 \text{ leading to } \sin\theta = \pm\dfrac{1}{2}\right]$ | DM1 |
| | Solutions $0, \dfrac{1}{6}\pi, \dfrac{1}{2}\pi, \dfrac{5}{6}\pi$ | A1 |

24) JUNE 2023_9709_12 Q8

| | | | |
|---|---|---|---|
| (a) |  | *B1 | The line $y = x$ correctly drawn. Can be implied by reasonably correct graph of $f^{-1}(x)$. |
| | | DB1 | Fully correct (needs to reach $y = 2\pi$ and $x$-axis and cross the line $y = x$ in the correct squares). |
| | | 2 | |

| | | | |
|---|---|---|---|
| (b) | $y = 3 + 2\sin\frac{1}{4}x$ leading to $\sin\frac{1}{4}x = \frac{y \pm 3}{2}$ | M1 | Attempting to arrive at an expression for $\sin\frac{1}{4}x$; condone ± sign errors. Variables may be interchanged initially.<br>M1 not implied by $x = \frac{y \pm 3}{2\sin\frac{1}{4}}$. |
| | $x = 4\sin^{-1}\left(\frac{y-3}{2}\right)$ leading to $[\mathrm{f}^{-1}(x) \text{or } y =]\ 4\sin^{-1}\left(\frac{x-3}{2}\right)$ | A1 | ISW Must clearly be $\sin^{-1}\left(\frac{x-3}{2}\right)$ NOT $\frac{\sin^{-1}(x-3)}{2}$.<br>Allow $\left(\frac{3-x}{-2}\right)$ but not $\div\frac{1}{4}$. |
| | | **2** | |
| (c) |  | B1 | Continuing given graph from y intercept to $-2\pi$.<br>The correct shape needed between 0 and $-2\pi$, including starting to level off (gradient in the final two squares needs to be reducing) as $-2\pi$ is approached.<br>The y co-ordinate at $-2\pi$ must be in the correct square. |
| | Yes it does have an inverse, because the graph is always increasing OR because it is one-one OR because it passes the horizontal line test OR it is not a many to one [function]. | B1 FT | If there is no graph to the left of the y axis, no mark is available.<br>FT an incorrect graph and if the answer is now 'No' provide an appropriate reason. |
| | | **2** | |
| (d) | **{ } indicates different elements throughout.** | | |
| | {Stretch} {factor 4} {in $x$-direction} | B2, 1, 0 | B2 for fully correct, B1 with two elements correct.<br>Condone use of 'sf' instead of factor and 'co-ordinates' stretched instead of graph stretched.<br>Allow any mention of $x$-axis, horizontally or y-axis invariant.<br>Wavelength or period increased by a factor of 4 for B2 or by 4 for B1. |
| | {Stretch} {factor 2} {in $y$-direction} | B2, 1, 0 | B2 for fully correct, B1 with two elements correct.<br>Condone use of 'sf' instead of factor and 'co-ordinates' stretched instead of graph stretched.<br>Allow any mention of $y$-axis, vertically or $x$-axis invariant.<br>Allow y 'co-ordinates' doubled or amplitude doubled for B2. |
| | {Translation} $\begin{pmatrix}\{0\}\\\{3\}\end{pmatrix}$ | B2, 1, 0 | B2 for fully correct, B1 with two elements correct.<br>Allow shift. Any mention of $y$ axis, $y$-direction or vertically implies $\{0\}$, so shift by 3 vertically is B2, but shift by a factor of 3 vertically or a translation of 3 'up' is B1. |
| | | **6** | After scoring B2, B2 the final transformation can only be awarded B2 if the order is fully correct i.e. the translation must not be applied before the y stretch. If all correct except the order award B2B2B1. |

25) JUNE 2023_9709_13 Q4

| | | | |
|---|---|---|---|
| (a) | $3\sin^2 x - 3\sin^2 x\cos^2 x - 4\cos^2 x\ [=0]$ | M1 | Replace $\tan^2 x$ with $\frac{\sin^2 x}{\cos^2 x}$ and multiply by $\cos^2 x$. |
| | $3(1-\cos^2 x) - 3(1-\cos^2 x)\cos^2 x - 4\cos^2 x\ [=0]$ | M1 | Replace $\sin^2 x$ by $1-\cos^2 x$ twice. |
| | $3\cos^4 x - 10\cos^2 x + 3 = 0$ or $-3\cos^4 x + 10\cos^2 x - 3 = 0$ | A1 | Or multiple of these equations. |
| | | 3 | |
| (b) | $(3\cos^2 x - 1)(\cos^2 x - 3)\ [=0]$ | M1 | OE, using *their* equation in the given form. Allow unusual notation if meaning is clear. |
| | $\cos x = [\pm]\frac{1}{\sqrt{3}}$ | A1 | SOI Answer only SC B1. |
| | 54.7°, | A1 | |
| | 125.3° | A1 FT | Only other answer and must be from correct factorisation for A1. FT for 180° – *their* first answer. Answers only SC B1, SC B1 FT. |
| | | 4 | |

26) OCT 2020_9709_11 Q4

| | |
|---|---|
| $(y=)[3]+[2]\left[\cos\frac{1}{2}\theta\right]$ | B1 B1 B1 |

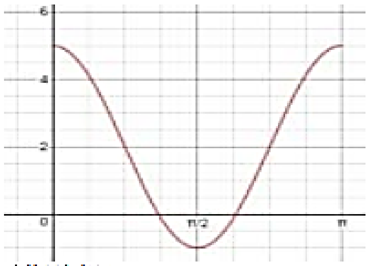
27) OCT 2020_9709_11 Q7

| | | | |
|---|---|---|---|
| (a) | $\left(\frac{\sin\theta}{1-\sin\theta} - \frac{\sin\theta}{1+\sin\theta} = \right)\frac{\sin\theta(1+\sin\theta) - \sin\theta(1-\sin\theta)}{1-\sin^2\theta}$ | *M1 | Put over a single common denominator |
| | $\frac{2\sin^2\theta}{\cos^2\theta}$ | DM1 | Replace $1-\sin^2\theta$ by $\cos^2\theta$ and simplify numerator |
| | $2\tan^2\theta$ | A1 | AG |
| | | 3 | |
| (b) | $2\tan^2\theta = 8 \rightarrow \tan\theta = (\pm)2$ | B1 | SOI |
| | $(\theta =)$ 63.4°, 116.6° | B1 B1 FT | FT on 180 – 1st solution (with justification) |
| | | 3 | |

28) OCT2020_9709_12 Q6

| | | | |
|---|---|---|---|
| (a) | $\left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)\left(\frac{1}{\sin x} + 1\right)$ | B1 | Uses "$\tan x = \sin x \div \cos x$" throughout |
| | $\left(\frac{1-\sin x}{\cos x}\right)\left(\frac{1+\sin x}{\sin x}\right)$ or $\left(\frac{1-\sin^2 x}{\cos x\sin x}\right)$ | M1 | Correct algebra leading to two or four terms |
| | $\left(\frac{\cos^2 x}{\cos x\sin x}\right)$ | A1 | OE. A correct expression which can be cancelled directly to $\frac{\cos x}{\sin x}$ e.g. $\frac{\cos x(1-\sin^2 x)}{\sin x(1-\sin^2 x)}$ |
| | $\left(\frac{\cos^2 x}{\cos x\sin x}\right) = \left(\frac{\cos x}{\sin x}\right) = \frac{1}{\tan x}$ | A1 | AG. Must show cancelling. If $x$ is missing throughout their working withhold this mark. |
| | | 4 | |
| (b) | Uses (a) $\rightarrow \frac{1}{\tan x} = 2\tan^2 x$ $\tan^3 x = \frac{1}{2}$ | M1 | Reducing to $\tan^3 x = k$. |
| | $(x=)$ 38.4° | A1 | AWRT. Ignore extra answers outside the range 0 to 180° but A0 if within. |
| | | 2 | |

29) OCT2020_9709_12 Q11

| | | | |
|---|---|---|---|
| (a) | 5, −1 | B1 B1 | Sight of each value |
| | | 2 | |
| (b) |  | *B1 | Needs to be a curve, not straight lines. One complete cycle starting and finishing at *their* largest value. |
| | | DB1 | One complete cycle starting and finishing at $y = 5$ and going down to $y = -1$ and starting to level off at least one end. |
| 1(c)(i) | 0 solution | B1 | |
| | | 1 | |
| 1(c)(ii) | 2 solutions | B1 | |
| | | 1 | |
| (c)(iii) | 1 solution | B1 | |
| | | 1 | |
| 1(d) | Stretch by (scale factor) $\frac{1}{2}$, parallel to $x$-axis or in $x$ direction (or horizontally) | B1 | |
| | Translation of $\begin{pmatrix}0\\4\end{pmatrix}$ | B1 | Accept translation/shift. Accept translation 4 units in positive $y$-direction. |
| | | 2 | |
| 1(e) | Translation of $\begin{pmatrix}-\frac{\pi}{2}\\0\end{pmatrix}$ | B1 | Accept translation/shift. Accept translation $-\frac{\pi}{2}$ units in $x$-direction. |
| | Stretch by (scale factor) 2 parallel to $y$-axis (or vertically). | B1 | |
| | | 2 | |

30) OCT 2020_9709_13 Q3

| | | |
|---|---|---|
| $3\tan^4\theta + \tan^2\theta - 2\ (=0)$ | M1 | SOI 3-term quartic, condone sign errors for this mark only |
| $(3\tan^2\theta - 2)(\tan^2\theta + 1)\ (=0)$ | M1 | Attempt to factorise or solve 3-term quadratic in $\tan^2\theta$. |
| $\tan\theta = (\pm)\sqrt{\frac{2}{3}}$ or $(\pm)0.816$ or $(\pm)0.817$ | A1 | SOI Implied by final answer = 39.2° after 1st M1 scored |
| 39.2°, 140.8° | A1 A1 FT | FT for 2nd solution =180° – 1st solution |
| | 5 | |

31) OCT 2021_9709_11 Q3

| | | |
|---|---|---|
| $3\cos\theta(2\tan\theta-1)+2(2\tan\theta-1)$ $[=0]$ | **M1** | Or similar partial factorisation; condone sign errors. |
| $(2\tan\theta-1)(3\cos\theta+2)$ $[=0]$<br>[leading to $\tan\theta=\frac{1}{2}$, $\cos\theta=-\frac{2}{3}$] | **M1** | OE. At least 2 out of 4 products correct. |
| 26.6°, 131.8° | **A1 A1** | WWW. Must be 1 d.p. or better.<br>Final A0 if extra solution within the interval.<br>**SC B1** No factorisation: Division by $2\tan\theta-1$ leading to 131.8° or division by $3\cos\theta+2$ or similar leading to 26.6°. |
| **Alternative method for question 3** | | |
| $6\cos\theta\left(\frac{\sin\theta}{\cos\theta}\right)-3\cos\theta+4\left(\frac{\sin\theta}{\cos\theta}\right)-2$ $[=0]$<br>$6\cos\theta\sin\theta-3\cos^2\theta+4\sin\theta-2\cos\theta$ $[=0]$<br>$2\sin\theta(3\cos\theta+2)-\cos\theta(3\cos\theta+2)$ $[=0]$ | **M1** | Using $\tan\theta=\frac{\sin\theta}{\cos\theta}$ and reaching a partial factorisation; condone sign errors. |
| $(2\sin\theta-\cos\theta)(3\cos\theta+2)$ $[=0]$<br>[leading to $\tan\theta=\frac{1}{2}$, $\cos\theta=-\frac{2}{3}$] | **M1** | At least 2 out of 4 products correct. |
| 26.6°, 131.8° | **A1 A1** | WWW. Must be 1 d.p. or better.<br>Final A0 if extra solution within the interval.<br>**SC B1** No factorisation: Division by $2\tan\theta-1$ leading to 131.8° or division by $3\cos\theta+2$ or similar leading to 26.6°. |
| | **4** | |

32) OCT 2021_9709_11 Q5

| | | |
|---|---|---|
| (a) | $a=5$ | **B1** |
| | $b=2$ | **B1** |
| | $c=3$ | **B1** |
| (b)(i) | 3 | **B1** |
| | | **1** |
| (b)(ii) | 2 | **B1** |
| | | **1** |

33) OCT 2021_9709_12 Q1

| | | |
|---|---|---|
| $2\cos^2\theta-7\cos\theta+3[=0]$ | **M1** | Forming a 3-term quadratic expression with all terms on the same side or correctly set up prior to completing the square. Allow ± sign errors. |
| $(2\cos\theta-1)(\cos\theta-3)=0$ | **DM1** | Solving *their* 3-term quadratic using factorisation, formula or completing the square. |
| [$\cos\theta=\frac{1}{2}$ or $\cos\theta=3$ leading to] $\theta=-60°$ or $\theta=60°$ | **A1** | |
| $\theta=-60°$ and $\theta=60°$ | **A1 FT** | FT for ± same answer between 0° and 90° or 0 and $\frac{\pi}{2}$.<br>$\pm\frac{\pi}{3}$ or $\pm1.05$ AWRT scores maximum M1M1A0A1FT.<br>**Special case:** If M1 DM0 scored then SC B1 for $\theta=-60°$ or $\theta=60°$, and SC B1 FT can be awarded for $\pm$(*their* 60°). |

34) OCT 2021_9709_13 Q7

| | | | |
|---|---|---|---|
| (a) | $\tan x + \cos x = k(\tan x - \cos x)$ leading to $\sin x + \cos^2 x = k(\sin x - \cos^2 x)$ | M1 | Use $\tan x = \frac{\sin x}{\cos x}$ and clear fraction. |
| | $\sin x + 1 - \sin^2 x = k\sin x - k + k\sin^2 x$ | *M1 | Use $\cos^2 x = 1 - \sin^2 x$ twice to obtain an equation in sine. |
| | $k\sin^2 x + \sin^2 x + k\sin x - \sin x - k - 1 = 0$ | DM1 | Gather like terms on one side of the equation. |
| | $(k+1)\sin^2 x + (k-1)\sin x - (k+1) = 0$ | A1 | AG. Factorise to obtain answer. |
| | | 4 | |
| (b) | $5\sin^2 x + 3\sin x - 5 = 0$ | B1 | |
| | $\sin x = \frac{-3 \pm \sqrt{9+100}}{10}$ | M1 | Use formula or complete the square. |
| | $x = 48.1°, 131.9°$ | A1<br>A1 FT | AWRT. Maximum A1 if extra solutions in range.<br>FT for 180 – *their* answer or 540 – *their* answer if sinx is negative<br>If M0 given and correct answers only **SCB1B1** available.<br>If answers in radians; 0.839, 2.30 can score **SCB1** for both. |

35) OCT 2022_9709_11 Q6

| | | | |
|---|---|---|---|
| (a) | $\frac{\sin\theta - \cos\theta + \sin\theta + \cos\theta}{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}\left[= \frac{\sin\theta - \cos\theta + \sin\theta + \cos\theta}{\sin^2\theta - \cos^2\theta}\right] = 1$ | *M1 | Use common denominator and equate to 1. |
| | $2\sin\theta\ [= \sin^2\theta - \cos^2\theta] = \sin^2\theta - (1 - \sin^2\theta)$ | DM1 | Multiply by common denominator and replace $\cos^2\theta$ by $1 - \sin^2\theta$. |
| | $2\sin^2\theta - 2\sin\theta - 1 = 0$ | A1 | OE In the given form. |
| | | 3 | |
| (b) | $[\sin\theta =]\frac{2 \pm \sqrt{(-2)^2 - 4(2)(-1)}}{4}\left[= \frac{2 \pm \sqrt{4+8}}{4} = \frac{1 \pm \sqrt{3}}{2}\right]$ | M1 | Use formula or complete the square to solve a quadratic equation of the correct form. |
| | 201.5° or 338.5° | A1 A1 FT | AWRT; A1 for either solution correct.<br>A1 FT for 540 – (first value).<br>If M0, allow **SC B1 B1FT** similarly. |

36) OCT 2022_9709_12 Q3b

| | | |
|---|---|---|
| $2(4\cos\theta - 1)(\cos\theta - 1)$ or $(4\cos\theta - 1)(\cos\theta - 1)$ | M1 | OE<br>Or use of formula or completing the square. Allow use of replacement variable. |
| $\cos\theta = \frac{2}{8}$, $\cos\theta = 1$ | A1 | OE For both answers.<br>**SC: If M0, SC B1** available for sight of $\cos\theta = \frac{2}{8}$ and 1 |
| $[\theta =]$ 0°, 75.5° | A1 | AWRT ISW rejection of 0°.<br>For both answers and no others in the range $0° \leqslant \theta \leqslant 180°$, must be in degrees.<br>**SC: If M0 B1** scored, **SC B1** available for correct answers.<br>SC: If M1 A0 scored, **SC B1** available for $\cos\theta = \frac{2}{8}$ and $\theta$ = 75.5° only, WWW. |

37) OCT 2022_9709_12 Q7

| | | | |
|---|---|---|---|
| (a) | $\dfrac{\sin\theta(\sin\theta-\cos\theta)+\cos\theta(\sin\theta+\cos\theta)}{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}\left[=\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}\right]$ | *M1 | Sight of a correct common denominator, either in one or two fractions, condone missing brackets if recovered. In the numerator condone $\pm$ sign errors only. |
| | $\dfrac{\dfrac{\sin^2\theta}{\cos^2\theta}+\dfrac{\cos^2\theta}{\cos^2\theta}}{\dfrac{\sin^2\theta}{\cos^2\theta}-\dfrac{\cos^2\theta}{\cos^2\theta}}$ | DM1 | Divide throughout by $\cos^2\theta$. |
| | $\dfrac{\tan^2\theta+1}{\tan^2\theta-1}$ AG | A1 | |
| | **Alternative method for Question 7(a)** | | |
| | $\dfrac{\dfrac{\sin^2\theta}{\cos^2\theta}+1}{\dfrac{\sin^2\theta}{\cos^2\theta}-1}\times\dfrac{\cos^2\theta}{\cos^2\theta}$ or the equivalent step $\left[=\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}\right]$ | *M1 | Replace $\tan^2\theta$ with $\dfrac{\sin^2\theta}{\cos^2\theta}$ and multiply top and bottom by $\cos^2\theta$. Condone $\pm$ sign errors. |
| | Sight of convincing use of partial fractions | DM1 | |
| | $\dfrac{\sin\theta}{\sin\theta+\cos\theta}+\dfrac{\cos\theta}{\sin\theta-\cos\theta}$ AG | A1 | |
| | | 3 | **Note:** M1 DM1 A1 for working on both sides at the same time and finishing at the same correct expression. M1 DM1 for starting separately and finishing at the same correct expression and A1 if there is a final conclusion e.g. QED. Do not allow cross multiplication.<br>Condone use of s, c and t and omission of $\theta$. |
| b) | $\dfrac{\tan^2\theta+1}{\tan^2\theta-1}=2\Rightarrow\tan^2\theta+1=2(\tan^2\theta-1)$ | *M1 | Equate expression from **(a)** to 2 and clear fraction. |
| | $\tan\theta=[\pm]\sqrt{3}$ | DM1 | Simplify as far as $\tan\theta=$ . May be implied by a correct final answer in degrees or radians. |
| | **Alternative method for first two marks of Question 7(b)** | | |
| | $\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}=2\Rightarrow\ 1=2\sin^2\theta-2(1-\sin^2\theta)$ | *M1 | Equate expression to 2, clear fraction and use trig identities to form an equation in $\sin\theta$ or $\cos\theta$ only. |
| | $\sin\theta=[\pm]\sqrt{\dfrac{3}{4}}$ or $\cos\theta=[\pm]\sqrt{\dfrac{1}{4}}$ | DM1 | Simplify as far as $\sin\theta=$, or $\cos\theta=$. |
| | $\theta=\dfrac{1}{3}\pi,\dfrac{2}{3}\pi$ | A1<br>A1 FT | A1 for either correct answer then A1FT For their second value being $\pi-$ (their first) and no others in range $0\leqslant\theta\leqslant\pi$ , both values must be exact and in radians.<br>SC: B1 for $\theta=60°,120°$ or $0.333\pi,0.667\pi$ AWRT.<br>or 1.05, 2.09 AWRT. |
| | | 4 | |

38) OCT 2022_9709_13 Q1

| | | |
|---|---|---|
| $8(1-\cos^2\theta)+6\cos\theta+1\ \ [=0]$ | M1 | Expect $8\cos^2\theta-6\cos\theta-9=0$. |
| $(4\cos\theta+3)(2\cos\theta-3)\ \ [=0]$ | A1 | Factors or formula or completing square must be shown. |
| $[\rightarrow\cos\theta=-0.75\rightarrow\theta=]\ 138.6°$ only, | A1 | AWRT, ignore solutions outside the given range, answer in radians A0. |
| | 3 | |

39) OCT 2022_9709_13 Q6

| | | |
|---|---|---|
| Use of $\sin^2\alpha + \cos^2\alpha = 1$ eg $\sin\alpha = [\pm]\sqrt{1-\left(\frac{8}{17}\right)^2}$ | *M1 | Or Pythagoras seen (may quote 8, 15, 17 triple). |
| $\sin\alpha = \frac{15}{17}$ | A1 | |
| $\tan\alpha = \frac{15}{8}$ | A1 | |
| $\frac{1}{\sin\alpha} + \frac{1}{\tan\alpha} = \frac{17}{15} + \frac{8}{15}$ | DM1 | Dealing with reciprocals and addition of fractions correctly. |
| $= \frac{5}{3}$ oe | A1 | Correct answer with no working shown scores 0. Extra answers from $\sin\alpha = -\frac{15}{17}$ are allowed. |
| | 5 | |